# Computer algebra independent integration tests

## 4-Trig-functions/4.7-Miscellaneous/4.7.3-c+d-x-^m-trig^n-trig^p

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# Contents

# Chapter 1

# Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [ 397 ]. This is test number [ 137 ].

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | solved | Failed |
|---|---|---|
| Rubi | % 100.00 ( 397 ) | % 0.00 ( 0 ) |
| Mathematica | % 99.75 ( 396 ) | % 0.25 ( 1 ) |
| Maple | % 90.43 ( 359 ) | % 9.57 ( 38 ) |
| Maxima | % 76.07 ( 302 ) | % 23.93 ( 95 ) |
| Fricas | % 91.94 ( 365 ) | % 8.06 ( 32 ) |
| Sympy | % 30.48 ( 121 ) | % 69.52 ( 276 ) |
| Giac | % 55.16 ( 219 ) | % 44.84 ( 178 ) |
| Mupad | % 39.04 ( 155 ) | % 60.96 ( 242 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

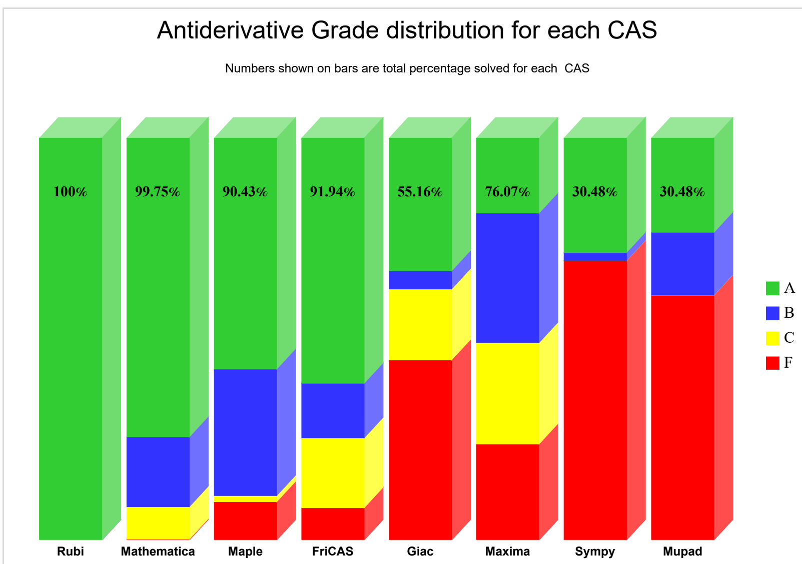
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

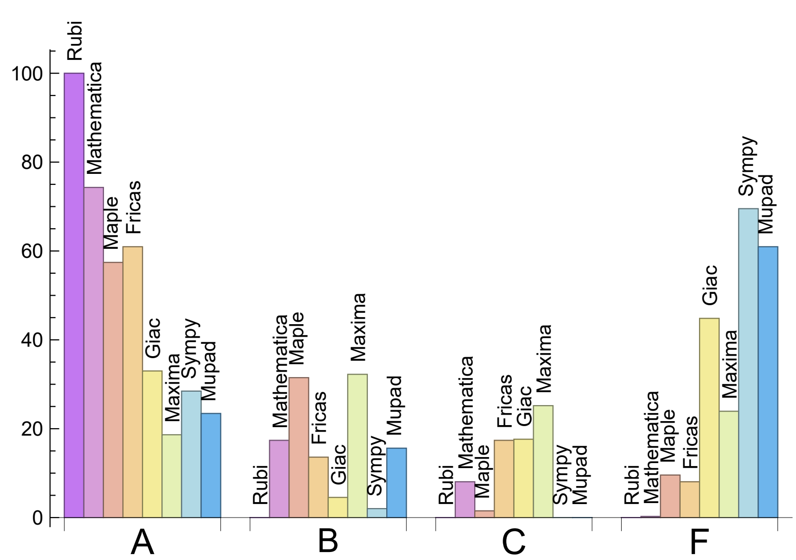
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 100.00 | 0.00 | 0.00 | 0.00 |
| Mathematica | 74.31 | 17.38 | 8.06 | 0.25 |
| Maple | 57.43 | 31.49 | 1.51 | 9.57 |
| Maxima | 18.64 | 32.24 | 25.19 | 23.93 |
| Fricas | 60.96 | 13.60 | 17.38 | 8.06 |
| Sympy | 28.46 | 2.02 | 0.00 | 69.52 |
| Giac | 33.00 | 4.53 | 17.63 | 44.84 |
| Mupad | 23.43 | 15.62 | 0.00 | 60.96 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the CAS systems for each grade level.




The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 0 | 0.00 % | 0.00 % | 0.00 % |
| Mathematica | 1 | 0.00 % | 100.00 % | 0.00 % |
| Maple | 38 | 100.00 % | 0.00 % | 0.00 % |
| Maxima | 95 | 48.42 % | 48.42 % | 3.16 % |
| Fricas | 32 | 0.00 % | 0.00 % | 100.00 % |
| Sympy | 276 | 60.51 % | 34.06 % | 5.43 % |
| Giac | 178 | 81.46 % | 18.54 % | 0.00 % |
| Mupad | 242 | 85.54 % | 14.46 % | 0.00 % |

Table 1.4: Time and leaf size performance for each CAS

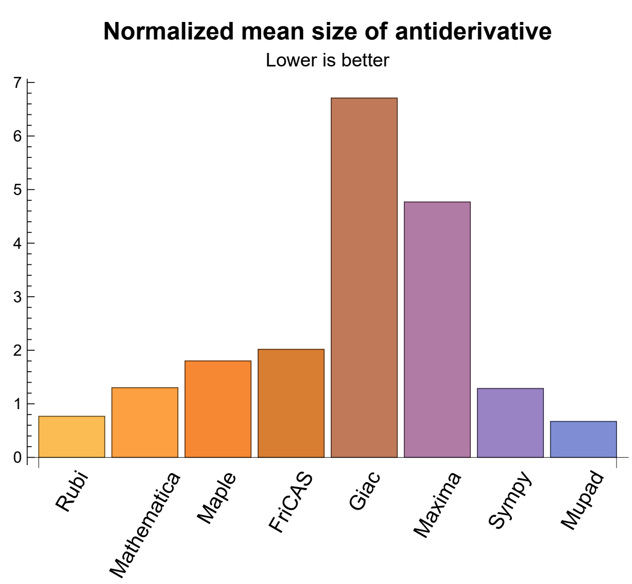
## 1.3 Performance

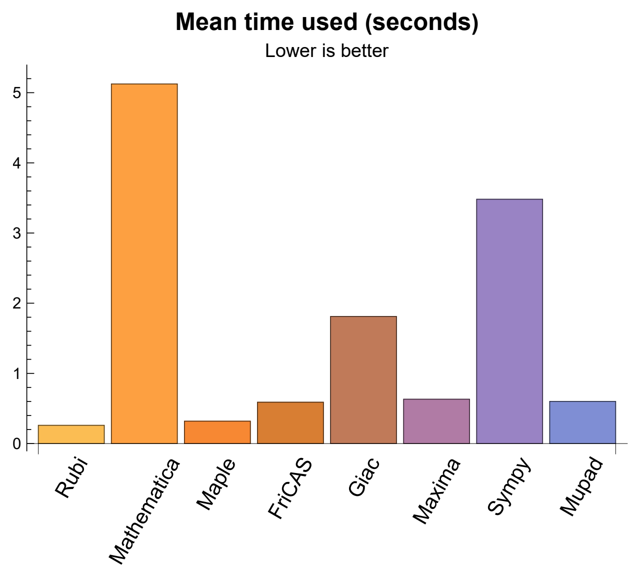
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|---|
| Rubi | 0.26 | 149.21 | 0.77 | 115.00 | 1.00 |
| Mathematica | 5.12 | 277.59 | 1.30 | 132.00 | 0.91 |
| Maple | 0.32 | 343.87 | 1.80 | 242.00 | 1.40 |
| Maxima | 0.63 | 908.93 | 4.77 | 405.50 | 1.67 |
| Fricas | 0.59 | 405.49 | 2.02 | 235.00 | 1.21 |
| Sympy | 3.48 | 177.93 | 1.29 | 0.00 | 0.00 |
| Giac | 1.81 | 946.22 | 6.71 | 145.00 | 1.23 |
| Mupad | 0.60 | 84.90 | 0.67 | -1.00 | -0.01 |

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.






## 1.4 list of integrals that has no closed form antiderivative

{31, 36, 37, 38, 43, 44, 45, 50, 51, 97, 102, 103, 104, 109, 110, 111, 116, 117, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 379, 380, 381, 386, 387, 388, 392, 393, 394}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {32, 33, 34, 35, 39, 41, 46, 47, 58, 63, 99, 105, 106, 107, 130, 131, 134, 135, 164, 165, 166, 171, 173, 178, 179, 180, 181, 190, 195, 222, 223, 236, 237, 241, 242, 249, 254, 255, 261, 267, 268, 280, 285, 291, 292, 300, 304, 305, 306, 310, 311, 312, 318, 319, 331, 376, 382, 383, 384, 385, 390}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

### 1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

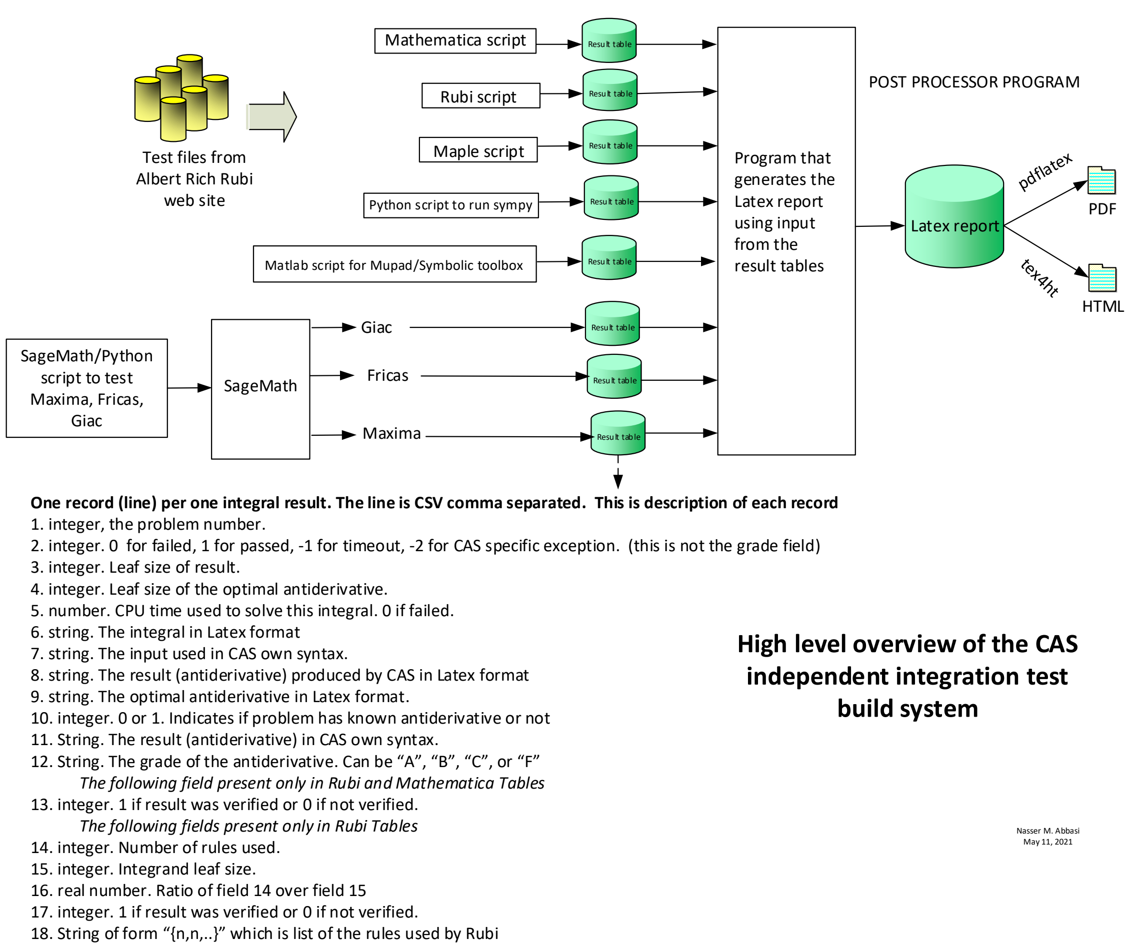
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



# Chapter 2

# detailed summary tables of results

## 2.1 List of integrals sorted by grade for each CAS

### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 43, 44, 45, 49, 50, 51, 52, 53, 54, 55, 56, 57, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 109, 110, 111, 113, 116, 117, 124, 125, 126, 127, 128, 129, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 174, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 219, 220, 221, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 238, 239, 240, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 256, 257, 258, 259, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 275, 276, 277, 278, 279, 282, 283, 284, 285, 288, 289, 290, 293, 294, 295, 296, 298, 301, 302, 303, 307, 308, 309, 313, 314, 315, 316, 317, 320, 321, 322, 323, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 343, 344, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 386, 387, 388, 391, 392, 393, 394, 395, 396 }

B grade: { 32, 33, 34, 35, 40, 41, 42, 46, 47, 98, 105, 106, 107, 112, 114, 115, 164, 165, 166, 171, 172, 173, 178, 179, 180, 181, 216, 218, 222, 223, 228, 235, 236, 241, 242, 250, 254, 255, 260, 261, 273, 274, 280, 281, 286, 287, 291, 292, 299, 300, 304, 305, 306, 310, 311, 312, 318, 324, 325, 331, 340, 342, 382, 383, 384, 385, 389, 390, 397 }

C grade: { 48, 58, 59, 60, 61, 62, 63, 108, 118, 119, 120, 121, 122, 123, 130, 131, 132, 133, 134, 135, 190, 191, 192, 193, 194, 195, 237, 319, 345, 347, 356, 358 }

F grade: { 297 }

### 2.1.3 Maple

A grade: { 5, 6, 7, 8, 9, 10, 11, 12, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 36, 37, 38, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 102, 103, 104, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 149, 150, 151, 152, 153, 159, 160, 161, 162, 163, 168, 169, 170, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 213, 214, 215, 219, 220, 221, 224, 225, 226, 227, 232, 233, 234, 237, 238, 239, 240, 244, 245, 246, 250, 251, 252, 253, 256, 257, 258, 259, 263, 264, 265, 269, 270, 271, 272, 276, 277, 278, 281, 282, 283, 284, 287, 288, 289, 290, 293, 294, 295, 296, 297, 301, 302, 303, 306, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 360, 362, 363, 364, 365, 366, 367, 371, 372, 373, 374, 375, 379, 380, 381, 385, 386, 387, 388, 391, 392, 393, 394, 395, 396 }

B grade: { 2, 3, 4, 14, 15, 16, 23, 24, 25, 32, 33, 34, 35, 39, 40, 41, 46, 47, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 137, 138, 139, 146, 147, 148, 155, 156, 157, 158, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 204, 209, 210, 211, 212, 216, 217, 218, 222, 223, 228, 229, 230, 231, 235, 236, 241, 242, 243, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 273, 274, 275, 279, 280, 286, 291, 292, 298, 299, 300, 304, 305, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 361, 368, 369, 370, 376, 377, 378, 382, 383, 384, 389, 390, 397 }

C grade: { 174, 262, 331, 340, 347, 356 }

F grade: { 1, 13, 22, 70, 79, 88, 136, 145, 154, 285, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 358, 359 }

### 2.1.4 Maxima

A grade: { 5, 17, 26, 31, 36, 37, 38, 43, 45, 74, 92, 97, 102, 103, 104, 109, 111, 116, 117, 140, 149, 158, 163, 168, 169, 170, 175, 177, 208, 213, 214, 215, 221, 224, 225, 226, 227, 232, 233, 234, 240, 246, 253, 257, 259, 265, 272, 278, 284, 290, 297, 303, 307, 308, 309, 314, 315, 316, 322, 360, 361, 362, 363, 364, 368, 369, 370, 371, 379, 380, 384, 386, 387, 388 }

B grade: { 2, 3, 4, 14, 15, 16, 23, 24, 25, 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 48, 49, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 98, 99, 100, 101, 105, 106, 107, 108, 112, 113, 114, 115, 137, 138, 139, 146, 147, 148, 155, 156, 157, 164, 165, 166, 167, 172, 173, 174, 178, 179, 180, 181, 205, 206, 207, 209, 210, 211, 212, 216, 217, 222, 223, 228, 229, 230, 231, 235, 236, 241, 242, 243, 247, 248, 250, 254, 255, 256, 260, 262, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 285, 286, 287, 291, 292, 293, 294, 298, 299, 304, 305, 306, 310, 311, 312, 313, 317, 318, 323, 324, 325, 376, 377, 382, 383, 391, 396 }

C grade: { 6, 7, 8, 9, 10, 11, 12, 18, 19, 20, 21, 27, 28, 29, 30, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 84, 85, 86, 87, 93, 94, 95, 96, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 365, 366, 367, 372, 373, 374, 375 }

F grade: { 1, 13, 22, 44, 50, 51, 70, 79, 88, 110, 136, 145, 154, 171, 176, 182, 183, 202, 203, 204, 218, 219, 220, 237, 238, 239, 244, 245, 249, 251, 252, 258, 261, 263, 264, 270, 271, 276, 277, 282, 283, 288, 289, 295, 296, 300, 301, 302, 319, 320, 321, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 378, 381, 385, 389, 390, 392, 393, 394, 395, 397 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 31, 36, 37, 38, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 97, 102, 103, 104, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 154, 157, 158, 159, 160, 161, 163, 168, 169, 170, 174, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 256, 257, 258, 259, 262, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 293, 294, 295, 296, 297, 301, 302, 303, 306, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 386, 387, 388, 391, 392, 393, 394, 396 }

B grade: { 8, 9, 21, 30, 35, 41, 42, 47, 78, 80, 81, 82, 86, 87, 96, 101, 107, 108, 115, 144, 153, 155, 156, 162, 167, 173, 181, 203, 207, 212, 218, 224, 231, 237, 243, 249, 250, 255, 261, 269, 274, 275, 281, 287, 292, 300, 313, 319, 325, 378, 385, 390, 395, 397 }

C grade: { 32, 33, 34, 39, 40, 46, 98, 99, 100, 105, 106, 112, 113, 114, 164, 165, 166, 171, 172, 178, 179, 180, 202, 205, 206, 209, 210, 211, 216, 217, 222, 223, 228, 229, 230, 235, 236, 241, 242, 247, 248, 254, 260, 266, 267, 268, 273, 279, 280, 285, 286, 291, 298, 299, 304, 305, 310, 311, 312, 317, 318, 323, 324, 376, 377, 382, 383, 384, 389 }

F grade: { 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359 }

### 2.1.6 Sympy

A grade: { 2, 3, 4, 5, 10, 11, 12, 14, 15, 16, 17, 23, 24, 25, 26, 31, 36, 37, 38, 43, 44, 50, 51, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 97, 102, 103, 104, 108, 109, 110, 111, 116, 117, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 219, 220, 227, 232, 233, 238, 239, 244, 245, 246, 251, 252, 253, 256, 257, 258, 259, 263, 264, 270, 271, 276, 277, 282, 283, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 314, 315, 320, 321, 326, 327, 360, 364 }

B grade: { 53, 54, 55, 56, 361, 362, 363, 396 }

C grade: { }

F grade: { 1, 6, 7, 8, 9, 13, 18, 19, 20, 21, 22, 27, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 42, 45, 46, 47, 48, 49, 52, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 159, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 174, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 215, 216, 217, 218, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 254, 255, 260, 261, 262, 265, 266, 267, 268, 269, 272, 273, 274, 275, 278, 279, 280, 281, 284, 285, 286, 287, 291, 292, 293, 294, 298, 299, 300, 304, 305, 306, 309, 310, 311, 312, 313, 316, 317, 318, 319, 322, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397 }

### 2.1.7 Giac

A grade: { 2, 3, 4, 5, 10, 11, 12, 14, 15, 16, 17, 23, 24, 25, 26, 31, 36, 37, 38, 43, 44, 45, 50, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 97, 102, 103, 104, 109, 110, 111, 116, 117, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 316, 322, 326, 360, 361, 362, 363, 364, 365, 366, 379, 380, 381, 386, 387, 388, 392, 393, 394 }

B grade: { 42, 48, 49, 108, 174, 204, 250, 256, 262, 293, 294, 367, 368, 369, 370, 371, 391, 396 }

C grade: { 6, 7, 8, 9, 18, 27, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 84, 85, 86, 87, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 141, 159, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 372, 373, 374 }

F grade: { 1, 13, 19, 20, 21, 22, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 46, 47, 51, 70, 76, 77, 78, 79, 88, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 136, 142, 143, 144, 145, 150, 151, 152, 153, 154, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 202, 203, 205, 206, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 245, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 282, 283, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 315, 317, 318, 319, 320, 321, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 375, 376, 377, 378, 382, 383, 384, 385, 389, 390, 395, 397 }

### 2.1.8 Mupad

A grade: { 31, 36, 37, 38, 43, 44, 45, 50, 51, 97, 102, 103, 104, 109, 110, 111, 116, 117, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 379, 380, 381, 386, 387, 388, 392, 393, 394 }

B grade: { 2, 3, 4, 5, 14, 15, 16, 17, 23, 24, 25, 26, 42, 48, 49, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 108, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 174, 204, 212, 250, 256, 262, 275, 293, 294, 360, 361, 362, 363, 364, 368, 369, 370, 371, 391, 395, 396, 397 }

C grade: { }

F grade: { 1, 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 27, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 46, 47, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 159, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 269, 273, 274, 279, 280, 281, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 365, 366, 367, 372, 373, 374, 375, 376, 377, 378, 382, 383, 384, 385, 389, 390 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

| Problem 1 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 138 | 0 | 0 | 94 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.01 | 0.00 | 0.00 | 0.69 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.156 | 0.082 | 0.353 | 0.000 | 0.479 | 0.000 | 0.000 | 0.000 |

| Problem 2 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 86 | 853 | 586 | 255 | 502 | 181 | 245 |
| normalized size | 1 | 1.00 | 0.55 | 5.47 | 3.76 | 1.63 | 3.22 | 1.16 | 1.57 |
| time (sec) | N/A | 0.107 | 0.479 | 0.056 | 0.385 | 0.656 | 4.252 | 0.196 | 0.491 |

| Problem 3 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 120 | 71 | 466 | 342 | 166 | 342 | 121 | 165 |
| normalized size | 1 | 1.00 | 0.59 | 3.88 | 2.85 | 1.38 | 2.85 | 1.01 | 1.38 |
| time (sec) | N/A | 0.083 | 0.298 | 0.009 | 0.364 | 0.440 | 2.385 | 0.221 | 0.854 |

| Problem 4 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 50 | 215 | 171 | 92 | 175 | 73 | 100 |
| normalized size | 1 | 1.00 | 0.56 | 2.42 | 1.92 | 1.03 | 1.97 | 0.82 | 1.12 |
| time (sec) | N/A | 0.054 | 0.229 | 0.007 | 0.363 | 0.495 | 1.050 | 0.205 | 0.159 |

| Problem 5 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 34 | 74 | 65 | 42 | 80 | 38 | 47 |
| normalized size | 1 | 1.00 | 0.68 | 1.48 | 1.30 | 0.84 | 1.60 | 0.76 | 0.94 |
| time (sec) | N/A | 0.026 | 0.100 | 0.007 | 0.335 | 0.634 | 0.473 | 1.720 | 0.699 |

| Problem 6 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 60 | 84 | 141 | 80 | 0 | 569 | -1 |
| normalized size | 1 | 1.00 | 0.92 | 1.29 | 2.17 | 1.23 | 0.00 | 8.75 | -0.02 |
| time (sec) | N/A | 0.139 | 0.126 | 0.017 | 0.408 | 0.611 | 0.000 | 0.847 | 0.000 |

| Problem 7 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 80 | 124 | 164 | 132 | 0 | 2870 | -1 |
| normalized size | 1 | 1.00 | 0.94 | 1.46 | 1.93 | 1.55 | 0.00 | 33.76 | -0.01 |
| time (sec) | N/A | 0.149 | 0.308 | 0.010 | 0.436 | 0.714 | 0.000 | 0.564 | 0.000 |

| Problem 8 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 114 | 102 | 162 | 199 | 230 | 0 | 5398 | -1 |
| normalized size | 1 | 1.00 | 0.89 | 1.42 | 1.75 | 2.02 | 0.00 | 47.35 | -0.01 |
| time (sec) | N/A | 0.175 | 1.084 | 0.012 | 0.506 | 0.805 | 0.000 | 0.534 | 0.000 |

| Problem 9 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 164 | 200 | 249 | 320 | 0 | 7592 | -1 |
| normalized size | 1 | 1.00 | 1.14 | 1.39 | 1.73 | 2.22 | 0.00 | 52.72 | -0.01 |
| time (sec) | N/A | 0.198 | 0.667 | 0.010 | 0.636 | 0.609 | 0.000 | 0.612 | 0.000 |

| Problem 10 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 8 | 8 | 8 | 7 | 13 | 6 | 5 | 6 | -1 |
| normalized size | 1 | 1.00 | 1.00 | 0.88 | 1.62 | 0.75 | 0.62 | 0.75 | -0.12 |
| time (sec) | N/A | 0.029 | 0.006 | 0.022 | 0.374 | 0.721 | 0.859 | 0.174 | 0.000 |

| Problem 11 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 15 | 15 | 24 | 22 | 19 | -1 |
| normalized size | 1 | 1.00 | 1.00 | 0.94 | 0.94 | 1.50 | 1.38 | 1.19 | -0.06 |
| time (sec) | N/A | 0.046 | 0.006 | 0.018 | 0.385 | 0.763 | 1.572 | 0.145 | 0.000 |

| Problem 12 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 29 | 26 | 15 | 30 | 24 | 26 | -1 |
| normalized size | 1 | 1.00 | 1.00 | 0.90 | 0.52 | 1.03 | 0.83 | 0.90 | -0.03 |
| time (sec) | N/A | 0.059 | 0.008 | 0.017 | 0.394 | 0.506 | 1.161 | 1.601 | 0.000 |

| Problem 13 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 275 | 237 | 0 | 0 | 186 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 0.68 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.330 | 0.692 | 0.348 | 0.000 | 0.615 | 0.000 | 0.000 | 0.000 |

| Problem 14 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 205 | 385 | 835 | 880 | 352 | 646 | 350 | 448 |
| normalized size | 1 | 1.00 | 1.88 | 4.07 | 4.29 | 1.72 | 3.15 | 1.71 | 2.19 |
| time (sec) | N/A | 0.200 | 1.449 | 0.091 | 0.416 | 0.544 | 7.476 | 0.230 | 1.409 |

| Problem 15 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 151 | 121 | 447 | 499 | 227 | 391 | 231 | 289 |
| normalized size | 1 | 1.00 | 0.80 | 2.96 | 3.30 | 1.50 | 2.59 | 1.53 | 1.91 |
| time (sec) | N/A | 0.135 | 0.941 | 0.011 | 0.371 | 0.494 | 4.010 | 0.224 | 1.149 |

| Problem 16 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 93 | 204 | 240 | 130 | 216 | 137 | 161 |
| normalized size | 1 | 1.00 | 0.90 | 1.98 | 2.33 | 1.26 | 2.10 | 1.33 | 1.56 |
| time (sec) | N/A | 0.077 | 0.580 | 0.010 | 0.373 | 0.485 | 2.094 | 0.212 | 0.868 |

| Problem 17 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 44 | 71 | 85 | 59 | 85 | 69 | 59 |
| normalized size | 1 | 1.00 | 0.86 | 1.39 | 1.67 | 1.16 | 1.67 | 1.35 | 1.16 |
| time (sec) | N/A | 0.033 | 0.169 | 0.009 | 0.340 | 0.801 | 0.872 | 0.188 | 0.152 |

| Problem 18 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 121 | 102 | 166 | 274 | 153 | 0 | 6059 | -1 |
| normalized size | 1 | 1.00 | 0.84 | 1.37 | 2.26 | 1.26 | 0.00 | 50.07 | -0.01 |
| time (sec) | N/A | 0.270 | 0.316 | 0.012 | 0.447 | 0.493 | 0.000 | 0.540 | 0.000 |

| Problem 19 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 139 | 242 | 302 | 236 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 1.44 | 1.80 | 1.40 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.301 | 1.370 | 0.013 | 0.518 | 0.597 | 0.000 | 0.000 | 0.000 |

| Problem 20 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 221 | 221 | 183 | 311 | 337 | 399 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 1.41 | 1.52 | 1.81 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.358 | 2.156 | 0.015 | 0.676 | 0.546 | 0.000 | 0.000 | 0.000 |

| Problem 21 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 270 | 298 | 384 | 387 | 564 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.10 | 1.42 | 1.43 | 2.09 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.420 | 1.682 | 0.012 | 0.871 | 0.726 | 0.000 | 0.000 | 0.000 |

| Problem 22 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 271 | 271 | 246 | 0 | 0 | 184 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.91 | 0.00 | 0.00 | 0.68 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.331 | 0.290 | 0.241 | 0.000 | 0.495 | 0.000 | 0.000 | 0.000 |

| Problem 23 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 260 | 260 | 158 | 1143 | 967 | 434 | 935 | 361 | 576 |
| normalized size | 1 | 1.00 | 0.61 | 4.40 | 3.72 | 1.67 | 3.60 | 1.39 | 2.22 |
| time (sec) | N/A | 0.241 | 1.699 | 0.120 | 0.421 | 0.594 | 13.311 | 2.893 | 1.944 |

| Problem 24 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 196 | 135 | 594 | 549 | 283 | 602 | 241 | 366 |
| normalized size | 1 | 1.00 | 0.69 | 3.03 | 2.80 | 1.44 | 3.07 | 1.23 | 1.87 |
| time (sec) | N/A | 0.165 | 0.893 | 0.019 | 0.380 | 0.486 | 7.281 | 3.966 | 1.714 |

| Problem 25 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 91 | 260 | 263 | 159 | 320 | 145 | 202 |
| normalized size | 1 | 1.00 | 0.68 | 1.94 | 1.96 | 1.19 | 2.39 | 1.08 | 1.51 |
| time (sec) | N/A | 0.092 | 0.491 | 0.019 | 0.362 | 0.455 | 3.625 | 0.199 | 1.249 |

| Problem 26 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 75 | 85 | 92 | 76 | 138 | 75 | 94 |
| normalized size | 1 | 1.00 | 1.04 | 1.18 | 1.28 | 1.06 | 1.92 | 1.04 | 1.31 |
| time (sec) | N/A | 0.045 | 0.109 | 0.017 | 0.337 | 0.439 | 1.832 | 3.714 | 0.248 |

| Problem 27 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 110 | 178 | 274 | 156 | 0 | 6046 | -1 |
| normalized size | 1 | 1.00 | 0.85 | 1.38 | 2.12 | 1.21 | 0.00 | 46.87 | -0.01 |
| time (sec) | N/A | 0.232 | 0.392 | 0.019 | 0.471 | 0.487 | 0.000 | 2.114 | 0.000 |

| Problem 28 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 151 | 256 | 301 | 245 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.84 | 1.43 | 1.68 | 1.37 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.280 | 1.229 | 0.023 | 0.527 | 0.513 | 0.000 | 0.000 | 0.000 |

| Problem 29 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 229 | 199 | 329 | 336 | 423 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.87 | 1.44 | 1.47 | 1.85 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.344 | 2.760 | 0.022 | 0.683 | 0.649 | 0.000 | 0.000 | 0.000 |

| Problem 30 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 287 | 316 | 404 | 386 | 588 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.10 | 1.41 | 1.34 | 2.05 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.389 | 2.248 | 0.023 | 0.933 | 0.695 | 0.000 | 0.000 | 0.000 |

| Problem 31 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.06 |
| time (sec) | N/A | 0.018 | 2.575 | 0.160 | 0.000 | 0.479 | 0.000 | 0.000 | 0.000 |

| Problem 32 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 151 | 799 | 1150 | 1262 | 1204 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 5.29 | 7.62 | 8.36 | 7.97 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.220 | 6.115 | 0.172 | 0.619 | 0.602 | 0.000 | 0.000 | 0.000 |

| Problem 33 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 560 | 783 | 747 | 814 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 4.41 | 6.17 | 5.88 | 6.41 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.192 | 2.667 | 0.095 | 0.516 | 0.580 | 0.000 | 0.000 | 0.000 |

| Problem 34 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 356 | 468 | 404 | 498 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 3.83 | 5.03 | 4.34 | 5.35 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.166 | 1.407 | 0.074 | 0.469 | 0.496 | 0.000 | 0.000 | 0.000 |

| Problem 35 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 188 | 215 | 189 | 250 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.89 | 3.31 | 2.91 | 3.85 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.096 | 5.112 | 0.073 | 0.459 | 0.505 | 0.000 | 0.000 | 0.000 |

| Problem 36 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.06 |
| time (sec) | N/A | 0.021 | 3.662 | 0.174 | 0.000 | 0.513 | 0.000 | 0.000 | 0.000 |

| Problem 37 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.06 |
| time (sec) | N/A | 0.021 | 6.983 | 0.200 | 0.000 | 0.441 | 0.000 | 0.000 | 0.000 |

| Problem 38 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.204 | 2.930 | 0.080 | 0.000 | 0.451 | 0.000 | 0.000 | 0.000 |

| Problem 39 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 208 | 308 | 716 | 2944 | 1021 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.48 | 3.44 | 14.15 | 4.91 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.172 | 1.358 | 0.125 | 0.839 | 0.553 | 0.000 | 0.000 | 0.000 |

| Problem 40 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 146 | 311 | 433 | 1770 | 669 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.13 | 2.97 | 12.12 | 4.58 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.116 | 1.168 | 0.089 | 0.561 | 0.547 | 0.000 | 0.000 | 0.000 |

| Problem 41 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 234 | 233 | 556 | 375 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.60 | 2.59 | 6.18 | 4.17 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.062 | 2.037 | 0.028 | 0.525 | 0.532 | 0.000 | 0.000 | 0.000 |

| Problem 42 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 131 | 52 | 259 | 62 | 0 | 801 | 88 |
| normalized size | 1 | 1.00 | 4.37 | 1.73 | 8.63 | 2.07 | 0.00 | 26.70 | 2.93 |
| time (sec) | N/A | 0.020 | 0.055 | 0.021 | 0.378 | 0.473 | 0.000 | 0.752 | 2.281 |

| Problem 43 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.115 | 16.759 | 0.218 | 0.000 | 0.493 | 0.000 | 0.000 | 0.000 |

| Problem 44 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.152 | 20.304 | 0.379 | 0.000 | 0.535 | 0.000 | 0.000 | 0.000 |

| Problem 45 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-2) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.217 | 6.314 | 0.099 | 0.000 | 0.492 | 0.000 | 0.000 | 0.000 |

| Problem 46 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 504 | 716 | 4540 | 1071 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 3.68 | 5.23 | 33.14 | 7.82 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.256 | 6.608 | 0.133 | 0.893 | 0.699 | 0.000 | 0.000 | 0.000 |

| Problem 47 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 277 | 409 | 1044 | 587 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.41 | 3.56 | 9.08 | 5.10 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.174 | 6.414 | 0.105 | 0.697 | 0.746 | 0.000 | 0.000 | 0.000 |

| Problem 48 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 94 | 95 | 1130 | 102 | 0 | 3482 | 147 |
| normalized size | 1 | 1.00 | 1.74 | 1.76 | 20.93 | 1.89 | 0.00 | 64.48 | 2.72 |
| time (sec) | N/A | 0.065 | 0.892 | 0.033 | 0.356 | 0.717 | 0.000 | 2.881 | 2.559 |

| Problem 49 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 48 | 61 | 287 | 44 | 0 | 526 | 53 |
| normalized size | 1 | 1.00 | 1.37 | 1.74 | 8.20 | 1.26 | 0.00 | 15.03 | 1.51 |
| time (sec) | N/A | 0.031 | 0.073 | 0.029 | 0.393 | 0.584 | 0.000 | 0.358 | 1.716 |

| Problem 50 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.137 | 11.329 | 0.366 | 0.000 | 1.245 | 0.000 | 0.000 | 0.000 |

| Problem 51 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.166 | 10.641 | 0.539 | 0.000 | 0.879 | 0.000 | 0.000 | 0.000 |

| Problem 52 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 196 | 179 | 234 | 275 | 222 | 0 | 1198 | -1 |
| normalized size | 1 | 1.00 | 0.91 | 1.19 | 1.40 | 1.13 | 0.00 | 6.11 | -0.01 |
| time (sec) | N/A | 0.448 | 2.239 | 0.028 | 0.548 | 0.805 | 0.000 | 3.063 | 0.000 |

| Problem 53 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | B | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 157 | 187 | 256 | 167 | 665 | 743 | -1 |
| normalized size | 1 | 1.00 | 0.93 | 1.11 | 1.52 | 0.99 | 3.96 | 4.42 | -0.01 |
| time (sec) | N/A | 0.295 | 0.842 | 0.025 | 0.556 | 0.803 | 41.479 | 0.509 | 0.000 |

| Problem 54 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | B | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 142 | 134 | 142 | 209 | 125 | 389 | 402 | -1 |
| normalized size | 1 | 1.00 | 0.94 | 1.00 | 1.47 | 0.88 | 2.74 | 2.83 | -0.01 |
| time (sec) | N/A | 0.231 | 0.258 | 0.023 | 0.452 | 0.749 | 6.139 | 0.560 | 0.000 |

| Problem 55 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | B | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 142 | 134 | 142 | 209 | 125 | 389 | 402 | -1 |
| normalized size | 1 | 1.00 | 0.94 | 1.00 | 1.47 | 0.88 | 2.74 | 2.83 | -0.01 |
| time (sec) | N/A | 0.221 | 0.024 | 0.000 | 0.601 | 0.469 | 6.213 | 0.403 | 0.000 |

| Problem 56 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | B | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 157 | 187 | 256 | 167 | 665 | 743 | -1 |
| normalized size | 1 | 1.00 | 0.93 | 1.11 | 1.52 | 0.99 | 3.96 | 4.42 | -0.01 |
| time (sec) | N/A | 0.275 | 0.040 | 0.000 | 0.547 | 0.501 | 42.222 | 1.025 | 0.000 |

| Problem 57 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 196 | 179 | 234 | 275 | 222 | 0 | 1198 | -1 |
| normalized size | 1 | 1.00 | 0.91 | 1.19 | 1.40 | 1.13 | 0.00 | 6.11 | -0.01 |
| time (sec) | N/A | 0.334 | 1.056 | 0.000 | 0.470 | 0.677 | 0.000 | 0.678 | 0.000 |

| Problem 58 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 406 | 406 | 1171 | 474 | 543 | 370 | 0 | 2453 | -1 |
| normalized size | 1 | 1.00 | 2.88 | 1.17 | 1.34 | 0.91 | 0.00 | 6.04 | -0.00 |
| time (sec) | N/A | 1.137 | 15.071 | 0.049 | 0.596 | 0.591 | 0.000 | 3.345 | 0.000 |

| Problem 59 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 353 | 353 | 677 | 386 | 495 | 298 | 0 | 1529 | -1 |
| normalized size | 1 | 1.00 | 1.92 | 1.09 | 1.40 | 0.84 | 0.00 | 4.33 | -0.00 |
| time (sec) | N/A | 0.684 | 9.140 | 0.042 | 0.592 | 0.632 | 0.000 | 2.766 | 0.000 |

| Problem 60 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 304 | 264 | 294 | 422 | 245 | 0 | 838 | -1 |
| normalized size | 1 | 1.00 | 0.87 | 0.97 | 1.39 | 0.81 | 0.00 | 2.76 | -0.00 |
| time (sec) | N/A | 0.470 | 5.364 | 0.042 | 0.569 | 0.561 | 0.000 | 2.852 | 0.000 |

| Problem 61 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 304 | 264 | 294 | 422 | 245 | 0 | 838 | -1 |
| normalized size | 1 | 1.00 | 0.87 | 0.97 | 1.39 | 0.81 | 0.00 | 2.76 | -0.00 |
| time (sec) | N/A | 0.468 | 3.049 | 0.000 | 0.579 | 0.629 | 0.000 | 3.872 | 0.000 |

| Problem 62 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 353 | 353 | 677 | 386 | 495 | 298 | 0 | 1529 | -1 |
| normalized size | 1 | 1.00 | 1.92 | 1.09 | 1.40 | 0.84 | 0.00 | 4.33 | -0.00 |
| time (sec) | N/A | 0.571 | 8.949 | 0.000 | 0.607 | 0.824 | 0.000 | 6.939 | 0.000 |

| Problem 63 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 406 | 406 | 1171 | 474 | 543 | 370 | 0 | 2453 | -1 |
| normalized size | 1 | 1.00 | 2.88 | 1.17 | 1.34 | 0.91 | 0.00 | 6.04 | -0.00 |
| time (sec) | N/A | 0.668 | 13.628 | 0.000 | 0.603 | 0.692 | 0.000 | 2.458 | 0.000 |

| Problem 64 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 550 | 470 | 547 | 406 | 0 | 2418 | -1 |
| normalized size | 1 | 1.00 | 1.35 | 1.15 | 1.34 | 1.00 | 0.00 | 5.94 | -0.00 |
| time (sec) | N/A | 1.051 | 14.246 | 0.039 | 0.539 | 0.880 | 0.000 | 3.276 | 0.000 |

| Problem 65 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 393 | 376 | 503 | 316 | 0 | 1503 | -1 |
| normalized size | 1 | 1.00 | 1.12 | 1.07 | 1.43 | 0.90 | 0.00 | 4.28 | -0.00 |
| time (sec) | N/A | 0.674 | 3.076 | 0.037 | 0.514 | 0.583 | 0.000 | 4.752 | 0.000 |

| Problem 66 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 264 | 286 | 425 | 244 | 0 | 818 | -1 |
| normalized size | 1 | 1.00 | 0.88 | 0.96 | 1.42 | 0.82 | 0.00 | 2.74 | -0.00 |
| time (sec) | N/A | 0.499 | 0.811 | 0.033 | 0.492 | 0.807 | 0.000 | 2.828 | 0.000 |

| Problem 67 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 264 | 286 | 425 | 244 | 0 | 818 | -1 |
| normalized size | 1 | 1.00 | 0.88 | 0.96 | 1.42 | 0.82 | 0.00 | 2.74 | -0.00 |
| time (sec) | N/A | 0.460 | 0.287 | 0.000 | 0.484 | 0.847 | 0.000 | 1.073 | 0.000 |

| Problem 68 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 393 | 376 | 503 | 316 | 0 | 1503 | -1 |
| normalized size | 1 | 1.00 | 1.12 | 1.07 | 1.43 | 0.90 | 0.00 | 4.28 | -0.00 |
| time (sec) | N/A | 0.566 | 2.367 | 0.000 | 0.497 | 0.679 | 0.000 | 6.767 | 0.000 |

| Problem 69 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 550 | 470 | 547 | 406 | 0 | 2418 | -1 |
| normalized size | 1 | 1.00 | 1.35 | 1.15 | 1.34 | 1.00 | 0.00 | 5.94 | -0.00 |
| time (sec) | N/A | 0.697 | 9.966 | 0.000 | 0.634 | 0.911 | 0.000 | 4.041 | 0.000 |

| Problem 70 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 267 | 267 | 250 | 0 | 0 | 184 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 0.69 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.283 | 0.455 | 0.184 | 0.000 | 0.683 | 0.000 | 0.000 | 0.000 |

| Problem 71 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 205 | 150 | 835 | 889 | 294 | 646 | 350 | 448 |
| normalized size | 1 | 1.00 | 0.73 | 4.07 | 4.34 | 1.43 | 3.15 | 1.71 | 2.19 |
| time (sec) | N/A | 0.203 | 1.550 | 0.057 | 0.471 | 0.487 | 7.187 | 0.247 | 1.898 |

| Problem 72 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 151 | 127 | 447 | 505 | 183 | 391 | 231 | 290 |
| normalized size | 1 | 1.00 | 0.84 | 2.96 | 3.34 | 1.21 | 2.59 | 1.53 | 1.92 |
| time (sec) | N/A | 0.133 | 0.890 | 0.010 | 0.370 | 0.702 | 3.951 | 1.910 | 1.338 |

| Problem 73 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 86 | 204 | 243 | 100 | 216 | 137 | 145 |
| normalized size | 1 | 1.00 | 0.83 | 1.98 | 2.36 | 0.97 | 2.10 | 1.33 | 1.41 |
| time (sec) | N/A | 0.079 | 0.490 | 0.010 | 0.374 | 0.719 | 2.034 | 4.947 | 1.131 |

| Problem 74 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 71 | 71 | 86 | 46 | 85 | 69 | 58 |
| normalized size | 1 | 1.00 | 1.39 | 1.39 | 1.69 | 0.90 | 1.67 | 1.35 | 1.14 |
| time (sec) | N/A | 0.034 | 0.145 | 0.010 | 0.386 | 0.681 | 0.867 | 0.171 | 0.949 |

| Problem 75 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 121 | 100 | 167 | 273 | 152 | 0 | 6279 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 1.38 | 2.26 | 1.26 | 0.00 | 51.89 | -0.01 |
| time (sec) | N/A | 0.224 | 0.296 | 0.010 | 0.432 | 0.617 | 0.000 | 3.832 | 0.000 |

| Problem 76 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 139 | 240 | 300 | 233 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 1.43 | 1.79 | 1.39 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.266 | 1.087 | 0.012 | 0.523 | 0.827 | 0.000 | 0.000 | 0.000 |

| Problem 77 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 221 | 221 | 181 | 313 | 335 | 393 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.82 | 1.42 | 1.52 | 1.78 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.324 | 2.558 | 0.013 | 0.650 | 0.526 | 0.000 | 0.000 | 0.000 |

| Problem 78 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 270 | 300 | 381 | 385 | 558 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.11 | 1.41 | 1.43 | 2.07 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.377 | 1.823 | 0.013 | 0.877 | 0.666 | 0.000 | 0.000 | 0.000 |

| Problem 79 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 213 | 0 | 0 | 134 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.31 | 0.00 | 0.00 | 0.83 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.205 | 1.038 | 0.151 | 0.000 | 0.591 | 0.000 | 0.000 | 0.000 |

| Problem 80 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 132 | 1915 | 735 | 466 | 1231 | 224 | 349 |
| normalized size | 1 | 1.00 | 1.01 | 14.62 | 5.61 | 3.56 | 9.40 | 1.71 | 2.66 |
| time (sec) | N/A | 0.164 | 1.270 | 0.092 | 0.373 | 0.618 | 13.619 | 1.113 | 1.684 |

| Problem 81 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 106 | 1074 | 442 | 308 | 835 | 153 | 329 |
| normalized size | 1 | 1.00 | 1.01 | 10.23 | 4.21 | 2.93 | 7.95 | 1.46 | 3.13 |
| time (sec) | N/A | 0.130 | 0.655 | 0.025 | 0.364 | 0.500 | 7.943 | 0.227 | 1.691 |

| Problem 82 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 77 | 519 | 232 | 180 | 493 | 94 | 179 |
| normalized size | 1 | 1.00 | 0.97 | 6.57 | 2.94 | 2.28 | 6.24 | 1.19 | 2.27 |
| time (sec) | N/A | 0.123 | 0.425 | 0.021 | 0.341 | 0.436 | 3.978 | 0.203 | 1.306 |

| Problem 83 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 54 | 194 | 96 | 85 | 238 | 48 | 57 |
| normalized size | 1 | 1.00 | 1.02 | 3.66 | 1.81 | 1.60 | 4.49 | 0.91 | 1.08 |
| time (sec) | N/A | 0.054 | 0.286 | 0.023 | 0.326 | 0.594 | 1.998 | 1.363 | 1.040 |

| Problem 84 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 65 | 105 | 160 | 88 | 0 | 669 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 1.35 | 2.05 | 1.13 | 0.00 | 8.58 | -0.01 |
| time (sec) | N/A | 0.140 | 0.160 | 0.026 | 0.410 | 0.494 | 0.000 | 0.223 | 0.000 |

| Problem 85 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 81 | 156 | 171 | 138 | 0 | 3218 | -1 |
| normalized size | 1 | 1.00 | 0.78 | 1.50 | 1.64 | 1.33 | 0.00 | 30.94 | -0.01 |
| time (sec) | N/A | 0.169 | 0.432 | 0.026 | 0.444 | 0.548 | 0.000 | 0.975 | 0.000 |

| Problem 86 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 105 | 193 | 206 | 255 | 0 | 5600 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 1.52 | 1.62 | 2.01 | 0.00 | 44.09 | -0.01 |
| time (sec) | N/A | 0.198 | 0.846 | 0.026 | 0.491 | 0.471 | 0.000 | 0.548 | 0.000 |

| Problem 87 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 123 | 230 | 256 | 406 | 0 | 8508 | -1 |
| normalized size | 1 | 1.00 | 0.78 | 1.46 | 1.62 | 2.57 | 0.00 | 53.85 | -0.01 |
| time (sec) | N/A | 0.228 | 1.651 | 0.027 | 0.610 | 0.543 | 0.000 | 0.640 | 0.000 |

| Problem 88 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 376 | 0 | 0 | 276 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.92 | 0.00 | 0.00 | 0.68 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.403 | 0.628 | 0.211 | 0.000 | 0.721 | 0.000 | 0.000 | 0.000 |

| Problem 89 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 330 | 330 | 238 | 1812 | 1339 | 471 | 1098 | 531 | 816 |
| normalized size | 1 | 1.00 | 0.72 | 5.49 | 4.06 | 1.43 | 3.33 | 1.61 | 2.47 |
| time (sec) | N/A | 0.391 | 3.172 | 0.115 | 0.420 | 0.811 | 20.156 | 0.871 | 4.606 |

| Problem 90 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 259 | 259 | 369 | 992 | 766 | 296 | 690 | 351 | 516 |
| normalized size | 1 | 1.00 | 1.42 | 3.83 | 2.96 | 1.14 | 2.66 | 1.36 | 1.99 |
| time (sec) | N/A | 0.279 | 1.453 | 0.023 | 0.379 | 0.533 | 11.140 | 0.280 | 2.573 |

| Problem 91 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 184 | 127 | 466 | 375 | 166 | 382 | 209 | 249 |
| normalized size | 1 | 1.00 | 0.69 | 2.53 | 2.04 | 0.90 | 2.08 | 1.14 | 1.35 |
| time (sec) | N/A | 0.197 | 0.879 | 0.023 | 0.348 | 0.471 | 5.959 | 0.251 | 0.809 |

| Problem 92 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 94 | 163 | 139 | 76 | 163 | 106 | 99 |
| normalized size | 1 | 1.00 | 0.86 | 1.50 | 1.28 | 0.70 | 1.50 | 0.97 | 0.91 |
| time (sec) | N/A | 0.097 | 0.289 | 0.023 | 0.337 | 0.931 | 3.051 | 1.171 | 1.240 |

| Problem 93 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 185 | 185 | 154 | 253 | 407 | 228 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 1.37 | 2.20 | 1.23 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.339 | 0.490 | 0.024 | 0.471 | 0.647 | 0.000 | 0.000 | 0.000 |

| Problem 94 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 257 | 257 | 213 | 365 | 438 | 347 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 1.42 | 1.70 | 1.35 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.416 | 1.468 | 0.027 | 0.584 | 0.910 | 0.000 | 0.000 | 0.000 |

| Problem 95 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 338 | 338 | 279 | 475 | 473 | 585 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 1.41 | 1.40 | 1.73 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.505 | 3.905 | 0.027 | 0.779 | 0.826 | 0.000 | 0.000 | 0.000 |

| Problem 96 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 413 | 413 | 457 | 580 | 523 | 824 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.11 | 1.40 | 1.27 | 2.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.590 | 2.896 | 0.027 | 1.104 | 0.618 | 0.000 | 0.000 | 0.000 |

| Problem 97 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.127 | 6.383 | 0.241 | 0.000 | 0.532 | 0.000 | 0.000 | 0.000 |

| Problem 98 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 333 | 333 | 837 | 1295 | 1538 | 1367 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.51 | 3.89 | 4.62 | 4.11 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.284 | 1.325 | 0.227 | 0.638 | 0.750 | 0.000 | 0.000 | 0.000 |

| Problem 99 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 254 | 330 | 847 | 919 | 921 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.30 | 3.33 | 3.62 | 3.63 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.198 | 0.938 | 0.151 | 0.517 | 0.785 | 0.000 | 0.000 | 0.000 |

| Problem 100 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 171 | 221 | 479 | 507 | 558 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.29 | 2.80 | 2.96 | 3.26 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.138 | 0.847 | 0.128 | 0.470 | 1.544 | 0.000 | 0.000 | 0.000 |

| Problem 101 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 176 | 199 | 199 | 277 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.87 | 2.12 | 2.12 | 2.95 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.062 | 0.171 | 0.076 | 0.454 | 0.648 | 0.000 | 0.000 | 0.000 |

| Problem 102 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.107 | 8.354 | 0.237 | 0.000 | 0.539 | 0.000 | 0.000 | 0.000 |

| Problem 103 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.131 | 4.020 | 0.471 | 0.000 | 0.691 | 0.000 | 0.000 | 0.000 |

| Problem 104 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.035 | 1.198 | 0.089 | 0.000 | 0.713 | 0.000 | 0.000 | 0.000 |

| Problem 105 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 155 | 795 | 913 | 3229 | 856 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 5.13 | 5.89 | 20.83 | 5.52 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.229 | 6.720 | 0.130 | 1.205 | 0.948 | 0.000 | 0.000 | 0.000 |

| Problem 106 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 374 | 573 | 1945 | 599 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.94 | 4.51 | 15.31 | 4.72 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.197 | 6.154 | 0.108 | 0.742 | 0.782 | 0.000 | 0.000 | 0.000 |

| Problem 107 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 198 | 297 | 646 | 384 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.04 | 3.06 | 6.66 | 3.96 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.130 | 6.026 | 0.093 | 0.742 | 0.724 | 0.000 | 0.000 | 0.000 |

| Problem 108 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 82 | 49 | 292 | 97 | 104 | 1375 | 67 |
| normalized size | 1 | 1.00 | 2.00 | 1.20 | 7.12 | 2.37 | 2.54 | 33.54 | 1.63 |
| time (sec) | N/A | 0.026 | 0.479 | 0.053 | 0.584 | 0.587 | 0.480 | 2.528 | 1.573 |

| Problem 109 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.036 | 4.521 | 0.223 | 0.000 | 0.797 | 0.000 | 0.000 | 0.000 |

| Problem 110 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.034 | 2.349 | 0.372 | 0.000 | 0.561 | 0.000 | 0.000 | 0.000 |

| Problem 111 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.075 | 38.567 | 0.104 | 0.000 | 0.636 | 0.000 | 0.000 | 0.000 |

| Problem 112 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 416 | 416 | 966 | 1673 | 6952 | 2762 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.32 | 4.02 | 16.71 | 6.64 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.502 | 8.295 | 0.187 | 5.492 | 2.058 | 0.000 | 0.000 | 0.000 |

| Problem 113 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 308 | 308 | 528 | 1056 | 3872 | 1734 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.71 | 3.43 | 12.57 | 5.63 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.344 | 4.744 | 0.144 | 1.920 | 1.190 | 0.000 | 0.000 | 0.000 |

| Problem 114 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 471 | 546 | 1932 | 966 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.63 | 3.05 | 10.79 | 5.40 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.223 | 7.578 | 0.120 | 0.809 | 0.706 | 0.000 | 0.000 | 0.000 |

| Problem 115 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 260 | 246 | 770 | 454 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.41 | 2.28 | 7.13 | 4.20 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.107 | 1.593 | 0.099 | 0.536 | 0.573 | 0.000 | 0.000 | 0.000 |

| Problem 116 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.081 | 35.887 | 2.760 | 0.000 | 0.733 | 0.000 | 0.000 | 0.000 |

| Problem 117 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.080 | 41.721 | 4.191 | 0.000 | 0.557 | 0.000 | 0.000 | 0.000 |

| Problem 118 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 406 | 406 | 1168 | 476 | 543 | 341 | 0 | 2465 | -1 |
| normalized size | 1 | 1.00 | 2.88 | 1.17 | 1.34 | 0.84 | 0.00 | 6.07 | -0.00 |
| time (sec) | N/A | 0.667 | 15.946 | 0.043 | 0.533 | 0.544 | 0.000 | 3.084 | 0.000 |

| Problem 119 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 353 | 353 | 676 | 384 | 499 | 280 | 0 | 1538 | -1 |
| normalized size | 1 | 1.00 | 1.92 | 1.09 | 1.41 | 0.79 | 0.00 | 4.36 | -0.00 |
| time (sec) | N/A | 0.526 | 9.036 | 0.038 | 0.510 | 0.695 | 0.000 | 4.780 | 0.000 |

| Problem 120 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 304 | 278 | 296 | 422 | 235 | 0 | 842 | -1 |
| normalized size | 1 | 1.00 | 0.91 | 0.97 | 1.39 | 0.77 | 0.00 | 2.77 | -0.00 |
| time (sec) | N/A | 0.416 | 6.616 | 0.034 | 0.499 | 0.727 | 0.000 | 2.783 | 0.000 |

| Problem 121 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 304 | 264 | 296 | 422 | 235 | 0 | 842 | -1 |
| normalized size | 1 | 1.00 | 0.87 | 0.97 | 1.39 | 0.77 | 0.00 | 2.77 | -0.00 |
| time (sec) | N/A | 0.419 | 6.389 | 0.000 | 0.497 | 0.720 | 0.000 | 4.918 | 0.000 |

| Problem 122 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 353 | 353 | 676 | 384 | 499 | 280 | 0 | 1538 | -1 |
| normalized size | 1 | 1.00 | 1.92 | 1.09 | 1.41 | 0.79 | 0.00 | 4.36 | -0.00 |
| time (sec) | N/A | 0.528 | 8.929 | 0.000 | 0.530 | 0.724 | 0.000 | 1.932 | 0.000 |

| Problem 123 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 406 | 406 | 1168 | 476 | 543 | 341 | 0 | 2465 | -1 |
| normalized size | 1 | 1.00 | 2.88 | 1.17 | 1.34 | 0.84 | 0.00 | 6.07 | -0.00 |
| time (sec) | N/A | 0.627 | 15.313 | 0.000 | 0.521 | 0.523 | 0.000 | 4.904 | 0.000 |

| Problem 124 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 228 | 206 | 251 | 285 | 347 | 0 | 1358 | -1 |
| normalized size | 1 | 1.00 | 0.90 | 1.10 | 1.25 | 1.52 | 0.00 | 5.96 | -0.00 |
| time (sec) | N/A | 0.398 | 3.419 | 0.049 | 0.473 | 0.508 | 0.000 | 3.297 | 0.000 |

| Problem 125 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 200 | 187 | 206 | 264 | 249 | 0 | 842 | -1 |
| normalized size | 1 | 1.00 | 0.94 | 1.03 | 1.32 | 1.24 | 0.00 | 4.21 | -0.00 |
| time (sec) | N/A | 0.329 | 2.797 | 0.049 | 0.480 | 0.745 | 0.000 | 1.995 | 0.000 |

| Problem 126 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 174 | 161 | 159 | 219 | 175 | 0 | 452 | -1 |
| normalized size | 1 | 1.00 | 0.93 | 0.91 | 1.26 | 1.01 | 0.00 | 2.60 | -0.01 |
| time (sec) | N/A | 0.270 | 0.822 | 0.046 | 0.462 | 0.519 | 0.000 | 0.991 | 0.000 |

| Problem 127 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 174 | 161 | 159 | 219 | 175 | 0 | 452 | -1 |
| normalized size | 1 | 1.00 | 0.93 | 0.91 | 1.26 | 1.01 | 0.00 | 2.60 | -0.01 |
| time (sec) | N/A | 0.249 | 0.112 | 0.000 | 0.462 | 0.751 | 0.000 | 1.006 | 0.000 |

| Problem 128 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 200 | 187 | 206 | 264 | 249 | 0 | 842 | -1 |
| normalized size | 1 | 1.00 | 0.94 | 1.03 | 1.32 | 1.24 | 0.00 | 4.21 | -0.00 |
| time (sec) | N/A | 0.318 | 1.268 | 0.000 | 0.782 | 0.516 | 0.000 | 4.123 | 0.000 |

| Problem 129 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 228 | 206 | 251 | 285 | 347 | 0 | 1358 | -1 |
| normalized size | 1 | 1.00 | 0.90 | 1.10 | 1.25 | 1.52 | 0.00 | 5.96 | -0.00 |
| time (sec) | N/A | 0.378 | 2.325 | 0.000 | 2.022 | 0.608 | 0.000 | 4.929 | 0.000 |

| Problem 130 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 615 | 615 | 3348 | 719 | 820 | 521 | 0 | 3689 | -1 |
| normalized size | 1 | 1.00 | 5.44 | 1.17 | 1.33 | 0.85 | 0.00 | 6.00 | -0.00 |
| time (sec) | N/A | 1.153 | 24.098 | 0.047 | 0.902 | 1.013 | 0.000 | 15.988 | 0.000 |

| Problem 131 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 534 | 534 | 1041 | 580 | 760 | 427 | 0 | 2300 | -1 |
| normalized size | 1 | 1.00 | 1.95 | 1.09 | 1.42 | 0.80 | 0.00 | 4.31 | -0.00 |
| time (sec) | N/A | 0.879 | 11.684 | 0.045 | 0.924 | 0.992 | 0.000 | 4.627 | 0.000 |

| Problem 132 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 459 | 459 | 432 | 447 | 674 | 356 | 0 | 1258 | -1 |
| normalized size | 1 | 1.00 | 0.94 | 0.97 | 1.47 | 0.78 | 0.00 | 2.74 | -0.00 |
| time (sec) | N/A | 0.671 | 7.338 | 0.045 | 0.836 | 1.013 | 0.000 | 2.743 | 0.000 |

| Problem 133 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 459 | 459 | 432 | 447 | 674 | 356 | 0 | 1258 | -1 |
| normalized size | 1 | 1.00 | 0.94 | 0.97 | 1.47 | 0.78 | 0.00 | 2.74 | -0.00 |
| time (sec) | N/A | 0.658 | 7.259 | 0.000 | 0.570 | 0.671 | 0.000 | 2.849 | 0.000 |

| Problem 134 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 534 | 534 | 1041 | 580 | 760 | 427 | 0 | 2300 | -1 |
| normalized size | 1 | 1.00 | 1.95 | 1.09 | 1.42 | 0.80 | 0.00 | 4.31 | -0.00 |
| time (sec) | N/A | 0.801 | 11.455 | 0.000 | 0.798 | 0.626 | 0.000 | 4.214 | 0.000 |

| Problem 135 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 615 | 615 | 3348 | 719 | 820 | 521 | 0 | 3689 | -1 |
| normalized size | 1 | 1.00 | 5.44 | 1.17 | 1.33 | 0.85 | 0.00 | 6.00 | -0.00 |
| time (sec) | N/A | 0.953 | 22.966 | 0.000 | 1.787 | 0.822 | 0.000 | 7.576 | 0.000 |

| Problem 136 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 273 | 273 | 245 | 0 | 0 | 184 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.90 | 0.00 | 0.00 | 0.67 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.288 | 0.237 | 0.176 | 0.000 | 0.601 | 0.000 | 0.000 | 0.000 |

| Problem 137 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 260 | 260 | 158 | 1150 | 967 | 378 | 935 | 361 | 576 |
| normalized size | 1 | 1.00 | 0.61 | 4.42 | 3.72 | 1.45 | 3.60 | 1.39 | 2.22 |
| time (sec) | N/A | 0.234 | 1.849 | 0.089 | 0.392 | 0.699 | 13.230 | 0.261 | 1.335 |

| Problem 138 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 196 | 135 | 594 | 549 | 238 | 602 | 241 | 366 |
| normalized size | 1 | 1.00 | 0.69 | 3.03 | 2.80 | 1.21 | 3.07 | 1.23 | 1.87 |
| time (sec) | N/A | 0.161 | 0.925 | 0.020 | 0.361 | 0.670 | 7.633 | 4.509 | 2.057 |

| Problem 139 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 89 | 260 | 263 | 130 | 320 | 145 | 202 |
| normalized size | 1 | 1.00 | 0.66 | 1.94 | 1.96 | 0.97 | 2.39 | 1.08 | 1.51 |
| time (sec) | N/A | 0.087 | 0.462 | 0.018 | 0.340 | 0.640 | 3.792 | 2.963 | 1.634 |

| Problem 140 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 75 | 85 | 92 | 58 | 138 | 75 | 94 |
| normalized size | 1 | 1.00 | 1.04 | 1.18 | 1.28 | 0.81 | 1.92 | 1.04 | 1.31 |
| time (sec) | N/A | 0.047 | 0.138 | 0.020 | 0.321 | 0.616 | 1.936 | 0.202 | 0.313 |

| Problem 141 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 110 | 178 | 274 | 155 | 0 | 6046 | -1 |
| normalized size | 1 | 1.00 | 0.85 | 1.38 | 2.12 | 1.20 | 0.00 | 46.87 | -0.01 |
| time (sec) | N/A | 0.212 | 0.341 | 0.022 | 0.494 | 0.423 | 0.000 | 1.922 | 0.000 |

| Problem 142 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 151 | 256 | 301 | 235 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.84 | 1.43 | 1.68 | 1.31 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.268 | 1.631 | 0.022 | 0.559 | 0.744 | 0.000 | 0.000 | 0.000 |

| Problem 143 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 231 | 197 | 329 | 336 | 397 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.85 | 1.42 | 1.45 | 1.72 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.329 | 3.754 | 0.022 | 0.687 | 0.752 | 0.000 | 0.000 | 0.000 |

| Problem 144 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 287 | 316 | 404 | 386 | 568 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.10 | 1.41 | 1.34 | 1.98 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.451 | 2.490 | 0.020 | 1.896 | 0.705 | 0.000 | 0.000 | 0.000 |

| Problem 145 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 419 | 419 | 409 | 0 | 0 | 276 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 0.66 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.435 | 0.573 | 0.316 | 0.000 | 0.872 | 0.000 | 0.000 | 0.000 |

| Problem 146 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 330 | 330 | 563 | 1842 | 1339 | 527 | 1098 | 531 | 816 |
| normalized size | 1 | 1.00 | 1.71 | 5.58 | 4.06 | 1.60 | 3.33 | 1.61 | 2.47 |
| time (sec) | N/A | 0.368 | 3.480 | 0.073 | 0.427 | 1.199 | 20.952 | 0.294 | 4.382 |

| Problem 147 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 259 | 259 | 195 | 1016 | 766 | 342 | 690 | 351 | 516 |
| normalized size | 1 | 1.00 | 0.75 | 3.92 | 2.96 | 1.32 | 2.66 | 1.36 | 1.99 |
| time (sec) | N/A | 0.273 | 2.181 | 0.022 | 0.852 | 0.765 | 11.354 | 0.360 | 2.426 |

| Problem 148 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 184 | 252 | 484 | 375 | 193 | 382 | 209 | 295 |
| normalized size | 1 | 1.00 | 1.37 | 2.63 | 2.04 | 1.05 | 2.08 | 1.14 | 1.60 |
| time (sec) | N/A | 0.190 | 0.960 | 0.020 | 0.600 | 0.507 | 6.228 | 2.110 | 0.845 |

| Problem 149 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 110 | 175 | 139 | 91 | 163 | 106 | 119 |
| normalized size | 1 | 1.00 | 1.01 | 1.61 | 1.28 | 0.83 | 1.50 | 0.97 | 1.09 |
| time (sec) | N/A | 0.094 | 0.357 | 0.022 | 0.947 | 0.760 | 3.101 | 2.966 | 0.469 |

| Problem 150 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 185 | 185 | 154 | 252 | 408 | 229 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 1.36 | 2.21 | 1.24 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.280 | 0.507 | 0.021 | 0.507 | 0.607 | 0.000 | 0.000 | 0.000 |

| Problem 151 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 257 | 257 | 212 | 367 | 439 | 339 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.82 | 1.43 | 1.71 | 1.32 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.345 | 2.105 | 0.023 | 0.566 | 0.508 | 0.000 | 0.000 | 0.000 |

| Problem 152 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 338 | 338 | 283 | 473 | 474 | 567 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.84 | 1.40 | 1.40 | 1.68 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.438 | 3.373 | 0.023 | 1.372 | 0.541 | 0.000 | 0.000 | 0.000 |

| Problem 153 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 413 | 413 | 451 | 583 | 524 | 811 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.09 | 1.41 | 1.27 | 1.96 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.538 | 3.402 | 0.023 | 1.272 | 0.628 | 0.000 | 0.000 | 0.000 |

| Problem 154 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 285 | 285 | 255 | 0 | 0 | 184 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 0.65 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.317 | 3.338 | 0.252 | 0.000 | 0.497 | 0.000 | 0.000 | 0.000 |

| Problem 155 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 233 | 233 | 153 | 2061 | 1033 | 546 | 1334 | 359 | 576 |
| normalized size | 1 | 1.00 | 0.66 | 8.85 | 4.43 | 2.34 | 5.73 | 1.54 | 2.47 |
| time (sec) | N/A | 0.266 | 1.537 | 0.132 | 0.391 | 0.503 | 31.302 | 1.126 | 2.585 |

| Problem 156 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 132 | 1100 | 602 | 349 | 857 | 241 | 366 |
| normalized size | 1 | 1.00 | 0.73 | 6.08 | 3.33 | 1.93 | 4.73 | 1.33 | 2.02 |
| time (sec) | N/A | 0.219 | 2.313 | 0.023 | 0.349 | 0.458 | 18.632 | 0.379 | 1.227 |

| Problem 157 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 91 | 498 | 303 | 194 | 461 | 145 | 202 |
| normalized size | 1 | 1.00 | 0.71 | 3.86 | 2.35 | 1.50 | 3.57 | 1.12 | 1.57 |
| time (sec) | N/A | 0.144 | 0.563 | 0.020 | 0.407 | 0.468 | 9.992 | 0.410 | 0.806 |

| Problem 158 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 63 | 176 | 119 | 87 | 201 | 75 | 84 |
| normalized size | 1 | 1.00 | 0.82 | 2.29 | 1.55 | 1.13 | 2.61 | 0.97 | 1.09 |
| time (sec) | N/A | 0.074 | 0.219 | 0.020 | 0.796 | 0.441 | 5.332 | 0.241 | 0.709 |

| Problem 159 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 110 | 178 | 274 | 156 | 0 | 6046 | -1 |
| normalized size | 1 | 1.00 | 0.85 | 1.38 | 2.12 | 1.21 | 0.00 | 46.87 | -0.01 |
| time (sec) | N/A | 0.246 | 0.308 | 0.023 | 0.724 | 0.440 | 0.000 | 0.568 | 0.000 |

| Problem 160 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 189 | 256 | 301 | 248 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.06 | 1.43 | 1.68 | 1.39 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.297 | 0.955 | 0.025 | 0.485 | 0.483 | 0.000 | 0.000 | 0.000 |

| Problem 161 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 235 | 239 | 329 | 336 | 434 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.02 | 1.40 | 1.43 | 1.85 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.353 | 1.039 | 0.025 | 0.686 | 0.539 | 0.000 | 0.000 | 0.000 |

| Problem 162 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | B | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 287 | 554 | 404 | 386 | 638 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.93 | 1.41 | 1.34 | 2.22 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.419 | 4.944 | 0.024 | 0.859 | 0.561 | 0.000 | 0.000 | 0.000 |

| Problem 163 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.172 | 7.774 | 0.125 | 0.000 | 0.459 | 0.000 | 0.000 | 0.000 |

| Problem 164 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 307 | 307 | 2828 | 1326 | 1635 | 1453 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 9.21 | 4.32 | 5.33 | 4.73 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.340 | 6.521 | 0.536 | 1.062 | 0.661 | 0.000 | 0.000 | 0.000 |

| Problem 165 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 246 | 246 | 1918 | 899 | 967 | 984 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 7.80 | 3.65 | 3.93 | 4.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.278 | 6.406 | 0.463 | 0.798 | 0.585 | 0.000 | 0.000 | 0.000 |

| Problem 166 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 564 | 535 | 522 | 594 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 3.12 | 2.96 | 2.88 | 3.28 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.227 | 2.904 | 0.493 | 0.460 | 0.562 | 0.000 | 0.000 | 0.000 |

| Problem 167 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 114 | 131 | 249 | 222 | 292 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.15 | 2.18 | 1.95 | 2.56 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.128 | 0.346 | 0.411 | 0.787 | 0.588 | 0.000 | 0.000 | 0.000 |

| Problem 168 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.144 | 0.813 | 0.302 | 0.000 | 0.462 | 0.000 | 0.000 | 0.000 |

| Problem 169 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.170 | 2.493 | 0.461 | 0.000 | 0.458 | 0.000 | 0.000 | 0.000 |

| Problem 170 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.239 | 9.893 | 0.096 | 0.000 | 0.427 | 0.000 | 0.000 | 0.000 |

| Problem 171 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-1) | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 798 | 1056 | 0 | 1233 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.67 | 3.53 | 0.00 | 4.12 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.293 | 1.713 | 0.161 | 0.000 | 0.634 | 0.000 | 0.000 | 0.000 |

| Problem 172 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 216 | 539 | 649 | 11018 | 797 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.50 | 3.00 | 51.01 | 3.69 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.222 | 1.424 | 0.129 | 1.962 | 0.559 | 0.000 | 0.000 | 0.000 |

| Problem 173 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 310 | 332 | 3284 | 448 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.23 | 2.39 | 23.63 | 3.22 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.149 | 3.915 | 0.131 | 1.665 | 0.497 | 0.000 | 0.000 | 0.000 |

| Problem 174 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 104 | 124 | 2110 | 95 | 0 | 1967 | 162 |
| normalized size | 1 | 1.00 | 1.79 | 2.14 | 36.38 | 1.64 | 0.00 | 33.91 | 2.79 |
| time (sec) | N/A | 0.063 | 0.679 | 0.141 | 0.394 | 0.464 | 0.000 | 5.159 | 2.305 |

| Problem 175 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.214 | 3.717 | 0.273 | 0.000 | 0.442 | 0.000 | 0.000 | 0.000 |

| Problem 176 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.256 | 4.041 | 0.435 | 0.000 | 0.450 | 0.000 | 0.000 | 0.000 |

| Problem 177 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.036 | 11.481 | 0.115 | 0.000 | 0.439 | 0.000 | 0.000 | 0.000 |

| Problem 178 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 302 | 302 | 1534 | 1868 | 7111 | 1747 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 5.08 | 6.19 | 23.55 | 5.78 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.462 | 7.120 | 0.204 | 4.852 | 0.583 | 0.000 | 0.000 | 0.000 |

| Problem 179 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 256 | 256 | 994 | 1194 | 3952 | 1135 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 3.88 | 4.66 | 15.44 | 4.43 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.368 | 6.893 | 0.139 | 1.737 | 0.550 | 0.000 | 0.000 | 0.000 |

| Problem 180 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 540 | 635 | 1966 | 655 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 3.21 | 3.78 | 11.70 | 3.90 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.266 | 6.681 | 0.112 | 0.709 | 0.474 | 0.000 | 0.000 | 0.000 |

| Problem 181 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 240 | 281 | 839 | 339 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.20 | 2.58 | 7.70 | 3.11 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.128 | 6.166 | 0.086 | 0.528 | 0.459 | 0.000 | 0.000 | 0.000 |

| Problem 182 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.040 | 8.080 | 2.646 | 0.000 | 0.438 | 0.000 | 0.000 | 0.000 |

| Problem 183 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.037 | 9.289 | 4.187 | 0.000 | 0.471 | 0.000 | 0.000 | 0.000 |

| Problem 184 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 550 | 470 | 547 | 376 | 0 | 2418 | -1 |
| normalized size | 1 | 1.00 | 1.35 | 1.15 | 1.34 | 0.92 | 0.00 | 5.94 | -0.00 |
| time (sec) | N/A | 0.797 | 12.148 | 0.045 | 0.563 | 0.567 | 0.000 | 3.319 | 0.000 |

| Problem 185 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 393 | 376 | 503 | 294 | 0 | 1503 | -1 |
| normalized size | 1 | 1.00 | 1.12 | 1.07 | 1.43 | 0.84 | 0.00 | 4.28 | -0.00 |
| time (sec) | N/A | 0.556 | 2.983 | 0.039 | 0.511 | 0.503 | 0.000 | 2.647 | 0.000 |

| Problem 186 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 264 | 286 | 425 | 233 | 0 | 818 | -1 |
| normalized size | 1 | 1.00 | 0.88 | 0.96 | 1.42 | 0.78 | 0.00 | 2.74 | -0.00 |
| time (sec) | N/A | 0.447 | 0.590 | 0.038 | 0.537 | 0.510 | 0.000 | 2.818 | 0.000 |

| Problem 187 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 264 | 286 | 425 | 233 | 0 | 818 | -1 |
| normalized size | 1 | 1.00 | 0.88 | 0.96 | 1.42 | 0.78 | 0.00 | 2.74 | -0.00 |
| time (sec) | N/A | 0.455 | 0.159 | 0.000 | 0.511 | 0.546 | 0.000 | 0.981 | 0.000 |

| Problem 188 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 393 | 376 | 503 | 294 | 0 | 1503 | -1 |
| normalized size | 1 | 1.00 | 1.12 | 1.07 | 1.43 | 0.84 | 0.00 | 4.28 | -0.00 |
| time (sec) | N/A | 0.572 | 1.199 | 0.001 | 0.535 | 0.507 | 0.000 | 3.867 | 0.000 |

| Problem 189 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 550 | 470 | 547 | 376 | 0 | 2418 | -1 |
| normalized size | 1 | 1.00 | 1.35 | 1.15 | 1.34 | 0.92 | 0.00 | 5.94 | -0.00 |
| time (sec) | N/A | 0.672 | 8.395 | 0.000 | 0.519 | 0.535 | 0.000 | 3.104 | 0.000 |

| Problem 190 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 615 | 615 | 1795 | 716 | 820 | 548 | 0 | 3677 | -1 |
| normalized size | 1 | 1.00 | 2.92 | 1.16 | 1.33 | 0.89 | 0.00 | 5.98 | -0.00 |
| time (sec) | N/A | 1.145 | 22.421 | 0.068 | 0.566 | 0.579 | 0.000 | 18.657 | 0.000 |

| Problem 191 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 534 | 534 | 1043 | 583 | 754 | 446 | 0 | 2293 | -1 |
| normalized size | 1 | 1.00 | 1.95 | 1.09 | 1.41 | 0.84 | 0.00 | 4.29 | -0.00 |
| time (sec) | N/A | 0.838 | 12.002 | 0.065 | 0.584 | 0.560 | 0.000 | 7.722 | 0.000 |

| Problem 192 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 459 | 459 | 435 | 444 | 674 | 365 | 0 | 1258 | -1 |
| normalized size | 1 | 1.00 | 0.95 | 0.97 | 1.47 | 0.80 | 0.00 | 2.74 | -0.00 |
| time (sec) | N/A | 0.693 | 6.942 | 0.062 | 0.519 | 0.541 | 0.000 | 2.148 | 0.000 |

| Problem 193 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 459 | 459 | 435 | 444 | 674 | 365 | 0 | 1258 | -1 |
| normalized size | 1 | 1.00 | 0.95 | 0.97 | 1.47 | 0.80 | 0.00 | 2.74 | -0.00 |
| time (sec) | N/A | 0.672 | 6.874 | 0.000 | 0.513 | 0.538 | 0.000 | 4.485 | 0.000 |

| Problem 194 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 534 | 534 | 1043 | 583 | 754 | 446 | 0 | 2293 | -1 |
| normalized size | 1 | 1.00 | 1.95 | 1.09 | 1.41 | 0.84 | 0.00 | 4.29 | -0.00 |
| time (sec) | N/A | 0.858 | 11.710 | 0.000 | 0.535 | 0.585 | 0.000 | 7.042 | 0.000 |

| Problem 195 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 615 | 615 | 1795 | 716 | 820 | 548 | 0 | 3677 | -1 |
| normalized size | 1 | 1.00 | 2.92 | 1.16 | 1.33 | 0.89 | 0.00 | 5.98 | -0.00 |
| time (sec) | N/A | 1.015 | 21.748 | 0.002 | 0.577 | 0.571 | 0.000 | 12.742 | 0.000 |

| Problem 196 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 550 | 477 | 557 | 445 | 0 | 2417 | -1 |
| normalized size | 1 | 1.00 | 1.35 | 1.17 | 1.37 | 1.09 | 0.00 | 5.94 | -0.00 |
| time (sec) | N/A | 0.897 | 5.073 | 0.046 | 0.534 | 0.594 | 0.000 | 13.771 | 0.000 |

| Problem 197 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 391 | 383 | 513 | 326 | 0 | 1502 | -1 |
| normalized size | 1 | 1.00 | 1.11 | 1.09 | 1.46 | 0.93 | 0.00 | 4.28 | -0.00 |
| time (sec) | N/A | 0.628 | 2.831 | 0.045 | 0.514 | 0.559 | 0.000 | 12.684 | 0.000 |

| Problem 198 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 264 | 293 | 435 | 242 | 0 | 818 | -1 |
| normalized size | 1 | 1.00 | 0.88 | 0.98 | 1.45 | 0.81 | 0.00 | 2.74 | -0.00 |
| time (sec) | N/A | 0.458 | 1.250 | 0.043 | 0.525 | 0.511 | 0.000 | 7.825 | 0.000 |

| Problem 199 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 264 | 293 | 435 | 242 | 0 | 818 | -1 |
| normalized size | 1 | 1.00 | 0.88 | 0.98 | 1.45 | 0.81 | 0.00 | 2.74 | -0.00 |
| time (sec) | N/A | 0.448 | 0.523 | 0.000 | 0.512 | 0.564 | 0.000 | 3.740 | 0.000 |

| Problem 200 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 391 | 383 | 513 | 326 | 0 | 1502 | -1 |
| normalized size | 1 | 1.00 | 1.11 | 1.09 | 1.46 | 0.93 | 0.00 | 4.28 | -0.00 |
| time (sec) | N/A | 0.559 | 0.202 | 0.000 | 0.542 | 0.580 | 0.000 | 13.119 | 0.000 |

| Problem 201 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 550 | 477 | 557 | 445 | 0 | 2417 | -1 |
| normalized size | 1 | 1.00 | 1.35 | 1.17 | 1.37 | 1.09 | 0.00 | 5.94 | -0.00 |
| time (sec) | N/A | 0.669 | 2.738 | 0.000 | 0.564 | 0.606 | 0.000 | 16.884 | 0.000 |

| Problem 202 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 104 | 150 | 0 | 244 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.93 | 1.34 | 0.00 | 2.18 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.185 | 0.171 | 0.122 | 0.000 | 0.510 | 0.000 | 0.000 | 0.000 |

| Problem 203 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 72 | 112 | 0 | 162 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.87 | 1.35 | 0.00 | 1.95 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.170 | 0.100 | 0.111 | 0.000 | 0.484 | 0.000 | 0.000 | 0.000 |

| Problem 204 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 33 | 76 | 0 | 45 | 0 | 206 | 56 |
| normalized size | 1 | 1.00 | 1.00 | 2.30 | 0.00 | 1.36 | 0.00 | 6.24 | 1.70 |
| time (sec) | N/A | 0.055 | 0.024 | 0.115 | 0.000 | 0.461 | 0.000 | 0.224 | 1.234 |

| Problem 205 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 180 | 159 | 240 | 3719 | 508 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.88 | 1.33 | 20.66 | 2.82 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.401 | 0.408 | 0.157 | 0.945 | 0.510 | 0.000 | 0.000 | 0.000 |

| Problem 206 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 108 | 170 | 2855 | 370 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.02 | 1.60 | 26.93 | 3.49 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.278 | 0.332 | 0.164 | 0.752 | 0.536 | 0.000 | 0.000 | 0.000 |

| Problem 207 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 62 | 109 | 1739 | 203 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.85 | 1.49 | 23.82 | 2.78 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.164 | 0.111 | 0.138 | 0.588 | 0.482 | 0.000 | 0.000 | 0.000 |

| Problem 208 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.06 |
| time (sec) | N/A | 0.018 | 2.554 | 0.131 | 0.000 | 0.424 | 0.000 | 0.000 | 0.000 |

| Problem 209 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 157 | 616 | 792 | 1402 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.99 | 3.90 | 5.01 | 8.87 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.210 | 0.068 | 0.109 | 0.604 | 0.569 | 0.000 | 0.000 | 0.000 |

| Problem 210 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 126 | 423 | 490 | 970 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.95 | 3.20 | 3.71 | 7.35 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.183 | 0.085 | 0.079 | 0.573 | 0.537 | 0.000 | 0.000 | 0.000 |

| Problem 211 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 100 | 257 | 280 | 594 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.04 | 2.68 | 2.92 | 6.19 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.153 | 0.041 | 0.064 | 0.529 | 0.496 | 0.000 | 0.000 | 0.000 |

| Problem 212 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 70 | 123 | 114 | 310 | 0 | 0 | 148 |
| normalized size | 1 | 1.00 | 1.06 | 1.86 | 1.73 | 4.70 | 0.00 | 0.00 | 2.24 |
| time (sec) | N/A | 0.093 | 0.014 | 0.071 | 0.525 | 0.494 | 0.000 | 0.000 | 1.572 |

| Problem 213 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.06 |
| time (sec) | N/A | 0.020 | 3.532 | 0.128 | 0.000 | 0.438 | 0.000 | 0.000 | 0.000 |

| Problem 214 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.06 |
| time (sec) | N/A | 0.020 | 5.299 | 0.135 | 0.000 | 0.440 | 0.000 | 0.000 | 0.000 |

| Problem 215 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 148 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.136 | 6.617 | 0.270 | 0.000 | 0.431 | 0.000 | 0.000 | 0.000 |

| Problem 216 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 275 | 557 | 901 | 924 | 1071 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.03 | 3.28 | 3.36 | 3.89 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.215 | 1.470 | 0.362 | 0.673 | 0.555 | 0.000 | 0.000 | 0.000 |

| Problem 217 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 186 | 315 | 512 | 510 | 656 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.69 | 2.75 | 2.74 | 3.53 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.146 | 0.832 | 0.260 | 0.597 | 0.518 | 0.000 | 0.000 | 0.000 |

| Problem 218 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-1) | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 213 | 209 | 0 | 331 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.07 | 2.03 | 0.00 | 3.21 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.068 | 0.402 | 0.057 | 0.000 | 1.036 | 0.000 | 0.000 | 0.000 |

| Problem 219 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.110 | 6.038 | 0.269 | 0.000 | 0.434 | 0.000 | 0.000 | 0.000 |

| Problem 220 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.139 | 7.080 | 0.422 | 0.000 | 0.428 | 0.000 | 0.000 | 0.000 |

| Problem 221 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-2) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.171 | 7.810 | 0.252 | 0.000 | 0.444 | 0.000 | 0.000 | 0.000 |

| Problem 222 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 251 | 251 | 1720 | 641 | 685 | 1134 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 6.85 | 2.55 | 2.73 | 4.52 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.298 | 7.120 | 0.218 | 0.561 | 0.615 | 0.000 | 0.000 | 0.000 |

| Problem 223 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 184 | 518 | 379 | 379 | 688 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.82 | 2.06 | 2.06 | 3.74 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.228 | 6.469 | 0.327 | 0.531 | 0.543 | 0.000 | 0.000 | 0.000 |

| Problem 224 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 134 | 179 | 145 | 346 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.17 | 1.56 | 1.26 | 3.01 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.128 | 0.300 | 0.339 | 0.516 | 0.495 | 0.000 | 0.000 | 0.000 |

| Problem 225 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-2) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.151 | 0.763 | 0.453 | 0.000 | 0.438 | 0.000 | 0.000 | 0.000 |

| Problem 226 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-2) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.176 | 2.401 | 0.693 | 0.000 | 0.422 | 0.000 | 0.000 | 0.000 |

| Problem 227 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.183 | 5.892 | 0.060 | 0.000 | 0.431 | 0.000 | 0.000 | 0.000 |

| Problem 228 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 247 | 578 | 1242 | 1779 | 2600 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.34 | 5.03 | 7.20 | 10.53 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.229 | 1.285 | 0.170 | 0.714 | 0.658 | 0.000 | 0.000 | 0.000 |

| Problem 229 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 197 | 350 | 816 | 1063 | 1778 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.78 | 4.14 | 5.40 | 9.03 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.166 | 0.946 | 0.119 | 0.610 | 0.604 | 0.000 | 0.000 | 0.000 |

| Problem 230 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 213 | 469 | 590 | 1090 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.68 | 3.69 | 4.65 | 8.58 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.117 | 0.589 | 0.093 | 0.596 | 0.559 | 0.000 | 0.000 | 0.000 |

| Problem 231 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 141 | 208 | 267 | 554 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.99 | 2.93 | 3.76 | 7.80 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.056 | 0.115 | 0.081 | 0.697 | 0.517 | 0.000 | 0.000 | 0.000 |

| Problem 232 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.045 | 4.533 | 0.099 | 0.000 | 0.441 | 0.000 | 0.000 | 0.000 |

| Problem 233 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.041 | 5.715 | 0.099 | 0.000 | 0.446 | 0.000 | 0.000 | 0.000 |

| Problem 234 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.197 | 17.786 | 0.082 | 0.000 | 0.459 | 0.000 | 0.000 | 0.000 |

| Problem 235 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 350 | 350 | 739 | 1158 | 3240 | 1753 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.11 | 3.31 | 9.26 | 5.01 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.640 | 6.410 | 0.501 | 1.411 | 0.651 | 0.000 | 0.000 | 0.000 |

| Problem 236 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 226 | 226 | 593 | 556 | 1632 | 1067 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.62 | 2.46 | 7.22 | 4.72 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.384 | 6.275 | 0.277 | 0.774 | 0.560 | 0.000 | 0.000 | 0.000 |

| Problem 237 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-1) | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 517 | 235 | 0 | 434 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 3.95 | 1.79 | 0.00 | 3.31 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.135 | 2.908 | 0.129 | 0.000 | 0.504 | 0.000 | 0.000 | 0.000 |

| Problem 238 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.160 | 11.546 | 2.634 | 0.000 | 0.462 | 0.000 | 0.000 | 0.000 |

| Problem 239 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.190 | 11.077 | 3.124 | 0.000 | 0.463 | 0.000 | 0.000 | 0.000 |

| Problem 240 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.237 | 18.288 | 0.111 | 0.000 | 0.422 | 0.000 | 0.000 | 0.000 |

| Problem 241 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 325 | 325 | 1477 | 1223 | 5140 | 3459 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 4.54 | 3.76 | 15.82 | 10.64 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.823 | 6.975 | 0.187 | 2.137 | 0.772 | 0.000 | 0.000 | 0.000 |

| Problem 242 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 201 | 872 | 632 | 2522 | 1987 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 4.34 | 3.14 | 12.55 | 9.89 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.443 | 6.757 | 0.164 | 0.823 | 0.642 | 0.000 | 0.000 | 0.000 |

| Problem 243 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 141 | 210 | 270 | 1035 | 942 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.49 | 1.91 | 7.34 | 6.68 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.139 | 0.898 | 0.125 | 0.625 | 0.541 | 0.000 | 0.000 | 0.000 |

| Problem 244 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.147 | 14.068 | 3.382 | 0.000 | 0.530 | 0.000 | 0.000 | 0.000 |

| Problem 245 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.192 | 16.231 | 5.592 | 0.000 | 0.638 | 0.000 | 0.000 | 0.000 |

| Problem 246 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.142 | 2.700 | 0.080 | 0.000 | 0.439 | 0.000 | 0.000 | 0.000 |

| Problem 247 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 227 | 227 | 428 | 767 | 2944 | 1186 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.89 | 3.38 | 12.97 | 5.22 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.186 | 1.168 | 0.196 | 0.792 | 0.575 | 0.000 | 0.000 | 0.000 |

| Problem 248 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 159 | 256 | 463 | 1774 | 779 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.61 | 2.91 | 11.16 | 4.90 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.126 | 0.807 | 0.128 | 0.621 | 0.530 | 0.000 | 0.000 | 0.000 |

| Problem 249 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-1) | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 174 | 239 | 0 | 446 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.79 | 2.46 | 0.00 | 4.60 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.068 | 1.655 | 0.023 | 0.000 | 0.504 | 0.000 | 0.000 | 0.000 |

| Problem 250 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 93 | 49 | 259 | 60 | 0 | 1537 | 78 |
| normalized size | 1 | 1.00 | 3.21 | 1.69 | 8.93 | 2.07 | 0.00 | 53.00 | 2.69 |
| time (sec) | N/A | 0.019 | 0.045 | 0.021 | 0.449 | 0.475 | 0.000 | 1.308 | 2.583 |

| Problem 251 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.094 | 11.050 | 0.169 | 0.000 | 0.441 | 0.000 | 0.000 | 0.000 |

| Problem 252 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.115 | 19.124 | 0.237 | 0.000 | 0.433 | 0.000 | 0.000 | 0.000 |

| Problem 253 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.036 | 2.877 | 0.083 | 0.000 | 0.430 | 0.000 | 0.000 | 0.000 |

| Problem 254 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 128 | 424 | 348 | 1363 | 373 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 3.31 | 2.72 | 10.65 | 2.91 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.210 | 6.545 | 0.131 | 0.630 | 0.462 | 0.000 | 0.000 | 0.000 |

| Problem 255 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 276 | 191 | 417 | 210 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.88 | 1.99 | 4.34 | 2.19 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.141 | 6.368 | 0.089 | 0.602 | 0.465 | 0.000 | 0.000 | 0.000 |

| Problem 256 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 76 | 47 | 237 | 53 | 65 | 223 | 52 |
| normalized size | 1 | 1.00 | 1.90 | 1.18 | 5.92 | 1.32 | 1.62 | 5.58 | 1.30 |
| time (sec) | N/A | 0.028 | 0.276 | 0.053 | 0.457 | 0.436 | 0.246 | 0.561 | 1.436 |

| Problem 257 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.040 | 3.833 | 0.168 | 0.000 | 0.439 | 0.000 | 0.000 | 0.000 |

| Problem 258 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.039 | 5.071 | 0.235 | 0.000 | 0.458 | 0.000 | 0.000 | 0.000 |

| Problem 259 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.157 | 23.697 | 0.090 | 0.000 | 0.431 | 0.000 | 0.000 | 0.000 |

| Problem 260 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 228 | 532 | 677 | 11054 | 892 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.33 | 2.97 | 48.48 | 3.91 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.211 | 1.545 | 0.125 | 2.377 | 0.566 | 0.000 | 0.000 | 0.000 |

| Problem 261 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-1) | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 362 | 345 | 0 | 511 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.50 | 2.38 | 0.00 | 3.52 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.131 | 3.118 | 0.126 | 0.000 | 0.533 | 0.000 | 0.000 | 0.000 |

| Problem 262 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 107 | 123 | 2123 | 93 | 0 | 2762 | 151 |
| normalized size | 1 | 1.00 | 1.91 | 2.20 | 37.91 | 1.66 | 0.00 | 49.32 | 2.70 |
| time (sec) | N/A | 0.054 | 0.299 | 0.158 | 0.483 | 0.480 | 0.000 | 7.901 | 1.161 |

| Problem 263 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.150 | 3.778 | 0.186 | 0.000 | 0.457 | 0.000 | 0.000 | 0.000 |

| Problem 264 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.181 | 4.069 | 0.280 | 0.000 | 0.491 | 0.000 | 0.000 | 0.000 |

| Problem 265 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.248 | 24.569 | 0.082 | 0.000 | 0.435 | 0.000 | 0.000 | 0.000 |

| Problem 266 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 469 | 469 | 694 | 1866 | 5695 | 2507 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.48 | 3.98 | 12.14 | 5.35 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.795 | 3.365 | 0.599 | 2.568 | 0.815 | 0.000 | 0.000 | 0.000 |

| Problem 267 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 343 | 343 | 473 | 1152 | 3205 | 1697 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.38 | 3.36 | 9.34 | 4.95 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.573 | 1.342 | 0.467 | 1.143 | 0.650 | 0.000 | 0.000 | 0.000 |

| Problem 268 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 219 | 317 | 568 | 1598 | 1031 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.45 | 2.59 | 7.30 | 4.71 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.377 | 2.472 | 0.266 | 0.687 | 0.569 | 0.000 | 0.000 | 0.000 |

| Problem 269 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 122 | 212 | 160 | 806 | 366 | 0 | 0 | -1 |
| normalized size | 1 | 1.08 | 1.88 | 1.42 | 7.13 | 3.24 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.130 | 0.485 | 0.106 | 0.619 | 0.471 | 0.000 | 0.000 | 0.000 |

| Problem 270 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.153 | 9.652 | 2.332 | 0.000 | 0.438 | 0.000 | 0.000 | 0.000 |

| Problem 271 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.180 | 9.758 | 2.560 | 0.000 | 0.446 | 0.000 | 0.000 | 0.000 |

| Problem 272 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.190 | 2.633 | 0.076 | 0.000 | 0.427 | 0.000 | 0.000 | 0.000 |

| Problem 273 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 118 | 285 | 687 | 2355 | 1627 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.42 | 5.82 | 19.96 | 13.79 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.280 | 2.131 | 0.133 | 0.745 | 0.619 | 0.000 | 0.000 | 0.000 |

| Problem 274 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 277 | 351 | 777 | 950 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 3.15 | 3.99 | 8.83 | 10.80 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.191 | 1.716 | 0.119 | 0.686 | 0.558 | 0.000 | 0.000 | 0.000 |

| Problem 275 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 32 | 182 | 308 | 75 | 0 | 0 | 55 |
| normalized size | 1 | 1.00 | 0.91 | 5.20 | 8.80 | 2.14 | 0.00 | 0.00 | 1.57 |
| time (sec) | N/A | 0.059 | 0.203 | 0.088 | 0.477 | 0.448 | 0.000 | 0.000 | 1.656 |

| Problem 276 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.094 | 7.065 | 0.431 | 0.000 | 0.431 | 0.000 | 0.000 | 0.000 |

| Problem 277 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.086 | 7.326 | 0.776 | 0.000 | 0.425 | 0.000 | 0.000 | 0.000 |

| Problem 278 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.215 | 25.514 | 0.136 | 0.000 | 0.432 | 0.000 | 0.000 | 0.000 |

| Problem 279 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 601 | 601 | 907 | 1613 | 8043 | 3173 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.51 | 2.68 | 13.38 | 5.28 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.313 | 8.346 | 0.540 | 6.030 | 0.813 | 0.000 | 0.000 | 0.000 |

| Problem 280 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 305 | 305 | 889 | 802 | 3820 | 1801 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.91 | 2.63 | 12.52 | 5.90 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.865 | 7.971 | 0.307 | 1.616 | 0.656 | 0.000 | 0.000 | 0.000 |

| Problem 281 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 174 | 520 | 267 | 1503 | 621 | 0 | 0 | -1 |
| normalized size | 1 | 1.13 | 3.38 | 1.73 | 9.76 | 4.03 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.191 | 4.923 | 0.158 | 0.900 | 0.517 | 0.000 | 0.000 | 0.000 |

| Problem 282 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.201 | 22.219 | 2.670 | 0.000 | 0.506 | 0.000 | 0.000 | 0.000 |

| Problem 283 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.224 | 30.370 | 5.195 | 0.000 | 0.584 | 0.000 | 0.000 | 0.000 |

| Problem 284 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.924 | 39.651 | 0.113 | 0.000 | 0.452 | 0.000 | 0.000 | 0.000 |

| Problem 285 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | C | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 387 | 387 | 672 | 0 | 3989 | 1735 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.74 | 0.00 | 10.31 | 4.48 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.960 | 7.210 | 1.624 | 1.241 | 0.619 | 0.000 | 0.000 | 0.000 |

| Problem 286 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 235 | 613 | 429 | 2219 | 1229 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.61 | 1.83 | 9.44 | 5.23 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.537 | 6.623 | 0.224 | 0.743 | 0.562 | 0.000 | 0.000 | 0.000 |

| Problem 287 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 282 | 182 | 1184 | 527 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.24 | 1.44 | 9.40 | 4.18 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.168 | 2.409 | 0.141 | 0.685 | 0.512 | 0.000 | 0.000 | 0.000 |

| Problem 288 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.500 | 47.009 | 0.779 | 0.000 | 0.439 | 0.000 | 0.000 | 0.000 |

| Problem 289 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.513 | 19.784 | 0.882 | 0.000 | 0.452 | 0.000 | 0.000 | 0.000 |

| Problem 290 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.152 | 5.405 | 0.095 | 0.000 | 0.448 | 0.000 | 0.000 | 0.000 |

| Problem 291 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 418 | 489 | 3438 | 888 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 3.01 | 3.52 | 24.73 | 6.39 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.258 | 6.561 | 0.111 | 0.684 | 0.572 | 0.000 | 0.000 | 0.000 |

| Problem 292 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 286 | 301 | 667 | 540 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.49 | 2.62 | 5.80 | 4.70 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.174 | 6.378 | 0.098 | 0.629 | 0.533 | 0.000 | 0.000 | 0.000 |

| Problem 293 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 66 | 95 | 988 | 86 | 0 | 4474 | 150 |
| normalized size | 1 | 1.00 | 1.20 | 1.73 | 17.96 | 1.56 | 0.00 | 81.35 | 2.73 |
| time (sec) | N/A | 0.062 | 0.510 | 0.030 | 0.523 | 0.451 | 0.000 | 1.815 | 3.063 |

| Problem 294 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 48 | 61 | 283 | 36 | 0 | 571 | 53 |
| normalized size | 1 | 1.00 | 1.37 | 1.74 | 8.09 | 1.03 | 0.00 | 16.31 | 1.51 |
| time (sec) | N/A | 0.032 | 0.061 | 0.030 | 0.351 | 0.409 | 0.000 | 1.844 | 2.187 |

| Problem 295 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.106 | 7.244 | 0.278 | 0.000 | 0.488 | 0.000 | 0.000 | 0.000 |

| Problem 296 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.128 | 10.385 | 0.292 | 0.000 | 0.448 | 0.000 | 0.000 | 0.000 |

| Problem 297 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F(-1) | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.084 | 180.018 | 0.090 | 0.000 | 0.422 | 0.000 | 0.000 | 0.000 |

| Problem 298 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 337 | 337 | 530 | 1127 | 3828 | 1311 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.57 | 3.34 | 11.36 | 3.89 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.408 | 3.400 | 0.208 | 2.009 | 0.639 | 0.000 | 0.000 | 0.000 |

| Problem 299 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 193 | 526 | 584 | 1893 | 791 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.73 | 3.03 | 9.81 | 4.10 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.271 | 7.183 | 0.174 | 0.851 | 0.548 | 0.000 | 0.000 | 0.000 |

| Problem 300 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-1) | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 117 | 555 | 267 | 0 | 435 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 4.74 | 2.28 | 0.00 | 3.72 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.129 | 6.504 | 0.116 | 0.000 | 0.525 | 0.000 | 0.000 | 0.000 |

| Problem 301 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.091 | 26.390 | 1.595 | 0.000 | 0.456 | 0.000 | 0.000 | 0.000 |

| Problem 302 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.089 | 29.120 | 2.605 | 0.000 | 0.449 | 0.000 | 0.000 | 0.000 |

| Problem 303 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.035 | 9.704 | 0.118 | 0.000 | 0.422 | 0.000 | 0.000 | 0.000 |

| Problem 304 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 259 | 259 | 803 | 720 | 2405 | 590 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 3.10 | 2.78 | 9.29 | 2.28 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.356 | 6.886 | 0.127 | 1.186 | 0.454 | 0.000 | 0.000 | 0.000 |

| Problem 305 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 169 | 454 | 400 | 1226 | 352 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.69 | 2.37 | 7.25 | 2.08 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.222 | 6.656 | 0.098 | 0.673 | 0.455 | 0.000 | 0.000 | 0.000 |

| Problem 306 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | A | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 240 | 183 | 519 | 168 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.22 | 1.69 | 4.81 | 1.56 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.117 | 6.153 | 0.076 | 0.583 | 0.430 | 0.000 | 0.000 | 0.000 |

| Problem 307 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.039 | 6.531 | 1.759 | 0.000 | 0.425 | 0.000 | 0.000 | 0.000 |

| Problem 308 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.037 | 6.961 | 2.733 | 0.000 | 0.417 | 0.000 | 0.000 | 0.000 |

| Problem 309 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.269 | 13.368 | 0.105 | 0.000 | 0.415 | 0.000 | 0.000 | 0.000 |

| Problem 310 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 399 | 399 | 2090 | 1729 | 8770 | 3308 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 5.24 | 4.33 | 21.98 | 8.29 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.971 | 7.470 | 0.247 | 6.051 | 1.046 | 0.000 | 0.000 | 0.000 |

| Problem 311 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 325 | 325 | 1486 | 1115 | 4918 | 2260 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 4.57 | 3.43 | 15.13 | 6.95 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.640 | 6.909 | 0.162 | 2.019 | 0.830 | 0.000 | 0.000 | 0.000 |

| Problem 312 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 201 | 875 | 614 | 2442 | 1396 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 4.35 | 3.05 | 12.15 | 6.95 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.414 | 6.757 | 0.132 | 0.815 | 0.654 | 0.000 | 0.000 | 0.000 |

| Problem 313 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 141 | 212 | 270 | 1035 | 760 | 0 | 0 | -1 |
| normalized size | 1 | 1.01 | 1.53 | 1.94 | 7.45 | 5.47 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.135 | 0.564 | 0.109 | 0.633 | 0.558 | 0.000 | 0.000 | 0.000 |

| Problem 314 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.189 | 9.076 | 3.352 | 0.000 | 0.525 | 0.000 | 0.000 | 0.000 |

| Problem 315 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.209 | 6.651 | 5.624 | 0.000 | 0.629 | 0.000 | 0.000 | 0.000 |

| Problem 316 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.233 | 30.513 | 0.139 | 0.000 | 0.462 | 0.000 | 0.000 | 0.000 |

| Problem 317 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 486 | 486 | 819 | 1629 | 8032 | 2218 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.69 | 3.35 | 16.53 | 4.56 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.206 | 7.897 | 0.560 | 6.459 | 0.837 | 0.000 | 0.000 | 0.000 |

| Problem 318 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 341 | 341 | 889 | 770 | 3819 | 1362 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.61 | 2.26 | 11.20 | 3.99 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.648 | 7.530 | 0.316 | 1.622 | 0.649 | 0.000 | 0.000 | 0.000 |

| Problem 319 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-1) | B | F | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 182 | 669 | 344 | 0 | 592 | 0 | 0 | -1 |
| normalized size | 1 | 1.12 | 4.13 | 2.12 | 0.00 | 3.65 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.196 | 6.588 | 0.164 | 0.000 | 0.537 | 0.000 | 0.000 | 0.000 |

| Problem 320 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.187 | 20.743 | 3.249 | 0.000 | 0.527 | 0.000 | 0.000 | 0.000 |

| Problem 321 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.197 | 25.884 | 4.286 | 0.000 | 0.578 | 0.000 | 0.000 | 0.000 |

| Problem 322 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.265 | 32.372 | 0.118 | 0.000 | 0.430 | 0.000 | 0.000 | 0.000 |

| Problem 323 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 318 | 318 | 483 | 1329 | 5610 | 4193 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.52 | 4.18 | 17.64 | 13.19 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.321 | 8.626 | 0.218 | 3.734 | 0.945 | 0.000 | 0.000 | 0.000 |

| Problem 324 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 190 | 190 | 381 | 716 | 2722 | 2387 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.01 | 3.77 | 14.33 | 12.56 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.213 | 8.177 | 0.157 | 1.102 | 0.700 | 0.000 | 0.000 | 0.000 |

| Problem 325 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 110 | 236 | 325 | 1078 | 1193 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.15 | 2.95 | 9.80 | 10.85 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.106 | 2.088 | 0.171 | 0.748 | 0.596 | 0.000 | 0.000 | 0.000 |

| Problem 326 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.087 | 28.703 | 1.049 | 0.000 | 0.512 | 0.000 | 0.000 | 0.000 |

| Problem 327 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.088 | 32.466 | 3.790 | 0.000 | 0.568 | 0.000 | 0.000 | 0.000 |

| Problem 328 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 73 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.054 | 0.330 | 0.080 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 329 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 51 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.040 | 0.375 | 0.042 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 330 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 52 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.039 | 0.152 | 0.039 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 331 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F(-2) | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 181 | 310 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 5.48 | 9.39 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.026 | 1.759 | 0.156 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 332 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 33 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.026 | 0.160 | 0.039 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 333 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 54 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.036 | 0.185 | 0.039 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 334 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 53 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.038 | 0.230 | 0.039 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 335 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 65 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.048 | 0.256 | 0.043 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 336 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 65 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.062 | 0.280 | 0.060 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 337 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 61 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.049 | 0.231 | 0.048 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 338 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 54 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.053 | 0.184 | 0.049 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 339 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 42 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.036 | 0.127 | 0.049 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 340 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 132 | 310 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.49 | 5.85 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.037 | 2.154 | 0.083 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 341 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 63 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.046 | 0.236 | 0.048 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 342 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 212 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.047 | 8.104 | 0.046 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 343 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 89 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.060 | 0.338 | 0.045 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 344 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 67 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.044 | 0.541 | 0.056 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 345 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 108 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.032 | 0.903 | 0.046 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 346 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 56 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.032 | 0.189 | 0.041 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 347 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 86 | 308 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.26 | 8.11 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.022 | 1.159 | 0.115 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 348 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 37 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.023 | 0.177 | 0.040 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 349 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 56 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.032 | 0.179 | 0.042 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 350 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 57 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.034 | 0.228 | 0.043 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 351 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 73 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.044 | 0.288 | 0.043 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 352 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 73 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.058 | 0.266 | 0.076 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 353 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 65 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.044 | 0.292 | 0.060 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 354 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 56 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.040 | 0.178 | 0.064 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 355 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 46 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.032 | 0.137 | 0.061 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 356 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 106 | 308 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.83 | 5.31 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.032 | 0.724 | 0.125 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 357 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 65 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.044 | 0.225 | 0.053 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 358 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 114 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.044 | 0.983 | 0.052 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 359 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 93 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.060 | 0.394 | 0.053 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 360 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 22 | 26 | 18 | 17 | 37 | 18 | 18 |
| normalized size | 1 | 1.00 | 0.71 | 0.84 | 0.58 | 0.55 | 1.19 | 0.58 | 0.58 |
| time (sec) | N/A | 0.041 | 0.018 | 0.055 | 0.318 | 0.449 | 1.884 | 0.150 | 0.092 |

| Problem 361 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 154 | 260 | 146 | 200 | 440 | 167 | 212 |
| normalized size | 1 | 1.00 | 1.18 | 1.98 | 1.11 | 1.53 | 3.36 | 1.27 | 1.62 |
| time (sec) | N/A | 0.188 | 0.224 | 0.078 | 0.346 | 0.447 | 25.349 | 2.643 | 2.261 |

| Problem 362 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 109 | 179 | 101 | 127 | 289 | 112 | 136 |
| normalized size | 1 | 1.00 | 0.95 | 1.56 | 0.88 | 1.10 | 2.51 | 0.97 | 1.18 |
| time (sec) | N/A | 0.141 | 0.163 | 0.058 | 0.338 | 0.471 | 13.049 | 0.294 | 0.343 |

| Problem 363 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 60 | 107 | 60 | 70 | 155 | 64 | 73 |
| normalized size | 1 | 1.00 | 0.82 | 1.47 | 0.82 | 0.96 | 2.12 | 0.88 | 1.00 |
| time (sec) | N/A | 0.103 | 0.109 | 0.056 | 0.333 | 0.455 | 6.972 | 0.163 | 1.815 |

| Problem 364 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 34 | 52 | 27 | 27 | 56 | 27 | 30 |
| normalized size | 1 | 1.00 | 0.83 | 1.27 | 0.66 | 0.66 | 1.37 | 0.66 | 0.73 |
| time (sec) | N/A | 0.056 | 0.018 | 0.056 | 0.342 | 0.439 | 3.852 | 1.506 | 1.715 |

| Problem 365 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 49 | 58 | 95 | 62 | 0 | 51 | -1 |
| normalized size | 1 | 1.00 | 0.86 | 1.02 | 1.67 | 1.09 | 0.00 | 0.89 | -0.02 |
| time (sec) | N/A | 0.252 | 0.061 | 0.056 | 0.382 | 0.440 | 0.000 | 2.954 | 0.000 |

| Problem 366 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 61 | 82 | 324 | 95 | 0 | 111 | -1 |
| normalized size | 1 | 1.00 | 0.78 | 1.05 | 4.15 | 1.22 | 0.00 | 1.42 | -0.01 |
| time (sec) | N/A | 0.238 | 0.138 | 0.054 | 0.401 | 0.479 | 0.000 | 0.196 | 0.000 |

| Problem 367 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 77 | 104 | 362 | 158 | 0 | 201 | -1 |
| normalized size | 1 | 1.00 | 0.78 | 1.05 | 3.66 | 1.60 | 0.00 | 2.03 | -0.01 |
| time (sec) | N/A | 0.329 | 0.243 | 0.058 | 0.422 | 0.443 | 0.000 | 2.727 | 0.000 |

| Problem 368 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 198 | 198 | 128 | 1000 | 244 | 283 | 0 | 4684 | 344 |
| normalized size | 1 | 1.00 | 0.65 | 5.05 | 1.23 | 1.43 | 0.00 | 23.66 | 1.74 |
| time (sec) | N/A | 0.252 | 0.683 | 0.056 | 0.401 | 0.449 | 0.000 | 0.553 | 0.650 |

| Problem 369 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 171 | 105 | 580 | 173 | 188 | 0 | 3139 | 216 |
| normalized size | 1 | 1.00 | 0.61 | 3.39 | 1.01 | 1.10 | 0.00 | 18.36 | 1.26 |
| time (sec) | N/A | 0.184 | 0.422 | 0.036 | 0.372 | 0.449 | 0.000 | 4.428 | 2.182 |

| Problem 370 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 73 | 294 | 108 | 111 | 0 | 1880 | 121 |
| normalized size | 1 | 1.00 | 0.65 | 2.62 | 0.96 | 0.99 | 0.00 | 16.79 | 1.08 |
| time (sec) | N/A | 0.136 | 0.400 | 0.033 | 0.353 | 0.446 | 0.000 | 5.766 | 0.313 |

| Problem 371 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 46 | 119 | 55 | 54 | 0 | 920 | 53 |
| normalized size | 1 | 1.00 | 0.70 | 1.80 | 0.83 | 0.82 | 0.00 | 13.94 | 0.80 |
| time (sec) | N/A | 0.068 | 0.143 | 0.032 | 0.345 | 0.436 | 0.000 | 1.992 | 0.205 |

| Problem 372 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 63 | 116 | 117 | 85 | 0 | 1118 | -1 |
| normalized size | 1 | 1.00 | 0.89 | 1.63 | 1.65 | 1.20 | 0.00 | 15.75 | -0.01 |
| time (sec) | N/A | 0.281 | 0.148 | 0.040 | 0.406 | 0.457 | 0.000 | 0.323 | 0.000 |

| Problem 373 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 81 | 169 | 118 | 131 | 0 | 5381 | -1 |
| normalized size | 1 | 1.00 | 0.79 | 1.66 | 1.16 | 1.28 | 0.00 | 52.75 | -0.01 |
| time (sec) | N/A | 0.276 | 0.532 | 0.043 | 0.421 | 0.446 | 0.000 | 10.367 | 0.000 |

| Problem 374 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 136 | 104 | 207 | 130 | 225 | 0 | 9416 | -1 |
| normalized size | 1 | 1.00 | 0.76 | 1.52 | 0.96 | 1.65 | 0.00 | 69.24 | -0.01 |
| time (sec) | N/A | 0.374 | 0.989 | 0.041 | 0.430 | 0.460 | 0.000 | 10.030 | 0.000 |

| Problem 375 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 205 | 125 | 243 | 141 | 343 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.61 | 1.19 | 0.69 | 1.67 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.380 | 1.142 | 0.040 | 0.447 | 0.495 | 0.000 | 0.000 | 0.000 |

| Problem 376 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 255 | 255 | 459 | 849 | 602 | 925 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.80 | 3.33 | 2.36 | 3.63 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.347 | 1.537 | 0.121 | 0.543 | 0.558 | 0.000 | 0.000 | 0.000 |

| Problem 377 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | C | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 172 | 223 | 481 | 409 | 562 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.30 | 2.80 | 2.38 | 3.27 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.229 | 1.096 | 0.204 | 0.491 | 0.541 | 0.000 | 0.000 | 0.000 |

| Problem 378 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 171 | 204 | 0 | 281 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.80 | 2.15 | 0.00 | 2.96 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.109 | 0.304 | 0.068 | 0.000 | 0.478 | 0.000 | 0.000 | 0.000 |

| Problem 379 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.210 | 6.230 | 0.280 | 0.000 | 0.435 | 0.000 | 0.000 | 0.000 |

| Problem 380 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.283 | 6.714 | 0.471 | 0.000 | 0.420 | 0.000 | 0.000 | 0.000 |

| Problem 381 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.328 | 7.140 | 0.753 | 0.000 | 0.438 | 0.000 | 0.000 | 0.000 |

| Problem 382 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 2482 | 956 | 607 | 1644 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 8.30 | 3.20 | 2.03 | 5.50 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.503 | 6.746 | 0.128 | 0.517 | 0.644 | 0.000 | 0.000 | 0.000 |

| Problem 383 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | C | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 242 | 242 | 1719 | 639 | 442 | 1122 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 7.10 | 2.64 | 1.83 | 4.64 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.446 | 6.638 | 0.115 | 0.480 | 0.603 | 0.000 | 0.000 | 0.000 |

| Problem 384 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | C | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 173 | 516 | 377 | 301 | 677 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.98 | 2.18 | 1.74 | 3.91 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.328 | 6.563 | 0.133 | 0.447 | 0.554 | 0.000 | 0.000 | 0.000 |

| Problem 385 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F | B | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 254 | 177 | 0 | 340 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.37 | 1.65 | 0.00 | 3.18 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.180 | 5.574 | 0.310 | 0.000 | 0.528 | 0.000 | 0.000 | 0.000 |

| Problem 386 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-2) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.299 | 3.226 | 0.305 | 0.000 | 0.436 | 0.000 | 0.000 | 0.000 |

| Problem 387 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-2) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.342 | 4.208 | 0.493 | 0.000 | 0.442 | 0.000 | 0.000 | 0.000 |

| Problem 388 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.398 | 6.019 | 0.643 | 0.000 | 0.443 | 0.000 | 0.000 | 0.000 |

| Problem 389 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | C | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 230 | 230 | 607 | 677 | 0 | 896 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.64 | 2.94 | 0.00 | 3.90 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.330 | 2.599 | 0.180 | 0.000 | 0.555 | 0.000 | 0.000 | 0.000 |

| Problem 390 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-1) | B | F(-2) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 364 | 328 | 0 | 513 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.48 | 2.23 | 0.00 | 3.49 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.212 | 3.829 | 0.073 | 0.000 | 0.535 | 0.000 | 0.000 | 0.000 |

| Problem 391 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F(-2) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 105 | 87 | 3330 | 93 | 0 | 2876 | 150 |
| normalized size | 1 | 1.00 | 1.84 | 1.53 | 58.42 | 1.63 | 0.00 | 50.46 | 2.63 |
| time (sec) | N/A | 0.091 | 0.483 | 0.041 | 1.121 | 0.444 | 0.000 | 1.789 | 1.305 |

| Problem 392 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.266 | 13.726 | 0.338 | 0.000 | 0.454 | 0.000 | 0.000 | 0.000 |

| Problem 393 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.344 | 16.576 | 0.442 | 0.000 | 1.607 | 0.000 | 0.000 | 0.000 |

| Problem 394 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.436 | 19.134 | 0.769 | 0.000 | 0.514 | 0.000 | 0.000 | 0.000 |

| Problem 395 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 77 | 66 | 0 | 106 | 0 | 0 | 46 |
| normalized size | 1 | 1.00 | 1.35 | 1.16 | 0.00 | 1.86 | 0.00 | 0.00 | 0.81 |
| time (sec) | N/A | 0.067 | 0.031 | 0.059 | 0.000 | 1.227 | 0.000 | 0.000 | 2.214 |

| Problem 396 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 15 | 111 | 26 | 144 | 118 | 31 |
| normalized size | 1 | 1.00 | 1.00 | 1.07 | 7.93 | 1.86 | 10.29 | 8.43 | 2.21 |
| time (sec) | N/A | 0.033 | 0.020 | 0.056 | 0.418 | 0.564 | 5.739 | 1.553 | 2.347 |

| Problem 397 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 146 | 102 | 0 | 144 | 0 | 0 | 63 |
| normalized size | 1 | 1.00 | 2.18 | 1.52 | 0.00 | 2.15 | 0.00 | 0.00 | 0.94 |
| time (sec) | N/A | 0.137 | 0.281 | 0.209 | 0.000 | 2.067 | 0.000 | 0.000 | 2.317 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [205] had the largest ratio of [1.250]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 2 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 3 | A | 5 | 5 | 1.00 | 20 | 0.250 |
| 4 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 5 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 6 | A | 5 | 5 | 1.00 | 20 | 0.250 |
| 7 | A | 6 | 6 | 1.00 | 20 | 0.300 |
| 8 | A | 7 | 6 | 1.00 | 20 | 0.300 |
| 9 | A | 8 | 6 | 1.00 | 20 | 0.300 |
| 10 | A | 3 | 3 | 1.00 | 8 | 0.375 |
| 11 | A | 4 | 4 | 1.00 | 8 | 0.500 |
| 12 | A | 5 | 4 | 1.00 | 8 | 0.500 |
| 13 | A | 8 | 3 | 1.00 | 22 | 0.136 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 14 | A | 9 | 5 | 1.00 | 22 | 0.227 |
| 15 | A | 7 | 5 | 1.00 | 22 | 0.227 |
| 16 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 17 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 18 | A | 8 | 4 | 1.00 | 22 | 0.182 |
| 19 | A | 10 | 5 | 1.00 | 22 | 0.227 |
| 20 | A | 12 | 5 | 1.00 | 22 | 0.227 |
| 21 | A | 14 | 5 | 1.00 | 22 | 0.227 |
| 22 | A | 8 | 3 | 1.00 | 22 | 0.136 |
| 23 | A | 9 | 4 | 1.00 | 22 | 0.182 |
| 24 | A | 9 | 5 | 1.00 | 22 | 0.227 |
| 25 | A | 4 | 2 | 1.00 | 22 | 0.091 |
| 26 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 27 | A | 8 | 4 | 1.00 | 22 | 0.182 |
| 28 | A | 10 | 5 | 1.00 | 22 | 0.227 |
| 29 | A | 12 | 5 | 1.00 | 22 | 0.227 |
| 30 | A | 14 | 5 | 1.00 | 22 | 0.227 |
| 31 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 32 | A | 7 | 6 | 1.00 | 14 | 0.429 |
| 33 | A | 6 | 6 | 1.00 | 14 | 0.429 |
| 34 | A | 5 | 5 | 1.00 | 14 | 0.357 |
| 35 | A | 4 | 4 | 1.00 | 12 | 0.333 |
| 36 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 37 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 38 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 39 | A | 10 | 6 | 1.00 | 20 | 0.300 |
| 40 | A | 8 | 5 | 1.00 | 20 | 0.250 |
| 41 | A | 6 | 4 | 1.00 | 20 | 0.200 |
| 42 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 43 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 44 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 45 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 46 | A | 7 | 7 | 1.00 | 22 | 0.318 |
| 47 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 48 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 49 | A | 3 | 3 | 1.00 | 20 | 0.150 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 50 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 51 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 52 | A | 10 | 8 | 1.00 | 22 | 0.364 |
| 53 | A | 9 | 8 | 1.00 | 22 | 0.364 |
| 54 | A | 8 | 8 | 1.00 | 22 | 0.364 |
| 55 | A | 8 | 8 | 1.00 | 22 | 0.364 |
| 56 | A | 9 | 8 | 1.00 | 22 | 0.364 |
| 57 | A | 10 | 8 | 1.00 | 22 | 0.364 |
| 58 | A | 18 | 7 | 1.00 | 24 | 0.292 |
| 59 | A | 16 | 7 | 1.00 | 24 | 0.292 |
| 60 | A | 14 | 7 | 1.00 | 24 | 0.292 |
| 61 | A | 14 | 7 | 1.00 | 24 | 0.292 |
| 62 | A | 16 | 7 | 1.00 | 24 | 0.292 |
| 63 | A | 18 | 7 | 1.00 | 24 | 0.292 |
| 64 | A | 18 | 7 | 1.00 | 24 | 0.292 |
| 65 | A | 16 | 7 | 1.00 | 24 | 0.292 |
| 66 | A | 14 | 7 | 1.00 | 24 | 0.292 |
| 67 | A | 14 | 7 | 1.00 | 24 | 0.292 |
| 68 | A | 16 | 7 | 1.00 | 24 | 0.292 |
| 69 | A | 18 | 7 | 1.00 | 24 | 0.292 |
| 70 | A | 8 | 3 | 1.00 | 22 | 0.136 |
| 71 | A | 9 | 5 | 1.00 | 22 | 0.227 |
| 72 | A | 7 | 5 | 1.00 | 22 | 0.227 |
| 73 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 74 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 75 | A | 8 | 4 | 1.00 | 22 | 0.182 |
| 76 | A | 10 | 5 | 1.00 | 22 | 0.227 |
| 77 | A | 12 | 5 | 1.00 | 22 | 0.227 |
| 78 | A | 14 | 5 | 1.00 | 22 | 0.227 |
| 79 | A | 5 | 3 | 1.00 | 24 | 0.125 |
| 80 | A | 7 | 3 | 1.00 | 24 | 0.125 |
| 81 | A | 6 | 3 | 1.00 | 24 | 0.125 |
| 82 | A | 5 | 3 | 1.00 | 24 | 0.125 |
| 83 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 84 | A | 5 | 4 | 1.00 | 24 | 0.167 |
| 85 | A | 6 | 5 | 1.00 | 24 | 0.208 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 86 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 87 | A | 8 | 5 | 1.00 | 24 | 0.208 |
| 88 | A | 11 | 3 | 1.00 | 24 | 0.125 |
| 89 | A | 17 | 3 | 1.00 | 24 | 0.125 |
| 90 | A | 14 | 3 | 1.00 | 24 | 0.125 |
| 91 | A | 11 | 3 | 1.00 | 24 | 0.125 |
| 92 | A | 8 | 3 | 1.00 | 22 | 0.136 |
| 93 | A | 11 | 4 | 1.00 | 24 | 0.167 |
| 94 | A | 14 | 5 | 1.00 | 24 | 0.208 |
| 95 | A | 17 | 5 | 1.00 | 24 | 0.208 |
| 96 | A | 20 | 5 | 1.00 | 24 | 0.208 |
| 97 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 98 | A | 17 | 8 | 1.00 | 20 | 0.400 |
| 99 | A | 14 | 8 | 1.00 | 20 | 0.400 |
| 100 | A | 11 | 7 | 1.00 | 20 | 0.350 |
| 101 | A | 8 | 6 | 1.00 | 18 | 0.333 |
| 102 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 103 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 104 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 105 | A | 8 | 8 | 1.00 | 16 | 0.500 |
| 106 | A | 7 | 7 | 1.00 | 16 | 0.438 |
| 107 | A | 6 | 6 | 1.00 | 16 | 0.375 |
| 108 | A | 3 | 2 | 1.00 | 14 | 0.143 |
| 109 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 110 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 111 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 112 | A | 31 | 7 | 1.00 | 22 | 0.318 |
| 113 | A | 25 | 9 | 1.00 | 22 | 0.409 |
| 114 | A | 17 | 7 | 1.00 | 22 | 0.318 |
| 115 | A | 12 | 5 | 1.00 | 20 | 0.250 |
| 116 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 117 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 118 | A | 18 | 7 | 1.00 | 24 | 0.292 |
| 119 | A | 16 | 7 | 1.00 | 24 | 0.292 |
| 120 | A | 14 | 7 | 1.00 | 24 | 0.292 |
| 121 | A | 14 | 7 | 1.00 | 24 | 0.292 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 122 | A | 16 | 7 | 1.00 | 24 | 0.292 |
| 123 | A | 18 | 7 | 1.00 | 24 | 0.292 |
| 124 | A | 10 | 7 | 1.00 | 26 | 0.269 |
| 125 | A | 9 | 7 | 1.00 | 26 | 0.269 |
| 126 | A | 8 | 7 | 1.00 | 26 | 0.269 |
| 127 | A | 8 | 7 | 1.00 | 26 | 0.269 |
| 128 | A | 9 | 7 | 1.00 | 26 | 0.269 |
| 129 | A | 10 | 7 | 1.00 | 26 | 0.269 |
| 130 | A | 26 | 7 | 1.00 | 26 | 0.269 |
| 131 | A | 23 | 7 | 1.00 | 26 | 0.269 |
| 132 | A | 20 | 7 | 1.00 | 26 | 0.269 |
| 133 | A | 20 | 7 | 1.00 | 26 | 0.269 |
| 134 | A | 23 | 7 | 1.00 | 26 | 0.269 |
| 135 | A | 26 | 7 | 1.00 | 26 | 0.269 |
| 136 | A | 8 | 3 | 1.00 | 22 | 0.136 |
| 137 | A | 9 | 4 | 1.00 | 22 | 0.182 |
| 138 | A | 9 | 5 | 1.00 | 22 | 0.227 |
| 139 | A | 4 | 2 | 1.00 | 22 | 0.091 |
| 140 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 141 | A | 8 | 4 | 1.00 | 22 | 0.182 |
| 142 | A | 10 | 5 | 1.00 | 22 | 0.227 |
| 143 | A | 12 | 5 | 1.00 | 22 | 0.227 |
| 144 | A | 14 | 5 | 1.00 | 22 | 0.227 |
| 145 | A | 11 | 3 | 1.00 | 24 | 0.125 |
| 146 | A | 17 | 3 | 1.00 | 24 | 0.125 |
| 147 | A | 14 | 3 | 1.00 | 24 | 0.125 |
| 148 | A | 11 | 3 | 1.00 | 24 | 0.125 |
| 149 | A | 8 | 3 | 1.00 | 22 | 0.136 |
| 150 | A | 11 | 4 | 1.00 | 24 | 0.167 |
| 151 | A | 14 | 5 | 1.00 | 24 | 0.208 |
| 152 | A | 17 | 5 | 1.00 | 24 | 0.208 |
| 153 | A | 20 | 5 | 1.00 | 24 | 0.208 |
| 154 | A | 8 | 3 | 1.00 | 24 | 0.125 |
| 155 | A | 12 | 3 | 1.00 | 24 | 0.125 |
| 156 | A | 10 | 3 | 1.00 | 24 | 0.125 |
| 157 | A | 8 | 3 | 1.00 | 24 | 0.125 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 158 | A | 6 | 3 | 1.00 | 22 | 0.136 |
| 159 | A | 8 | 4 | 1.00 | 24 | 0.167 |
| 160 | A | 10 | 5 | 1.00 | 24 | 0.208 |
| 161 | A | 12 | 5 | 1.00 | 24 | 0.208 |
| 162 | A | 14 | 5 | 1.00 | 24 | 0.208 |
| 163 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 164 | A | 13 | 11 | 1.00 | 22 | 0.500 |
| 165 | A | 12 | 12 | 1.00 | 22 | 0.546 |
| 166 | A | 9 | 8 | 1.00 | 22 | 0.364 |
| 167 | A | 8 | 8 | 1.00 | 20 | 0.400 |
| 168 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 169 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 170 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 171 | A | 16 | 9 | 1.00 | 22 | 0.409 |
| 172 | A | 13 | 8 | 1.00 | 22 | 0.364 |
| 173 | A | 10 | 7 | 1.00 | 22 | 0.318 |
| 174 | A | 5 | 5 | 1.00 | 20 | 0.250 |
| 175 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 176 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 177 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 178 | A | 15 | 8 | 1.00 | 16 | 0.500 |
| 179 | A | 13 | 10 | 1.00 | 16 | 0.625 |
| 180 | A | 9 | 7 | 1.00 | 16 | 0.438 |
| 181 | A | 7 | 7 | 1.00 | 14 | 0.500 |
| 182 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 183 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 184 | A | 18 | 7 | 1.00 | 24 | 0.292 |
| 185 | A | 16 | 7 | 1.00 | 24 | 0.292 |
| 186 | A | 14 | 7 | 1.00 | 24 | 0.292 |
| 187 | A | 14 | 7 | 1.00 | 24 | 0.292 |
| 188 | A | 16 | 7 | 1.00 | 24 | 0.292 |
| 189 | A | 18 | 7 | 1.00 | 24 | 0.292 |
| 190 | A | 26 | 7 | 1.00 | 26 | 0.269 |
| 191 | A | 23 | 7 | 1.00 | 26 | 0.269 |
| 192 | A | 20 | 7 | 1.00 | 26 | 0.269 |
| 193 | A | 20 | 7 | 1.00 | 26 | 0.269 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 194 | A | 23 | 7 | 1.00 | 26 | 0.269 |
| 195 | A | 26 | 7 | 1.00 | 26 | 0.269 |
| 196 | A | 18 | 7 | 1.00 | 26 | 0.269 |
| 197 | A | 16 | 7 | 1.00 | 26 | 0.269 |
| 198 | A | 14 | 7 | 1.00 | 26 | 0.269 |
| 199 | A | 14 | 7 | 1.00 | 26 | 0.269 |
| 200 | A | 16 | 7 | 1.00 | 26 | 0.269 |
| 201 | A | 18 | 7 | 1.00 | 26 | 0.269 |
| 202 | A | 12 | 10 | 1.00 | 12 | 0.833 |
| 203 | A | 11 | 10 | 1.00 | 12 | 0.833 |
| 204 | A | 6 | 5 | 1.00 | 10 | 0.500 |
| 205 | A | 26 | 15 | 1.00 | 12 | 1.250 |
| 206 | A | 19 | 11 | 1.00 | 12 | 0.917 |
| 207 | A | 16 | 10 | 1.00 | 10 | 1.000 |
| 208 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 209 | A | 7 | 6 | 1.00 | 14 | 0.429 |
| 210 | A | 6 | 6 | 1.00 | 14 | 0.429 |
| 211 | A | 5 | 5 | 1.00 | 14 | 0.357 |
| 212 | A | 4 | 4 | 1.00 | 12 | 0.333 |
| 213 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 214 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 215 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 216 | A | 14 | 8 | 1.00 | 20 | 0.400 |
| 217 | A | 11 | 7 | 1.00 | 20 | 0.350 |
| 218 | A | 8 | 6 | 1.00 | 18 | 0.333 |
| 219 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 220 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 221 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 222 | A | 12 | 12 | 1.00 | 22 | 0.546 |
| 223 | A | 9 | 8 | 1.00 | 22 | 0.364 |
| 224 | A | 8 | 8 | 1.00 | 20 | 0.400 |
| 225 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 226 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 227 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 228 | A | 12 | 6 | 1.00 | 20 | 0.300 |
| 229 | A | 10 | 6 | 1.00 | 20 | 0.300 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 230 | A | 8 | 5 | 1.00 | 20 | 0.250 |
| 231 | A | 6 | 4 | 1.00 | 18 | 0.222 |
| 232 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 233 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 234 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 235 | A | 23 | 14 | 1.00 | 22 | 0.636 |
| 236 | A | 19 | 15 | 1.00 | 22 | 0.682 |
| 237 | A | 10 | 10 | 1.00 | 20 | 0.500 |
| 238 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 239 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 240 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 241 | A | 22 | 18 | 1.00 | 22 | 0.818 |
| 242 | A | 17 | 13 | 1.00 | 22 | 0.591 |
| 243 | A | 11 | 10 | 1.00 | 20 | 0.500 |
| 244 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 245 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 246 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 247 | A | 10 | 6 | 1.00 | 20 | 0.300 |
| 248 | A | 8 | 5 | 1.00 | 20 | 0.250 |
| 249 | A | 6 | 4 | 1.00 | 20 | 0.200 |
| 250 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 251 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 252 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 253 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 254 | A | 7 | 7 | 1.00 | 16 | 0.438 |
| 255 | A | 6 | 6 | 1.00 | 16 | 0.375 |
| 256 | A | 3 | 2 | 1.00 | 14 | 0.143 |
| 257 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 258 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 259 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 260 | A | 13 | 8 | 1.00 | 22 | 0.364 |
| 261 | A | 10 | 7 | 1.00 | 22 | 0.318 |
| 262 | A | 5 | 5 | 1.00 | 20 | 0.250 |
| 263 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 264 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 265 | A | 0 | 0 | 0.00 | 0 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 266 | A | 27 | 14 | 1.00 | 22 | 0.636 |
| 267 | A | 23 | 14 | 1.00 | 22 | 0.636 |
| 268 | A | 19 | 15 | 1.00 | 22 | 0.682 |
| 269 | A | 10 | 10 | 1.08 | 20 | 0.500 |
| 270 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 271 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 272 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 273 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 274 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 275 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 276 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 277 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 278 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 279 | A | 64 | 24 | 1.00 | 24 | 1.000 |
| 280 | A | 36 | 22 | 1.00 | 24 | 0.917 |
| 281 | A | 13 | 12 | 1.13 | 22 | 0.546 |
| 282 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 283 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 284 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 285 | A | 40 | 18 | 1.00 | 20 | 0.900 |
| 286 | A | 29 | 19 | 1.00 | 20 | 0.950 |
| 287 | A | 13 | 12 | 1.00 | 18 | 0.667 |
| 288 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 289 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 290 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 291 | A | 7 | 7 | 1.00 | 22 | 0.318 |
| 292 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 293 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 294 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 295 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 296 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 297 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 298 | A | 25 | 9 | 1.00 | 22 | 0.409 |
| 299 | A | 17 | 7 | 1.00 | 22 | 0.318 |
| 300 | A | 12 | 5 | 1.00 | 20 | 0.250 |
| 301 | A | 0 | 0 | 0.00 | 0 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 302 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 303 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 304 | A | 13 | 10 | 1.00 | 16 | 0.625 |
| 305 | A | 9 | 7 | 1.00 | 16 | 0.438 |
| 306 | A | 7 | 7 | 1.00 | 14 | 0.500 |
| 307 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 308 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 309 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 310 | A | 25 | 16 | 1.00 | 22 | 0.727 |
| 311 | A | 22 | 18 | 1.00 | 22 | 0.818 |
| 312 | A | 17 | 13 | 1.00 | 22 | 0.591 |
| 313 | A | 11 | 10 | 1.01 | 20 | 0.500 |
| 314 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 315 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 316 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 317 | A | 44 | 19 | 1.00 | 24 | 0.792 |
| 318 | A | 31 | 19 | 1.00 | 24 | 0.792 |
| 319 | A | 13 | 12 | 1.12 | 22 | 0.546 |
| 320 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 321 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 322 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 323 | A | 16 | 9 | 1.00 | 24 | 0.375 |
| 324 | A | 10 | 7 | 1.00 | 24 | 0.292 |
| 325 | A | 7 | 5 | 1.00 | 22 | 0.227 |
| 326 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 327 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 328 | A | 4 | 3 | 1.00 | 18 | 0.167 |
| 329 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 330 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 331 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 332 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 333 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 334 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 335 | A | 4 | 3 | 1.00 | 18 | 0.167 |
| 336 | A | 5 | 4 | 1.00 | 18 | 0.222 |
| 337 | A | 4 | 4 | 1.00 | 18 | 0.222 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 338 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 339 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 340 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 341 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 342 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 343 | A | 5 | 4 | 1.00 | 18 | 0.222 |
| 344 | A | 4 | 3 | 1.00 | 18 | 0.167 |
| 345 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 346 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 347 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 348 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 349 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 350 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 351 | A | 4 | 3 | 1.00 | 18 | 0.167 |
| 352 | A | 5 | 4 | 1.00 | 18 | 0.222 |
| 353 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 354 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 355 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 356 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 357 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 358 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 359 | A | 5 | 4 | 1.00 | 18 | 0.222 |
| 360 | A | 6 | 3 | 1.00 | 8 | 0.375 |
| 361 | A | 14 | 5 | 1.00 | 14 | 0.357 |
| 362 | A | 10 | 4 | 1.00 | 14 | 0.286 |
| 363 | A | 10 | 5 | 1.00 | 14 | 0.357 |
| 364 | A | 6 | 2 | 1.00 | 12 | 0.167 |
| 365 | A | 12 | 5 | 1.00 | 14 | 0.357 |
| 366 | A | 12 | 6 | 1.00 | 14 | 0.429 |
| 367 | A | 16 | 7 | 1.00 | 14 | 0.500 |
| 368 | A | 14 | 5 | 1.00 | 23 | 0.217 |
| 369 | A | 10 | 4 | 1.00 | 23 | 0.174 |
| 370 | A | 10 | 5 | 1.00 | 23 | 0.217 |
| 371 | A | 6 | 2 | 1.00 | 21 | 0.095 |
| 372 | A | 12 | 5 | 1.00 | 23 | 0.217 |
| 373 | A | 12 | 6 | 1.00 | 23 | 0.261 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 374 | A | 16 | 7 | 1.00 | 23 | 0.304 |
| 375 | A | 16 | 8 | 1.00 | 23 | 0.348 |
| 376 | A | 20 | 9 | 1.00 | 25 | 0.360 |
| 377 | A | 16 | 8 | 1.00 | 25 | 0.320 |
| 378 | A | 12 | 7 | 1.00 | 23 | 0.304 |
| 379 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 380 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 381 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 382 | A | 20 | 12 | 1.00 | 23 | 0.522 |
| 383 | A | 19 | 13 | 1.00 | 23 | 0.565 |
| 384 | A | 14 | 9 | 1.00 | 23 | 0.391 |
| 385 | A | 13 | 9 | 1.00 | 21 | 0.429 |
| 386 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 387 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 388 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 389 | A | 19 | 9 | 1.00 | 25 | 0.360 |
| 390 | A | 15 | 8 | 1.00 | 25 | 0.320 |
| 391 | A | 9 | 6 | 1.00 | 23 | 0.261 |
| 392 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 393 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 394 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 395 | A | 12 | 7 | 1.00 | 8 | 0.875 |
| 396 | A | 5 | 4 | 1.00 | 10 | 0.400 |
| 397 | A | 19 | 6 | 1.00 | 10 | 0.600 |

# Chapter 3

# Listing of integrals

## 3.1 $\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=137

$$-\frac{2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2ib}{}\right.}{b}$$

```
[Out] -2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b
*(d*x+c)/d)^m)-2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b
*c/d))/((I*b*(d*x+c)/d)^m)
```

**Rubi** [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4406, 12, 3308, 2181}

$$-\frac{2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gam}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -((2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-3 - m)*(c + d*x)^m*Gamma[
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^m \sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int (c+dx)^m \sin(2a+2bx)\,dx$$
$$= \frac{1}{4}i\int e^{-i(2a+2bx)}(c+dx)^m\,dx - \frac{1}{4}i\int e^{i(2a+2bx)}(c+dx)^m\,dx$$
$$= -\frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-3-m}e^{-2i}\ldots}{\ldots}$$

**Mathematica [A]** time = 0.08, size = 138, normalized size = 1.01

$$-\frac{2^{-m-3}e^{-\frac{2i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{4ia}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)+e^{\frac{4ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right)\ldots\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -((2^(-3 - m)*(c + d*x)^m*(E^((4*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, (
(-2*I)*b*(c + d*x))/d] + E^(((4*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1
 + m, ((2*I)*b*(c + d*x))/d]))/(b*E^(((2*I)*(b*c + a*d))/d)*((b^2*(c + d*x)
^2)/d^2)^m))
```

**fricas [A]** time = 0.48, size = 94, normalized size = 0.69

$$-\frac{e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2ibc+2iad}{d}\right)}\Gamma\left(m+1,\frac{2ibdx+2ibc}{d}\right)+e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}\Gamma\left(m+1,\frac{-2ibdx-2ibc}{d}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m + 1, (2*I*b*d*x
 + 2*I*b*c)/d) + e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1
, (-2*I*b*d*x - 2*I*b*c)/d))/b
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a), x)
```

**maple [F]** time = 0.35, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\ \sin(a+b\,x)\ (c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^m,x)
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^m, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a),x)
[Out] Integral((c + d*x)**m*sin(a + b*x)*cos(a + b*x), x)
```

## 3.2 $\int (c+dx)^4 \cos(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=156

$$\frac{3d^4\sin^2(a+bx)}{4b^5}-\frac{3d^3(c+dx)\sin(a+bx)\cos(a+bx)}{2b^4}-\frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3}+\frac{d(c+dx)^3\sin(a+bx)\cos(a+$$

```
[Out] 3/2*c*d^3*x/b^3+3/4*d^4*x^2/b^3-1/4*(d*x+c)^4/b-3/2*d^3*(d*x+c)*cos(b*x+a)*
sin(b*x+a)/b^4+d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+3/4*d^4*sin(b*x+a)^2/b
^5-3/2*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3+1/2*(d*x+c)^4*sin(b*x+a)^2/b
```

**Rubi [A]** time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4404, 3311, 32, 3310}

$$-\frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3}-\frac{3d^3(c+dx)\sin(a+bx)\cos(a+bx)}{2b^4}+\frac{d(c+dx)^3\sin(a+bx)\cos(a+bx)}{b^2}+\frac{3d^4\sin^2(a}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (3*c*d^3*x)/(2*b^3) + (3*d^4*x^2)/(4*b^3) - (c + d*x)^4/(4*b) - (3*d^3*(c +
 d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b^4) + (d*(c + d*x)^3*Cos[a + b*x]*Sin[
a + b*x])/b^2 + (3*d^4*Sin[a + b*x]^2)/(4*b^5) - (3*d^2*(c + d*x)^2*Sin[a +
 b*x]^2)/(2*b^3) + ((c + d*x)^4*Sin[a + b*x]^2)/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx = \frac{(c+dx)^4\sin^2(a+bx)}{2b} - \frac{(2d)\int (c+dx)^3\sin^2(a+bx)\,dx}{b}$$
$$= \frac{d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2} - \frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3} + \frac{(c\ \cdots}{}$$
$$= -\frac{(c+dx)^4}{4b} - \frac{3d^3(c+dx)\cos(a+bx)\sin(a+bx)}{2b^4} + \frac{d(c+dx)^3\cos(a\ \cdots}{b\ \cdots}$$
$$= \frac{3cd^3x}{2b^3} + \frac{3d^4x^2}{4b^3} - \frac{(c+dx)^4}{4b} - \frac{3d^3(c+dx)\cos(a+bx)\sin(a+bx)}{2b^4} + \cdots$$

**Mathematica [A]** time = 0.48, size = 86, normalized size = 0.55

$$\frac{4bd(c+dx)\sin(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)-2\cos(2(a+bx))\left(2b^4(c+dx)^4-6b^2d^2(c+dx)^2+3d^4\right)}{16b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] +
4*b*d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(16*b^5)
```

**fricas [A]** time = 0.66, size = 255, normalized size = 1.63

$$\frac{b^4d^4x^4 + 4\,b^4cd^3x^3 + 3\left(2\,b^4c^2d^2 - b^2d^4\right)x^2 - \left(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 2\,b^4c^4 - 6\,b^2c^2d^2 + 3\,d^4 + 6\left(2\,b^4c^2d^2 - b\ \cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 3*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - (2*b
^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4
*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 +
 2*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^
2*d^2 - b*d^4)*x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3*d - 3*b^2*c*d^3)
*x)/b^5
```

**giac [A]** time = 0.20, size = 181, normalized size = 1.16

$$-\frac{\left(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 + 8\,b^4c^3dx + 2\,b^4c^4 - 6\,b^2d^4x^2 - 12\,b^2cd^3x - 6\,b^2c^2d^2 + 3\,d^4\right)\cos\left(2\,bx + \cdots\right.}{8\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x
+ 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x - 6*b^2*c^2*d^2 + 3*d^4)*cos(2
*b*x + 2*a)/b^5 + 1/4*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x +
2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

**maple [B]** time = 0.06, size = 853, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^4*d^4*(-1/2*(b*x+a)^4*cos(b*x+a)^2+2*(b*x+a)^3*(1/2*cos(b*x+a)*sin
(b*x+a)+1/2*b*x+1/2*a)+3/2*(b*x+a)^2*cos(b*x+a)^2-3*(b*x+a)*(1/2*cos(b*x+a)
*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)^2+3/4*sin(b*x+a)^2-3/4*(b*x+a)^4)-4/
b^4*a*d^4*(-1/2*(b*x+a)^3*cos(b*x+a)^2+3/2*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*
x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)*cos(b*x+a)^2-3/8*cos(b*x+a)*sin(b*x+a)-3/8*
b*x-3/8*a-1/2*(b*x+a)^3)+4/b^3*c*d^3*(-1/2*(b*x+a)^3*cos(b*x+a)^2+3/2*(b*x+
a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)*cos(b*x+a)^2-3/8
*cos(b*x+a)*sin(b*x+a)-3/8*b*x-3/8*a-1/2*(b*x+a)^3)+6/b^4*a^2*d^4*(-1/2*(b*
x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(
b*x+a)^2-1/4*sin(b*x+a)^2)-12/b^3*a*c*d^3*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x
+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^
2)+6/b^2*c^2*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b
*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-4/b^4*a^3*d^4*(-1/2*(b
*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+12/b^3*a^2*c*d^
3*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-12/b^
2*a*c^2*d^2*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/
4*a)+4/b*c^3*d*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x
+1/4*a)-1/2/b^4*a^4*d^4*cos(b*x+a)^2+2/b^3*a^3*c*d^3*cos(b*x+a)^2-3/b^2*a^2
*c^2*d^2*cos(b*x+a)^2+2/b*a*c^3*d*cos(b*x+a)^2-1/2*c^4*cos(b*x+a)^2)
```

**maxima [B]** time = 0.39, size = 586, normalized size = 3.76

$$-\frac{4\,c^4\cos\left(bx+a\right)^2-\frac{16\,ac^3d\cos(bx+a)^2}{b}+\frac{24\,a^2c^2d^2\cos(bx+a)^2}{b^2}-\frac{16\,a^3cd^3\cos(bx+a)^2}{b^3}+\frac{4\,a^4d^4\cos(bx+a)^2}{b^4}+\frac{4\,(2\,(bx+a)\cos(2\,bx+2\,a)-}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(4*c^4*cos(b*x + a)^2 - 16*a*c^3*d*cos(b*x + a)^2/b + 24*a^2*c^2*d^2*c
os(b*x + a)^2/b^2 - 16*a^3*c*d^3*cos(b*x + a)^2/b^3 + 4*a^4*d^4*cos(b*x + a
)^2/b^4 + 4*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*c^3*d/b - 12*
(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 12*(2*(b*
x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a^2*c*d^3/b^3 - 4*(2*(b*x + a)*
cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a^3*d^4/b^4 + 6*((2*(b*x + a)^2 - 1)*c
os(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 12*((2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 6
*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*a^2*
d^4/b^4 + 2*(2*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(2*(b*x +
 a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 2*(2*(2*(b*x + a)^3 - 3*b*x - 3*a)
*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^4/b^4 + ((2
*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b
*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b
```

**mupad [B]** time = 0.49, size = 245, normalized size = 1.57

$$\frac{3\,x^2\,\cos\left(2\,a+2\,b\,x\right)\,\left(d^4-2\,b^2\,c^2\,d^2\right)}{4\,b^3}-\frac{\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{b^4\,c^4}{2}-\frac{3\,b^2\,c^2\,d^2}{2}+\frac{3\,d^4}{4}\right)}{2\,b^5}-\frac{3\,x\,\sin\left(2\,a+2\,b\,x\right)\,\left(d^4-2\,b^2\,c^2\right.}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^4,x)
```

```
[Out] (3*x^2*cos(2*a + 2*b*x)*(d^4 - 2*b^2*c^2*d^2))/(4*b^3) - (cos(2*a + 2*b*x)*
((3*d^4)/4 + (b^4*c^4)/2 - (3*b^2*c^2*d^2)/2))/(2*b^5) - (3*x*sin(2*a + 2*b
*x)*(d^4 - 2*b^2*c^2*d^2))/(4*b^4) - (d^4*x^4*cos(2*a + 2*b*x))/(4*b) - (si
n(2*a + 2*b*x)*(3*c*d^3 - 2*b^2*c^3*d))/(4*b^4) + (x*cos(2*a + 2*b*x)*(3*c*
d^3 - 2*b^2*c^3*d))/(2*b^3) + (d^4*x^3*sin(2*a + 2*b*x))/(2*b^2) - (c*d^3*x
^3*cos(2*a + 2*b*x))/b + (3*c*d^3*x^2*sin(2*a + 2*b*x))/(2*b^2)
```

**sympy** [**A**] time = 4.25, size = 502, normalized size = 3.22

$$\begin{cases} -\frac{c^4\cos^2(a+bx)}{2b} + \frac{c^3dx\sin^2(a+bx)}{b} - \frac{c^3dx\cos^2(a+bx)}{b} + \frac{3c^2d^2x^2\sin^2(a+bx)}{2b} - \frac{3c^2d^2x^2\cos^2(a+bx)}{2b} + \frac{cd^3x^3\sin^2(a+bx)}{b} - \frac{cd^3x^3\cos^2(a}{b} \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sin(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**4*cos(a + b*x)**2/(2*b) + c**3*d*x*sin(a + b*x)**2/b - c**3*
d*x*cos(a + b*x)**2/b + 3*c**2*d**2*x**2*sin(a + b*x)**2/(2*b) - 3*c**2*d**
2*x**2*cos(a + b*x)**2/(2*b) + c*d**3*x**3*sin(a + b*x)**2/b - c*d**3*x**3*
cos(a + b*x)**2/b + d**4*x**4*sin(a + b*x)**2/(4*b) - d**4*x**4*cos(a + b*x
)**2/(4*b) + c**3*d*sin(a + b*x)*cos(a + b*x)/b**2 + 3*c**2*d**2*x*sin(a +
b*x)*cos(a + b*x)/b**2 + 3*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)/b**2 + d**
4*x**3*sin(a + b*x)*cos(a + b*x)/b**2 + 3*c**2*d**2*cos(a + b*x)**2/(2*b**3
) - 3*c*d**3*x*sin(a + b*x)**2/(2*b**3) + 3*c*d**3*x*cos(a + b*x)**2/(2*b**
3) - 3*d**4*x**2*sin(a + b*x)**2/(4*b**3) + 3*d**4*x**2*cos(a + b*x)**2/(4*
b**3) - 3*c*d**3*sin(a + b*x)*cos(a + b*x)/(2*b**4) - 3*d**4*x*sin(a + b*x)
*cos(a + b*x)/(2*b**4) - 3*d**4*cos(a + b*x)**2/(4*b**5), Ne(b, 0)), ((c**4
*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*c
os(a), True))
```

## 3.3 $\int (c+dx)^3 \cos(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=120

$$-\frac{3d^3\sin(a+bx)\cos(a+bx)}{8b^4}-\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}+\frac{3d(c+dx)^2\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)^3\sin^2(a+}{2b}$$

```
[Out] 3/8*d^3*x/b^3-1/4*(d*x+c)^3/b-3/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4+3/4*d*(d*x+
c)^2*cos(b*x+a)*sin(b*x+a)/b^2-3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3+1/2*(d*x+c)
^3*sin(b*x+a)^2/b
```

**Rubi [A]** time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4404, 3311, 32, 2635, 8}

$$-\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}+\frac{3d(c+dx)^2\sin(a+bx)\cos(a+bx)}{4b^2}-\frac{3d^3\sin(a+bx)\cos(a+bx)}{8b^4}+\frac{(c+dx)^3\sin^2(a+}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (3*d^3*x)/(8*b^3) - (c + d*x)^3/(4*b) - (3*d^3*Cos[a + b*x]*Sin[a + b*x])/(
8*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) - (3*d^2*(c +
d*x)*Sin[a + b*x]^2)/(4*b^3) + ((c + d*x)^3*Sin[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx = \frac{(c+dx)^3\sin^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)^2\sin^2(a+bx)\,dx}{2b}$$

$$= \frac{3d(c+dx)^2\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3} + \frac{(c\ \ldots}{}$$

$$= -\frac{(c+dx)^3}{4b} - \frac{3d^3\cos(a+bx)\sin(a+bx)}{8b^4} + \frac{3d(c+dx)^2\cos(a+bx)\ldots}{4b^2}$$

$$= \frac{3d^3x}{8b^3} - \frac{(c+dx)^3}{4b} - \frac{3d^3\cos(a+bx)\sin(a+bx)}{8b^4} + \frac{3d(c+dx)^2\cos(a\ldots}{4b\ldots}$$

**Mathematica [A]** time = 0.30, size = 71, normalized size = 0.59

$$\frac{3d\sin(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)-2b(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 3*d*(-d^2 +
 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(16*b^4)
```

**fricas [A]** time = 0.44, size = 166, normalized size = 1.38

$$\frac{2\,b^3d^3x^3+6\,b^3cd^2x^2-2\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+2\,b^3c^3-3\,bcd^2+3\left(2\,b^3c^2d-bd^3\right)x\right)\cos\left(bx+a\right)^2+3\left(2\,b^2d\ldots\right.}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 +
 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b
^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) +
 3*(2*b^3*c^2*d - b*d^3)*x)/b^4
```

**giac [A]** time = 0.22, size = 121, normalized size = 1.01

$$-\frac{\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b^3c^3-3\,bd^3x-3\,bcd^2\right)\cos\left(2\,bx+2\,a\right)}{8\,b^4}+\frac{3\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c\ldots\right.}{16\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3
*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/b^4 + 3/16*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x
+ 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a)/b^4
```

**maple [B]** time = 0.01, size = 466, normalized size = 3.88

$$\frac{d^3\left(-\frac{(bx+a)^3\left(\cos^2(bx+a)\right)}{2}+\frac{3(bx+a)^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{2}+\frac{3(bx+a)\left(\cos^2(bx+a)\right)}{4}-\frac{3\cos(bx+a)\sin(bx+a)}{8}-\frac{3bx}{8}-\frac{3a}{8}-\frac{(bx+a)^3}{2}\right)}{b^3} - \frac{3a\,d^3\left(-\frac{(bx+a)^2\left(\cos^2(bx+\ldots\right.}{2}\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^3*d^3*(-1/2*(b*x+a)^3*cos(b*x+a)^2+3/2*(b*x+a)^2*(1/2*cos(b*x+a)*s
in(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)*cos(b*x+a)^2-3/8*cos(b*x+a)*sin(b*x+a)
-3/8*b*x-3/8*a-1/2*(b*x+a)^3)-3/b^3*a*d^3*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x
+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^
2)+3/b^2*c*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x
+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+3/b^3*a^2*d^3*(-1/2*(b*x
+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-6/b^2*a*c*d^2*(-1
/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+3/b*c^2*d*
(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+1/2/b^3
*a^3*d^3*cos(b*x+a)^2-3/2/b^2*a^2*c*d^2*cos(b*x+a)^2+3/2/b*a*c^2*d*cos(b*x+
a)^2-1/2*c^3*cos(b*x+a)^2)
```

**maxima [B]** time = 0.36, size = 342, normalized size = 2.85

$$-\frac{8\,c^3\cos\left(bx+a\right)^2 - \frac{24\,ac^2d\cos(bx+a)^2}{b} + \frac{24\,a^2cd^2\cos(bx+a)^2}{b^2} - \frac{8\,a^3d^3\cos(bx+a)^2}{b^3} + \frac{6\,(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))c^2d}{b} - \frac{12\,(2 \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/16*(8*c^3*cos(b*x + a)^2 - 24*a*c^2*d*cos(b*x + a)^2/b + 24*a^2*c*d^2*co
s(b*x + a)^2/b^2 - 8*a^3*d^3*cos(b*x + a)^2/b^3 + 6*(2*(b*x + a)*cos(2*b*x
+ 2*a) - sin(2*b*x + 2*a))*c^2*d/b - 12*(2*(b*x + a)*cos(2*b*x + 2*a) - sin
(2*b*x + 2*a))*a*c*d^2/b^2 + 6*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x +
2*a))*a^2*d^3/b^3 + 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*s
in(2*b*x + 2*a))*c*d^2/b^2 - 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b
*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (2*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(
2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

**mupad [B]** time = 0.85, size = 165, normalized size = 1.38

$$\frac{\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{3\,c\,d^2}{4}-\frac{b^2\,c^3}{2}\right)}{2\,b^3}-\frac{3\,\sin\left(2\,a+2\,b\,x\right)\,\left(d^3-2\,b^2\,c^2\,d\right)}{16\,b^4}-\frac{d^3\,x^3\,\cos\left(2\,a+2\,b\,x\right)}{4\,b}+\frac{3\,d^3\,x^2\,\sin\left(2\,a+2\,b\,x \cdots\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^3,x)
```

```
[Out] (cos(2*a + 2*b*x)*((3*c*d^2)/4 - (b^2*c^3)/2))/(2*b^3) - (3*sin(2*a + 2*b*x
)*(d^3 - 2*b^2*c^2*d))/(16*b^4) - (d^3*x^3*cos(2*a + 2*b*x))/(4*b) + (3*d^3
*x^2*sin(2*a + 2*b*x))/(8*b^2) + (3*x*cos(2*a + 2*b*x)*(d^3 - 2*b^2*c^2*d))
/(8*b^3) + (3*c*d^2*x*sin(2*a + 2*b*x))/(4*b^2) - (3*c*d^2*x^2*cos(2*a + 2*
b*x))/(4*b)
```

**sympy [A]** time = 2.39, size = 342, normalized size = 2.85

$$\begin{cases} -\frac{c^3\cos^2(a+bx)}{2b} + \frac{3c^2dx\sin^2(a+bx)}{4b} - \frac{3c^2dx\cos^2(a+bx)}{4b} + \frac{3cd^2x^2\sin^2(a+bx)}{4b} - \frac{3cd^2x^2\cos^2(a+bx)}{4b} + \frac{d^3x^3\sin^2(a+bx)}{4b} - \frac{d^3x^3\cos^2(a+bx)}{4b} \cdots \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**3*cos(a + b*x)**2/(2*b) + 3*c**2*d*x*sin(a + b*x)**2/(4*b) -
 3*c**2*d*x*cos(a + b*x)**2/(4*b) + 3*c*d**2*x**2*sin(a + b*x)**2/(4*b) - 3
*c*d**2*x**2*cos(a + b*x)**2/(4*b) + d**3*x**3*sin(a + b*x)**2/(4*b) - d**3
*x**3*cos(a + b*x)**2/(4*b) + 3*c**2*d*sin(a + b*x)*cos(a + b*x)/(4*b**2) +
```

```
 3*c*d**2*x*sin(a + b*x)*cos(a + b*x)/(2*b**2) + 3*d**3*x**2*sin(a + b*x)*c
os(a + b*x)/(4*b**2) + 3*c*d**2*cos(a + b*x)**2/(4*b**3) - 3*d**3*x*sin(a +
 b*x)**2/(8*b**3) + 3*d**3*x*cos(a + b*x)**2/(8*b**3) - 3*d**3*sin(a + b*x)
*cos(a + b*x)/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3
 + d**3*x**4/4)*sin(a)*cos(a), True))
```

## 3.4 $\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=89

$$-\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}+\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{cdx}{2b}-\frac{d^2x^2}{4b}$$

```
[Out] -1/2*c*d*x/b-1/4*d^2*x^2/b+1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-1/4*d^2*
sin(b*x+a)^2/b^3+1/2*(d*x+c)^2*sin(b*x+a)^2/b
```

**Rubi [A]** time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4404, 3310}

$$\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}-\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{cdx}{2b}-\frac{d^2x^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -(c*d*x)/(2*b) - (d^2*x^2)/(4*b) + (d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/
(2*b^2) - (d^2*Sin[a + b*x]^2)/(4*b^3) + ((c + d*x)^2*Sin[a + b*x]^2)/(2*b)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx &= \frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{d\int (c+dx)\sin^2(a+bx)\,dx}{b}\\ &= \frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2}-\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{(c+dx)^2\sin^2(a+\\ &= -\frac{cdx}{2b}-\frac{d^2x^2}{4b}+\frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2}-\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{(c\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 50, normalized size = 0.56

$$\frac{\cos(2(a+bx))\left(d^2-2b^2(c+dx)^2\right)+2bd(c+dx)\sin(2(a+bx))}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] ((d^2 - 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 2*b*d*(c + d*x)*Sin[2*(a + b*
x)])/(8*b^3)
```

**fricas** [A]   time = 0.50, size = 92, normalized size = 1.03

$$\frac{b^2d^2x^2 + 2\,b^2cdx - \left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(bx + a\right)^2 + 2\left(bd^2x + bcd\right)\cos\left(bx + a\right)\sin\left(bx + a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 -
 d^2)*cos(b*x + a)^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/b^3
```

**giac** [A]   time = 0.20, size = 73, normalized size = 0.82

$$-\frac{\left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(2\,bx + 2\,a\right)}{8\,b^3} + \frac{\left(bd^2x + bcd\right)\sin\left(2\,bx + 2\,a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b^3 +
 1/4*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a)/b^3
```

**maple** [B]   time = 0.01, size = 215, normalized size = 2.42

$$\frac{\frac{d^2\left(-\frac{(bx+a)^2\left(\cos^2(bx+a)\right)}{2}+(bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\frac{\left(\sin^2(bx+a)\right)}{4}\right)}{b^2}-\frac{2a\,d^2\left(-\frac{(bx+a)\left(\cos^2(bx+a)\right)}{2}+\frac{\cos(bx+a)\sin(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^2}+\frac{2cd}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^2*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x
+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-2/b^2*a*d^2*(-1/2*(b*x+a
)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+2/b*c*d*(-1/2*(b*x+
a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-1/2/b^2*a^2*d^2*co
s(b*x+a)^2+1/b*a*c*d*cos(b*x+a)^2-1/2*c^2*cos(b*x+a)^2)
```

**maxima** [B]   time = 0.36, size = 171, normalized size = 1.92

$$-\frac{4\,c^2\cos\left(bx + a\right)^2 - \frac{8\,acd\cos(bx+a)^2}{b} + \frac{4\,a^2d^2\cos(bx+a)^2}{b^2} + \frac{2\,(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))cd}{b} - \frac{2\,(2\,(bx+a)\cos(2\,bx+2\,a)-\mathrm{si}}{b^2}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(4*c^2*cos(b*x + a)^2 - 8*a*c*d*cos(b*x + a)^2/b + 4*a^2*d^2*cos(b*x +
 a)^2/b^2 + 2*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*c*d/b - 2*(
2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a*d^2/b^2 + ((2*(b*x + a)^
2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*d^2/b^2)/b
```

**mupad** [B]   time = 0.16, size = 100, normalized size = 1.12

$$\frac{\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{d^2}{4}-\frac{b^2\,c^2}{2}\right)}{2\,b^3}+\frac{d^2\,x\,\sin\left(2\,a+2\,b\,x\right)}{4\,b^2}-\frac{d^2\,x^2\,\cos\left(2\,a+2\,b\,x\right)}{4\,b}+\frac{c\,d\,\sin\left(2\,a+2\,b\,x\right)}{4\,b^2}-\frac{c\,d\,x\,\cos\left(2\,a\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2,x)
```

```
[Out] (cos(2*a + 2*b*x)*(d^2/4 - (b^2*c^2)/2))/(2*b^3) + (d^2*x*sin(2*a + 2*b*x))
/(4*b^2) - (d^2*x^2*cos(2*a + 2*b*x))/(4*b) + (c*d*sin(2*a + 2*b*x))/(4*b^2
) - (c*d*x*cos(2*a + 2*b*x))/(2*b)
```

**sympy** [**A**] time = 1.05, size = 175, normalized size = 1.97

$$\begin{cases} -\frac{c^2\cos^2(a+bx)}{2b} + \frac{cdx\sin^2(a+bx)}{2b} - \frac{cdx\cos^2(a+bx)}{2b} + \frac{d^2x^2\sin^2(a+bx)}{4b} - \frac{d^2x^2\cos^2(a+bx)}{4b} + \frac{cd\sin(a+bx)\cos(a+bx)}{2b^2} + \frac{d^2x\sin(a+bx)\cos}{2b^2} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**2*cos(a + b*x)**2/(2*b) + c*d*x*sin(a + b*x)**2/(2*b) - c*d*
x*cos(a + b*x)**2/(2*b) + d**2*x**2*sin(a + b*x)**2/(4*b) - d**2*x**2*cos(a
 + b*x)**2/(4*b) + c*d*sin(a + b*x)*cos(a + b*x)/(2*b**2) + d**2*x*sin(a +
b*x)*cos(a + b*x)/(2*b**2) + d**2*cos(a + b*x)**2/(4*b**3), Ne(b, 0)), ((c*
*2*x + c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a), True))
```

## 3.5 $\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=50

$$\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{dx}{4b}$$

```
[Out] -1/4*d*x/b+1/4*d*cos(b*x+a)*sin(b*x+a)/b^2+1/2*(d*x+c)*sin(b*x+a)^2/b
```

**Rubi** [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4404, 2635, 8}

$$\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{dx}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -(d*x)/(4*b) + (d*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) + ((c + d*x)*Sin[a + b
*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx&=\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{d\int \sin^2(a+bx)\,dx}{2b}\\&=\frac{d\cos(a+bx)\sin(a+bx)}{4b^2}+\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{d\int 1\,dx}{4b}\\&=-\frac{dx}{4b}+\frac{d\cos(a+bx)\sin(a+bx)}{4b^2}+\frac{(c+dx)\sin^2(a+bx)}{2b}\end{aligned}$$

**Mathematica** [A] time = 0.10, size = 34, normalized size = 0.68

$$\frac{d\sin(2(a+bx))-2b(c+dx)\cos(2(a+bx))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)])/(8*b^2)
```

**fricas [A]** time = 0.63, size = 42, normalized size = 0.84

$$\frac{bdx - 2\,(bdx + bc)\cos\left(bx + a\right)^{2} + d\cos\left(bx + a\right)\sin\left(bx + a\right)}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a))/
b^2
```

**giac [A]** time = 1.72, size = 38, normalized size = 0.76

$$-\frac{(bdx + bc)\cos\left(2\,bx + 2\,a\right)}{4\,b^{2}} + \frac{d\sin\left(2\,bx + 2\,a\right)}{8\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(b*d*x + b*c)*cos(2*b*x + 2*a)/b^2 + 1/8*d*sin(2*b*x + 2*a)/b^2
```

**maple [A]** time = 0.01, size = 74, normalized size = 1.48

$$\frac{\frac{d\left(-\frac{(bx+a)\left(\cos^{2}(bx+a)\right)}{2}+\frac{\cos(bx+a)\sin(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b}+\frac{da\left(\cos^{2}(bx+a)\right)}{2b}-\frac{c\left(\cos^{2}(bx+a)\right)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b*d*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4
*a)+1/2/b*d*a*cos(b*x+a)^2-1/2*c*cos(b*x+a)^2)
```

**maxima [A]** time = 0.33, size = 65, normalized size = 1.30

$$-\frac{4\,c\cos\left(bx + a\right)^{2} - \frac{4\,ad\cos(bx+a)^{2}}{b} + \frac{(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))d}{b}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(4*c*cos(b*x + a)^2 - 4*a*d*cos(b*x + a)^2/b + (2*(b*x + a)*cos(2*b*x
+ 2*a) - sin(2*b*x + 2*a))*d/b)/b
```

**mupad [B]** time = 0.70, size = 47, normalized size = 0.94

$$\frac{d\,\sin\left(2\,a+2\,b\,x\right)}{8\,b^2}-\frac{c\,\cos\left(2\,a+2\,b\,x\right)}{4\,b}-\frac{d\,x\,\cos\left(2\,a+2\,b\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x),x)
```

```
[Out] (d*sin(2*a + 2*b*x))/(8*b^2) - (c*cos(2*a + 2*b*x))/(4*b) - (d*x*cos(2*a +
2*b*x))/(4*b)
```

**sympy [A]** time = 0.47, size = 80, normalized size = 1.60

$$\begin{cases} -\frac{c\cos^2(a+bx)}{2b} + \frac{dx\sin^2(a+bx)}{4b} - \frac{dx\cos^2(a+bx)}{4b} + \frac{d\sin(a+bx)\cos(a+bx)}{4b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sin(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((-c*cos(a + b*x)**2/(2*b) + d*x*sin(a + b*x)**2/(4*b) - d*x*cos(a
 + b*x)**2/(4*b) + d*sin(a + b*x)*cos(a + b*x)/(4*b**2), Ne(b, 0)), ((c*x +
 d*x**2/2)*sin(a)*cos(a), True))
```

## 3.6 $\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=65

$$\frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] 1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d+1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*
c/d)/d
```

**Rubi [A]** time = 0.14, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4406, 12, 3303, 3299, 3302}

$$\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) + (Cos[2*a - (2
*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx$$
$$= \frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx$$
$$= \frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx + \frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}$$
$$= \frac{\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

**Mathematica [A]** time = 0.13, size = 60, normalized size = 0.92

$$\frac{\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)+\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d] + Cos[2*a - (2*b*c)/d]
*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

**fricas [A]** time = 0.61, size = 80, normalized size = 1.23

$$\frac{\left(\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+2\,\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*((cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*s
in(-2*(b*c - a*d)/d) + 2*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)
/d))/d
```

**giac [C]** time = 0.85, size = 569, normalized size = 8.75

$$\Im\left(\operatorname{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan(a)^2\tan\left(\frac{bc}{d}\right)^2-\Im\left(\operatorname{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan(a)^2\tan\left(\frac{bc}{d}\right)^2+2\,\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right)\tan(a)^2\tan\left(\frac{b}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral
(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(2*b*x
+ 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*real_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(a)^2*tan(b*c/d) - 2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*
tan(b*c/d)^2 - 2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d
)^2 - imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + imag_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*sin_integral(2*(b*d*x + b*c)/d)*tan(a
)^2 + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 4*imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*sin_integral(2*
```

```
(b*d*x + b*c)/d)*tan(a)*tan(b*c/d) - imag_part(cos_integral(2*b*x + 2*b*c/d
))*tan(b*c/d)^2 + imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 -
2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(2
*b*x + 2*b*c/d))*tan(a) + 2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a
) - 2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*real_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_integral(2*b*x + 2*
b*c/d)) - imag_part(cos_integral(-2*b*x - 2*b*c/d)) + 2*sin_integral(2*(b*d
*x + b*c)/d))/(d*tan(a)^2*tan(b*c/d)^2 + d*tan(a)^2 + d*tan(b*c/d)^2 + d)
```

**maple [A]** time = 0.02, size = 84, normalized size = 1.29

$$\frac{\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{2d}-\frac{\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

```
[Out] 1/2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-1/2*Ci(2*b*x+2*a+2*(
-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d
```

**maxima [C]** time = 0.41, size = 141, normalized size = 2.17

$$-\frac{b\left(i\,E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-i\,E_1\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_1\left(-\frac{2i\,bc+2\cdots}{}\right.\right.}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*(b*(I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I*e
xp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c - a
*d)/d) + b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + ex
p_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c - a*
d)/d))/(b*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x))/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x))/(c + d*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x), x)
```

## 3.7 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=85

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sin(2a+2bx)}{2d(c+dx)}$$

```
[Out] b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2-b*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/
d)/d^2-1/2*sin(2*b*x+2*a)/d/(d*x+c)
```

**Rubi [A]** time = 0.15, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4406, 12, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sin(2a+2bx)}{2d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2 - Sin[2*a + 2*b
*x]/(2*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x
])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx &= \int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx\\ &= \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\\ &= -\frac{\sin(2a+2bx)}{2d(c+dx)}+\frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d}\\ &= -\frac{\sin(2a+2bx)}{2d(c+dx)}+\frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}-\frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin(}{}}{d}\\ &= \frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}\end{aligned}$$

**Mathematica [A]** time = 0.31, size = 80, normalized size = 0.94

$$\frac{2b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right)-2b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)-\frac{d\sin(2(a+bx))}{c+dx}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (2*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - (d*Sin[2*(a + b*
x)])/(c + d*x) - 2*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(
2*d^2)
```

**fricas [A]** time = 0.71, size = 132, normalized size = 1.55

$$-\frac{2\,d\cos(bx+a)\sin(bx+a)+2\,(bdx+bc)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\text{Si}\left(\frac{2\,(bdx+bc)}{d}\right)-\left((bdx+bc)\,\text{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(bdx+bc)\right.}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*cos(b*x + a)*sin(b*x + a) + 2*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)
*sin_integral(2*(b*d*x + b*c)/d) - ((b*d*x + b*c)*cos_integral(2*(b*d*x + b
*c)/d) + (b*d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)
/d))/(d^3*x + c*d^2)
```

**giac [C]** time = 0.56, size = 2870, normalized size = 33.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan
(b*c/d)^2 + b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
```

```
a)^2*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*
tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b
*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 4*b*d*x*sin_integral(2*(b*d
*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b*c*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + b*c*real_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b*d*x*real_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - b*d*x*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b*d*x*real_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*b*d*x*real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 2*b*c*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b*c*imag_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b*c*s
in_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - b*d*x*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - b*d*x*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b*c*imag_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b*c*ima
g_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 4*b
*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b*d*x*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b*d*x*real_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*b*d*x*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b*d*x*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 4*b*d*x*sin_integral(2*(b
*d*x + b*c)/d)*tan(b*x)^2*tan(a) - b*c*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(a)^2 - b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)^2 + 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x
)^2*tan(b*c/d) + 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c
/d) + 4*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(
b*c/d) + 4*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*
tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(
b*c/d) + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c
/d) - 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) - b*c*rea
l_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - b*c*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b*d*x*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b*d*x*imag_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*b*d*x*sin_integral(
2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + b*c*real_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b*c*real_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(a)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*x)^2 + b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 -
 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b*c*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 4*b*c*sin_inte
gral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - b*d*x*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(a)^2 - b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(a)^2 + 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*
c/d) - 2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d
) + 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) + 4*b*d*x*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 4*b*d*x*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) - 2*b*c*imag_part(cos_
integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*b*c*imag_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) - 4*b*c*sin_integral(2*(b*d*x + b
*c)/d)*tan(a)^2*tan(b*c/d) - b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))
*tan(b*c/d)^2 - b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^
2 + 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*
b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*b*c*s
in_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*d*tan(b*x)^2*tan(a)*
```

```
tan(b*c/d)^2 + 2*d*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + b*c*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2 + b*c*real_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(b*x)^2 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)
+ 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - 4*b*d*x*sin_in
tegral(2*(b*d*x + b*c)/d)*tan(a) - b*c*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(a)^2 - b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 + 2*
b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b*d*x*imag_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b*d*x*sin_integral(2*(b*d
*x + b*c)/d)*tan(b*c/d) + 4*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)*tan(b*c/d) + 4*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*ta
n(b*c/d) - b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - b*c*
real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 + b*d*x*real_part(co
s_integral(2*b*x + 2*b*c/d)) + b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/
d)) - 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*b*c*imag_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - 4*b*c*sin_integral(2*(b*d*x + b
*c)/d)*tan(a) + 2*d*tan(b*x)^2*tan(a) + 2*d*tan(b*x)*tan(a)^2 + 2*b*c*imag_
part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b*c*imag_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*ta
n(b*c/d) - 2*d*tan(b*x)*tan(b*c/d)^2 - 2*d*tan(a)*tan(b*c/d)^2 + b*c*real_p
art(cos_integral(2*b*x + 2*b*c/d)) + b*c*real_part(cos_integral(-2*b*x - 2*
b*c/d)) - 2*d*tan(b*x) - 2*d*tan(a))/(d^3*x*tan(b*x)^2*tan(a)^2*tan(b*c/d)^
2 + c*d^2*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + d^3*x*tan(b*x)^2*tan(a)^2 + d^
3*x*tan(b*x)^2*tan(b*c/d)^2 + d^3*x*tan(a)^2*tan(b*c/d)^2 + c*d^2*tan(b*x)^
2*tan(a)^2 + c*d^2*tan(b*x)^2*tan(b*c/d)^2 + c*d^2*tan(a)^2*tan(b*c/d)^2 +
d^3*x*tan(b*x)^2 + d^3*x*tan(a)^2 + d^3*x*tan(b*c/d)^2 + c*d^2*tan(b*x)^2 +
 c*d^2*tan(a)^2 + c*d^2*tan(b*c/d)^2 + d^3*x + c*d^2)
```

**maple [A]** time = 0.01, size = 124, normalized size = 1.46

$$\frac{b\left(-\frac{2\sin(2bx+2a)}{((bx+a)d-da+cb)d}+\frac{\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}}{d}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x)
```

```
[Out] 1/4*b*(-2*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*
c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)
/d)/d)/d)
```

**maxima [C]** time = 0.44, size = 164, normalized size = 1.93

$$-\frac{b^2\left(i\,E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-i\,E_2\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^2\left(E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_2\left(-\frac{2i\,bc}{}\right.\right.}{4\left(bcd+(bx+a)d^2-ad^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I
*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) + b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c
- a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^2, x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**2, x)
```

## 3.8 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=114

$$-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b\cos(2a+2bx)}{2d^2(c+dx)}-\frac{\sin(2a+2bx)}{4d(c+dx)^2}$$

```
[Out] -1/2*b*cos(2*b*x+2*a)/d^2/(d*x+c)-b^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^
3-b^2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3-1/4*sin(2*b*x+2*a)/d/(d*x+c)^2
```

**Rubi [A]** time = 0.17, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4406, 12, 3297, 3303, 3299, 3302}

$$-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b\cos(2a+2bx)}{2d^2(c+dx)}-\frac{\sin(2a+2bx)}{4d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(2*d^2*(c + d*x)) - (b^2*CosIntegral[(2*b*c)/d + 2*b*
x]*Sin[2*a - (2*b*c)/d])/d^3 - Sin[2*a + 2*b*x]/(4*d*(c + d*x)^2) - (b^2*Co
s[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx &= \int \frac{\sin(2a+2bx)}{2(c+dx)^3}\,dx\\
&= \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx\\
&= -\frac{\sin(2a+2bx)}{4d(c+dx)^2} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{2d}\\
&= -\frac{b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{b^2\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d^2}\\
&= -\frac{b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{\left(b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d^2} - \frac{(b
\\
&= -\frac{b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{b^2\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{b^2\cos
\end{aligned}$$

**Mathematica [A]** time = 1.08, size = 102, normalized size = 0.89

$$-\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right)+4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+\frac{d(2b(c+dx)\cos(2(a+bx))+d\sin(2(a+bx)))}{(c+dx)^2}}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -1/4*(4*b^2*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*(2*b*(
c + d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + 4*b^2*Cos[2*
a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^3
```

**fricas [B]** time = 0.81, size = 230, normalized size = 2.02

$$\frac{bd^2x - d^2\cos(bx+a)\sin(bx+a) + bcd - 2\left(bd^2x+bcd\right)\cos(bx+a)^2 - 2\left(b^2d^2x^2+2b^2cdx+b^2c^2\right)\cos\left(-\frac{2(}{}\right.}{2\left(d^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^2*x - d^2*cos(b*x + a)*sin(b*x + a) + b*c*d - 2*(b*d^2*x + b*c*d)*
cos(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)
/d)*sin_integral(2*(b*d*x + b*c)/d) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos_integral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*c
os_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*
x + c^2*d^3)
```

**giac [C]** time = 0.53, size = 5398, normalized size = 47.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
)^2*tan(b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)
/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*real_part
(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^2*d^2
*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^
2 - 2*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
a)*tan(b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^2*c*d*x*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 4*b^2*c*d*x*sin_integral(2*(b*
d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_
integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + b^2*d^2*x^2*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^2*d^2*x^2*sin_integra
l(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 + 4*b^2*d^2*x^2*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b^2*d^2*x^2*imag_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 8*b^2*d^2
*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*b^2*c
*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d
) + 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)
^2*tan(b*c/d) - b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(b*x)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(
b*x)^2*tan(b*c/d)^2 - 4*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*
tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 4*b^2*c*d*x*real_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + b^2*c^2*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*c^2*imag_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c^2*s
in_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2
*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b^2*d^2
*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 2*b^2*c*
d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 2*b^2*c*
d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 4*b^2*c
*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 - 2*b^2*d^2*x^2*re
al_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 2*b^2*d^2*x^
2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 8*b^2*c
*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)
- 8*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*t
an(b*c/d) + 16*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*
tan(b*c/d) + 2*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^
2*tan(b*c/d) + 2*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
a)^2*tan(b*c/d) + 2*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^2*c*d*x*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 4*b^2*c*d*x*sin_integral(2*(b
*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_integr
al(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*real_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^2*c*d*
x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*b^2*c*
d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 4*b^2
*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + b*d^2*x*tan(
b*x)^2*tan(a)^2*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2
```

```
*b*c/d))*tan(b*x)^2 - b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(b*x)^2 + 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 4*
b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^
2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - b^2*d
^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + b^2*d^2*x^2*imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*b^2*d^2*x^2*sin_integral
(2*(b*d*x + b*c)/d)*tan(a)^2 - b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(a)^2 + b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(a)^2 - 2*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^
2*tan(a)^2 - 4*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^
2*tan(b*c/d) - 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*
x)^2*tan(b*c/d) + 4*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)*tan(b*c/d) - 4*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(a)*tan(b*c/d) + 8*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan
(b*c/d) + 4*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan
(a)*tan(b*c/d) - 4*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*
x)^2*tan(a)*tan(b*c/d) + 8*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)
^2*tan(a)*tan(b*c/d) + 4*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))
*tan(a)^2*tan(b*c/d) + 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(a)^2*tan(b*c/d) - b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
b*c/d)^2 - 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 - b^2
*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + b^2
*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*
b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 4*b^2*c*d
*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*b^2*c*d
*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + b^2*c^2*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - b^2*c^2*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c^2*s
in_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + b*c*d*tan(b*x)^2*tan
(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*t
an(b*x)^2 - 2*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^
2 + 4*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 2*b^2*d^2*x^2*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*b^2*d^2*x^2*real_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(a) + 2*b^2*c^2*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b^2*c^2*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*x)^2*tan(a) - 2*b^2*c*d*x*imag_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a)^2 + 2*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(a)^2 - 4*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + b*d^2*x*
tan(b*x)^2*tan(a)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*c/d) - 2*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
b*c/d) - 2*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
b*c/d) - 2*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(b*c/d) + 8*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b
*c/d) - 8*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*
c/d) + 16*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d) + 2*b
^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*b^2
*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) - 2*b^2*
c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + 2*b^2*c*d*x*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 4*b^2*c*d*x*sin_int
egral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 - b*d^2*x*tan(b*x)^2*tan(b*c/d)^2 - 2
*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b
^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*b*
d^2*x*tan(b*x)*tan(a)*tan(b*c/d)^2 - b*d^2*x*tan(a)^2*tan(b*c/d)^2 + b^2*d^
2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d)) - b^2*d^2*x^2*imag_part(cos_
integral(-2*b*x - 2*b*c/d)) + 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)
 + b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 - b^2*c^2*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 + 2*b^2*c^2*sin_integral
(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 4*b^2*c*d*x*real_part(cos_integral(2*b*x +
```

```
 2*b*c/d))*tan(a) + 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(a) - b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + b^2*c^2
*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*b^2*c^2*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)^2 + b*c*d*tan(b*x)^2*tan(a)^2 - 4*b^2*c*d*x*rea
l_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 4*b^2*c*d*x*real_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b^2*c^2*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 4*b^2*c^2*imag_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b^2*c^2*sin_integral(2*(b*d*x + b*c)/
d)*tan(a)*tan(b*c/d) - b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*c/d)^2 + b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2
- 2*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 - b*c*d*tan(b*x)^2
*tan(b*c/d)^2 - 4*b*c*d*tan(b*x)*tan(a)*tan(b*c/d)^2 - d^2*tan(b*x)^2*tan(a
)*tan(b*c/d)^2 - b*c*d*tan(a)^2*tan(b*c/d)^2 - d^2*tan(b*x)*tan(a)^2*tan(b*
c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d)) - 2*b^2*c*d*x
*imag_part(cos_integral(-2*b*x - 2*b*c/d)) + 4*b^2*c*d*x*sin_integral(2*(b*
d*x + b*c)/d) - b*d^2*x*tan(b*x)^2 + 2*b^2*c^2*real_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(a) + 2*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(a) - 4*b*d^2*x*tan(b*x)*tan(a) - b*d^2*x*tan(a)^2 - 2*b^2*c^2*real_part(
cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b^2*c^2*real_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(b*c/d) + b*d^2*x*tan(b*c/d)^2 + b^2*c^2*imag_part(
cos_integral(2*b*x + 2*b*c/d)) - b^2*c^2*imag_part(cos_integral(-2*b*x - 2*
b*c/d)) + 2*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d) - b*c*d*tan(b*x)^2 - 4*
b*c*d*tan(b*x)*tan(a) - d^2*tan(b*x)^2*tan(a) - b*c*d*tan(a)^2 - d^2*tan(b*
x)*tan(a)^2 + b*c*d*tan(b*c/d)^2 + d^2*tan(b*x)*tan(b*c/d)^2 + d^2*tan(a)*t
an(b*c/d)^2 + b*d^2*x + b*c*d + d^2*tan(b*x) + d^2*tan(a))/(d^5*x^2*tan(b*x
)^2*tan(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + d^
5*x^2*tan(b*x)^2*tan(a)^2 + d^5*x^2*tan(b*x)^2*tan(b*c/d)^2 + d^5*x^2*tan(a
)^2*tan(b*c/d)^2 + c^2*d^3*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan
(b*x)^2*tan(a)^2 + 2*c*d^4*x*tan(b*x)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(a)^2*t
an(b*c/d)^2 + d^5*x^2*tan(b*x)^2 + d^5*x^2*tan(a)^2 + c^2*d^3*tan(b*x)^2*ta
n(a)^2 + d^5*x^2*tan(b*c/d)^2 + c^2*d^3*tan(b*x)^2*tan(b*c/d)^2 + c^2*d^3*t
an(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2 + 2*c*d^4*x*tan(a)^2 + 2*c*d^4*
x*tan(b*c/d)^2 + d^5*x^2 + c^2*d^3*tan(b*x)^2 + c^2*d^3*tan(a)^2 + c^2*d^3*
tan(b*c/d)^2 + 2*c*d^4*x + c^2*d^3)
```

**maple** [A] time = 0.01, size = 162, normalized size = 1.42

$$\frac{b^{2}\left(-\frac{\sin\left(2bx+2a\right)}{\left(\left(bx+a\right)d-da+cb\right)^{2}d}+\frac{-\frac{2\cos\left(2bx+2a\right)}{\left(\left(bx+a\right)d-da+cb\right)d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}\right)}{d}}{d}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x)
```

```
[Out] 1/4*b^2*(-sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a
)*d-d*a+c*b)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci
(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)
```

**maxima** [C] time = 0.51, size = 199, normalized size = 1.75

$$-\frac{b^{3}\left(i\,E_{3}\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-i\,E_{3}\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^{3}\left(E_{3}\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_{3}\left(-\frac{2i\,bc}{}\right.\right.}{4\left(b^{2}c^{2}d-2\,abcd^{2}+(bx+a)^{2}d^{3}+a^{2}d^{3}+2\left(bcd^{2}-ad^{3}\right)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I
*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) + b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c
- a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d
^2 - a*d^3)*(b*x + a))*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,\sin(a+b\,x)}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**3, x)
```

## 3.9 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

**Optimal**. Leaf size=144

$$-\frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{b^2\sin(2a+2bx)}{3d^3(c+dx)}-\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2}-\frac{\sin(2a}{6d(c}$$

```
[Out] -2/3*b^3*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^4-1/6*b*cos(2*b*x+2*a)/d^2/(d
*x+c)^2+2/3*b^3*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/6*sin(2*b*x+2*a)/d
/(d*x+c)^3+1/3*b^2*sin(2*b*x+2*a)/d^3/(d*x+c)
```

**Rubi [A]** time = 0.20, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4406, 12, 3297, 3303, 3299, 3302}

$$-\frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{b^2\sin(2a+2bx)}{3d^3(c+dx)}-\frac{b\cos(2a+2b}{6d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(6*d^2*(c + d*x)^2) - (2*b^3*Cos[2*a - (2*b*c)/d]*Cos
Integral[(2*b*c)/d + 2*b*x])/(3*d^4) - Sin[2*a + 2*b*x]/(6*d*(c + d*x)^3) +
 (b^2*Sin[2*a + 2*b*x])/(3*d^3*(c + d*x)) + (2*b^3*Sin[2*a - (2*b*c)/d]*Sin
Integral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin(2a+2bx)}{2(c+dx)^4}\,dx$$
$$= \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx$$
$$= -\frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{3d}$$
$$= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} - \frac{b^2\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx}{3d^2}$$
$$= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{3d^3(c+dx)} - \frac{\left(2b^3\right)\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{3d^3}$$
$$= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{3d^3(c+dx)} - \frac{\left(2b^3\cos\left(2a-\frac{2bc}{d}\right)\right)}{3d^3}$$
$$= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b^2 s}{3}$$

**Mathematica [A]** time = 0.67, size = 164, normalized size = 1.14

$$\frac{-4b^3(c+dx)^3\left(\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right)-\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)\right)-d\cos(2bx)\left(\sin(2a)\left(d^2-2b^2(c+d\right.\right.}{6d^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] (-(d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2*a
])) + d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2*a])*Sin[
2*b*x] - 4*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x)
)/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]))/(6*d^4*(c + d*
x)^3)
```

**fricas [B]** time = 0.61, size = 320, normalized size = 2.22

$$\frac{bd^3x + bcd^2 - 2\left(bd^3x + bcd^2\right)\cos\left(bx+a\right)^2 + 2\left(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3\right)\cos\left(bx+a\right)\sin\left(bx+a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b*d^3*x + b*c*d^2 - 2*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 2*(2*b^2*d^
3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 4*(b
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/
d)*sin_integral(2*(b*d*x + b*c)/d) - 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
b^3*c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b
```

```
^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-2*(b*d*x + b*c)/d))*c
os(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

**giac [C]** time = 0.61, size = 7592, normalized size = 52.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan
(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(2
*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 4*b^3*d^3*x^3*imag_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 8*b^3*d^3*x
^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 4*b^3*d
^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d
)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*ta
n(a)*tan(b*c/d)^2 + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^
2*tan(a)*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*
c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*re
al_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^3*d^3*x^3*
real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 8*b^3*d^3*x
^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) +
8*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*t
an(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b
*x)^2*tan(a)^2*tan(b*c/d) + 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 24*b^3*c*d^2*x^2*sin_integral(
2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^3*d^3*x^3*real_part
(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 12*b^3*c*d
^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d
)^2 - 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2
*tan(a)*tan(b*c/d)^2 + 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan
(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x +
2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 4*b^3
*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^3
*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 8*b^
3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 6*b^3*c*d^2*x
^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 6*b^3*c*d
^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b^
3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) -
4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/
d) + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) +
24*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)
*tan(b*c/d) + 24*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)*tan(b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(a)^2*tan(b*c/d) + 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(a)^2*tan(b*c/d) - 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c
)/d)*tan(a)^2*tan(b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 12*b^3*c^2*d*x*imag_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 24*b^3*c^2*d*x*sin_
integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 6*b^3*c*d^2*x^
2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*
c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2
 + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)
```

```
^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c
/d)^2 + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 +
 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*
tan(b*c/d)^2 - 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(a)*tan(b*c/d)^2 + 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d
)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2
*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^3*c^3*real_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*c^3*real_p
art(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^
3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b^3*d^3*x
^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 12*b^3*c*d^2*x^2*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 12*b^3*c*d^2*x
^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 24*b^3*c*d
^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 2*b^3*d^3*x^3*re
al_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 2*b^3*d^3*x^3*real_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 6*b^3*c^2*d*x*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 6*b^3*c^2*d*x*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 12*b^3*c*d^2*x^2*imag_part(c
os_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 12*b^3*c*d^2*x^2*imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 24*b^3*c*d^2*
x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) + 8*b^3*d^3*x^3*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b^3*d^3*x^3*r
eal_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 24*b^3*c^2*d*x
*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 24
*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*ta
n(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)
^2*tan(b*c/d) + 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(a)^2*tan(b*c/d) - 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(
a)^2*tan(b*c/d) - 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d) + 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)
/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^3*d^3*x^3*real_part(cos_integral(2
*b*x + 2*b*c/d))*tan(b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 12*b^3*c*d^2*x^2*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 12*b^3*c*d^2*x^2*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 24*b^3*c*d^2*x^2*sin_in
tegral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 4*b^2*d^3*x^2*tan(b*x)^2*ta
n(a)*tan(b*c/d)^2 + 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(a)*tan(b*c/d)^2 - 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 8*b^3*c^3*sin_integral(2*(b*d*x + b*
c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 4*b^2*d^3*x^2*tan(b*x)*tan(
a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*x)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) +
4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - 8*b^3*d^3*
x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(a) - 12*b^3*c^2*d*x*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 12*b^3*c^2*d*x*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 24*b^3*c^2*d*x*sin_integral
(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 6*b^3*c*d^2*x^2*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(a)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(a)^2 - 2*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d
))*tan(b*x)^2*tan(a)^2 - 2*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(a)^2 + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/
d))*tan(b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*ta
```

```
n(b*c/d) + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 12*b^
3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) -
12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c
/d) + 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)
+ 24*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/
d) + 24*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(
b*c/d) + 8*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
a)*tan(b*c/d) + 8*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x
)^2*tan(a)*tan(b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c
/d))*tan(a)^2*tan(b*c/d) + 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(a)^2*tan(b*c/d) - 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/
d)*tan(a)^2*tan(b*c/d) - 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b
*c/d))*tan(b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(b*c/d)^2 - 2*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(b*c/d)^2 - 2*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(b*x)^2*tan(b*c/d)^2 + 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*
c/d))*tan(a)*tan(b*c/d)^2 - 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(a)*tan(b*c/d)^2 + 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)
/d)*tan(a)*tan(b*c/d)^2 + 8*b^2*c*d^2*x*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*
b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*
b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 8
*b^2*c*d^2*x*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + b*d^3*x*tan(b*x)^2*tan(a)^2*t
an(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d)) + 2*b^
3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d)) + 6*b^3*c^2*d*x*real_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 6*b^3*c^2*d*x*real_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 12*b^3*c*d^2*x^2*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(a) + 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2
*b*x - 2*b*c/d))*tan(a) - 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*
tan(a) + 4*b^2*d^3*x^2*tan(b*x)^2*tan(a) - 4*b^3*c^3*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^3*c^3*imag_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/
d)*tan(b*x)^2*tan(a) - 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d
))*tan(a)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a
)^2 + 4*b^2*d^3*x^2*tan(b*x)*tan(a)^2 + 12*b^3*c*d^2*x^2*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*c/d) + 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b
*c)/d)*tan(b*c/d) + 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(b*c/d) - 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(b*c/d) + 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*t
an(b*c/d) + 24*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*
tan(b*c/d) + 24*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a
)*tan(b*c/d) - 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*
tan(b*c/d) + 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*t
an(b*c/d) - 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) -
 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 6*b^
3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 4*b^2*d^
3*x^2*tan(b*x)*tan(b*c/d)^2 - 4*b^2*d^3*x^2*tan(a)*tan(b*c/d)^2 + 4*b^3*c^3
*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*b^3*c^3*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 8*b^3*c^3*si
n_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 4*b^2*c^2*d*tan(b*x)^2*
tan(a)*tan(b*c/d)^2 + 4*b^2*c^2*d*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + b*c*d^2*
tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2
*b*x + 2*b*c/d)) + 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d)
) + 2*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b^3*c
^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 12*b^3*c^2*d*x*im
ag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 12*b^3*c^2*d*x*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(a) - 24*b^3*c^2*d*x*sin_integral(2*(b*d*x
 + b*c)/d)*tan(a) + 8*b^2*c*d^2*x*tan(b*x)^2*tan(a) - 2*b^3*c^3*real_part(c
os_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 2*b^3*c^3*real_part(cos_integral(-
```

```
2*b*x - 2*b*c/d))*tan(a)^2 + 8*b^2*c*d^2*x*tan(b*x)*tan(a)^2 + b*d^3*x*tan(
b*x)^2*tan(a)^2 + 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*t
an(b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*
c/d) + 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 8*b^3*c^
3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b^3*c^3*re
al_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) - 2*b^3*c^3*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 2*b^3*c^3*real_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 8*b^2*c*d^2*x*tan(b*x)*tan(b*c/d
)^2 - b*d^3*x*tan(b*x)^2*tan(b*c/d)^2 - 8*b^2*c*d^2*x*tan(a)*tan(b*c/d)^2 -
 4*b*d^3*x*tan(b*x)*tan(a)*tan(b*c/d)^2 - b*d^3*x*tan(a)^2*tan(b*c/d)^2 + 6
*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d)) + 6*b^3*c^2*d*x*real_
part(cos_integral(-2*b*x - 2*b*c/d)) - 4*b^2*d^3*x^2*tan(b*x) - 4*b^2*d^3*x
^2*tan(a) - 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 4*b
^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - 8*b^3*c^3*sin_int
egral(2*(b*d*x + b*c)/d)*tan(a) + 4*b^2*c^2*d*tan(b*x)^2*tan(a) + 4*b^2*c^2
*d*tan(b*x)*tan(a)^2 + b*c*d^2*tan(b*x)^2*tan(a)^2 + 4*b^3*c^3*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 4*b^3*c^3*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(b*c/d) + 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*t
an(b*c/d) - 4*b^2*c^2*d*tan(b*x)*tan(b*c/d)^2 - b*c*d^2*tan(b*x)^2*tan(b*c/
d)^2 - 4*b^2*c^2*d*tan(a)*tan(b*c/d)^2 - 4*b*c*d^2*tan(b*x)*tan(a)*tan(b*c/
d)^2 - 2*d^3*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - b*c*d^2*tan(a)^2*tan(b*c/d)^2
 - 2*d^3*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + 2*b^3*c^3*real_part(cos_integral(
2*b*x + 2*b*c/d)) + 2*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d)) - 8
*b^2*c*d^2*x*tan(b*x) - b*d^3*x*tan(b*x)^2 - 8*b^2*c*d^2*x*tan(a) - 4*b*d^3
*x*tan(b*x)*tan(a) - b*d^3*x*tan(a)^2 + b*d^3*x*tan(b*c/d)^2 - 4*b^2*c^2*d*
tan(b*x) - b*c*d^2*tan(b*x)^2 - 4*b^2*c^2*d*tan(a) - 4*b*c*d^2*tan(b*x)*tan
(a) - 2*d^3*tan(b*x)^2*tan(a) - b*c*d^2*tan(a)^2 - 2*d^3*tan(b*x)*tan(a)^2
+ b*c*d^2*tan(b*c/d)^2 + 2*d^3*tan(b*x)*tan(b*c/d)^2 + 2*d^3*tan(a)*tan(b*c
/d)^2 + b*d^3*x + b*c*d^2 + 2*d^3*tan(b*x) + 2*d^3*tan(a))/(d^7*x^3*tan(b*x
)^2*tan(a)^2*tan(b*c/d)^2 + 3*c*d^6*x^2*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 +
d^7*x^3*tan(b*x)^2*tan(a)^2 + d^7*x^3*tan(b*x)^2*tan(b*c/d)^2 + d^7*x^3*tan
(a)^2*tan(b*c/d)^2 + 3*c^2*d^5*x*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 3*c*d^6
*x^2*tan(b*x)^2*tan(a)^2 + 3*c*d^6*x^2*tan(b*x)^2*tan(b*c/d)^2 + 3*c*d^6*x^
2*tan(a)^2*tan(b*c/d)^2 + c^3*d^4*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + d^7*x^
3*tan(b*x)^2 + d^7*x^3*tan(a)^2 + 3*c^2*d^5*x*tan(b*x)^2*tan(a)^2 + d^7*x^3
*tan(b*c/d)^2 + 3*c^2*d^5*x*tan(b*x)^2*tan(b*c/d)^2 + 3*c^2*d^5*x*tan(a)^2*
tan(b*c/d)^2 + 3*c*d^6*x^2*tan(b*x)^2 + 3*c*d^6*x^2*tan(a)^2 + c^3*d^4*tan(
b*x)^2*tan(a)^2 + 3*c*d^6*x^2*tan(b*c/d)^2 + c^3*d^4*tan(b*x)^2*tan(b*c/d)^
2 + c^3*d^4*tan(a)^2*tan(b*c/d)^2 + d^7*x^3 + 3*c^2*d^5*x*tan(b*x)^2 + 3*c^
2*d^5*x*tan(a)^2 + 3*c^2*d^5*x*tan(b*c/d)^2 + 3*c*d^6*x^2 + c^3*d^4*tan(b*x
)^2 + c^3*d^4*tan(a)^2 + c^3*d^4*tan(b*c/d)^2 + 3*c^2*d^5*x + c^3*d^4)
```

**maple [A]** time = 0.01, size = 200, normalized size = 1.39

$$\frac{b^3\left(-\frac{2\sin(2bx+2a)}{3((bx+a)d-da+cb)^3 d}+\frac{-\frac{2\cos(2bx+2a)}{3((bx+a)d-da+cb)^2 d}-\frac{2\left(-\frac{2\sin(2bx+2a)}{((bx+a)d-da+cb)d}+\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}\right)}{3d}}{d}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x)
```

```
[Out] 1/4*b^3*(-2/3*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^3/d+2/3*(-cos(2*b*x+2*a)/(
(b*x+a)*d-d*a+c*b)^2/d-(-2*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si(2*b
```

```
*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)
*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d)
```

**maxima [C]** time = 0.64, size = 249, normalized size = 1.73

$$-\frac{b^4\left(i E_4\left(\frac{2i bc+2i (bx+a)d-2i ad}{d}\right) - i E_4\left(-\frac{2i bc+2i (bx+a)d-2i ad}{d}\right)\right)\cos\left(-\frac{2 (bc-ad)}{d}\right) + b^4\left(E_4\left(\frac{2i bc+2i (bx+a)d-2i ad}{d}\right) + E_4\left(-\frac{2i bc}{\cdots}\right.\right.}{4\left(b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 + (bx+a)^3d^4 - a^3d^4 + 3\left(bcd^3 - ad^4\right)(bx+a)^2 + 3\left(b^2c^2d^2 - 2\,abcd\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(b^4*(I*exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I
*exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) + b^4*(exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c
- a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4
- a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3
+ a^2*d^4)*(b*x + a))*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{\left(c+dx\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**4, x)
```

# 3.10 $\int \frac{\cos(x)\sin(x)}{x}\,dx$

**Optimal**. Leaf size=8

$$\frac{\text{Si}(2x)}{2}$$

```
[Out] 1/2*Si(2*x)
```

**Rubi [A]** time = 0.03, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4406, 12, 3299}

$$\frac{\text{Si}(2x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/x,x]
```

```
[Out] SinIntegral[2*x]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{x}\,dx &= \int \frac{\sin(2x)}{2x}\,dx\\ &= \frac{1}{2}\int \frac{\sin(2x)}{x}\,dx\\ &= \frac{\text{Si}(2x)}{2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 8, normalized size = 1.00

$$\frac{\text{Si}(2x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/x,x]
```

```
[Out] SinIntegral[2*x]/2
```

**fricas [A]** time = 0.72, size = 6, normalized size = 0.75

$$\frac{1}{2}\,\mathrm{Si}\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x, algorithm="fricas")
[Out] 1/2*sin_integral(2*x)
```

**giac [A]** time = 0.17, size = 6, normalized size = 0.75

$$\frac{1}{2}\,\mathrm{Si}\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x, algorithm="giac")
[Out] 1/2*sin_integral(2*x)
```

**maple [A]** time = 0.02, size = 7, normalized size = 0.88

$$\frac{\mathrm{Si}\left(2x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/x,x)
[Out] 1/2*Si(2*x)
```

**maxima [C]** time = 0.37, size = 13, normalized size = 1.62

$$-\frac{1}{4}i\,\mathrm{Ei}\left(2i\,x\right)+\frac{1}{4}i\,\mathrm{Ei}\left(-2i\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x, algorithm="maxima")
[Out] -1/4*I*Ei(2*I*x) + 1/4*I*Ei(-2*I*x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.12

$$\int\frac{\cos(x)\,\sin(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/x,x)
[Out] int((cos(x)*sin(x))/x, x)
```

**sympy [A]** time = 0.86, size = 5, normalized size = 0.62

$$\frac{\mathrm{Si}\left(2x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x)
[Out] Si(2*x)/2
```

## 3.11 $\int \frac{\cos(x)\sin(x)}{x^2}\,dx$

**Optimal**. Leaf size=16

$$\text{Ci}(2x) - \frac{\sin(2x)}{2x}$$

```
[Out] Ci(2*x)-1/2*sin(2*x)/x
```

**Rubi** [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4406, 12, 3297, 3302}

$$\text{CosIntegral}(2x) - \frac{\sin(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/x^2,x]
```

```
[Out] CosIntegral[2*x] - Sin[2*x]/(2*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)\sin(x)}{x^2}\,dx &= \int \frac{\sin(2x)}{2x^2}\,dx \\
&= \frac{1}{2}\int \frac{\sin(2x)}{x^2}\,dx \\
&= -\frac{\sin(2x)}{2x} + \int \frac{\cos(2x)}{x}\,dx \\
&= \text{Ci}(2x) - \frac{\sin(2x)}{2x}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 16, normalized size = 1.00

$$\text{Ci}(2x) - \frac{\sin(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/x^2,x]
[Out] CosIntegral[2*x] - Sin[2*x]/(2*x)
```

**fricas** [A] time = 0.76, size = 24, normalized size = 1.50

$$\frac{x\,\text{Ci}\,(2\,x) + x\,\text{Ci}\,(-2\,x) - 2\,\cos(x)\sin(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^2,x, algorithm="fricas")
[Out] 1/2*(x*cos_integral(2*x) + x*cos_integral(-2*x) - 2*cos(x)*sin(x))/x
```

**giac** [A] time = 0.14, size = 19, normalized size = 1.19

$$\frac{2\,x\,\text{Ci}\,(2\,x) - \sin\,(2\,x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^2,x, algorithm="giac")
[Out] 1/2*(2*x*cos_integral(2*x) - sin(2*x))/x
```

**maple** [A] time = 0.02, size = 15, normalized size = 0.94

$$\text{Ci}\,(2x) - \frac{\sin\,(2x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/x^2,x)
[Out] Ci(2*x)-1/2*sin(2*x)/x
```

**maxima** [C] time = 0.38, size = 15, normalized size = 0.94

$$\frac{1}{2}\,\Gamma\,(-1, 2i\,x) + \frac{1}{2}\,\Gamma\,(-1, -2i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^2,x, algorithm="maxima")
[Out] 1/2*gamma(-1, 2*I*x) + 1/2*gamma(-1, -2*I*x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\cos(x)\,\sin(x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/x^2,x)
[Out] int((cos(x)*sin(x))/x^2, x)
```

**sympy** [**A**] time = 1.57, size = 22, normalized size = 1.38

$$-\log(x) + \frac{\log(x^2)}{2} + \operatorname{Ci}(2x) - \frac{\sin(2x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x**2,x)
```

```
[Out] -log(x) + log(x**2)/2 + Ci(2*x) - sin(2*x)/(2*x)
```

# 3.12 $\int \frac{\cos(x)\sin(x)}{x^3}\,dx$

**Optimal**. Leaf size=29

$$-\text{Si}(2x)-\frac{\sin(2x)}{4x^2}-\frac{\cos(2x)}{2x}$$

```
[Out] -1/2*cos(2*x)/x-Si(2*x)-1/4*sin(2*x)/x^2
```

**Rubi [A]** time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4406, 12, 3297, 3299}

$$-\text{Si}(2x)-\frac{\sin(2x)}{4x^2}-\frac{\cos(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/x^3,x]
```

```
[Out] -Cos[2*x]/(2*x) - Sin[2*x]/(4*x^2) - SinIntegral[2*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)\sin(x)}{x^3}\,dx &= \int \frac{\sin(2x)}{2x^3}\,dx\\
&= \frac{1}{2}\int \frac{\sin(2x)}{x^3}\,dx\\
&= -\frac{\sin(2x)}{4x^2}+\frac{1}{2}\int \frac{\cos(2x)}{x^2}\,dx\\
&= -\frac{\cos(2x)}{2x}-\frac{\sin(2x)}{4x^2}-\int \frac{\sin(2x)}{x}\,dx\\
&= -\frac{\cos(2x)}{2x}-\frac{\sin(2x)}{4x^2}-\text{Si}(2x)
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 1.00

$$-\text{Si}(2x) - \frac{\sin(2x)}{4x^2} - \frac{\cos(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/x^3,x]
[Out] -1/2*Cos[2*x]/x - Sin[2*x]/(4*x^2) - SinIntegral[2*x]
```

**fricas [A]** time = 0.51, size = 30, normalized size = 1.03

$$-\frac{2\,x\cos(x)^2 + 2\,x^2\,\text{Si}\,(2\,x) + \cos(x)\sin(x) - x}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="fricas")
[Out] -1/2*(2*x*cos(x)^2 + 2*x^2*sin_integral(2*x) + cos(x)*sin(x) - x)/x^2
```

**giac [A]** time = 1.60, size = 26, normalized size = 0.90

$$-\frac{4\,x^2\,\text{Si}\,(2\,x) + 2\,x\cos\,(2\,x) + \sin\,(2\,x)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="giac")
[Out] -1/4*(4*x^2*sin_integral(2*x) + 2*x*cos(2*x) + sin(2*x))/x^2
```

**maple [A]** time = 0.02, size = 26, normalized size = 0.90

$$-\frac{\cos\,(2x)}{2x} - \text{Si}\,(2x) - \frac{\sin\,(2x)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/x^3,x)
[Out] -1/2*cos(2*x)/x-Si(2*x)-1/4*sin(2*x)/x^2
```

**maxima [C]** time = 0.39, size = 15, normalized size = 0.52

$$i\,\Gamma\,(-2, 2i\,x) - i\,\Gamma\,(-2, -2i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="maxima")
[Out] I*gamma(-2, 2*I*x) - I*gamma(-2, -2*I*x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(x)\,\sin(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/x^3,x)
[Out] int((cos(x)*sin(x))/x^3, x)
```

**sympy [A]** time = 1.16, size = 24, normalized size = 0.83

$$-\operatorname{Si}(2x) - \frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x**3,x)
```

```
[Out] -Si(2*x) - cos(2*x)/(2*x) - sin(2*x)/(4*x**2)
```

# 3.13 $\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=275

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right.}{8b}$$

```
[Out] -1/8*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c
)/d)^m)+1/8*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(
d*x+c)/d)^m)+1/8*I*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(
d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/8*I*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(
d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

**Rubi** [A] time = 0.33, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4406, 3307, 2181}

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/8)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])
/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/8)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d
*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/8)*3^(-1 - m)*
E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(
b*(((-I)*b*(c + d*x))/d)^m) - ((I/8)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((
3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^m\cos(a+bx) - \frac{1}{4}(c+dx)^m\cos(3a+3bx)\right)dx$$
$$= \frac{1}{4}\int (c+dx)^m\cos(a+bx)\,dx - \frac{1}{4}\int (c+dx)^m\cos(3a+3bx)\,dx$$
$$= \frac{1}{8}\int e^{-i(a+bx)}(c+dx)^m\,dx + \frac{1}{8}\int e^{i(a+bx)}(c+dx)^m\,dx - \frac{1}{8}\int e^{-i(3a+3bx)}($$
$$= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+d}{}$$

**Mathematica** [A] time = 0.69, size = 237, normalized size = 0.86

$$-\frac{ie^{-\frac{3i(ad+bc)}{d}}(c+dx)^m\left(\left(\frac{ib(c+dx)}{d}\right)^{-m}\left(3^{-m}\left(e^{\frac{6ibc}{d}}\Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right)-e^{6ia}\left(\frac{ib(c+dx)}{d}\right)^{2m}\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx}{d}\right.\right.\right.\right.}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/24*I)*(c + d*x)^m*((3*E^((2*I)*(2*a + (b*c)/d))*Gamma[1 + m, ((-I)*b*(
c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m + (-3*E^((2*I)*a + ((4*I)*b*c)/d)*Ga
mma[1 + m, (I*b*(c + d*x))/d] + (-((E^((6*I)*a)*((I*b*(c + d*x))/d)^(2*m)*G
amma[1 + m, ((-3*I)*b*(c + d*x))/d])/((b^2*(c + d*x)^2)/d^2)^m) + E^(((6*I)
*b*c)/d)*Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/3^m)/((I*b*(c + d*x))/d)^m))/
(b*E^(((3*I)*(b*c + a*d))/d))
```

**fricas** [A] time = 0.61, size = 186, normalized size = 0.68

$$\frac{-i\,e^{\left(-\frac{dm\log\left(\frac{3i\,b}{d}\right)-3i\,bc+3i\,ad}{d}\right)}\Gamma\left(m+1,\frac{3i\,bdx+3i\,bc}{d}\right)+3i\,e^{\left(-\frac{dm\log\left(\frac{i\,b}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right)-3i\,e^{\left(-\frac{dm\log\left(-\frac{i\,b}{d}\right)+i\,bc-i\,ad}{d}\right)}\Gamma\left(m\right.}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(-I*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gamma(m + 1, (3*I*b*
d*x + 3*I*b*c)/d) + 3*I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1
, (I*b*d*x + I*b*c)/d) - 3*I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma
(m + 1, (-I*b*d*x - I*b*c)/d) + I*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*
d)/d)*gamma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d))/b
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^2, x)
```

**maple** [F] time = 0.35, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\left(\sin^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a) \sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^2, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^m, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin^2(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x)**2*cos(a + b*x), x)
```

## 3.14 $\int (c+dx)^4 \cos(a+bx) \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=205

$$\frac{8d^4\sin^3(a+bx)}{81b^5}+\frac{160d^4\sin(a+bx)}{27b^5}-\frac{160d^3(c+dx)\cos(a+bx)}{27b^4}-\frac{8d^3(c+dx)\sin^2(a+bx)\cos(a+bx)}{27b^4}-\frac{4d^2(c+d}{}$$

```
[Out] -160/27*d^3*(d*x+c)*cos(b*x+a)/b^4+8/9*d*(d*x+c)^3*cos(b*x+a)/b^2+160/27*d^
4*sin(b*x+a)/b^5-8/3*d^2*(d*x+c)^2*sin(b*x+a)/b^3-8/27*d^3*(d*x+c)*cos(b*x+
a)*sin(b*x+a)^2/b^4+4/9*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2/b^2+8/81*d^4*si
n(b*x+a)^3/b^5-4/9*d^2*(d*x+c)^2*sin(b*x+a)^3/b^3+1/3*(d*x+c)^4*sin(b*x+a)^
3/b
```

**Rubi [A]** time = 0.20, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4404, 3311, 3296, 2637, 3310}

$$-\frac{4d^2(c+dx)^2\sin^3(a+bx)}{9b^3}-\frac{8d^2(c+dx)^2\sin(a+bx)}{3b^3}-\frac{160d^3(c+dx)\cos(a+bx)}{27b^4}-\frac{8d^3(c+dx)\sin^2(a+bx)\cos(a}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (-160*d^3*(c + d*x)*Cos[a + b*x])/(27*b^4) + (8*d*(c + d*x)^3*Cos[a + b*x])
/(9*b^2) + (160*d^4*Sin[a + b*x])/(27*b^5) - (8*d^2*(c + d*x)^2*Sin[a + b*x
])/(3*b^3) - (8*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(27*b^4) + (4*d*
(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^2) + (8*d^4*Sin[a + b*x]^3)/(
81*b^5) - (4*d^2*(c + d*x)^2*Sin[a + b*x]^3)/(9*b^3) + ((c + d*x)^4*Sin[a +
 b*x]^3)/(3*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^4\cos(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^4\sin^3(a+bx)}{3b} - \frac{(4d)\int (c+dx)^3\sin^3(a+bx)\,dx}{3b}$$
$$= \frac{4d(c+dx)^3\cos(a+bx)\sin^2(a+bx)}{9b^2} - \frac{4d^2(c+dx)^2\sin^3(a+bx)}{9b^3} +$$
$$= \frac{8d(c+dx)^3\cos(a+bx)}{9b^2} - \frac{8d^3(c+dx)\cos(a+bx)\sin^2(a+bx)}{27b^4} + \frac{4}{}$$
$$= -\frac{16d^3(c+dx)\cos(a+bx)}{27b^4} + \frac{8d(c+dx)^3\cos(a+bx)}{9b^2} - \frac{8d^2(c+dx)}{3}$$
$$= -\frac{160d^3(c+dx)\cos(a+bx)}{27b^4} + \frac{8d(c+dx)^3\cos(a+bx)}{9b^2} + \frac{16d^4\sin(a}{27b^5}$$
$$= -\frac{160d^3(c+dx)\cos(a+bx)}{27b^4} + \frac{8d(c+dx)^3\cos(a+bx)}{9b^2} + \frac{160d^4\sin(}{27b}$$

**Mathematica [A]** time = 1.45, size = 385, normalized size = 1.88

$$\frac{81b^4c^4\sin(a+bx) - 27b^4c^4\sin(3(a+bx)) + 324b^4c^3dx\sin(a+bx) - 108b^4c^3dx\sin(3(a+bx)) + 486b^4c^2d^2x^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (324*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 12*b*d*(c + d*
x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 81*b^4*c^4*Sin[a + b*x]
- 972*b^2*c^2*d^2*Sin[a + b*x] + 1944*d^4*Sin[a + b*x] + 324*b^4*c^3*d*x*Si
n[a + b*x] - 1944*b^2*c*d^3*x*Sin[a + b*x] + 486*b^4*c^2*d^2*x^2*Sin[a + b*
x] - 972*b^2*d^4*x^2*Sin[a + b*x] + 324*b^4*c*d^3*x^3*Sin[a + b*x] + 81*b^4
*d^4*x^4*Sin[a + b*x] - 27*b^4*c^4*Sin[3*(a + b*x)] + 36*b^2*c^2*d^2*Sin[3*
(a + b*x)] - 8*d^4*Sin[3*(a + b*x)] - 108*b^4*c^3*d*x*Sin[3*(a + b*x)] + 72
*b^2*c*d^3*x*Sin[3*(a + b*x)] - 162*b^4*c^2*d^2*x^2*Sin[3*(a + b*x)] + 36*b
^2*d^4*x^2*Sin[3*(a + b*x)] - 108*b^4*c*d^3*x^3*Sin[3*(a + b*x)] - 27*b^4*d
^4*x^4*Sin[3*(a + b*x)])/(324*b^5)
```

**fricas [A]** time = 0.54, size = 352, normalized size = 1.72

$$-\frac{12\left(3b^3d^4x^3 + 9b^3cd^3x^2 + 3b^3c^3d - 2bcd^3 + \left(9b^3c^2d^2 - 2bd^4\right)x\right)\cos(bx+a)^3 - 36\left(3b^3d^4x^3 + 9b^3cd^3x^2 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/81*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (9*b
^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^3 - 36*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x
^2 + 3*b^3*c^3*d - 14*b*c*d^3 + (9*b^3*c^2*d^2 - 14*b*d^4)*x)*cos(b*x + a)
- (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 252*b^2*c^2*d^2 + 488*
d^4 + 18*(9*b^4*c^2*d^2 - 14*b^2*d^4)*x^2 - (27*b^4*d^4*x^4 + 108*b^4*c*d^3
*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)
```

```
*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)*x)*cos(b*x + a)^2 + 36*(3*b^4*c^3*d -
 14*b^2*c*d^3)*x)*sin(b*x + a))/b^5
```

**giac [A]** time = 0.23, size = 350, normalized size = 1.71

$$-\frac{\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+9\,b^3c^2d^2x+3\,b^3c^3d-2\,bd^4x-2\,bcd^3\right)\cos\left(3\,bx+3\,a\right)}{27\,b^5}+\frac{\left(b^3d^4x^3+3\,b^3cd^3x^2+3\,b^3c^2d^2\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*
b*d^4*x - 2*b*c*d^3)*cos(3*b*x + 3*a)/b^5 + (b^3*d^4*x^3 + 3*b^3*c*d^3*x^2
+ 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5 - 1
/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c^
3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*d
^4)*sin(3*b*x + 3*a)/b^5 + 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d
^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2
*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

**maple [B]** time = 0.09, size = 835, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/3*(b*x+a)^4*sin(b*x+a)^3+4/9*(b*x+a)^3*(2+sin(b*x+a)^2)*c
os(b*x+a)-8/3*(b*x+a)^2*sin(b*x+a)+160/27*sin(b*x+a)-16/3*(b*x+a)*cos(b*x+a
)-4/9*(b*x+a)^2*sin(b*x+a)^3-8/27*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+8/81*
sin(b*x+a)^3)-4/b^4*a*d^4*(1/3*(b*x+a)^3*sin(b*x+a)^3+1/3*(b*x+a)^2*(2+sin(
b*x+a)^2)*cos(b*x+a)-4/3*cos(b*x+a)-4/3*(b*x+a)*sin(b*x+a)-2/9*(b*x+a)*sin(
b*x+a)^3-2/27*(2+sin(b*x+a)^2)*cos(b*x+a))+4/b^3*c*d^3*(1/3*(b*x+a)^3*sin(b
*x+a)^3+1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)-4/3*cos(b*x+a)-4/3*(b*x+a
)*sin(b*x+a)-2/9*(b*x+a)*sin(b*x+a)^3-2/27*(2+sin(b*x+a)^2)*cos(b*x+a))+6/b
^4*a^2*d^4*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x
+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))-12/b^3*a*c*d^3*(1/3*(b*x+a)^2*sin(b*x
+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x
+a))+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)
*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))-4/b^4*a^3*d^4*(1/3*(b*x+a)*si
n(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+12/b^3*a^2*c*d^3*(1/3*(b*x+a)*s
in(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))-12/b^2*a*c^2*d^2*(1/3*(b*x+a)*
sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+4/b*c^3*d*(1/3*(b*x+a)*sin(b*
x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+1/3/b^4*a^4*d^4*sin(b*x+a)^3-4/3/b^
3*a^3*c*d^3*sin(b*x+a)^3+2/b^2*a^2*c^2*d^2*sin(b*x+a)^3-4/3/b*a*c^3*d*sin(b
*x+a)^3+1/3*c^4*sin(b*x+a)^3)
```

**maxima [B]** time = 0.42, size = 880, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/324*(108*c^4*sin(b*x + a)^3 - 432*a*c^3*d*sin(b*x + a)^3/b + 648*a^2*c^2*
d^2*sin(b*x + a)^3/b^2 - 432*a^3*c*d^3*sin(b*x + a)^3/b^3 + 108*a^4*d^4*sin
(b*x + a)^3/b^4 - 36*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x +
a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*c^3*d/b + 108*(3*(b*x + a)*sin(3*b*
x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a*
```

```
c^2*d^2/b^2 - 108*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a)
+ cos(3*b*x + 3*a) - 9*cos(b*x + a))*a^2*c*d^3/b^3 + 36*(3*(b*x + a)*sin(3*
b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*
a^3*d^4/b^4 - 18*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a)
+ (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))
*c^2*d^2/b^2 + 36*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a)
 + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a)
)*a*c*d^3/b^3 - 18*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a
) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a
))*a^2*d^4/b^4 - 12*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2
 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*
((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*c*d^3/b^3 + 12*((9*(b*x + a)^2 -
2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3
- 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x +
a))*a*d^4/b^4 - (12*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 324*((
b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) + (27*(b*x + a)^4 - 36*(b*x + a)^2 +
 8)*sin(3*b*x + 3*a) - 81*((b*x + a)^4 - 12*(b*x + a)^2 + 24)*sin(b*x + a))
*d^4/b^4)/b
```

**mupad [B]** time = 1.41, size = 448, normalized size = 2.19

$$\frac{\sin(a+b\,x)^3\,\left(27\,b^4\,c^4-252\,b^2\,c^2\,d^2+488\,d^4\right)}{81\,b^5}-\frac{8\cos(a+b\,x)^3\,\left(20\,c\,d^3-3\,b^2\,c^3\,d\right)}{27\,b^4}+\frac{8\cos(a+b\,x)^2\,\sin(a+}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] (sin(a + b*x)^3*(488*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(81*b^5) - (8*cos
(a + b*x)^3*(20*c*d^3 - 3*b^2*c^3*d))/(27*b^4) + (8*cos(a + b*x)^2*sin(a +
b*x)*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^5) - (4*cos(a + b*x)*sin(a + b*x)^2*(1
4*c*d^3 - 3*b^2*c^3*d))/(9*b^4) + (8*d^4*x^3*cos(a + b*x)^3)/(9*b^2) - (8*x
*cos(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^4) + (d^4*x^4*sin(a + b*x)^
3)/(3*b) - (4*x*sin(a + b*x)^3*(14*c*d^3 - 3*b^2*c^3*d))/(9*b^3) - (2*x^2*s
in(a + b*x)^3*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^3) + (8*c*d^3*x^2*cos(a + b*x)
^3)/(3*b^2) + (4*d^4*x^3*cos(a + b*x)*sin(a + b*x)^2)/(3*b^2) - (8*d^4*x^2*
cos(a + b*x)^2*sin(a + b*x))/(3*b^3) + (4*c*d^3*x^3*sin(a + b*x)^3)/(3*b) -
 (4*x*cos(a + b*x)*sin(a + b*x)^2*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^4) + (4*c*
d^3*x^2*cos(a + b*x)*sin(a + b*x)^2)/b^2 - (16*c*d^3*x*cos(a + b*x)^2*sin(a
 + b*x))/(3*b^3)
```

**sympy [A]** time = 7.48, size = 646, normalized size = 3.15

$$\begin{cases}\frac{c^4\sin^3(a+bx)}{3b}+\frac{4c^3dx\sin^3(a+bx)}{3b}+\frac{2c^2d^2x^2\sin^3(a+bx)}{b}+\frac{4cd^3x^3\sin^3(a+bx)}{3b}+\frac{d^4x^4\sin^3(a+bx)}{3b}+\frac{4c^3d\sin^2(a+bx)\cos(a+bx)}{3b^2}+\frac{8c^3d}{} \\ \left(c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}\right)\sin^2(a)\cos(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)**3/(3*b) + 4*c**3*d*x*sin(a + b*x)**3/(3*b) +
2*c**2*d**2*x**2*sin(a + b*x)**3/b + 4*c*d**3*x**3*sin(a + b*x)**3/(3*b) +
d**4*x**4*sin(a + b*x)**3/(3*b) + 4*c**3*d*sin(a + b*x)**2*cos(a + b*x)/(3*
b**2) + 8*c**3*d*cos(a + b*x)**3/(9*b**2) + 4*c**2*d**2*x*sin(a + b*x)**2*c
os(a + b*x)/b**2 + 8*c**2*d**2*x*cos(a + b*x)**3/(3*b**2) + 4*c*d**3*x**2*s
in(a + b*x)**2*cos(a + b*x)/b**2 + 8*c*d**3*x**2*cos(a + b*x)**3/(3*b**2) +
 4*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 8*d**4*x**3*cos(a + b*
x)**3/(9*b**2) - 28*c**2*d**2*sin(a + b*x)**3/(9*b**3) - 8*c**2*d**2*sin(a
+ b*x)*cos(a + b*x)**2/(3*b**3) - 56*c*d**3*x*sin(a + b*x)**3/(9*b**3) - 16
```

```
*c*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 28*d**4*x**2*sin(a + b*x)
**3/(9*b**3) - 8*d**4*x**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 56*c*d**
3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 160*c*d**3*cos(a + b*x)**3/(27*b*
*4) - 56*d**4*x*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 160*d**4*x*cos(a +
b*x)**3/(27*b**4) + 488*d**4*sin(a + b*x)**3/(81*b**5) + 160*d**4*sin(a + b
*x)*cos(a + b*x)**2/(27*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2
*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**2*cos(a), True))
```

# 3.15 $\int (c+dx)^3 \cos(a+bx) \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=151

$$\frac{2d^3\cos^3(a+bx)}{27b^4}-\frac{14d^3\cos(a+bx)}{9b^4}-\frac{2d^2(c+dx)\sin^3(a+bx)}{9b^3}-\frac{4d^2(c+dx)\sin(a+bx)}{3b^3}+\frac{2d(c+dx)^2\cos(a+b}{3b^2}$$

```
[Out] -14/9*d^3*cos(b*x+a)/b^4+2/3*d*(d*x+c)^2*cos(b*x+a)/b^2+2/27*d^3*cos(b*x+a)
^3/b^4-4/3*d^2*(d*x+c)*sin(b*x+a)/b^3+1/3*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)
^2/b^2-2/9*d^2*(d*x+c)*sin(b*x+a)^3/b^3+1/3*(d*x+c)^3*sin(b*x+a)^3/b
```

**Rubi [A]** time = 0.13, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4404, 3311, 3296, 2638, 2633}

$$-\frac{2d^2(c+dx)\sin^3(a+bx)}{9b^3}-\frac{4d^2(c+dx)\sin(a+bx)}{3b^3}+\frac{2d(c+dx)^2\cos(a+bx)}{3b^2}+\frac{d(c+dx)^2\sin^2(a+bx)\cos(a+}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (-14*d^3*Cos[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Cos[a + b*x])/(3*b^2) + (
2*d^3*Cos[a + b*x]^3)/(27*b^4) - (4*d^2*(c + d*x)*Sin[a + b*x])/(3*b^3) + (
d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b^2) - (2*d^2*(c + d*x)*Sin[a
 + b*x]^3)/(9*b^3) + ((c + d*x)^3*Sin[a + b*x]^3)/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^3\cos(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\int (c+dx)^2\sin^3(a+bx)\,dx}{b}$$
$$= \frac{d(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{3b^2} - \frac{2d^2(c+dx)\sin^3(a+bx)}{9b^3} + \frac{(c+$$
$$= \frac{2d(c+dx)^2\cos(a+bx)}{3b^2} + \frac{d(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{3b^2} - \frac{2d^2(c$$
$$= -\frac{2d^3\cos(a+bx)}{9b^4} + \frac{2d(c+dx)^2\cos(a+bx)}{3b^2} + \frac{2d^3\cos^3(a+bx)}{27b^4} - \frac{4d^2}{}$$
$$= -\frac{14d^3\cos(a+bx)}{9b^4} + \frac{2d(c+dx)^2\cos(a+bx)}{3b^2} + \frac{2d^3\cos^3(a+bx)}{27b^4} - \frac{4d}{}$$

**Mathematica** [A] time = 0.94, size = 121, normalized size = 0.80

$$-\frac{-81d\cos(a+bx)\left(b^2(c+dx)^2-2d^2\right)+d\cos(3(a+bx))\left(9b^2(c+dx)^2-2d^2\right)+6b(c+dx)\sin(a+bx)\left(\cos(2(a\right.}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -1/108*(-81*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + d*(-2*d^2 + 9*b^2*(
c + d*x)^2)*Cos[3*(a + b*x)] + 6*b*(c + d*x)*(26*d^2 - 3*b^2*(c + d*x)^2 +
(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/b^4
```

**fricas** [A] time = 0.49, size = 227, normalized size = 1.50

$$-\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^3-3\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-14\,d^3\right)\cos\left(bx+a\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/27*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^
3 - 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 14*d^3)*cos(b*x + a)
- 3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 14*b*c*d^2 - (3*b^3*d^3*
x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*c^2*d - 2*b*d^3)*x)*
cos(b*x + a)^2 + (9*b^3*c^2*d - 14*b*d^3)*x)*sin(b*x + a))/b^4
```

**giac** [A] time = 0.22, size = 231, normalized size = 1.53

$$-\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(3\,bx+3\,a\right)}{108\,b^4}+\frac{3\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)}{4\,b^4}-\frac{(3\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/108*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(3*b*x + 3
*a)/b^4 + 3/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a
)/b^4 - 1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 -
 2*b*d^3*x - 2*b*c*d^2)*sin(3*b*x + 3*a)/b^4 + 1/4*(b^3*d^3*x^3 + 3*b^3*c*d
^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4
```

**maple [B]** time = 0.01, size = 447, normalized size = 2.96

$$\frac{\frac{d^{3}\left(\frac{(bx+a)^{3}\left(\sin^{3}(bx+a)\right)}{3}+\frac{(bx+a)^{2}\left(2+\sin^{2}(bx+a)\right)\cos(bx+a)}{3}-\frac{4\cos(bx+a)}{3}-\frac{4(bx+a)\sin(bx+a)}{3}-\frac{2(bx+a)\left(\sin^{3}(bx+a)\right)}{9}-\frac{2\left(2+\sin^{2}(bx+a)\right)\cos(bx+a)}{27}\right)}{b^{3}}-\frac{3a\,d^{3}\left(\frac{(bx+a)^{2}(s}{}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/3*(b*x+a)^3*sin(b*x+a)^3+1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*c
os(b*x+a)-4/3*cos(b*x+a)-4/3*(b*x+a)*sin(b*x+a)-2/9*(b*x+a)*sin(b*x+a)^3-2/
27*(2+sin(b*x+a)^2)*cos(b*x+a))-3/b^3*a*d^3*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9
*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))+3/b^
2*c*d^2*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)
-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))+3/b^3*a^2*d^3*(1/3*(b*x+a)*sin(b*x+a)^3+
1/9*(2+sin(b*x+a)^2)*cos(b*x+a))-6/b^2*a*c*d^2*(1/3*(b*x+a)*sin(b*x+a)^3+1/
9*(2+sin(b*x+a)^2)*cos(b*x+a))+3/b*c^2*d*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+s
in(b*x+a)^2)*cos(b*x+a))-1/3/b^3*a^3*d^3*sin(b*x+a)^3+1/b^2*a^2*c*d^2*sin(b
*x+a)^3-1/b*a*c^2*d*sin(b*x+a)^3+1/3*c^3*sin(b*x+a)^3)
```

**maxima [B]** time = 0.37, size = 499, normalized size = 3.30

$$\frac{36\,c^{3}\sin\left(bx+a\right)^{3}-\frac{108\,ac^{2}d\sin(bx+a)^{3}}{b}+\frac{108\,a^{2}cd^{2}\sin(bx+a)^{3}}{b^{2}}-\frac{36\,a^{3}d^{3}\sin(bx+a)^{3}}{b^{3}}-\frac{9\,(3\,(bx+a)\sin(3\,bx+3\,a)-9\,(bx+a)\sin(bx+a)+}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/108*(36*c^3*sin(b*x + a)^3 - 108*a*c^2*d*sin(b*x + a)^3/b + 108*a^2*c*d^2
*sin(b*x + a)^3/b^2 - 36*a^3*d^3*sin(b*x + a)^3/b^3 - 9*(3*(b*x + a)*sin(3*
b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*
c^2*d/b + 18*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos
(3*b*x + 3*a) - 9*cos(b*x + a))*a*c*d^2/b^2 - 9*(3*(b*x + a)*sin(3*b*x + 3*
a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a^2*d^3/
b^3 - 3*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x
 + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^2/b^
2 + 3*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x +
 a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*a*d^3/b^3
 - ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a)
 + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*((b*x + a)^3 - 6*b
*x - 6*a)*sin(b*x + a))*d^3/b^3)/b
```

**mupad [B]** time = 1.15, size = 289, normalized size = 1.91

$$\frac{2\,d^{3}\,x^{2}\cos\left(a+b\,x\right)^{3}}{3\,b^{2}}-\frac{\sin\left(a+b\,x\right)^{3}\,\left(14\,c\,d^{2}-3\,b^{2}\,c^{3}\right)}{9\,b^{3}}-\frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^{2}\,\left(14\,d^{3}-9\,b^{2}\,c^{2}\,d\right)}{9\,b^{4}}-\frac{x\sin\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] (2*d^3*x^2*cos(a + b*x)^3)/(3*b^2) - (sin(a + b*x)^3*(14*c*d^2 - 3*b^2*c^3)
)/(9*b^3) - (cos(a + b*x)*sin(a + b*x)^2*(14*d^3 - 9*b^2*c^2*d))/(9*b^4) -
(x*sin(a + b*x)^3*(14*d^3 - 9*b^2*c^2*d))/(9*b^3) - (2*cos(a + b*x)^3*(20*d
^3 - 9*b^2*c^2*d))/(27*b^4) + (d^3*x^3*sin(a + b*x)^3)/(3*b) - (4*c*d^2*cos
(a + b*x)^2*sin(a + b*x))/(3*b^3) + (4*c*d^2*x*cos(a + b*x)^3)/(3*b^2) - (4
*d^3*x*cos(a + b*x)^2*sin(a + b*x))/(3*b^3) + (d^3*x^2*cos(a + b*x)*sin(a +
```

```
b*x)^2)/b^2 + (c*d^2*x^2*sin(a + b*x)^3)/b + (2*c*d^2*x*cos(a + b*x)*sin(a
+ b*x)^2)/b^2
```

**sympy [A]** time = 4.01, size = 391, normalized size = 2.59

$$\begin{cases} \frac{c^3\sin^3(a+bx)}{3b} + \frac{c^2dx\sin^3(a+bx)}{b} + \frac{cd^2x^2\sin^3(a+bx)}{b} + \frac{d^3x^3\sin^3(a+bx)}{3b} + \frac{c^2d\sin^2(a+bx)\cos(a+bx)}{b^2} + \frac{2c^2d\cos^3(a+bx)}{3b^2} + \frac{2cd^2x\sin^2(a+b}{b^2} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin^2(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)**3/(3*b) + c**2*d*x*sin(a + b*x)**3/b + c*d**2
*x**2*sin(a + b*x)**3/b + d**3*x**3*sin(a + b*x)**3/(3*b) + c**2*d*sin(a +
b*x)**2*cos(a + b*x)/b**2 + 2*c**2*d*cos(a + b*x)**3/(3*b**2) + 2*c*d**2*x*
sin(a + b*x)**2*cos(a + b*x)/b**2 + 4*c*d**2*x*cos(a + b*x)**3/(3*b**2) + d
**3*x**2*sin(a + b*x)**2*cos(a + b*x)/b**2 + 2*d**3*x**2*cos(a + b*x)**3/(3
*b**2) - 14*c*d**2*sin(a + b*x)**3/(9*b**3) - 4*c*d**2*sin(a + b*x)*cos(a +
 b*x)**2/(3*b**3) - 14*d**3*x*sin(a + b*x)**3/(9*b**3) - 4*d**3*x*sin(a + b
*x)*cos(a + b*x)**2/(3*b**3) - 14*d**3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4
) - 40*d**3*cos(a + b*x)**3/(27*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/
2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**2*cos(a), True))
```

# 3.16 $\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx) \, dx$

**Optimal**. Leaf size=103

$$-\frac{2d^2\sin^3(a+bx)}{27b^3}-\frac{4d^2\sin(a+bx)}{9b^3}+\frac{4d(c+dx)\cos(a+bx)}{9b^2}+\frac{2d(c+dx)\sin^2(a+bx)\cos(a+bx)}{9b^2}+\frac{(c+dx)^2\sin^3(a+bx)}{3b}$$

```
[Out] 4/9*d*(d*x+c)*cos(b*x+a)/b^2-4/9*d^2*sin(b*x+a)/b^3+2/9*d*(d*x+c)*cos(b*x+a
)*sin(b*x+a)^2/b^2-2/27*d^2*sin(b*x+a)^3/b^3+1/3*(d*x+c)^2*sin(b*x+a)^3/b
```

**Rubi** [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4404, 3310, 3296, 2637}

$$\frac{4d(c+dx)\cos(a+bx)}{9b^2}+\frac{2d(c+dx)\sin^2(a+bx)\cos(a+bx)}{9b^2}-\frac{2d^2\sin^3(a+bx)}{27b^3}-\frac{4d^2\sin(a+bx)}{9b^3}+\frac{(c+dx)^2\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (4*d*(c + d*x)*Cos[a + b*x])/(9*b^2) - (4*d^2*Sin[a + b*x])/(9*b^3) + (2*d*
(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^2) - (2*d^2*Sin[a + b*x]^3)/(27
*b^3) + ((c + d*x)^2*Sin[a + b*x]^3)/(3*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^2\cos(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^2\sin^3(a+bx)}{3b} - \frac{(2d)\int (c+dx)\sin^3(a+bx)\,dx}{3b}$$
$$= \frac{2d(c+dx)\cos(a+bx)\sin^2(a+bx)}{9b^2} - \frac{2d^2\sin^3(a+bx)}{27b^3} + \frac{(c+dx)^2\sin^{3}}{3b}$$
$$= \frac{4d(c+dx)\cos(a+bx)}{9b^2} + \frac{2d(c+dx)\cos(a+bx)\sin^2(a+bx)}{9b^2} - \frac{2d^2\sin}{2}$$
$$= \frac{4d(c+dx)\cos(a+bx)}{9b^2} - \frac{4d^2\sin(a+bx)}{9b^3} + \frac{2d(c+dx)\cos(a+bx)\sin^2}{9b^2}$$

**Mathematica [A]** time = 0.58, size = 93, normalized size = 0.90

$$\frac{-2\sin(a+bx)\left(\cos(2(a+bx))\left(9b^2(c+dx)^2-2d^2\right)-9b^2(c+dx)^2+26d^2\right)+54bd(c+dx)\cos(a+bx)-6bd(c+}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (54*b*d*(c + d*x)*Cos[a + b*x] - 6*b*d*(c + d*x)*Cos[3*(a + b*x)] - 2*(26*d
^2 - 9*b^2*(c + d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin
[a + b*x])/(108*b^3)
```

**fricas [A]** time = 0.49, size = 130, normalized size = 1.26

$$-\frac{6\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3-18\left(bd^2x+bcd\right)\cos\left(bx+a\right)-\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-\left(9\,b^2d^2x^2+18\,b^2c\right.\right.}{27\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/27*(6*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 18*(b*d^2*x + b*c*d)*cos(b*x +
a) - (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - (9*b^2*d^2*x^2 + 18*b^2*c*
d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 14*d^2)*sin(b*x + a))/b^3
```

**giac [A]** time = 0.21, size = 137, normalized size = 1.33

$$-\frac{\left(bd^2x+bcd\right)\cos\left(3\,bx+3\,a\right)}{18\,b^3}+\frac{\left(bd^2x+bcd\right)\cos\left(bx+a\right)}{2\,b^3}-\frac{\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2\right)\sin\left(3\,bx+3\,a\right)}{108\,b^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/18*(b*d^2*x + b*c*d)*cos(3*b*x + 3*a)/b^3 + 1/2*(b*d^2*x + b*c*d)*cos(b*
x + a)/b^3 - 1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3
*b*x + 3*a)/b^3 + 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x
 + a)/b^3
```

**maple [B]** time = 0.01, size = 204, normalized size = 1.98

$$\frac{\frac{d^2\left(\frac{(bx+a)^2\left(\sin^3(bx+a)\right)}{3}+\frac{2(bx+a)\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{9}-\frac{2\left(\sin^3(bx+a)\right)}{27}-\frac{4\sin(bx+a)}{9}\right)}{b^2}-\frac{2a\,d^2\left(\frac{(bx+a)\left(\sin^3(bx+a)\right)}{3}+\frac{\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{9}\right)}{b^2}+2cd\left(\frac{(bx+a)(\sin}{3}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos
(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))-2/b^2*a*d^2*(1/3*(b*x+a)*sin(b*x+
a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+2/b*c*d*(1/3*(b*x+a)*sin(b*x+a)^3+1/9
*(2+sin(b*x+a)^2)*cos(b*x+a))+1/3/b^2*a^2*d^2*sin(b*x+a)^3-2/3/b*a*c*d*sin(
b*x+a)^3+1/3*c^2*sin(b*x+a)^3)
```

**maxima [B]** time = 0.37, size = 240, normalized size = 2.33

$$\frac{36\,c^2\sin\left(bx+a\right)^3 - \frac{72\,acd\sin(bx+a)^3}{b} + \frac{36\,a^2d^2\sin(bx+a)^3}{b^2} - \frac{6\,(3\,(bx+a)\sin(3\,bx+3\,a) - 9\,(bx+a)\sin(bx+a) + \cos(3\,bx+3\,a) - 9\,\cos(bx+a)}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/108*(36*c^2*sin(b*x + a)^3 - 72*a*c*d*sin(b*x + a)^3/b + 36*a^2*d^2*sin(b
*x + a)^3/b^2 - 6*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a)
+ cos(3*b*x + 3*a) - 9*cos(b*x + a))*c*d/b + 6*(3*(b*x + a)*sin(3*b*x + 3*a
) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a*d^2/b^2
 - (6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x + a)
^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2)/b
```

**mupad [B]** time = 0.87, size = 161, normalized size = 1.56

$$\frac{4\,d^2\,x\cos\left(a+b\,x\right)^3}{9\,b^2} - \frac{4\,d^2\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{9\,b^3} - \frac{\sin\left(a+b\,x\right)^3\,\left(14\,d^2-9\,b^2\,c^2\right)}{27\,b^3} + \frac{d^2\,x^2\sin\left(a+b\,x\right)^3}{3\,b} + 4\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] (4*d^2*x*cos(a + b*x)^3)/(9*b^2) - (4*d^2*cos(a + b*x)^2*sin(a + b*x))/(9*b
^3) - (sin(a + b*x)^3*(14*d^2 - 9*b^2*c^2))/(27*b^3) + (d^2*x^2*sin(a + b*x
)^3)/(3*b) + (4*c*d*cos(a + b*x)^3)/(9*b^2) + (2*c*d*cos(a + b*x)*sin(a + b
*x)^2)/(3*b^2) + (2*c*d*x*sin(a + b*x)^3)/(3*b) + (2*d^2*x*cos(a + b*x)*sin
(a + b*x)^2)/(3*b^2)
```

**sympy [A]** time = 2.09, size = 216, normalized size = 2.10

$$\begin{cases} \frac{c^2\sin^3{(a+bx)}}{3b} + \frac{2cdx\sin^3{(a+bx)}}{3b} + \frac{d^2x^2\sin^3{(a+bx)}}{3b} + \frac{2cd\sin^2{(a+bx)}\cos{(a+bx)}}{3b^2} + \frac{4cd\cos^3{(a+bx)}}{9b^2} + \frac{2d^2x\sin^2{(a+bx)}\cos{(a+bx)}}{3b^2} + 4a \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^2{(a)}\cos{(a)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)**3/(3*b) + 2*c*d*x*sin(a + b*x)**3/(3*b) + d**
2*x**2*sin(a + b*x)**3/(3*b) + 2*c*d*sin(a + b*x)**2*cos(a + b*x)/(3*b**2)
+ 4*c*d*cos(a + b*x)**3/(9*b**2) + 2*d**2*x*sin(a + b*x)**2*cos(a + b*x)/(3
*b**2) + 4*d**2*x*cos(a + b*x)**3/(9*b**2) - 14*d**2*sin(a + b*x)**3/(27*b*
*3) - 4*d**2*sin(a + b*x)*cos(a + b*x)**2/(9*b**3), Ne(b, 0)), ((c**2*x + c
*d*x**2 + d**2*x**3/3)*sin(a)**2*cos(a), True))
```

## 3.17 $\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=51

$$-\frac{d\cos^3(a+bx)}{9b^2}+\frac{d\cos(a+bx)}{3b^2}+\frac{(c+dx)\sin^3(a+bx)}{3b}$$

```
[Out] 1/3*d*cos(b*x+a)/b^2-1/9*d*cos(b*x+a)^3/b^2+1/3*(d*x+c)*sin(b*x+a)^3/b
```

**Rubi** [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4404, 2633}

$$-\frac{d\cos^3(a+bx)}{9b^2}+\frac{d\cos(a+bx)}{3b^2}+\frac{(c+dx)\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (d*Cos[a + b*x])/(3*b^2) - (d*Cos[a + b*x]^3)/(9*b^2) + ((c + d*x)*Sin[a +
b*x]^3)/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx &= \frac{(c+dx)\sin^3(a+bx)}{3b}-\frac{d\int \sin^3(a+bx)\,dx}{3b}\\
&= \frac{(c+dx)\sin^3(a+bx)}{3b}+\frac{d\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{3b^2}\\
&= \frac{d\cos(a+bx)}{3b^2}-\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin^3(a+bx)}{3b}
\end{aligned}$$

**Mathematica** [A] time = 0.17, size = 44, normalized size = 0.86

$$\frac{12b(c+dx)\sin^3(a+bx)+9d\cos(a+bx)-d\cos(3(a+bx))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (9*d*Cos[a + b*x] - d*Cos[3*(a + b*x)] + 12*b*(c + d*x)*Sin[a + b*x]^3)/(36
*b^2)
```

**fricas** [A] time = 0.80, size = 59, normalized size = 1.16

$$-\frac{d\cos\left(bx+a\right)^{3}-3\,d\cos\left(bx+a\right)-3\left(bdx-\left(bdx+bc\right)\cos\left(bx+a\right)^{2}+bc\right)\sin\left(bx+a\right)}{9\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(d*cos(b*x + a)^3 - 3*d*cos(b*x + a) - 3*(b*d*x - (b*d*x + b*c)*cos(b*
x + a)^2 + b*c)*sin(b*x + a))/b^2
```

**giac** [A] time = 0.19, size = 69, normalized size = 1.35

$$-\frac{d\cos\left(3\,bx+3\,a\right)}{36\,b^{2}}+\frac{d\cos\left(bx+a\right)}{4\,b^{2}}-\frac{\left(bdx+bc\right)\sin\left(3\,bx+3\,a\right)}{12\,b^{2}}+\frac{\left(bdx+bc\right)\sin\left(bx+a\right)}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/36*d*cos(3*b*x + 3*a)/b^2 + 1/4*d*cos(b*x + a)/b^2 - 1/12*(b*d*x + b*c)*
sin(3*b*x + 3*a)/b^2 + 1/4*(b*d*x + b*c)*sin(b*x + a)/b^2
```

**maple** [A] time = 0.01, size = 71, normalized size = 1.39

$$\frac{\frac{d\left(\frac{(bx+a)\left(\sin^{3}(bx+a)\right)}{3}+\frac{\left(2+\sin^{2}(bx+a)\right)\cos(bx+a)}{9}\right)}{b}-\frac{da\left(\sin^{3}(bx+a)\right)}{3b}+\frac{c\left(\sin^{3}(bx+a)\right)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b*d*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))-1/3/b
*d*a*sin(b*x+a)^3+1/3*c*sin(b*x+a)^3)
```

**maxima** [A] time = 0.34, size = 85, normalized size = 1.67

$$\frac{12\,c\sin\left(bx+a\right)^{3}-\frac{12\,ad\sin(bx+a)^{3}}{b}-\frac{\left(3\,(bx+a)\sin(3\,bx+3\,a)-9\,(bx+a)\sin(bx+a)+\cos(3\,bx+3\,a)-9\,\cos(bx+a)\right)d}{b}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/36*(12*c*sin(b*x + a)^3 - 12*a*d*sin(b*x + a)^3/b - (3*(b*x + a)*sin(3*b*
x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*d/
b)/b
```

**mupad** [B] time = 0.15, size = 59, normalized size = 1.16

$$\frac{\frac{2\,d\cos(a+b\,x)^{3}}{9}+b\left(\frac{c\sin(a+b\,x)^{3}}{3}+\frac{d\,x\sin(a+b\,x)^{3}}{3}\right)+\frac{d\,\cos(a+b\,x)\sin(a+b\,x)^{2}}{3}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x),x)
```

```
[Out] ((2*d*cos(a + b*x)^3)/9 + b*((c*sin(a + b*x)^3)/3 + (d*x*sin(a + b*x)^3)/3)
 + (d*cos(a + b*x)*sin(a + b*x)^2)/3)/b^2
```

**sympy** [**A**] time = 0.87, size = 85, normalized size = 1.67

$$\begin{cases} \frac{c\sin^3(a+bx)}{3b} + \frac{dx\sin^3(a+bx)}{3b} + \frac{d\sin^2(a+bx)\cos(a+bx)}{3b^2} + \frac{2d\cos^3(a+bx)}{9b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sin^2(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c*sin(a + b*x)**3/(3*b) + d*x*sin(a + b*x)**3/(3*b) + d*sin(a +
b*x)**2*cos(a + b*x)/(3*b**2) + 2*d*cos(a + b*x)**3/(9*b**2), Ne(b, 0)), ((
c*x + d*x**2/2)*sin(a)**2*cos(a), True))
```

## 3.18 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=121

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3b\right.}{4d}$$

```
[Out] -1/4*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d+1/4*Ci(b*c/d+b*x)*cos(a-b*c/d)/d+
1/4*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d-1/4*Si(b*c/d+b*x)*sin(a-b*c/d)/d
```

**Rubi [A]** time = 0.27, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d]
*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c
)/d + b*x])/(4*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(
4*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \left(\frac{\cos(a+bx)}{4(c+dx)} - \frac{\cos(3a+3bx)}{4(c+dx)}\right) dx$$
$$= \frac{1}{4}\int \frac{\cos(a+bx)}{c+dx}\,dx - \frac{1}{4}\int \frac{\cos(3a+3bx)}{c+dx}\,dx$$
$$= -\left(\frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx\right) + \frac{1}{4}\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,d$$
$$= \frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{4d} - \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d} - \frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+\right.}{4d}$$

**Mathematica [A]** time = 0.32, size = 102, normalized size = 0.84

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right) - \cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right) - \sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right) + \sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Cos[3*a - (3*b*c)/d]*CosIntegr
al[(3*b*(c + d*x))/d] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Sin[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

**fricas [A]** time = 0.49, size = 153, normalized size = 1.26

$$\frac{\left(\text{Ci}\left(\frac{bdx+bc}{d}\right)+\text{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right) - \left(\text{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\text{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right) + 2\,\sin\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*((cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*cos(-
(b*c - a*d)/d) - (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d*x
+ b*c)/d))*cos(-3*(b*c - a*d)/d) + 2*sin(-3*(b*c - a*d)/d)*sin_integral(3*(
b*d*x + b*c)/d) - 2*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

**giac [C]** time = 0.54, size = 6059, normalized size = 50.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(b*x + b*c/d))
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(co
s_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d) + 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(
```

```
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b
*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2
*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integr
al(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
+ 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 + real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(b*x + b*c/d))*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(-b*x - b*c/d)
)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 4*real_part(cos_
integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d) - 4*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d) - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3
/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d)
)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(-b*x
- b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*real_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*t
an(1/2*b*c/d)^2 + 4*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*ta
n(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + real_part(cos_integral(3*b*x +
 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + real_part(cos_i
ntegral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + real
_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))
*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(-b
*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos
_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d) + 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(
1/2*a)^2*tan(3/2*b*c/d) - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d) + 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*
a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 4*sin_integral((b*d*x + b*c)/d)*
tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x +
 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 4*sin_int
egral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 2*imag_
part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) -
2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b
*c/d) + 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b
*c/d) - 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(b*x + b*c/d))*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(-b*x
 - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 4*sin_integral((b
*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(c
os_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 2*imag
_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2
- 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2
- 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/
```

```
2*b*c/d)^2 + 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2
*a)^2*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3
/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 4*sin_integral(3*
(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part
(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^
2 - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)
*tan(1/2*b*c/d)^2 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*
b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/
2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*sin_integral(3*(
b*d*x + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(
cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 - 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d)^2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/
2*a)^2 + real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 + r
eal_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 - real_part(
cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 + 4*real_part(cos
_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 4*real
_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d
) + real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2
- real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - real
_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + real_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - real_part(
cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(co
s_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - real_part(cos_integral(
-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 4*real_part(cos_integral
(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*real_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*real_part(c
os_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*re
al_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d) - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^
2 + real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + re
al_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - real_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + real_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(
cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_in
tegral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + real_part(cos_integra
l(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*real_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 4*real_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)
^2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 + real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a) - 2*i
mag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a) + 4*sin_integr
al((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a) - 2*imag_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2 + 2*imag_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*a)*tan(1/2*a)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/
2*a)*tan(1/2*a)^2 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2
*tan(3/2*b*c/d) + 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*
tan(3/2*b*c/d) - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c
/d) + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d
) - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)
 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*imag_p
```

```
art(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 - 2*imag_par
t(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 + 4*sin_integ
ral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d)^2 + 2*imag_part(cos_integr
al(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integral(-b*
x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 + 4*sin_integral((b*d*x + b*c)/d)*t
an(1/2*a)*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2
*a)^2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2
*tan(1/2*b*c/d) - 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*b*c/
d) + 2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 2
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 4*sin_
integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*imag_part(cos_int
egral(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*imag_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*sin_integral((b*d*x
+ b*c)/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x
+ 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*
tan(3/2*a)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(b*x + b*c/d))*tan(1/
2*a)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*
tan(1/2*b*c/d)^2 - 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d
)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*b*c
/d)^2 - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*
b*c/d)^2 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^
2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2 - real_part(cos_i
ntegral(b*x + b*c/d))*tan(3/2*a)^2 - real_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2 + rea
l_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2 + real_part(cos_integral
(b*x + b*c/d))*tan(1/2*a)^2 + real_part(cos_integral(-b*x - b*c/d))*tan(1/2
*a)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2 + 4*real_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) + 4*real_part(co
s_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) - real_part(cos_int
egral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c
/d))*tan(3/2*b*c/d)^2 - real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d
)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2 - 4*real_p
art(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 4*real_part(cos_
integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + real_part(cos_integral(
3*b*x + 3*b*c/d))*tan(1/2*b*c/d)^2 + real_part(cos_integral(b*x + b*c/d))*t
an(1/2*b*c/d)^2 + real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 +
real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*c/d)^2 - 2*imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a) + 2*imag_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(3/2*a) - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a) + 2*im
ag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) - 2*imag_part(cos_integral(-b
*x - b*c/d))*tan(1/2*a) + 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a) + 2*im
ag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d) - 2*imag_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d) + 4*sin_integral(3*(b*d*x + b*c)/d)
*tan(3/2*b*c/d) - 2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) + 2
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) - 4*sin_integral((b*d
*x + b*c)/d)*tan(1/2*b*c/d) + real_part(cos_integral(3*b*x + 3*b*c/d)) - re
al_part(cos_integral(b*x + b*c/d)) - real_part(cos_integral(-b*x - b*c/d))
+ real_part(cos_integral(-3*b*x - 3*b*c/d)))/(d*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + d*tan
(1/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)
^2*tan(1/2*b*c/d)^2 + d*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2
+ d*tan(1/2*a)^2 + d*tan(3/2*b*c/d)^2 + d*tan(1/2*b*c/d)^2 + d)
```

**maple [A]** time = 0.01, size = 166, normalized size = 1.37

$$\frac{\frac{b\left(\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}\right)}{4}-\frac{b\left(\frac{3\,\mathrm{Si}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\sin\left(\frac{-3da+3cb}{d}\right)}{d}+\frac{3\,\mathrm{Ci}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}\right)}{12}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] 1/b*(1/4*b*(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/
d)*cos((-a*d+b*c)/d)/d)-1/12*b*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+
b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d))
```

**maxima [C]** time = 0.45, size = 274, normalized size = 2.26

$$-\frac{b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(E_1\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_1\left(-\frac{3i\,bc+3i\,(bx+a)d-3i\,}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integra
l_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(exp_in
tegral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1, -(
3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*(-I*exp_
integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(1, -(I*
b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(I*exp_integral_e(
1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_integral_e(1, -(3*I*b*c
 + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x), x)
```

# 3.19 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=168

$$\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}}{4d^2}$$

```
[Out] -1/4*cos(b*x+a)/d/(d*x+c)+1/4*cos(3*b*x+3*a)/d/(d*x+c)-1/4*b*cos(a-b*c/d)*S
i(b*c/d+b*x)/d^2+3/4*b*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^2+3/4*b*Ci(3*b*
c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2-1/4*b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

**Rubi [A]** time = 0.30, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -Cos[a + b*x]/(4*d*(c + d*x)) + Cos[3*a + 3*b*x]/(4*d*(c + d*x)) + (3*b*Cos
Integral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d^2) - (b*CosIntegral[
(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d^2) - (b*Cos[a - (b*c)/d]*SinIntegral[
(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d +
 3*b*x])/(4*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \left(\frac{\cos(a+bx)}{4(c+dx)^2} - \frac{\cos(3a+3bx)}{4(c+dx)^2}\right) dx$$
$$= \frac{1}{4}\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx - \frac{1}{4}\int \frac{\cos(3a+3bx)}{(c+dx)^2}\,dx$$
$$= -\frac{\cos(a+bx)}{4d(c+dx)} + \frac{\cos(3a+3bx)}{4d(c+dx)} - \frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{4d} + \frac{(3b)\int \frac{\sin(3a+3bx)}{c+dx}\,dx}{4d}$$
$$= -\frac{\cos(a+bx)}{4d(c+dx)} + \frac{\cos(3a+3bx)}{4d(c+dx)} + \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d} - \frac{\left(b\cos\right.}{}$$
$$= -\frac{\cos(a+bx)}{4d(c+dx)} + \frac{\cos(3a+3bx)}{4d(c+dx)} + \frac{3b\text{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d^2} - \frac{b\text{Ci}\left(\frac{bc}{d}+b\right.}{}$$

**Mathematica [A]** time = 1.37, size = 139, normalized size = 0.83

$$-\frac{-3b\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right)+b\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)-3b\cos\left(3a-\frac{3bc}{d}\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -1/4*((d*Cos[a + b*x])/(c + d*x) - (d*Cos[3*(a + b*x)])/(c + d*x) - 3*b*Cos
Integral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + b*CosIntegral[b*(c/d + x
)]*Sin[a - (b*c)/d] + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 3*b*Cos
[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^2
```

**fricas [A]** time = 0.60, size = 236, normalized size = 1.40

$$\frac{8d\cos(bx+a)^3 + 6(bdx+bc)\cos\left(-\frac{3(bc-ad)}{d}\right)\text{Si}\left(\frac{3(bdx+bc)}{d}\right) - 2(bdx+bc)\cos\left(-\frac{bc-ad}{d}\right)\text{Si}\left(\frac{bdx+bc}{d}\right) - 8d\cos(bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*d*cos(b*x + a)^3 + 6*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*sin_integra
l(3*(b*d*x + b*c)/d) - 2*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b*
d*x + b*c)/d) - 8*d*cos(b*x + a) - ((b*d*x + b*c)*cos_integral((b*d*x + b*c
)/d) + (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) +
3*((b*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integr
al(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

[Out] Timed out

**maple [A]** time = 0.01, size = 242, normalized size = 1.44

$$\frac{\frac{b^2\left(-\frac{\cos(bx+a)}{((bx+a)d-da+cb)d}-\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}}{d}\right)}{4}-\frac{b^2\left(-\frac{3\cos(3bx+3a)}{((bx+a)d-da+cb)d}-\frac{3\left(\frac{3\,\mathrm{Si}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}-\right.}{d}\right.}{12}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/4*b^2*(-cos(b*x+a)/((b*x+a)*d-d*a+c*b)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos
((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)-1/12*b^2*(-
3*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d-3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*co
s(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d
))
```

**maxima [C]** time = 0.52, size = 302, normalized size = 1.80

$$-\frac{8192\,b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-8192\,b^2\left(E_2\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_2\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/65536*(8192*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
- 8192*b^2*(exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + ex
p_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^2*(-8192*I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
 8192*I*exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c -
a*d)/d) + b^2*(8192*I*exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*
d)/d) - 8192*I*exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*sin(-3*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}{\left(a + bx \right)} \cos{\left(a + bx \right)}}{\left(c + dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**2, x)
```

## 3.20 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3} dx$

**Optimal**. Leaf size=221

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)}{8d^3}$$

```
[Out] 9/8*b^2*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^3-1/8*b^2*Ci(b*c/d+b*x)*cos(a-
b*c/d)/d^3-1/8*cos(b*x+a)/d/(d*x+c)^2+1/8*cos(3*b*x+3*a)/d/(d*x+c)^2-9/8*b^
2*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3+1/8*b^2*Si(b*c/d+b*x)*sin(a-b*c/d)
/d^3+1/8*b*sin(b*x+a)/d^2/(d*x+c)-3/8*b*sin(3*b*x+3*a)/d^2/(d*x+c)
```

**Rubi [A]** time = 0.36, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] -Cos[a + b*x]/(8*d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (b^2
*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(8*d^3) + (b*Sin[a + b*x])/(8*d^2*
(c + d*x)) - (3*b*Sin[3*a + 3*b*x])/(8*d^2*(c + d*x)) + (b^2*Sin[a - (b*c)/
d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) - (9*b^2*Sin[3*a - (3*b*c)/d]*SinInt
egral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\cos(a+bx)}{4(c+dx)^3} - \frac{\cos(3a+3bx)}{4(c+dx)^3}\right) dx$$

$$= \frac{1}{4}\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx - \frac{1}{4}\int \frac{\cos(3a+3bx)}{(c+dx)^3}\,dx$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx}{8d} + \frac{(3b)\int \frac{\sin(3a+3bx)}{(c+dx)^2}\,dx}{8d}$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} + \frac{b\sin(a+bx)}{8d^2(c+dx)} - \frac{3b\sin(3a+3bx)}{8d^2(c+dx)} - \frac{b^2\int \frac{\mathrm{co}}{}}{8}$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} + \frac{b\sin(a+bx)}{8d^2(c+dx)} - \frac{3b\sin(3a+3bx)}{8d^2(c+dx)} + \frac{\left(9b^2\,\mathrm{c}\right.}{}$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{8d^3} + \frac{9b^2\cos\left(3a\right.}{}$$

**Mathematica [A]** time = 2.16, size = 183, normalized size = 0.83

$$\frac{b^2\left(-\cos\left(a-\frac{bc}{d}\right)\right)\mathrm{Ci}\left(b\left(\frac{c}{d}+x\right)\right) + 9b^2\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3b(c+dx)}{d}\right) + b^2\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right) - 9b^2\sin\left(3\right.}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] (-(b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) + 9*b^2*Cos[3*a - (3*b*c)
/d]*CosIntegral[(3*b*(c + d*x))/d] + (d*(-(d*Cos[a + b*x]) + b*(c + d*x)*Si
n[a + b*x]))/(c + d*x)^2 + (d*(d*Cos[3*(a + b*x)] - 3*b*(c + d*x)*Sin[3*(a
+ b*x)]))/(c + d*x)^2 + b^2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 9*b
^2*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(8*d^3)
```

**fricas [A]** time = 0.55, size = 399, normalized size = 1.81

$$\frac{8\,d^2\cos\left(bx+a\right)^3 - 8\,d^2\cos\left(bx+a\right) - 18\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right) + 2\left(b^2d^2x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(8*d^2*cos(b*x + a)^3 - 8*d^2*cos(b*x + a) - 18*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*c - a*d)/d)*sin_integral((b*d*
x + b*c)/d) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x +
b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x + b*c)
/d))*cos(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_int
egral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integr
al(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) + 8*(b*d^2*x + b*c*d - 3*(b*d
^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.02, size = 311, normalized size = 1.41

$$\frac{\frac{b^3\left(-\frac{\cos(bx+a)}{2((bx+a)d-da+cb)^2 d}-\frac{-\frac{\sin(bx+a)}{((bx+a)d-da+cb)d}+\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}}{d}}{2d}\right)}{4}-\frac{b^3\left(-\frac{3\cos(3bx+3a)}{2((bx+a)d-da+cb)^2 d}-\frac{3\left(-\frac{3\sin(3bx+3a)}{((bx+a)d-da+cb)d}\right.}{}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/4*b^3*(-1/2*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d-1/2*(-sin(b*x+a)/((b*
x+a)*d-d*a+c*b)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*
d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)-1/12*b^3*(-3/2*cos(3*b*x+3*a)/((b*x+a)
*d-d*a+c*b)^2/d-3/2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d+3*(3*Si(3*b*x+
3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*co
s(3*(-a*d+b*c)/d)/d)/d)/d))
```

**maxima [C]** time = 0.68, size = 337, normalized size = 1.52

$$-\frac{8192\,b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-8192\,b^3\left(E_3\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_3\left(-\frac{3i\,bc}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/65536*(8192*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
- 8192*b^3*(exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + ex
p_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^3*(-8192*I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
 8192*I*exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c -
a*d)/d) + b^3*(8192*I*exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*
d)/d) - 8192*I*exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**3, x)
```

## 3.21 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

**Optimal**. Leaf size=270

$$-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{24d^4}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)}{8d^4}$$

```
[Out] -1/12*cos(b*x+a)/d/(d*x+c)^3+1/24*b^2*cos(b*x+a)/d^3/(d*x+c)+1/12*cos(3*b*x
+3*a)/d/(d*x+c)^3-3/8*b^2*cos(3*b*x+3*a)/d^3/(d*x+c)+1/24*b^3*cos(a-b*c/d)*
Si(b*c/d+b*x)/d^4-9/8*b^3*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^4-9/8*b^3*Ci
(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/24*b^3*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^
4+1/24*b*sin(b*x+a)/d^2/(d*x+c)^2-1/8*b*sin(3*b*x+3*a)/d^2/(d*x+c)^2
```

**Rubi [A]** time = 0.42, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -Cos[a + b*x]/(12*d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(24*d^3*(c + d*x)) +
Cos[3*a + 3*b*x]/(12*d*(c + d*x)^3) - (3*b^2*Cos[3*a + 3*b*x])/(8*d^3*(c +
d*x)) - (9*b^3*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^4)
 + (b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(24*d^4) + (b*Sin[a +
b*x])/(24*d^2*(c + d*x)^2) - (b*Sin[3*a + 3*b*x])/(8*d^2*(c + d*x)^2) + (b^
3*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(24*d^4) - (9*b^3*Cos[3*a -
(3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\cos(a+bx)}{4(c+dx)^4} - \frac{\cos(3a+3bx)}{4(c+dx)^4}\right)dx$$

$$= \frac{1}{4}\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx - \frac{1}{4}\int \frac{\cos(3a+3bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx}{12d} + \frac{b\int \frac{\sin(3a+3bx)}{(c+dx)^3}\,dx}{4d}$$

$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} + \frac{b\sin(a+bx)}{24d^2(c+dx)^2} - \frac{b\sin(3a+3bx)}{8d^2(c+dx)^2} - \frac{b^2\int \ldots}{2\ldots}$$

$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2\cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)} + \frac{b\,\mathrm{s}\ldots}{24\ldots}$$

$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2\cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)} + \frac{b\,\mathrm{s}\ldots}{24\ldots}$$

$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2\cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)} - \frac{9b^3\ldots}{\ldots}$$

**Mathematica [A]** time = 1.68, size = 298, normalized size = 1.10

$$\frac{b^3(c+dx)^3\left(\sin\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right)-27b^3(c+dx)^3\left(\sin\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3b(c+d\ldots}{d}\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] (d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) - d*
Cos[3*b*x]*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin[3*a
]) + d*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x]
- d*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*Sin[3*a])*Sin[
3*b*x] + b^3*(c + d*x)^3*(CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + Cos[a
 - (b*c)/d]*SinIntegral[b*(c/d + x)]) - 27*b^3*(c + d*x)^3*(CosIntegral[(3*
b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + Cos[3*a - (3*b*c)/d]*SinIntegral[(3*
b*(c + d*x))/d]))/(24*d^4*(c + d*x)^3)
```

**fricas [B]** time = 0.73, size = 564, normalized size = 2.09

$$-\frac{8\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^3+54\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\cos\left(-\frac{3(\ldots}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(8*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a
)^3 + 54*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-3*(
```

```
b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x +
b*c)/d) - 8*(7*b^2*d^3*x^2 + 14*b^2*c*d^2*x + 7*b^2*c^2*d - 2*d^3)*cos(b*x
+ a) - 8*(b*d^3*x + b*c*d^2 - 3*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*sin(b*x
 + a) - ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_inte
gral((b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^
3*c^3)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) + 27*((b^3*d^3*x
^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x + b*c
)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integr
al(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c
^2*d^5*x + c^3*d^4)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 384, normalized size = 1.42

$$\frac{\frac{b^4\left(-\frac{\cos(bx+a)}{3((bx+a)d-da+cb)^3 d}-\frac{-\frac{\sin(bx+a)}{2((bx+a)d-da+cb)^2 d}+\frac{-\frac{\cos(bx+a)}{((bx+a)d-da+cb)d}-\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}}{d}}{2d}}{3d}\right)}{4}-\frac{b^4\left(-\frac{\cos(3bx+3a)}{((bx+a)d-da+cb)^3 d}-\cdots\right.}{\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x)
```

```
[Out] 1/b*(1/4*b^4*(-1/3*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^3/d-1/3*(-1/2*sin(b*x+a)/
((b*x+a)*d-d*a+c*b)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-d*a+c*b)/d-(Si(b*x+a+(-
a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)
/d)/d)/d)-1/12*b^4*(-cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)^3/d-(-3/2*sin(3*b*x
+3*a)/((b*x+a)*d-d*a+c*b)^2/d+3/2*(-3*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d-
3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-
a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d)/d)/d))
```

**maxima [C]** time = 0.87, size = 387, normalized size = 1.43

$$-\frac{8192\,b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-8192\,b^4\left(E_4\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_4\left(-\frac{3i\,bc\cdots}{}\right.\right.}{65536\left(b^3c^3d-3\,ab^2c\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/65536*(8192*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
- 8192*b^4*(exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + ex
p_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^4*(-8192*I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
 8192*I*exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c -
```

```
a*d)/d) + b^4*(8192*I*exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*
d)/d) - 8192*I*exp_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*sin(-3*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^4,x)
[Out] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^4, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}{\left(a + bx \right)} \cos{\left(a + bx \right)}}{\left(c + dx\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**4,x)
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**4, x)
```

## 3.22 $\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=271

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{4}{}\right.}{b}$$

```
[Out] -2^(-4-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b
*(d*x+c)/d)^m)-2^(-4-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b
*c/d))/((I*b*(d*x+c)/d)^m)+exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b*(d
*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)+(d*x+c)^m*GAMMA(1+m,4*I*b*(d*x+
c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

**Rubi [A]** time = 0.33, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4406, 3308, 2181}

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gam}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-4 - m)*(c + d*x)^m*Gamma[
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m) + (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c + d*
x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 +
 m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*((I*b*
(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^m \sin(2a+2bx) - \frac{1}{8}(c+dx)^m \sin(4a+4bx)\right) dx$$
$$= -\left(\frac{1}{8}\int (c+dx)^m \sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^m \sin(2a+2bx)\,d$$
$$= -\left(\frac{1}{16}i\int e^{-i(4a+4bx)}(c+dx)^m\,dx\right) + \frac{1}{16}i\int e^{i(4a+4bx)}(c+dx)^m\,dx + \frac{1}{8}$$
$$= -\frac{2^{-4-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-4-m}}{}$$

**Mathematica [A]** time = 0.29, size = 246, normalized size = 0.91

$$\frac{4^{-m-3}e^{-\frac{4i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-2^{m+2}e^{2i\left(a+\frac{3bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right)-2^{m+2}e^{2i\left(3a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right.\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (4^(-3 - m)*(c + d*x)^m*(-(2^(2 + m)*E^((2*I)*(3*a + (b*c)/d))*((I*b*(c + d
*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]) - 2^(2 + m)*E^((2*I)*(a + (
3*b*c)/d))*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d] + E
^((8*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d] + E^(
((8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d]
))/(b*E^(((4*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

**fricas [A]** time = 0.50, size = 184, normalized size = 0.68

$$\frac{e^{\left(-\frac{dm\log\left(\frac{4ib}{d}\right)-4ibc+4iad}{d}\right)}\Gamma\left(m+1,\frac{4ibdx+4ibc}{d}\right)-4e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2ibc+2iad}{d}\right)}\Gamma\left(m+1,\frac{2ibdx+2ibc}{d}\right)-4e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(e^(-(d*m*log(4*I*b/d) - 4*I*b*c + 4*I*a*d)/d)*gamma(m + 1, (4*I*b*d*x
 + 4*I*b*c)/d) - 4*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m +
1, (2*I*b*d*x + 2*I*b*c)/d) - 4*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)
/d)*gamma(m + 1, (-2*I*b*d*x - 2*I*b*c)/d) + e^(-(d*m*log(-4*I*b/d) + 4*I*b
*c - 4*I*a*d)/d)*gamma(m + 1, (-4*I*b*d*x - 4*I*b*c)/d))/b
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^3, x)
```

**maple [F]** time = 0.24, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\left(\sin^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^3, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^m, x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

## 3.23 $\int (c+dx)^4 \cos(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=260

$$\frac{3d^4\sin^4(a+bx)}{128b^5}+\frac{45d^4\sin^2(a+bx)}{128b^5}-\frac{3d^3(c+dx)\sin^3(a+bx)\cos(a+bx)}{32b^4}-\frac{45d^3(c+dx)\sin(a+bx)\cos(a+b}{64b^4}$$

```
[Out] 45/64*c*d^3*x/b^3+45/128*d^4*x^2/b^3-3/32*(d*x+c)^4/b-45/64*d^3*(d*x+c)*cos
(b*x+a)*sin(b*x+a)/b^4+3/8*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+45/128*d^4
*sin(b*x+a)^2/b^5-9/16*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3-3/32*d^3*(d*x+c)*cos(
b*x+a)*sin(b*x+a)^3/b^4+1/4*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3/b^2+3/128*d
^4*sin(b*x+a)^4/b^5-3/16*d^2*(d*x+c)^2*sin(b*x+a)^4/b^3+1/4*(d*x+c)^4*sin(b
*x+a)^4/b
```

**Rubi** [A] time = 0.24, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4404, 3311, 32, 3310}

$$-\frac{3d^2(c+dx)^2\sin^4(a+bx)}{16b^3}-\frac{9d^2(c+dx)^2\sin^2(a+bx)}{16b^3}-\frac{3d^3(c+dx)\sin^3(a+bx)\cos(a+bx)}{32b^4}-\frac{45d^3(c+dx)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (45*c*d^3*x)/(64*b^3) + (45*d^4*x^2)/(128*b^3) - (3*(c + d*x)^4)/(32*b) - (
45*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(64*b^4) + (3*d*(c + d*x)^3*Cos
[a + b*x]*Sin[a + b*x])/(8*b^2) + (45*d^4*Sin[a + b*x]^2)/(128*b^5) - (9*d^
2*(c + d*x)^2*Sin[a + b*x]^2)/(16*b^3) - (3*d^3*(c + d*x)*Cos[a + b*x]*Sin[
a + b*x]^3)/(32*b^4) + (d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3)/(4*b^2)
+ (3*d^4*Sin[a + b*x]^4)/(128*b^5) - (3*d^2*(c + d*x)^2*Sin[a + b*x]^4)/(16
*b^3) + ((c + d*x)^4*Sin[a + b*x]^4)/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
```

```
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^4 \cos(a+bx)\sin^3(a+bx)\,dx = \frac{(c+dx)^4\sin^4(a+bx)}{4b} - \frac{d\int (c+dx)^3\sin^4(a+bx)\,dx}{b}$$
$$= \frac{d(c+dx)^3\cos(a+bx)\sin^3(a+bx)}{4b^2} - \frac{3d^2(c+dx)^2\sin^4(a+bx)}{16b^3} + \frac{(c+$$
$$= \frac{3d(c+dx)^3\cos(a+bx)\sin(a+bx)}{8b^2} - \frac{9d^2(c+dx)^2\sin^2(a+bx)}{16b^3} - \frac{3d^3($$
$$= -\frac{3(c+dx)^4}{32b} - \frac{45d^3(c+dx)\cos(a+bx)\sin(a+bx)}{64b^4} + \frac{3d(c+dx)^3\cos($$
$$= \frac{45cd^3x}{64b^3} + \frac{45d^4x^2}{128b^3} - \frac{3(c+dx)^4}{32b} - \frac{45d^3(c+dx)\cos(a+bx)\sin(a+bx)}{64b^4}$$

**Mathematica [A]** time = 1.70, size = 158, normalized size = 0.61

$$\frac{-8bd(c+dx)\sin(2(a+bx))\left(\cos(2(a+bx))\left(8b^2(c+dx)^2-3d^2\right)-16\left(2b^2(c+dx)^2-3d^2\right)\right)-64\cos(2(a+bx))\left(}{1024b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-64*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] +
 (3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Cos[4*(a + b*x)] - 8
*b*d*(c + d*x)*(-16*(-3*d^2 + 2*b^2*(c + d*x)^2) + (-3*d^2 + 8*b^2*(c + d*x
)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)])/(1024*b^5)
```

**fricas [A]** time = 0.59, size = 434, normalized size = 1.67

$$\frac{20\,b^4d^4x^4 + 80\,b^4cd^3x^3 + \left(32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 32\,b^4c^4 - 24\,b^2c^2d^2 + 3\,d^4 + 24\left(8\,b^4c^2d^2 - b^2d^4\right)x^2 + 16\left(8\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(20*b^4*d^4*x^4 + 80*b^4*c*d^3*x^3 + (32*b^4*d^4*x^4 + 128*b^4*c*d^3*
x^3 + 32*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^
2 + 16*(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^4 + 3*(40*b^4*c^2*d^2 -
17*b^2*d^4)*x^2 - (64*b^4*d^4*x^4 + 256*b^4*c*d^3*x^3 + 64*b^4*c^4 - 120*b^
2*c^2*d^2 + 51*d^4 + 24*(16*b^4*c^2*d^2 - 5*b^2*d^4)*x^2 + 16*(16*b^4*c^3*d
 - 15*b^2*c*d^3)*x)*cos(b*x + a)^2 + 2*(40*b^4*c^3*d - 51*b^2*c*d^3)*x - 2*
(2*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c
^2*d^2 - b*d^4)*x)*cos(b*x + a)^3 - (40*b^3*d^4*x^3 + 120*b^3*c*d^3*x^2 + 4
0*b^3*c^3*d - 51*b*c*d^3 + 3*(40*b^3*c^2*d^2 - 17*b*d^4)*x)*cos(b*x + a))*s
in(b*x + a))/b^5
```

**giac [A]** time = 2.89, size = 361, normalized size = 1.39

$$\frac{\left(32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 192\,b^4c^2d^2x^2 + 128\,b^4c^3dx + 32\,b^4c^4 - 24\,b^2d^4x^2 - 48\,b^2cd^3x - 24\,b^2c^2d^2 + 3\,d^4\right)\cos}{1024\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/1024*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4*
c^3*d*x + 32*b^4*c^4 - 24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 + 3
*d^4)*cos(4*b*x + 4*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4
*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x -
 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 - 1/256*(8*b^3*d^4*x^3 + 24*b^
3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(4
*b*x + 4*a)/b^5 + 1/8*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x +
2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

**maple [B]** time = 0.12, size = 1143, normalized size = 4.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/4*(b*x+a)^4*sin(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/
2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^4+3/8*(b*
x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+27/128*(
b*x+a)^2+3/128*sin(b*x+a)^4+45/128*sin(b*x+a)^2+9/16*(b*x+a)^2*cos(b*x+a)^2
-9/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+9/32*(b*x+a)^4)-4/b^
4*a*d^4*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*s
in(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)*sin(b*x+a)^4-3/128*(sin(b
*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-27/256*b*x-27/256*a+9/32*(b*x+a)*cos(b*x
+a)^2-9/64*cos(b*x+a)*sin(b*x+a)+3/16*(b*x+a)^3)+4/b^3*c*d^3*(1/4*(b*x+a)^3
*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+
3/8*b*x+3/8*a)-3/32*(b*x+a)*sin(b*x+a)^4-3/128*(sin(b*x+a)^3+3/2*sin(b*x+a)
)*cos(b*x+a)-27/256*b*x-27/256*a+9/32*(b*x+a)*cos(b*x+a)^2-9/64*cos(b*x+a)*
sin(b*x+a)+3/16*(b*x+a)^3)+6/b^4*a^2*d^4*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b
*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b
*x+a)^2-1/32*sin(b*x+a)^4-3/32*sin(b*x+a)^2)-12/b^3*a*c*d^3*(1/4*(b*x+a)^2*
sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8
*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-3/32*sin(b*x+a)^2)+6/b^2*c^2*d
^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+
a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-3/32*sin(b*x
+a)^2)-4/b^4*a^3*d^4*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b
*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+12/b^3*a^2*c*d^3*(1/4*(b*x+a)*sin(b*x+a)
^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)-12/b^2*a*
c^2*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a)-3/32*b*x-3/32*a)+4/b*c^3*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^
3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+1/4/b^4*a^4*d^4*sin(b*x+a)^4-
1/b^3*a^3*c*d^3*sin(b*x+a)^4+3/2/b^2*a^2*c^2*d^2*sin(b*x+a)^4-1/b*a*c^3*d*s
in(b*x+a)^4+1/4*c^4*sin(b*x+a)^4)
```

**maxima [B]** time = 0.42, size = 967, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(256*c^4*sin(b*x + a)^4 - 1024*a*c^3*d*sin(b*x + a)^4/b + 1536*a^2*c
^2*d^2*sin(b*x + a)^4/b^2 - 1024*a^3*c*d^3*sin(b*x + a)^4/b^3 + 256*a^4*d^4
*sin(b*x + a)^4/b^4 + 32*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2
*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*c^3*d/b - 96*(4*(b*x +
 a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8
*sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 96*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b
*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a^2*c*d^3
/b^3 - 32*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - s
in(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a^3*d^4/b^4 + 24*((8*(b*x + a)^2 - 1)
```

```
*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*s
in(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 48*((8*(b*x
+ a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*
(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 2
4*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x
+ 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a^2*
d^4/b^4 + 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x
+ a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x +
4*a) + 96*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 4*(4*(8*(b*x +
a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos
(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 96*(2*(b*x + a)^2
- 1)*sin(2*b*x + 2*a))*a*d^4/b^4 + ((32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*c
os(4*b*x + 4*a) - 64*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) -
 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a) + 128*(2*(b*x + a)^3 - 3*
b*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b
```

**mupad [B]** time = 1.94, size = 576, normalized size = 2.22

$$-\frac{192\,d^4\,\cos\left(2\,a+2\,b\,x\right)-3\,d^4\,\cos\left(4\,a+4\,b\,x\right)+128\,b^4\,c^4\,\cos\left(2\,a+2\,b\,x\right)-32\,b^4\,c^4\,\cos\left(4\,a+4\,b\,x\right)-256\,b^3\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] -(192*d^4*cos(2*a + 2*b*x) - 3*d^4*cos(4*a + 4*b*x) + 128*b^4*c^4*cos(2*a +
 2*b*x) - 32*b^4*c^4*cos(4*a + 4*b*x) - 256*b^3*c^3*d*sin(2*a + 2*b*x) + 32
*b^3*c^3*d*sin(4*a + 4*b*x) - 384*b^2*c^2*d^2*cos(2*a + 2*b*x) + 24*b^2*c^2
*d^2*cos(4*a + 4*b*x) - 384*b^2*d^4*x^2*cos(2*a + 2*b*x) + 24*b^2*d^4*x^2*c
os(4*a + 4*b*x) + 128*b^4*d^4*x^4*cos(2*a + 2*b*x) - 32*b^4*d^4*x^4*cos(4*a
 + 4*b*x) - 256*b^3*d^4*x^3*sin(2*a + 2*b*x) + 32*b^3*d^4*x^3*sin(4*a + 4*b
*x) + 384*b*c*d^3*sin(2*a + 2*b*x) - 12*b*c*d^3*sin(4*a + 4*b*x) + 384*b*d^
4*x*sin(2*a + 2*b*x) - 12*b*d^4*x*sin(4*a + 4*b*x) + 768*b^4*c^2*d^2*x^2*co
s(2*a + 2*b*x) - 192*b^4*c^2*d^2*x^2*cos(4*a + 4*b*x) - 768*b^2*c*d^3*x*cos
(2*a + 2*b*x) + 512*b^4*c^3*d*x*cos(2*a + 2*b*x) + 48*b^2*c*d^3*x*cos(4*a +
 4*b*x) - 128*b^4*c^3*d*x*cos(4*a + 4*b*x) + 512*b^4*c*d^3*x^3*cos(2*a + 2*
b*x) - 128*b^4*c*d^3*x^3*cos(4*a + 4*b*x) - 768*b^3*c^2*d^2*x*sin(2*a + 2*b
*x) - 768*b^3*c*d^3*x^2*sin(2*a + 2*b*x) + 96*b^3*c^2*d^2*x*sin(4*a + 4*b*x
) + 96*b^3*c*d^3*x^2*sin(4*a + 4*b*x))/(1024*b^5)
```

**sympy [A]** time = 13.31, size = 935, normalized size = 3.60

$$\begin{cases} \frac{c^4\sin^4(a+bx)}{4b}+\frac{5c^3dx\sin^4(a+bx)}{8b}-\frac{3c^3dx\sin^2(a+bx)\cos^2(a+bx)}{4b}-\frac{3c^3dx\cos^4(a+bx)}{8b}+\frac{15c^2d^2x^2\sin^4(a+bx)}{16b}-\frac{9c^2d^2x^2\sin^2(a+bx)\cos^2(a}{8b} \\ \left(c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}\right)\sin^3(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)**4/(4*b) + 5*c**3*d*x*sin(a + b*x)**4/(8*b) -
3*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 3*c**3*d*x*cos(a + b*x)*
*4/(8*b) + 15*c**2*d**2*x**2*sin(a + b*x)**4/(16*b) - 9*c**2*d**2*x**2*sin(
a + b*x)**2*cos(a + b*x)**2/(8*b) - 9*c**2*d**2*x**2*cos(a + b*x)**4/(16*b)
 + 5*c*d**3*x**3*sin(a + b*x)**4/(8*b) - 3*c*d**3*x**3*sin(a + b*x)**2*cos(
a + b*x)**2/(4*b) - 3*c*d**3*x**3*cos(a + b*x)**4/(8*b) + 5*d**4*x**4*sin(a
 + b*x)**4/(32*b) - 3*d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*
d**4*x**4*cos(a + b*x)**4/(32*b) + 5*c**3*d*sin(a + b*x)**3*cos(a + b*x)/(8
*b**2) + 3*c**3*d*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 15*c**2*d**2*x*si
n(a + b*x)**3*cos(a + b*x)/(8*b**2) + 9*c**2*d**2*x*sin(a + b*x)*cos(a + b*
```

```
x)**3/(8*b**2) + 15*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 9*c
*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 5*d**4*x**3*sin(a + b*x)
**3*cos(a + b*x)/(8*b**2) + 3*d**4*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b**
2) - 15*c**2*d**2*sin(a + b*x)**4/(32*b**3) + 9*c**2*d**2*cos(a + b*x)**4/(
32*b**3) - 51*c*d**3*x*sin(a + b*x)**4/(64*b**3) + 9*c*d**3*x*sin(a + b*x)*
*2*cos(a + b*x)**2/(32*b**3) + 45*c*d**3*x*cos(a + b*x)**4/(64*b**3) - 51*d
**4*x**2*sin(a + b*x)**4/(128*b**3) + 9*d**4*x**2*sin(a + b*x)**2*cos(a + b
*x)**2/(64*b**3) + 45*d**4*x**2*cos(a + b*x)**4/(128*b**3) - 51*c*d**3*sin(
a + b*x)**3*cos(a + b*x)/(64*b**4) - 45*c*d**3*sin(a + b*x)*cos(a + b*x)**3
/(64*b**4) - 51*d**4*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 45*d**4*x*s
in(a + b*x)*cos(a + b*x)**3/(64*b**4) + 51*d**4*sin(a + b*x)**4/(256*b**5)
- 45*d**4*cos(a + b*x)**4/(256*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 +
 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**3*cos(a), True))
```

# 3.24 $\int (c+dx)^3 \cos(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=196

$$-\frac{3d^3\sin^3(a+bx)\cos(a+bx)}{128b^4}-\frac{45d^3\sin(a+bx)\cos(a+bx)}{256b^4}-\frac{3d^2(c+dx)\sin^4(a+bx)}{32b^3}-\frac{9d^2(c+dx)\sin^2(a+bx)}{32b^3}$$

```
[Out] 45/256*d^3*x/b^3-3/32*(d*x+c)^3/b-45/256*d^3*cos(b*x+a)*sin(b*x+a)/b^4+9/32
*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-9/32*d^2*(d*x+c)*sin(b*x+a)^2/b^3-3/
128*d^3*cos(b*x+a)*sin(b*x+a)^3/b^4+3/16*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)^
3/b^2-3/32*d^2*(d*x+c)*sin(b*x+a)^4/b^3+1/4*(d*x+c)^3*sin(b*x+a)^4/b
```

**Rubi [A]** time = 0.17, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = $\{4404, 3311, 32, 2635, 8\}$

$$-\frac{3d^2(c+dx)\sin^4(a+bx)}{32b^3}-\frac{9d^2(c+dx)\sin^2(a+bx)}{32b^3}+\frac{3d(c+dx)^2\sin^3(a+bx)\cos(a+bx)}{16b^2}+\frac{9d(c+dx)^2\sin(a+b}{32b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (45*d^3*x)/(256*b^3) - (3*(c + d*x)^3)/(32*b) - (45*d^3*Cos[a + b*x]*Sin[a
+ b*x])/(256*b^4) + (9*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) -
(9*d^2*(c + d*x)*Sin[a + b*x]^2)/(32*b^3) - (3*d^3*Cos[a + b*x]*Sin[a + b*x
]^3)/(128*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3)/(16*b^2) - (
3*d^2*(c + d*x)*Sin[a + b*x]^4)/(32*b^3) + ((c + d*x)^3*Sin[a + b*x]^4)/(4*
b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
```

```
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^3\cos(a+bx)\sin^3(a+bx)\,dx = \frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{(3d)\int (c+dx)^2\sin^4(a+bx)\,dx}{4b}$$
$$= \frac{3d(c+dx)^2\cos(a+bx)\sin^3(a+bx)}{16b^2} - \frac{3d^2(c+dx)\sin^4(a+bx)}{32b^3} + \frac{(}{}$$
$$= \frac{9d(c+dx)^2\cos(a+bx)\sin(a+bx)}{32b^2} - \frac{9d^2(c+dx)\sin^2(a+bx)}{32b^3} - \frac{3a}{}$$
$$= -\frac{3(c+dx)^3}{32b} - \frac{45d^3\cos(a+bx)\sin(a+bx)}{256b^4} + \frac{9d(c+dx)^2\cos(a+b}{32b^2}$$
$$= \frac{45d^3x}{256b^3} - \frac{3(c+dx)^3}{32b} - \frac{45d^3\cos(a+bx)\sin(a+bx)}{256b^4} + \frac{9d(c+dx)^2 c}{}$$

**Mathematica [A]** time = 0.89, size = 135, normalized size = 0.69

$$\frac{-64b(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)+4b(c+dx)\cos(4(a+bx))\left(8b^2(c+dx)^2-3d^2\right)-6d\sin(2(}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-64*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 4*b*(c + d
*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 6*d*(-16*(-d^2 + 2*b^2*
(c + d*x)^2) + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)
])/(1024*b^4)
```

**fricas [A]** time = 0.49, size = 283, normalized size = 1.44

$$\frac{40\,b^3d^3x^3+120\,b^3cd^2x^2+8\left(8\,b^3d^3x^3+24\,b^3cd^2x^2+8\,b^3c^3-3\,bcd^2+3\left(8\,b^3c^2d-bd^3\right)x\right)\cos\left(bx+a\right)^4-8\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(40*b^3*d^3*x^3 + 120*b^3*c*d^2*x^2 + 8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2
*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^4 -
8*(16*b^3*d^3*x^3 + 48*b^3*c*d^2*x^2 + 16*b^3*c^3 - 15*b*c*d^2 + 3*(16*b^3*
c^2*d - 5*b*d^3)*x)*cos(b*x + a)^2 + 3*(40*b^3*c^2*d - 17*b*d^3)*x - 3*(2*(
8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 - (40*b^
2*d^3*x^2 + 80*b^2*c*d^2*x + 40*b^2*c^2*d - 17*d^3)*cos(b*x + a))*sin(b*x +
 a))/b^4
```

**giac [A]** time = 3.97, size = 241, normalized size = 1.23

$$\frac{\left(8\,b^3d^3x^3+24\,b^3cd^2x^2+24\,b^3c^2dx+8\,b^3c^3-3\,bd^3x-3\,bcd^2\right)\cos\left(4\,bx+4\,a\right)}{256\,b^4}-\frac{\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b*
d^3*x - 3*b*c*d^2)*cos(4*b*x + 4*a)/b^4 - 1/16*(2*b^3*d^3*x^3 + 6*b^3*c*d^2
```

```
*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/
b^4 - 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*sin(4*b*x
 + 4*a)/b^4 + 3/32*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin(
2*b*x + 2*a)/b^4
```

**maple [B]** time = 0.02, size = 594, normalized size = 3.03

$$\frac{d^3\left(\frac{(bx+a)^3\left(\sin^4(bx+a)\right)}{4}-\frac{3(bx+a)^2\left(-\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)}{4}-\frac{3(bx+a)\left(\sin^4(bx+a)\right)}{32}-\frac{3\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{128}-\frac{27bx}{256}-\frac{27a}{256}+\frac{9(bx+a)(c\cdots}{\cdots}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^
3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)*sin(b*x+a)^4-3/128
*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-27/256*b*x-27/256*a+9/32*(b*x+a)*
cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x+a)+3/16*(b*x+a)^3)-3/b^3*a*d^3*(1/4*(b
*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-3/32*sin(b*x+a)^2)+3/b
^2*c*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*si
n(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-3/32*s
in(b*x+a)^2)+3/b^3*a^2*d^3*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2
*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)-6/b^2*a*c*d^2*(1/4*(b*x+a)*sin(b*x
+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+3/b*c^
2*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)
-3/32*b*x-3/32*a)-1/4/b^3*a^3*d^3*sin(b*x+a)^4+3/4/b^2*a^2*c*d^2*sin(b*x+a)
^4-3/4/b*a*c^2*d*sin(b*x+a)^4+1/4*c^3*sin(b*x+a)^4)
```

**maxima [B]** time = 0.38, size = 549, normalized size = 2.80

$$256\,c^3\sin\left(bx+a\right)^4-\frac{768\,ac^2d\sin(bx+a)^4}{b}+\frac{768\,a^2cd^2\sin(bx+a)^4}{b^2}-\frac{256\,a^3d^3\sin(bx+a)^4}{b^3}+\frac{24\,(4\,(bx+a)\cos(4\,bx+4\,a)-16\,(bx+a)\cos(2\,bx+\cdots}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(256*c^3*sin(b*x + a)^4 - 768*a*c^2*d*sin(b*x + a)^4/b + 768*a^2*c*d
^2*sin(b*x + a)^4/b^2 - 256*a^3*d^3*sin(b*x + a)^4/b^3 + 24*(4*(b*x + a)*co
s(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2
*b*x + 2*a))*c^2*d/b - 48*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(
2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a*c*d^2/b^2 + 24*(4*(
b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a
) + 8*sin(2*b*x + 2*a))*a^2*d^3/b^3 + 12*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4
*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a
) + 32*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 12*((8*(b*x + a)^2 - 1)*cos(
4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*
b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (4*(8*(b*x + a)^3 -
 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x
 + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 96*(2*(b*x + a)^2 - 1)*s
in(2*b*x + 2*a))*d^3/b^3)/b
```

**mupad [B]** time = 1.71, size = 366, normalized size = 1.87

$$-\frac{24\,d^3\,\sin\left(2\,a+2\,b\,x\right)-\frac{3\,d^3\,\sin(4\,a+4\,b\,x)}{4}+32\,b^3\,c^3\,\cos\left(2\,a+2\,b\,x\right)-8\,b^3\,c^3\,\cos\left(4\,a+4\,b\,x\right)-48\,b^2\,c^2\,d\,\sin\left(2\,a\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] -(24*d^3*sin(2*a + 2*b*x) - (3*d^3*sin(4*a + 4*b*x))/4 + 32*b^3*c^3*cos(2*a
 + 2*b*x) - 8*b^3*c^3*cos(4*a + 4*b*x) - 48*b^2*c^2*d*sin(2*a + 2*b*x) + 6*
b^2*c^2*d*sin(4*a + 4*b*x) + 32*b^3*d^3*x^3*cos(2*a + 2*b*x) - 8*b^3*d^3*x^
3*cos(4*a + 4*b*x) - 48*b^2*d^3*x^2*sin(2*a + 2*b*x) + 6*b^2*d^3*x^2*sin(4*
a + 4*b*x) - 48*b*c*d^2*cos(2*a + 2*b*x) + 3*b*c*d^2*cos(4*a + 4*b*x) - 48*
b*d^3*x*cos(2*a + 2*b*x) + 3*b*d^3*x*cos(4*a + 4*b*x) + 96*b^3*c^2*d*x*cos(
2*a + 2*b*x) - 24*b^3*c^2*d*x*cos(4*a + 4*b*x) - 96*b^2*c*d^2*x*sin(2*a + 2
*b*x) + 12*b^2*c*d^2*x*sin(4*a + 4*b*x) + 96*b^3*c*d^2*x^2*cos(2*a + 2*b*x)
 - 24*b^3*c*d^2*x^2*cos(4*a + 4*b*x))/(256*b^4)
```

**sympy [A]** time = 7.28, size = 602, normalized size = 3.07

$$\begin{cases} \frac{c^3\sin^4(a+bx)}{4b} + \frac{15c^2dx\sin^4(a+bx)}{32b} - \frac{9c^2dx\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{9c^2dx\cos^4(a+bx)}{32b} + \frac{15cd^2x^2\sin^4(a+bx)}{32b} - \frac{9cd^2x^2\sin^2(a+bx)\cos^2 \cdots}{16b} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin^3(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)**4/(4*b) + 15*c**2*d*x*sin(a + b*x)**4/(32*b)
- 9*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 9*c**2*d*x*cos(a + b*
x)**4/(32*b) + 15*c*d**2*x**2*sin(a + b*x)**4/(32*b) - 9*c*d**2*x**2*sin(a
+ b*x)**2*cos(a + b*x)**2/(16*b) - 9*c*d**2*x**2*cos(a + b*x)**4/(32*b) + 5
*d**3*x**3*sin(a + b*x)**4/(32*b) - 3*d**3*x**3*sin(a + b*x)**2*cos(a + b*x
)**2/(16*b) - 3*d**3*x**3*cos(a + b*x)**4/(32*b) + 15*c**2*d*sin(a + b*x)**
3*cos(a + b*x)/(32*b**2) + 9*c**2*d*sin(a + b*x)*cos(a + b*x)**3/(32*b**2)
+ 15*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 9*c*d**2*x*sin(a + b
*x)*cos(a + b*x)**3/(16*b**2) + 15*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(
32*b**2) + 9*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) - 15*c*d**2*s
in(a + b*x)**4/(64*b**3) + 9*c*d**2*cos(a + b*x)**4/(64*b**3) - 51*d**3*x*s
in(a + b*x)**4/(256*b**3) + 9*d**3*x*sin(a + b*x)**2*cos(a + b*x)**2/(128*b
**3) + 45*d**3*x*cos(a + b*x)**4/(256*b**3) - 51*d**3*sin(a + b*x)**3*cos(a
 + b*x)/(256*b**4) - 45*d**3*sin(a + b*x)*cos(a + b*x)**3/(256*b**4), Ne(b,
 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**3*cos
(a), True))
```

## 3.25 $\int (c+dx)^2 \cos(a+bx) \sin^3(a+bx) \, dx$

**Optimal**. Leaf size=134

$$-\frac{d^2\sin^4(a+bx)}{32b^3}-\frac{3d^2\sin^2(a+bx)}{32b^3}+\frac{d(c+dx)\sin^3(a+bx)\cos(a+bx)}{8b^2}+\frac{3d(c+dx)\sin(a+bx)\cos(a+bx)}{16b^2}+\frac{(c+$$

```
[Out] -3/16*c*d*x/b-3/32*d^2*x^2/b+3/16*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-3/32*
d^2*sin(b*x+a)^2/b^3+1/8*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)^3/b^2-1/32*d^2*sin
(b*x+a)^4/b^3+1/4*(d*x+c)^2*sin(b*x+a)^4/b
```

**Rubi [A]** time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4404, 3310}

$$\frac{d(c+dx)\sin^3(a+bx)\cos(a+bx)}{8b^2}+\frac{3d(c+dx)\sin(a+bx)\cos(a+bx)}{16b^2}-\frac{d^2\sin^4(a+bx)}{32b^3}-\frac{3d^2\sin^2(a+bx)}{32b^3}+\frac{(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-3*c*d*x)/(16*b) - (3*d^2*x^2)/(32*b) + (3*d*(c + d*x)*Cos[a + b*x]*Sin[a
+ b*x])/(16*b^2) - (3*d^2*Sin[a + b*x]^2)/(32*b^3) + (d*(c + d*x)*Cos[a + b
*x]*Sin[a + b*x]^3)/(8*b^2) - (d^2*Sin[a + b*x]^4)/(32*b^3) + ((c + d*x)^2*
Sin[a + b*x]^4)/(4*b)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\cos(a+bx)\sin^3(a+bx)\,dx &= \frac{(c+dx)^2\sin^4(a+bx)}{4b}-\frac{d\int (c+dx)\sin^4(a+bx)\,dx}{2b}\\ &= \frac{d(c+dx)\cos(a+bx)\sin^3(a+bx)}{8b^2}-\frac{d^2\sin^4(a+bx)}{32b^3}+\frac{(c+dx)^2\sin^4(a\\ &= \frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{16b^2}-\frac{3d^2\sin^2(a+bx)}{32b^3}+\frac{d(c+dx)\cos(\\ &= -\frac{3cdx}{16b}-\frac{3d^2x^2}{32b}+\frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{16b^2}-\frac{3d^2\sin^2(a+b\end{aligned}$$

**Mathematica [A]** time = 0.49, size = 91, normalized size = 0.68

$$\frac{-16\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+\cos(4(a+bx))\left(8b^2(c+dx)^2-d^2\right)-4bd(c+dx)(\sin(4(a+bx))-8\sin(}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-16*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (-d^2 + 8*b^2*(c + d*x)^
2)*Cos[4*(a + b*x)] - 4*b*d*(c + d*x)*(-8*Sin[2*(a + b*x)] + Sin[4*(a + b*x
)]))/(256*b^3)
```

**fricas** [A] time = 0.46, size = 159, normalized size = 1.19

$$\frac{5\,b^2d^2x^2 + 10\,b^2cdx + \left(8\,b^2d^2x^2 + 16\,b^2cdx + 8\,b^2c^2 - d^2\right)\cos\left(bx + a\right)^4 - \left(16\,b^2d^2x^2 + 32\,b^2cdx + 16\,b^2c^2 - 5\,\cdots}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(5*b^2*d^2*x^2 + 10*b^2*c*d*x + (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*
c^2 - d^2)*cos(b*x + a)^4 - (16*b^2*d^2*x^2 + 32*b^2*c*d*x + 16*b^2*c^2 - 5
*d^2)*cos(b*x + a)^2 - 2*(2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 5*(b*d^2*x +
 b*c*d)*cos(b*x + a))*sin(b*x + a))/b^3
```

**giac** [A] time = 0.20, size = 145, normalized size = 1.08

$$\frac{\left(8\,b^2d^2x^2 + 16\,b^2cdx + 8\,b^2c^2 - d^2\right)\cos\left(4\,bx + 4\,a\right)}{256\,b^3} - \frac{\left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(2\,bx + 2\,a\right)}{16\,b^3} - \frac{\left(bd^2x \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(4*b*x + 4*a)/b^3
 - 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b^
3 - 1/64*(b*d^2*x + b*c*d)*sin(4*b*x + 4*a)/b^3 + 1/8*(b*d^2*x + b*c*d)*sin
(2*b*x + 2*a)/b^3
```

**maple** [B] time = 0.02, size = 260, normalized size = 1.94

$$\frac{\frac{d^2\left(\frac{(bx+a)^2\left(\sin^4(bx+a)\right)}{4} - \frac{(bx+a)\left(-\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4} + \frac{3bx}{8} + \frac{3a}{8}\right)}{2} + \frac{3(bx+a)^2}{32} - \frac{\left(\sin^4(bx+a)\right)}{32} - \frac{3\left(\sin^2(bx+a)\right)}{32}\right)}{b^2} - \frac{2a\,d^2\left(\frac{(bx+a)\left(\sin^4(bx+a)\right)}{4} + \frac{\left(\sin^3(\cdots\right.}{}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+
3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-
3/32*sin(b*x+a)^2)-2/b^2*a*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3
+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+2/b*c*d*(1/4*(b*x+a)*sin(b*x+a
)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+1/4/b^2*
a^2*d^2*sin(b*x+a)^4-1/2/b*a*c*d*sin(b*x+a)^4+1/4*c^2*sin(b*x+a)^4)
```

**maxima** [B] time = 0.36, size = 263, normalized size = 1.96

$$\frac{64\,c^2\sin\left(bx + a\right)^4 - \frac{128\,acd\sin(bx+a)^4}{b} + \frac{64\,a^2d^2\sin(bx+a)^4}{b^2} + \frac{4\,(4\,(bx+a)\cos(4\,bx+4\,a) - 16\,(bx+a)\cos(2\,bx+2\,a) - \sin(4\,bx+4\,a) + 8\,\sin(\cdots}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/256*(64*c^2*sin(b*x + a)^4 - 128*a*c*d*sin(b*x + a)^4/b + 64*a^2*d^2*sin(
b*x + a)^4/b^2 + 4*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x +
 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*c*d/b - 4*(4*(b*x + a)*cos(4
*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*
x + 2*a))*a*d^2/b^2 + ((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x +
a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*si
n(2*b*x + 2*a))*d^2/b^2)/b
```

**mupad [B]** time = 1.25, size = 202, normalized size = 1.51

$$\frac{8\,d^2\,\cos\left(2\,a+2\,b\,x\right)-\frac{d^2\,\cos\left(4\,a+4\,b\,x\right)}{2}-16\,b^2\,c^2\,\cos\left(2\,a+2\,b\,x\right)+4\,b^2\,c^2\,\cos\left(4\,a+4\,b\,x\right)+16\,b\,c\,d\,\sin\left(2\,a+2\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] (8*d^2*cos(2*a + 2*b*x) - (d^2*cos(4*a + 4*b*x))/2 - 16*b^2*c^2*cos(2*a + 2
*b*x) + 4*b^2*c^2*cos(4*a + 4*b*x) + 16*b*c*d*sin(2*a + 2*b*x) - 2*b*c*d*si
n(4*a + 4*b*x) - 16*b^2*d^2*x^2*cos(2*a + 2*b*x) + 4*b^2*d^2*x^2*cos(4*a +
4*b*x) + 16*b*d^2*x*sin(2*a + 2*b*x) - 2*b*d^2*x*sin(4*a + 4*b*x) - 32*b^2*
c*d*x*cos(2*a + 2*b*x) + 8*b^2*c*d*x*cos(4*a + 4*b*x))/(128*b^3)
```

**sympy [A]** time = 3.62, size = 320, normalized size = 2.39

$$\begin{cases} \frac{c^2\sin^4(a+bx)}{4b} + \frac{5cdx\sin^4(a+bx)}{16b} - \frac{3cdx\sin^2(a+bx)\cos^2(a+bx)}{8b} - \frac{3cdx\cos^4(a+bx)}{16b} + \frac{5d^2x^2\sin^4(a+bx)}{32b} - \frac{3d^2x^2\sin^2(a+bx)\cos^2(a+bx)}{16b} - \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^3(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)**4/(4*b) + 5*c*d*x*sin(a + b*x)**4/(16*b) - 3*
c*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(8*b) - 3*c*d*x*cos(a + b*x)**4/(16*b
) + 5*d**2*x**2*sin(a + b*x)**4/(32*b) - 3*d**2*x**2*sin(a + b*x)**2*cos(a
+ b*x)**2/(16*b) - 3*d**2*x**2*cos(a + b*x)**4/(32*b) + 5*c*d*sin(a + b*x)*
*3*cos(a + b*x)/(16*b**2) + 3*c*d*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) +
5*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 3*d**2*x*sin(a + b*x)*cos
(a + b*x)**3/(16*b**2) - 5*d**2*sin(a + b*x)**4/(64*b**3) + 3*d**2*cos(a +
b*x)**4/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*
cos(a), True))
```

## 3.26 $\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=72

$$\frac{d\sin^3(a+bx)\cos(a+bx)}{16b^2}+\frac{3d\sin(a+bx)\cos(a+bx)}{32b^2}+\frac{(c+dx)\sin^4(a+bx)}{4b}-\frac{3dx}{32b}$$

```
[Out] -3/32*d*x/b+3/32*d*cos(b*x+a)*sin(b*x+a)/b^2+1/16*d*cos(b*x+a)*sin(b*x+a)^3
/b^2+1/4*(d*x+c)*sin(b*x+a)^4/b
```

**Rubi** [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4404, 2635, 8}

$$\frac{d\sin^3(a+bx)\cos(a+bx)}{16b^2}+\frac{3d\sin(a+bx)\cos(a+bx)}{32b^2}+\frac{(c+dx)\sin^4(a+bx)}{4b}-\frac{3dx}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-3*d*x)/(32*b) + (3*d*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) + (d*Cos[a + b*x
]*Sin[a + b*x]^3)/(16*b^2) + ((c + d*x)*Sin[a + b*x]^4)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx &= \frac{(c+dx)\sin^4(a+bx)}{4b}-\frac{d\int \sin^4(a+bx)\,dx}{4b}\\
&= \frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2}+\frac{(c+dx)\sin^4(a+bx)}{4b}-\frac{(3d)\int \sin^2(a+ \ldots}{16b}\\
&= \frac{3d\cos(a+bx)\sin(a+bx)}{32b^2}+\frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2}+\frac{(c+dx)\,\mathrm{si}\ldots}{4\ldots}\\
&= -\frac{3dx}{32b}+\frac{3d\cos(a+bx)\sin(a+bx)}{32b^2}+\frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2}+\frac{(c\ldots}{\ldots}
\end{aligned}$$

**Mathematica** [A] time = 0.11, size = 75, normalized size = 1.04

$$\frac{d(\sin(2(a+bx))-2bx\cos(2(a+bx)))}{16b^2}-\frac{d(\sin(4(a+bx))-4bx\cos(4(a+bx)))}{128b^2}+\frac{c\sin^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (c*Sin[a + b*x]^4)/(4*b) + (d*(-2*b*x*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]))
/(16*b^2) - (d*(-4*b*x*Cos[4*(a + b*x)] + Sin[4*(a + b*x)]))/(128*b^2)
```

**fricas** [A] time = 0.44, size = 76, normalized size = 1.06

$$\frac{8\,(bdx+bc)\cos\left(bx+a\right)^{4}+5\,bdx-16\,(bdx+bc)\cos\left(bx+a\right)^{2}-\left(2\,d\cos\left(bx+a\right)^{3}-5\,d\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{32\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(8*(b*d*x + b*c)*cos(b*x + a)^4 + 5*b*d*x - 16*(b*d*x + b*c)*cos(b*x +
 a)^2 - (2*d*cos(b*x + a)^3 - 5*d*cos(b*x + a))*sin(b*x + a))/b^2
```

**giac** [A] time = 3.71, size = 75, normalized size = 1.04

$$\frac{(bdx+bc)\cos\left(4\,bx+4\,a\right)}{32\,b^{2}}-\frac{(bdx+bc)\cos\left(2\,bx+2\,a\right)}{8\,b^{2}}-\frac{d\sin\left(4\,bx+4\,a\right)}{128\,b^{2}}+\frac{d\sin\left(2\,bx+2\,a\right)}{16\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(b*d*x + b*c)*cos(4*b*x + 4*a)/b^2 - 1/8*(b*d*x + b*c)*cos(2*b*x + 2*a
)/b^2 - 1/128*d*sin(4*b*x + 4*a)/b^2 + 1/16*d*sin(2*b*x + 2*a)/b^2
```

**maple** [A] time = 0.02, size = 85, normalized size = 1.18

$$\frac{\frac{d\left(\frac{(bx+a)\left(\sin^{4}(bx+a)\right)}{4}+\frac{\left(\sin^{3}(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{16}-\frac{3bx}{32}-\frac{3a}{32}\right)}{b}-\frac{da\left(\sin^{4}(bx+a)\right)}{4b}+\frac{c\left(\sin^{4}(bx+a)\right)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos
(b*x+a)-3/32*b*x-3/32*a)-1/4/b*d*a*sin(b*x+a)^4+1/4*c*sin(b*x+a)^4)
```

**maxima** [A] time = 0.34, size = 92, normalized size = 1.28

$$\frac{32\,c\sin\left(bx+a\right)^{4}-\frac{32\,ad\sin(bx+a)^{4}}{b}+\frac{(4\,(bx+a)\cos(4\,bx+4\,a)-16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)+8\,\sin(2\,bx+2\,a))d}{b}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/128*(32*c*sin(b*x + a)^4 - 32*a*d*sin(b*x + a)^4/b + (4*(b*x + a)*cos(4*b
*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x
+ 2*a))*d/b)/b
```

**mupad** [B] time = 0.25, size = 94, normalized size = 1.31

$$\frac{2\,d\,\sin\left(2\,a+2\,b\,x\right)-\frac{d\,\sin(4\,a+4\,b\,x)}{4}-2\,b\,c\,\sin\left(2\,a+2\,b\,x\right)^{2}+8\,b\,c\,\sin\left(a+b\,x\right)^{2}+4\,b\,d\,x\,\left(2\,\sin\left(a+b\,x\right)^{2}-1\right)-b}{32\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x),x)
```

```
[Out] (2*d*sin(2*a + 2*b*x) - (d*sin(4*a + 4*b*x))/4 - 2*b*c*sin(2*a + 2*b*x)^2 +
 8*b*c*sin(a + b*x)^2 + 4*b*d*x*(2*sin(a + b*x)^2 - 1) - b*d*x*(2*sin(2*a +
 2*b*x)^2 - 1))/(32*b^2)
```

**sympy [A]** time = 1.83, size = 138, normalized size = 1.92

$$\begin{cases} \frac{c\sin^4(a+bx)}{4b} + \frac{5dx\sin^4(a+bx)}{32b} - \frac{3dx\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{3dx\cos^4(a+bx)}{32b} + \frac{5d\sin^3(a+bx)\cos(a+bx)}{32b^2} + \frac{3d\sin(a+bx)\cos^3(a+bx)}{32b^2} \\ \left(cx + \frac{dx^2}{2}\right)\sin^3(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c*sin(a + b*x)**4/(4*b) + 5*d*x*sin(a + b*x)**4/(32*b) - 3*d*x*s
in(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*d*x*cos(a + b*x)**4/(32*b) + 5*d*
sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 3*d*sin(a + b*x)*cos(a + b*x)**3/(
32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)**3*cos(a), True))
```

## 3.27 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=129

$$-\frac{\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}-\frac{\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\right.}{8d}$$

```
[Out] 1/4*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/8*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*
b*x)/d-1/8*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d+1/4*Ci(2*b*c/d+2*b*x)*sin(2
*a-2*b*c/d)/d
```

**Rubi** [A] time = 0.23, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4406, 3303, 3299, 3302}

$$-\frac{\sin\left(4a-\frac{4bc}{d}\right)\mathrm{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -(CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral
[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*Sin
Integral[(2*b*c)/d + 2*b*x])/(4*d) - (Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b
*c)/d + 4*b*x])/(8*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)} - \frac{\sin(4a+4bx)}{8(c+dx)}\right) dx$$

$$= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{c+dx}\,dx\right) + \frac{1}{4}\int \frac{\sin(2a+2bx)}{c+dx}\,dx$$

$$= -\left(\frac{1}{8}\cos\left(4a-\frac{4bc}{d}\right)\int \frac{\sin\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx\right) + \frac{1}{4}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+\right.}{c+d}$$

$$= -\frac{\text{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{8d} + \frac{\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{4d} + \frac{\cos\left(2a-\right.}{}$$

**Mathematica [A]** time = 0.39, size = 110, normalized size = 0.85

$$-\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4b(c+dx)}{d}\right) - 2\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right) - 2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + \cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -1/8*(CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] - 2*CosIntegral[(
2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 2*Cos[2*a - (2*b*c)/d]*SinIntegral
[(2*b*(c + d*x))/d] + Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/
d
```

**fricas [A]** time = 0.49, size = 156, normalized size = 1.21

$$\frac{2\left(\text{Ci}\left(\frac{2(bdx+bc)}{d}\right) + \text{Ci}\left(-\frac{2(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right) - \left(\text{Ci}\left(\frac{4(bdx+bc)}{d}\right) + \text{Ci}\left(-\frac{4(bdx+bc)}{d}\right)\right)\sin\left(-\frac{4(bc-ad)}{d}\right) - 2\cos}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/16*(2*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d)
)*sin(-2*(b*c - a*d)/d) - (cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-
4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d) - 2*cos(-4*(b*c - a*d)/d)*sin_int
egral(4*(b*d*x + b*c)/d) + 4*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
b*c)/d))/d
```

**giac [C]** time = 2.11, size = 6046, normalized size = 46.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/16*(imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^
2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_i
ntegral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2
+ 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(
b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/
d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*t
```

```
an(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integr
al(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real
_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(
b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan
(2*b*c/d)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2
+ imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)
^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*t
an(2*b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)
^2*tan(2*b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*t
an(2*b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2
*b*c/d)^2 - 8*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*ta
n(2*b*c/d)^2*tan(b*c/d) + 8*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2
*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 16*sin_integral(2*(b*d*x + b*c)/d)
*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4*b*x
 + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*
b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + imag_part(cos_integr
al(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(
b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x
+ b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*imag_part(cos_integral(4*b*x
 + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*imag_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 8
*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^
2 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2*tan(
b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/
d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*
tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan
(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/
d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_int
egral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*sin_integ
ral(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*real_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) + 2*real_part(
cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) - 4*real_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 - 4*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 - 2
*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2
- 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d
)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*
c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(
b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)
^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(
2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2
*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d
))*tan(2*a)^2*tan(a)*tan(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(2*a)^2*tan(a)*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*
c/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(-4
```

```
*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_int
egral(4*b*x + 4*b*c/d))*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_p
art(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2
*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2*tan(b*c/
d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan
(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d)
^2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a
)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2 - 2*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2 + imag_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2 - 2*sin_integral(4*(b*d*x
+ b*c)/d)*tan(2*a)^2*tan(a)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^
2*tan(a)^2 + 4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*t
an(2*b*c/d) - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2
*tan(2*b*c/d) + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b
*c/d) + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2
- 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*
imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - imag_
part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*sin_inte
gral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 - 4*sin_integral(2*(b*d*x
 + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c
/d))*tan(a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*t
an(a)^2*tan(2*b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)
^2*tan(2*b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(
2*b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2 + 4*
sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2 - 8*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(b*c/d) + 8*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(b*c/d) - 16*sin_integral(2*
(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)*tan(b*c/d) - 8*imag_part(cos_integral(2*
b*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 8*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 16*sin_integral(2*(b
*d*x + b*c)/d)*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4*
b*x + 4*b*c/d))*tan(2*a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x +
2*b*c/d))*tan(2*a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(2*a)^2*tan(b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*ta
n(2*a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(b*
c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(b*c/d)^2 + imag_p
art(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_
integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x -
 4*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)
^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 +
 4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)*tan(b*c/d
)^2 - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)*tan
(b*c/d)^2 + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)*tan(b*c
/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^
2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2
- 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*
sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*sin_integra
l(2*(b*d*x + b*c)/d)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a) - 4*real_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(2*a)^2*tan(a) + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*a)*tan(a)^2 + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)
^2 + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 2
*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 2*real
_part(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(2*b*c/d) - 2*real_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d) - 2*real_part(cos_integ
ral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 2*real_part(cos_integral(-4
```

```
*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 4*real_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a)*tan(2*b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(a)*tan(2*b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(
2*a)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*
tan(b*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)
 - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) + 4*real
_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d) + 4*real_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(c
os_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(b*c/d)^2 + 2*real_part(cos_integ
ral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(b*c/d)^2 + 4*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(a)*tan(b*c/d)^2 - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b
*c/d)*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 -
2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2 + 2*imag_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(2*a)^2 + imag_part(cos_integral(-4*b*x - 4*b*c
/d))*tan(2*a)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 - 4*sin_inte
gral(2*(b*d*x + b*c)/d)*tan(2*a)^2 + imag_part(cos_integral(4*b*x + 4*b*c/d
))*tan(a)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 2*imag_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - imag_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(a)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^2 + 4*si
n_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + 4*imag_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*a)*tan(2*b*c/d) + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*
b*c/d) - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 - 2*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2 + 2*imag_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c
/d))*tan(2*b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 - 4*
sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 - 8*imag_part(cos_integral(2
*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 8*imag_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(a)*tan(b*c/d) - 16*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c
/d) + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(b*c/d)^2 + 2*imag_part(c
os_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 4*sin_integral(
2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*c/d)
)*tan(2*a) + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) - 4*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(a) - 4*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(a) - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c
/d) - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d) + 4*real_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 4*real_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_integral(4*b*x + 4*b*c/d)) - 2*im
ag_part(cos_integral(2*b*x + 2*b*c/d)) + 2*imag_part(cos_integral(-2*b*x -
2*b*c/d)) - imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 2*sin_integral(4*(b
*d*x + b*c)/d) - 4*sin_integral(2*(b*d*x + b*c)/d))/(d*tan(2*a)^2*tan(a)^2*
tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + d*tan(
2*a)^2*tan(a)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + d
*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(a)^2 + d*tan(2*a)^
2*tan(2*b*c/d)^2 + d*tan(a)^2*tan(2*b*c/d)^2 + d*tan(2*a)^2*tan(b*c/d)^2 +
d*tan(a)^2*tan(b*c/d)^2 + d*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2 + d*
tan(a)^2 + d*tan(2*b*c/d)^2 + d*tan(b*c/d)^2 + d)
```

**maple [A]** time = 0.02, size = 178, normalized size = 1.38

$$\frac{-\frac{b\left(\frac{4\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}-\frac{4\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}\right)}{32}+\frac{b\left(\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2\cdots}{}\right)}{d}\cdots\right)}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/b*(-1/32*b*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b
*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)+1/8*b*(2*Si(2*b*x+2*a+2*(-a*d
+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b
*c)/d)/d))
```

**maxima** [C] time = 0.47, size = 274, normalized size = 2.12

$$\frac{b\left(-2i\,E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+2i\,E_1\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(i\,E_1\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_1\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/16*(b*(-2*I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) +
2*I*exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*
c - a*d)/d) + b*(I*exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/
d) - I*exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*
(b*c - a*d)/d) - 2*b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*
d)/d) + exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2
*(b*c - a*d)/d) + b*(exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d
)/d) + exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*
(b*c - a*d)/d))/(b*d)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^3}{c+d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x), x)
```

# 3.28 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=179

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\sin\left(4a-\frac{4bc}{d}\right)}{2d}$$

```
[Out] -1/2*b*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^2+1/2*b*Ci(2*b*c/d+2*b*x)*cos(2
*a-2*b*c/d)/d^2+1/2*b*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^2-1/2*b*Si(2*b*c
/d+2*b*x)*sin(2*a-2*b*c/d)/d^2-1/4*sin(2*b*x+2*a)/d/(d*x+c)+1/8*sin(4*b*x+4
*a)/d/(d*x+c)
```

**Rubi [A]** time = 0.28, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2b\right.}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Cos[4*
a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]/(
4*d*(c + d*x)) + Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]
*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Sin[4*a - (4*b*c)/d]*SinInteg
ral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^2} - \frac{\sin(4a+4bx)}{8(c+dx)^2}\right)dx$$
$$= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^2}\,dx\right) + \frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx$$
$$= -\frac{\sin(2a+2bx)}{4d(c+dx)} + \frac{\sin(4a+4bx)}{8d(c+dx)} + \frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{2d} - \frac{b\int \frac{\cos(4a+4bx)}{c+dx}\,dx}{2d}$$
$$= -\frac{\sin(2a+2bx)}{4d(c+dx)} + \frac{\sin(4a+4bx)}{8d(c+dx)} - \frac{\left(b\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\cos\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx}{2d} + \frac{(b}{}$$
$$= \frac{b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^2} - \frac{b\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)}{2d^2} - \frac{\sin(2a+}{4d(c+}$$

**Mathematica [A]** time = 1.23, size = 151, normalized size = 0.84

$$\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2b(c+dx)}{d}\right) - 4b\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4b(c+dx)}{d}\right) - 4b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right) + 4b\sin\left(4a-\right.}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 4*b*Cos[4*a - (4
*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] - (2*d*Sin[2*(a + b*x)])/(c + d*x)
+ (d*Sin[4*(a + b*x)])/(c + d*x) - 4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*
b*(c + d*x))/d] + 4*b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/
(8*d^2)
```

**fricas [A]** time = 0.51, size = 245, normalized size = 1.37

$$\frac{2\,(bdx+bc)\sin\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right) - 2\,(bdx+bc)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right) + \left((bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*d*x + b*c)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d)
- 2*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + (
(b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(
-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) - ((b*d*x + b*c)*cos_integral(4*
(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(
b*c - a*d)/d) + 4*(d*cos(b*x + a)^3 - d*cos(b*x + a))*sin(b*x + a))/(d^3*x
+ c*d^2)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.02, size = 256, normalized size = 1.43

$$\frac{-\frac{b^2\left(-\frac{4\sin(4bx+4a)}{((bx+a)d-da+cb)d}+\frac{\frac{16\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}+\frac{16\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}}{d}\right)}{32}+\frac{b^2\left(-\frac{2\sin(2bx+2a)}{((bx+a)d-da+cb)d}+\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\mathrm{s}\cdots}{d}\cdots\right.}{\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/b*(-1/32*b^2*(-4*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)/d+4*(4*Si(4*b*x+4*a+4
*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(
-a*d+b*c)/d)/d)/d)+1/8*b^2*(-2*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si
(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c
)/d)*cos(2*(-a*d+b*c)/d)/d)/d))
```

**maxima [C]** time = 0.53, size = 301, normalized size = 1.68

$$\frac{b^2\left(-2i\,E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+2i\,E_2\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^2\left(i\,E_2\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_2\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(b^2*(-2*I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 2*I*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^2*(I*exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) - 2*b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^2*(exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^3}{{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{3}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{\left(c + d x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**2, x)
```

## 3.29 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=229

$$\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3}+\frac{b^2\cos\left(4a\right.}{}$$

```
[Out] -1/4*b*cos(2*b*x+2*a)/d^2/(d*x+c)+1/4*b*cos(4*b*x+4*a)/d^2/(d*x+c)-1/2*b^2*
cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3+b^2*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*b*
x)/d^3+b^2*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^3-1/2*b^2*Ci(2*b*c/d+2*b*x)
*sin(2*a-2*b*c/d)/d^3-1/8*sin(2*b*x+2*a)/d/(d*x+c)^2+1/16*sin(4*b*x+4*a)/d/
(d*x+c)^2
```

**Rubi** [A] time = 0.34, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b}{}\right.}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(4*d^2*(c + d*x)) + (b*Cos[4*a + 4*b*x])/(4*d^2*(c +
d*x)) + (b^2*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d^3 - (b^
2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d^3) - Sin[2*a +
2*b*x]/(8*d*(c + d*x)^2) + Sin[4*a + 4*b*x]/(16*d*(c + d*x)^2) - (b^2*Cos[2
*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^3) + (b^2*Cos[4*a - (4
*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/d^3
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^3} - \frac{\sin(4a+4bx)}{8(c+dx)^3}\right)dx$$
$$= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^3}\,dx\right) + \frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx$$
$$= -\frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{4d} - \frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx}{4d}$$
$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)} + \frac{b\cos(4a+4bx)}{4d^2(c+dx)} - \frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} - \frac{b^2\int \frac{\mathrm{si}}{}}{}$$
$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)} + \frac{b\cos(4a+4bx)}{4d^2(c+dx)} - \frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} + \frac{(b^2\,\mathrm{co}}{}$$
$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)} + \frac{b\cos(4a+4bx)}{4d^2(c+dx)} + \frac{b^2\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{d^3} - \frac{b^2\mathrm{Ci}\big(}{}$$

**Mathematica** [A] time = 2.76, size = 199, normalized size = 0.87

$$\frac{-2\left(4b^2\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2b(c+dx)}{d}\right)+4b^2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right)+\frac{d(2b(c+dx)\cos(2(a+bx))+d\sin(2(a+bx)))}{(c+dx)^2}\right)+16b^2\,\mathrm{si}}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (16*b^2*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + (d*(4*b*(c +
d*x)*Cos[4*(a + b*x)] + d*Sin[4*(a + b*x)]))/(c + d*x)^2 - 2*(4*b^2*CosInte
gral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*(2*b*(c + d*x)*Cos[2*(a +
 b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + 4*b^2*Cos[2*a - (2*b*c)/d]*SinI
ntegral[(2*b*(c + d*x))/d]) + 16*b^2*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*
(c + d*x))/d])/(16*d^3)
```

**fricas** [A] time = 0.65, size = 423, normalized size = 1.85

$$\frac{8\left(bd^2x+bcd\right)\cos(bx+a)^4+2\,bd^2x+2\,bcd-10\left(bd^2x+bcd\right)\cos(bx+a)^2+4\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\big(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*(b*d^2*x + b*c*d)*cos(b*x + a)^4 + 2*b*d^2*x + 2*b*c*d - 10*(b*d^2*x
 + b*c*d)*cos(b*x + a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-4*(
b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + 2*(d^2
*cos(b*x + a)^3 - d^2*cos(b*x + a))*sin(b*x + a) - ((b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x
```

```
 + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) + 2*((b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(4*(b*d*x + b*c)/d) + (b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(
b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.02, size = 329, normalized size = 1.44

$$\frac{-\frac{b^3\left(-\frac{2\sin(4bx+4a)}{((bx+a)d-da+cb)^2 d}+\frac{-\frac{8\cos(4bx+4a)}{((bx+a)d-da+cb)d}-\frac{8\left(\frac{4\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}-\frac{4\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}\right)}{d}}{d}\right)}{32}+\frac{b^3\left(-\frac{\sin(2bx+2a)}{((bx+a)d-da+cb)^2 \cdots}\right.}{\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] 1/b*(-1/32*b^3*(-2*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)^2/d+2*(-4*cos(4*b*x+4
*a)/((b*x+a)*d-d*a+c*b)/d-4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c
)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d)+1/8*b^3*
(-sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a)*d-d*a+
c*b)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2
*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d))
```

**maxima [C]** time = 0.68, size = 336, normalized size = 1.47

$$\frac{b^3\left(-2i\,E_3\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+2i\,E_3\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^3\left(i\,E_3\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(b^3*(-2*I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 2*I*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^3*(I*exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) - 2*b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^3*(exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^3}{{\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^3, x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**3, x)
```

## 3.30 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

**Optimal**. Leaf size=287

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\sin}{}$$

```
[Out] 4/3*b^3*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^4-1/3*b^3*Ci(2*b*c/d+2*b*x)*co
s(2*a-2*b*c/d)/d^4-1/12*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2+1/12*b*cos(4*b*x+4*a
)/d^2/(d*x+c)^2-4/3*b^3*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^4+1/3*b^3*Si(2
*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/12*sin(2*b*x+2*a)/d/(d*x+c)^3+1/6*b^2*
sin(2*b*x+2*a)/d^3/(d*x+c)+1/24*sin(4*b*x+4*a)/d/(d*x+c)^3-1/3*b^2*sin(4*b*
x+4*a)/d^3/(d*x+c)
```

**Rubi [A]** time = 0.39, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\mathrm{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(12*d^2*(c + d*x)^2) + (b*Cos[4*a + 4*b*x])/(12*d^2*(
c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) + (4*b^3*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
- Sin[2*a + 2*b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(6*d^3*(c +
d*x)) + Sin[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*Sin[4*a + 4*b*x])/(3*d^3
*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) - (4*b^3*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^4} - \frac{\sin(4a+4bx)}{8(c+dx)^4}\right)dx \\
&= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^4}\,dx\right) + \frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx \\
&= -\frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{\sin(4a+4bx)}{24d(c+dx)^3} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{6d} - \frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^3}\,dx}{6d} \\
&= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{\sin(4a+4bx)}{24d(c+dx)^3} - \frac{b^2\int \text{s}}{} \\
&= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{6d^3(c+dx)} + \frac{\sin}{24} \\
&= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{6d^3(c+dx)} + \frac{\sin}{24} \\
&= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4} + \frac{4b^3\text{c}}{}
\end{aligned}$$

**Mathematica [A]** time = 2.25, size = 316, normalized size = 1.10

$$\frac{-8b^3(c+dx)^3\left(\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right)-\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)\right)+32b^3(c+dx)^3\left(\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4b(c+}{d}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] (-2*d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2*
a]) + d*Cos[4*b*x]*(2*b*d*(c + d*x)*Cos[4*a] + (d^2 - 8*b^2*(c + d*x)^2)*Si
n[4*a]) + 2*d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2*a]
)*Sin[2*b*x] - d*((-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*a] + 2*b*d*(c + d*x)*Sin
[4*a])*Sin[4*b*x] - 8*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*
b*(c + d*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]) + 32
*b^3*(c + d*x)^3*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] - Sin
[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d]))/(24*d^4*(c + d*x)^3)
```

**fricas [B]** time = 0.70, size = 588, normalized size = 2.05

$$\frac{bd^3x+4\left(bd^3x+bcd^2\right)\cos(bx+a)^4+bcd^2-5\left(bd^3x+bcd^2\right)\cos(bx+a)^2-8\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b*d^3*x + 4*(b*d^3*x + b*c*d^2)*cos(b*x + a)^4 + b*c*d^2 - 5*(b*d^3*x
+ b*c*d^2)*cos(b*x + a)^2 - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + b^3*c^3)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + 2*(b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/d)
*sin_integral(2*(b*d*x + b*c)/d) - ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*
c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-
2*(b*c - a*d)/d) + 4*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*
c^3)*cos_integral(4*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b
^3*c^2*d*x + b^3*c^3)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/
d) - 2*((8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3
 - (5*b^2*d^3*x^2 + 10*b^2*c*d^2*x + 5*b^2*c^2*d - d^3)*cos(b*x + a))*sin(b
*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.02, size = 404, normalized size = 1.41

$$\frac{-\frac{b^4\left(-\frac{4\sin(4bx+4a)}{3((bx+a)d-da+cb)^3 d}+\frac{-\frac{8\cos(4bx+4a)}{3((bx+a)d-da+cb)^2 d}-\frac{8\left(-\frac{4\sin(4bx+4a)}{((bx+a)d-da+cb)d}+\frac{\frac{16\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}+\frac{16\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}}{d}\right)}{3d}}{d}\right)}{32}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x)
```

```
[Out] 1/b*(-1/32*b^4*(-4/3*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)^3/d+4/3*(-2*cos(4*b
*x+4*a)/((b*x+a)*d-d*a+c*b)^2/d-2*(-4*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)/d+
4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-
a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)/d)/d)+1/8*b^4*(-2/3*sin(2*b*x+2*a)/((
b*x+a)*d-d*a+c*b)^3/d+2/3*(-cos(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d-(-2*sin(
2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-
a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d
))
```

**maxima [C]** time = 0.93, size = 386, normalized size = 1.34

$$\frac{b^4\left(-2i\,E_4\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+2i\,E_4\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^4\left(i\,E_4\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E\right.}{16\left(b^3c^3d-3\,ab^2c^2d^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/16*(b^4*(-2*I*exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 2*I*exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
```

```
b*c - a*d)/d) + b^4*(I*exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) - 2*b^4*(exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^4*(exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^3}{{\left(c+d\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{3}{\left(a + b x \right)} \cos{\left(a + b x \right)}}{\left(c + d x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**4, x)
```

## 3.31 $\int (c+dx)^m \cot(a+bx)\,dx$

**Optimal**. Leaf size=17

$$\text{Int}\left(\cot(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable((d*x+c)^m*cot(b*x+a),x)
```

**Rubi** [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \cot(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\,dx$$

**Mathematica** [A] time = 2.58, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Cot[a + b*x], x]
```

**fricas** [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m\cos(bx+a)\csc(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos(bx+a)\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

**maple** [A] time = 0.16, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos(bx+a)\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cos(bx + a) \csc(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\cos(a + b\,x)\,(c + d\,x)^m}{\sin(a + b\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x),x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \cos(a + bx) \csc(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*cos(a + b*x)*csc(a + b*x), x)
```

## 3.32 $\int (c+dx)^4 \cot(a+bx)\,dx$

**Optimal**. Leaf size=151

$$-\frac{3d^4\text{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}+\frac{3id^3(c+dx)\text{Li}_4\left(e^{2i(a+bx)}\right)}{b^4}+\frac{3d^2(c+dx)^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^3}-\frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx}{}$$

```
[Out] -1/5*I*(d*x+c)^5/d+(d*x+c)^4*ln(1-exp(2*I*(b*x+a)))/b-2*I*d*(d*x+c)^3*polyl
og(2,exp(2*I*(b*x+a)))/b^2+3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b^3+
3*I*d^3*(d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4-3/2*d^4*polylog(5,exp(2*I*(
b*x+a)))/b^5
```

**Rubi [A]** time = 0.22, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{3d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x],x]
```

```
[Out] ((-I/5)*(c + d*x)^5)/d + ((c + d*x)^4*Log[1 - E^((2*I)*(a + b*x))])/b - ((2
*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2
*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^(
(2*I)*(a + b*x))])/b^4 - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \cot(a+bx)\,dx = -\frac{i(c+dx)^5}{5d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1-e^{2i(a+bx)}}\,dx$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{(4d)\int (c+dx)^3\log\left(1-e^{2i(a+bx)}\right)dx}{b}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{\left(6id^2\right)\int(c}{}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(c+dx}{}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(c+dx}{}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(c+dx}{}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(c+dx}{}$$

**Mathematica [B]** time = 6.11, size = 799, normalized size = 5.29

$$\frac{1}{5}id^4x^5 + icd^3x^4 + \frac{d^4\log\left(1-e^{-i(a+bx)}\right)x^4}{b} + \frac{d^4\log\left(1+e^{-i(a+bx)}\right)x^4}{b} + 2ic^2d^2x^3 + \frac{4cd^3\log\left(1-e^{-i(a+bx)}\right)x^3}{b} + \frac{4cd^3\log}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x],x]
```

```
[Out] ((2*I)*c^3*d*Pi*x)/b + (2*I)*c^2*d^2*x^3 + I*c*d^3*x^4 + (I/5)*d^4*x^5 - ((
4*I)*c^3*d*x*ArcTan[Tan[a]])/b + 2*c^3*d*x^2*Cot[a] + (2*c^3*d*Pi*Log[1 + E
^((-2*I)*b*x)])/b^2 + (6*c^2*d^2*x^2*Log[1 - E^((-I)*(a + b*x))])/b + (4*c*
d^3*x^3*Log[1 - E^((-I)*(a + b*x))])/b + (d^4*x^4*Log[1 - E^((-I)*(a + b*x)
)])/b + (6*c^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))])/b + (4*c*d^3*x^3*Log[1
+ E^((-I)*(a + b*x))])/b + (d^4*x^4*Log[1 + E^((-I)*(a + b*x))])/b + (4*c^3
*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))])/b + (4*c^3*d*ArcTan[Tan[a]]
*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))])/b^2 - (2*c^3*d*Pi*Log[Cos[b*x]]
)/b^2 + (c^4*Log[Sin[a + b*x]])/b - (4*c^3*d*ArcTan[Tan[a]]*Log[Sin[b*x + A
rcTan[Tan[a]]]])/b^2 + ((4*I)*d^2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*PolyLog[2,
-E^((-I)*(a + b*x))])/b^2 + ((4*I)*d^2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*PolyLo
g[2, E^((-I)*(a + b*x))])/b^2 - ((2*I)*c^3*d*PolyLog[2, E^((2*I)*(b*x + Arc
Tan[Tan[a]]))])/b^2 + (12*c^2*d^2*PolyLog[3, -E^((-I)*(a + b*x))])/b^3 + (2
4*c*d^3*x*PolyLog[3, -E^((-I)*(a + b*x))])/b^3 + (12*d^4*x^2*PolyLog[3, -E^
((-I)*(a + b*x))])/b^3 + (12*c^2*d^2*PolyLog[3, E^((-I)*(a + b*x))])/b^3 +
```

```
(24*c*d^3*x*PolyLog[3, E^((-I)*(a + b*x))])/b^3 + (12*d^4*x^2*PolyLog[3, E^
((-I)*(a + b*x))])/b^3 - ((24*I)*c*d^3*PolyLog[4, -E^((-I)*(a + b*x))])/b^4
 - ((24*I)*d^4*x*PolyLog[4, -E^((-I)*(a + b*x))])/b^4 - ((24*I)*c*d^3*PolyL
og[4, E^((-I)*(a + b*x))])/b^4 - ((24*I)*d^4*x*PolyLog[4, E^((-I)*(a + b*x)
)])/b^4 - (24*d^4*PolyLog[5, -E^((-I)*(a + b*x))])/b^5 - (24*d^4*PolyLog[5,
 E^((-I)*(a + b*x))])/b^5 - 2*c^3*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Se
c[a]^2]
```

**fricas [C]** time = 0.60, size = 1204, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
cos(b*x + a) - I*sin(b*x + a)) + 24*d^4*polylog(5, -cos(b*x + a) + I*sin(b*
x + a)) + 24*d^4*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) - (-4*I*b^3*d^4
*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(cos(b
*x + a) + I*sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^
3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (4*I*b^
3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(
-cos(b*x + a) + I*sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 -
12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) -
 (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c
^4)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3
 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*sin(b*
x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(b*
x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4
*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*
c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4
*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*
b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) +
 1) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, cos(b*x + a) + I*sin(b*x + a
)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, cos(b*x + a) - I*sin(b*x + a
)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) + I*sin(b*x +
a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) - I*sin(b*x +
a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(b*x + a
) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylo
g(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2
*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/b^5
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^4 \cos(bx + a) \csc(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a), x)
```

**maple [B]** time = 0.17, size = 1150, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] 1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))+12/b^3*
c^2*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))
-1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^
2+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+8/5*I/b^5*d^4*a^5-I*c*d^3*x^4-2
*I*c^2*d^2*x^3-2*I*c^3*d*x^2-24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*p
olylog(5,exp(I*(b*x+a)))/b^5+I*c^4*x-2/b*c^4*ln(exp(I*(b*x+a)))+1/b*c^4*ln(
exp(I*(b*x+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)-1/5*I*d^4*x^5+4/b*c^3*d*ln(e
xp(I*(b*x+a))+1)*x+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*
(b*x+a)))*a+6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+24/b^3*c*d^3*polylog(3,-ex
p(I*(b*x+a)))*x-6/b^3*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))+6/b*c^2*d^2*ln(1-exp
(I*(b*x+a)))*x^2+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x+24*I/b^4*c*d^3*po
lylog(4,-exp(I*(b*x+a)))+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+2*I/b^4*d
^4*a^4*x-4*I/b^2*c^3*d*a^2+8*I/b^3*c^2*d^2*a^3-6*I/b^4*c*d^3*a^4-4*I/b^2*d^
4*polylog(2,exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*
I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,-exp(I*(b*x
+a)))*x-4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,ex
p(I*(b*x+a)))+8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(I*(b*
x+a))-1)+8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x
+a))-1)-12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))-4/b^2*c^3*d*a*ln(exp(I*(b*x+a
))-1)+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-8*I
/b^3*c*d^3*a^3*x+12*I/b^2*c^2*d^2*a^2*x-8*I/b*c^3*d*a*x-12*I/b^2*c*d^3*poly
log(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-12
*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I/b^2*c*d^3*polylog(2,exp(I*(
b*x+a)))*x^2+4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+4/b*c*d^3*ln(1-exp(I*(b*x+a
)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3
```

**maxima [B]** time = 0.62, size = 1262, normalized size = 8.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/10*(10*c^4*log(sin(b*x + a)) - 40*a*c^3*d*log(sin(b*x + a))/b + 60*a^2*c^
2*d^2*log(sin(b*x + a))/b^2 - 40*a^3*c*d^3*log(sin(b*x + a))/b^3 + 10*a^4*d
^4*log(sin(b*x + a))/b^4 + (-2*I*(b*x + a)^5*d^4 + (-10*I*b*c*d^3 + 10*I*a*
d^4)*(b*x + a)^4 - 240*d^4*polylog(5, -e^(I*b*x + I*a)) - 240*d^4*polylog(5
, e^(I*b*x + I*a)) + (-20*I*b^2*c^2*d^2 + 40*I*a*b*c*d^3 - 20*I*a^2*d^4)*(b
*x + a)^3 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 - 60*I*a^2*b*c*d^3 + 20*I
*a^3*d^4)*(b*x + a)^2 + (10*I*(b*x + a)^4*d^4 + (40*I*b*c*d^3 - 40*I*a*d^4)
*(b*x + a)^3 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + 60*I*a^2*d^4)*(b*x + a
)^2 + (40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + 120*I*a^2*b*c*d^3 - 40*I*a^3*
d^4)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (-10*I*(b*x + a)^
4*d^4 + (-40*I*b*c*d^3 + 40*I*a*d^4)*(b*x + a)^3 + (-60*I*b^2*c^2*d^2 + 120
*I*a*b*c*d^3 - 60*I*a^2*d^4)*(b*x + a)^2 + (-40*I*b^3*c^3*d + 120*I*a*b^2*c
^2*d^2 - 120*I*a^2*b*c*d^3 + 40*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a),
 -cos(b*x + a) + 1) + (-40*I*b^3*c^3*d + 120*I*a*b^2*c^2*d^2 - 120*I*a^2*b*
c*d^3 - 40*I*(b*x + a)^3*d^4 + 40*I*a^3*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4
)*(b*x + a)^2 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*a^2*d^4)*(b*x
 + a))*dilog(-e^(I*b*x + I*a)) + (-40*I*b^3*c^3*d + 120*I*a*b^2*c^2*d^2 - 1
20*I*a^2*b*c*d^3 - 40*I*(b*x + a)^3*d^4 + 40*I*a^3*d^4 + (-120*I*b*c*d^3 +
120*I*a*d^4)*(b*x + a)^2 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*a^
2*d^4)*(b*x + a))*dilog(e^(I*b*x + I*a)) + 5*((b*x + a)^4*d^4 + 4*(b*c*d^3
- a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2
+ 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(
cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 5*((b*x + a)^4*d^4
+ 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b
```

```
*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (240*I
*b*c*d^3 + 240*I*(b*x + a)*d^4 - 240*I*a*d^4)*polylog(4, -e^(I*b*x + I*a))
+ (240*I*b*c*d^3 + 240*I*(b*x + a)*d^4 - 240*I*a*d^4)*polylog(4, e^(I*b*x +
 I*a)) + 120*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*
c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(I*b*x + I*a)) + 120*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*p
olylog(3, e^(I*b*x + I*a)))/b^4)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+bx)\,(c+dx)^4}{\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x),x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cos(a+bx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*cos(a + b*x)*csc(a + b*x), x)
```

# 3.33 $\int (c+dx)^3 \cot(a+bx)\,dx$

**Optimal**. Leaf size=127

$$\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}+\frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^4}{4d}$$

```
[Out] -1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1-exp(2*I*(b*x+a)))/b-3/2*I*d*(d*x+c)^2*pol
ylog(2,exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^
3+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

**Rubi [A]** time = 0.19, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}+\frac{(c+dx)^3\log(\ldots}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x],x]
```

```
[Out] ((-I/4)*(c + d*x)^4)/d + ((c + d*x)^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((
3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*
x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, E^(
(2*I)*(a + b*x))])/b^4
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\cot(a+bx)\,dx = -\frac{i(c+dx)^4}{4d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1-e^{2i(a+bx)}}\,dx$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{(3d)\int (c+dx)^2\log\left(1-e^{2i(a+bx)}\right)dx}{b}$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{\left(3id^2\right)\int}{}$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2(c+}{}$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2(c+}{}$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2(c+}{}$$

**Mathematica [B]** time = 2.67, size = 560, normalized size = 4.41

$$6b^4c^2dx^2\cot(a) - 6b^4c^2dx^2e^{i\tan^{-1}(\tan(a))}\cot(a)\sqrt{\sec^2(a)} + 4b^3c^3\log(\sin(a+bx)) - 12ib^3c^2dx\tan^{-1}(\tan(a)) +$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x],x]
```

```
[Out] ((6*I)*b^3*c^2*d*Pi*x + (4*I)*b^4*c*d^2*x^3 + I*b^4*d^3*x^4 - (12*I)*b^3*c^
2*d*x*ArcTan[Tan[a]] + 6*b^4*c^2*d*x^2*Cot[a] + 6*b^2*c^2*d*Pi*Log[1 + E^((
-2*I)*b*x)] + 12*b^3*c*d^2*x^2*Log[1 - E^((-I)*(a + b*x))] + 4*b^3*d^3*x^3*
Log[1 - E^((-I)*(a + b*x))] + 12*b^3*c*d^2*x^2*Log[1 + E^((-I)*(a + b*x))]
+ 4*b^3*d^3*x^3*Log[1 + E^((-I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 - E^((2*
I)*(b*x + ArcTan[Tan[a]]))] + 12*b^2*c^2*d*ArcTan[Tan[a]]*Log[1 - E^((2*I)*
(b*x + ArcTan[Tan[a]]))] - 6*b^2*c^2*d*Pi*Log[Cos[b*x]] + 4*b^3*c^3*Log[Sin
[a + b*x]] - 12*b^2*c^2*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + (
12*I)*b^2*d^2*x*(2*c + d*x)*PolyLog[2, -E^((-I)*(a + b*x))] + (12*I)*b^2*d^
2*x*(2*c + d*x)*PolyLog[2, E^((-I)*(a + b*x))] - (6*I)*b^2*c^2*d*PolyLog[2,
 E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 24*b*c*d^2*PolyLog[3, -E^((-I)*(a + b*
x))] + 24*b*d^3*x*PolyLog[3, -E^((-I)*(a + b*x))] + 24*b*c*d^2*PolyLog[3, E
^((-I)*(a + b*x))] + 24*b*d^3*x*PolyLog[3, E^((-I)*(a + b*x))] - (24*I)*d^3
*PolyLog[4, -E^((-I)*(a + b*x))] - (24*I)*d^3*PolyLog[4, E^((-I)*(a + b*x))
] - 6*b^4*c^2*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2])/(4*b^4)
```

**fricas [C]** time = 0.58, size = 814, normalized size = 6.41

$$6i\,d^3\mathrm{polylog}\left(4, \cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right) - 6i\,d^3\mathrm{polylog}\left(4, \cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right) - 6i\,d^3\mathrm{polylog}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4,
 cos(b*x + a) - I*sin(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(
b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2*
d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x +
 a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(cos(b*x +
a) - I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*
dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x
 - 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a)
+ 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*
x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
+ a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
+ a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(b*d^3*x + b*c*d^2)*
polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3
, cos(b*x + a) - I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*
x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) -
 I*sin(b*x + a)))/b^4
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cos (bx+a) \csc (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a), x)
```

**maple [B]** time = 0.10, size = 783, normalized size = 6.17

$$\frac{6icd^2a^2x}{b^2}-\frac{6ic^2dax}{b}-\frac{6icd^2\operatorname{polylog}\left(2,e^{i(bx+a)}\right)x}{b^2}-\frac{6icd^2\operatorname{polylog}\left(2,-e^{i(bx+a)}\right)x}{b^2}-\frac{d^3a^3\ln\left(e^{i(bx+a)}-1\right)}{b^4}+\frac{2d^3a^3\ln}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] 6*I/b^2*c*d^2*a^2*x-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^2
*polylog(2,-exp(I*(b*x+a)))*x-6*I/b*c^2*d*a*x+6*I*d^3*polylog(4,exp(I*(b*x+
a)))/b^4+I*c^3*x-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+2/b^4*d^3*a^3*ln(exp(I*
(b*x+a)))+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,exp(
I*(b*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,-exp(
I*(b*x+a)))*x-3/2*I/b^4*a^4*d^3+6*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-I*c*
d^2*x^3-3/2*I*c^2*d*x^2-1/4*I*d^3*x^4+1/b*c^3*ln(exp(I*(b*x+a))-1)+1/b*c^3*
ln(exp(I*(b*x+a))+1)-2/b*c^3*ln(exp(I*(b*x+a)))+3/b^3*c*d^2*a^2*ln(exp(I*(b
*x+a))-1)-6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,exp(I*
(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-3*I/b^2*c^2*d*a^2-2*I/b^
3*a^3*d^3*x+4*I/b^3*c*d^2*a^3-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-3*I
/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/
b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-3/b^3*c*d
^2*a^2*ln(1-exp(I*(b*x+a)))+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+3/b*c*d^2*ln
(exp(I*(b*x+a))+1)*x^2-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+6/b^2*c^2*d*a*ln(
```

```
exp(I*(b*x+a))+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp(I*(b*x+
a)))*a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3
```

**maxima [B]** time = 0.52, size = 747, normalized size = 5.88

$$4\,c^3\log\left(\sin\left(bx+a\right)\right) - \frac{12\,ac^2d\log(\sin(bx+a))}{b} + \frac{12\,a^2cd^2\log(\sin(bx+a))}{b^2} - \frac{4\,a^3d^3\log(\sin(bx+a))}{b^3} + \frac{-i\,(bx+a)^4d^3+\left(-4i\,bcd^2+4i\,ad^3\right)(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(4*c^3*log(sin(b*x + a)) - 12*a*c^2*d*log(sin(b*x + a))/b + 12*a^2*c*d^
2*log(sin(b*x + a))/b^2 - 4*a^3*d^3*log(sin(b*x + a))/b^3 + (-I*(b*x + a)^4
*d^3 + (-4*I*b*c*d^2 + 4*I*a*d^3)*(b*x + a)^3 + 24*I*d^3*polylog(4, -e^(I*b
*x + I*a)) + 24*I*d^3*polylog(4, e^(I*b*x + I*a)) + (-6*I*b^2*c^2*d + 12*I*
a*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a)^2 + (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2
 - 12*I*a*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^
3)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (-4*I*(b*x + a)^3*d
^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b
*c*d^2 - 12*I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1)
+ (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 +
 (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) + (-12*I*b
^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b*
c*d^2 + 24*I*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I*a)) + 2*((b*x + a)^3*d^3
+ 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(
b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 2*((b
*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a
) + 1) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -e^(I*b*x + I*a))
+ 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, e^(I*b*x + I*a)))/b^3)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)^3}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x),x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3\cos\left(a+bx\right)\csc\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)*csc(a + b*x), x)
```

## 3.34 $\int (c+dx)^2 \cot(a+bx)\,dx$

**Optimal**. Leaf size=93

$$\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^3}{3d}$$

```
[Out] -1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b-I*d*(d*x+c)*polylog(2
,exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,exp(2*I*(b*x+a)))/b^3
```

**Rubi [A]** time = 0.17, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3717, 2190, 2531, 2282, 6589}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x],x]
```

```
[Out] ((-I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b - (I*
d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (d^2*PolyLog[3, E^((2*I)
*(a + b*x))])/(2*b^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2\cot(a+bx)\,dx &= -\frac{i(c+dx)^3}{3d}-2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx\\
&= -\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(2d)\int (c+dx)\log\left(1-e^{2i(a+bx)}\right)dx}{b}\\
&= -\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id(c+dx)\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{\left(id^2\right)\int \mathrm{Li}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id(c+dx)\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\,\mathrm{Subst}\left(\right.}{}\\
&= -\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id(c+dx)\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\mathrm{Li}_3\left(e^{2i(\right.}}{2b^3}
\end{aligned}
$$

**Mathematica [B]** time = 1.41, size = 356, normalized size = 3.83

$$
3b^3cdx^2\cot(a)-3b^3cdx^2e^{i\tan^{-1}(\tan(a))}\cot(a)\sqrt{\sec^2(a)}+3b^2c^2\log(\sin(a+bx))-6ib^2cdx\tan^{-1}(\tan(a))+6b^2c
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x],x]
```

```
[Out] ((3*I)*b^2*c*d*Pi*x + I*b^3*d^2*x^3 - (6*I)*b^2*c*d*x*ArcTan[Tan[a]] + 3*b^
3*c*d*x^2*Cot[a] + 3*b*c*d*Pi*Log[1 + E^((-2*I)*b*x)] + 3*b^2*d^2*x^2*Log[1
 - E^((-I)*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))] + 6*b^2*
c*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 6*b*c*d*ArcTan[Tan[a]]*Lo
g[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - 3*b*c*d*Pi*Log[Cos[b*x]] + 3*b^2*
c^2*Log[Sin[a + b*x]] - 6*b*c*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]
]] + (6*I)*b*d^2*x*PolyLog[2, -E^((-I)*(a + b*x))] + (6*I)*b*d^2*x*PolyLog[
2, E^((-I)*(a + b*x))] - (3*I)*b*c*d*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[
a]]))] + 6*d^2*PolyLog[3, -E^((-I)*(a + b*x))] + 6*d^2*PolyLog[3, E^((-I)*(
a + b*x))] - 3*b^3*c*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2])/(3*b
^3)
```

**fricas [C]** time = 0.50, size = 498, normalized size = 5.35

$$
2\,d^2\mathrm{polylog}\left(3,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+2\,d^2\mathrm{polylog}\left(3,\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)+2\,d^2\mathrm{polylog}\left(3,\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) + 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a)) + 2*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*
I*b*c*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*d
ilog(cos(b*x + a) - I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-cos(
b*x + a) + I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-cos(b*x + a)
 - I*sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a)
 + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x
+ a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b
```

```
*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-
1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x +
 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^
2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) +
 1))/b^3
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \cos(bx+a)\csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*csc(b*x + a), x)
```

**maple [B]** time = 0.07, size = 468, normalized size = 5.03

$$\frac{d^2a^2\ln\left(e^{i(bx+a)}-1\right)}{b^3}-\frac{2d^2a^2\ln\left(e^{i(bx+a)}\right)}{b^3}+\frac{d^2\ln\left(1-e^{i(bx+a)}\right)x^2}{b}-\frac{d^2\ln\left(1-e^{i(bx+a)}\right)a^2}{b^3}+\frac{d^2\ln\left(e^{i(bx+a)}+1\right)x^2}{b}+\frac{4ia}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] 1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+1/b*d^2
*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(exp
(I*(b*x+a))+1)*x^2+4/3*I/b^3*d^2*a^3-I*c*d*x^2-4*I/b*c*d*a*x+2*d^2*polylog(
3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3+I*c^2*x-2/b*c^2*
ln(exp(I*(b*x+a)))+1/b*c^2*ln(exp(I*(b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1
)-1/3*I*d^2*x^3+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)
))*a+2/b*c*d*ln(exp(I*(b*x+a))+1)*x+4/b^2*c*d*a*ln(exp(I*(b*x+a)))-2/b^2*c*
d*a*ln(exp(I*(b*x+a))-1)+2*I/b^2*d^2*a^2*x-2*I/b^2*c*d*a^2-2*I/b^2*d^2*poly
log(2,-exp(I*(b*x+a)))*x-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*c*
d*polylog(2,-exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))
```

**maxima [B]** time = 0.47, size = 404, normalized size = 4.34

$$6c^2\log(\sin(bx+a)) - \frac{12acd\log(\sin(bx+a))}{b} + \frac{6a^2d^2\log(\sin(bx+a))}{b^2} + \frac{-2i(bx+a)^3d^2+\left(-6ibcd+6iad^2\right)(bx+a)^2+12d^2\mathrm{Li}_3\left(-e^{(ibx+ia)}\right)+12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(6*c^2*log(sin(b*x + a)) - 12*a*c*d*log(sin(b*x + a))/b + 6*a^2*d^2*log
(sin(b*x + a))/b^2 + (-2*I*(b*x + a)^3*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x
+ a)^2 + 12*d^2*polylog(3, -e^(I*b*x + I*a)) + 12*d^2*polylog(3, e^(I*b*x +
 I*a)) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a))*arctan
2(sin(b*x + a), cos(b*x + a) + 1) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d +
12*I*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (-12*I*b*
c*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*dilog(-e^(I*b*x + I*a)) + (-12*I*b*c
*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*dilog(e^(I*b*x + I*a)) + 3*((b*x + a)
^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
 2*cos(b*x + a) + 1) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,{\left(c+d\,x\right)}^2}{\sin(a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x),x)
[Out] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x), x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \cos(a + bx) \csc(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a),x)
[Out] Integral((c + d*x)**2*cos(a + b*x)*csc(a + b*x), x)
```

## 3.35 $\int (c+dx)\cot(a+bx)\,dx$

**Optimal**. Leaf size=65

$$-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^2}{2d}$$

```
[Out] -1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b-1/2*I*d*polylog(2,exp(2
*I*(b*x+a)))/b^2
```

**Rubi** [A] time = 0.10, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3717, 2190, 2279, 2391}

$$-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x],x]
```

```
[Out] ((-I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b - ((I/2
)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)\cot(a+bx)\,dx &= -\frac{i(c+dx)^2}{2d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx \\
&= -\frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d\int \log\left(1-e^{2i(a+bx)}\right)\,dx}{b} \\
&= -\frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{(id)\,\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i(a+bx)}\right)}{2b^2} \\
&= -\frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id\,\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}
\end{aligned}
$$

**Mathematica [B]** time = 5.11, size = 188, normalized size = 2.89

$$
-\frac{d\csc(a)\sec(a)\left(b^2x^2e^{i\tan^{-1}(\tan(a))} + \frac{\tan(a)\left(i\mathrm{Li}_2\left(e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right)+ibx\left(2\tan^{-1}(\tan(a))-\pi\right)-2\left(\tan^{-1}(\tan(a))+bx\right)\log\left(1-e^{2i\left(\tan^{-1}(\mathrm{ta}\right.}\right.\right.}{\sqrt{\tan^2(a)\cdots}}\right.}{2b^2\sqrt{\sec^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Cot[a + b*x],x]
```

```
[Out] (d*x^2*Cot[a])/2 + (c*(Log[Cos[a + b*x]] + Log[Tan[a + b*x]]))/b - (d*Csc[a
]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) -
 Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*
x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + A
rcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/S
qrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

**fricas [B]** time = 0.50, size = 250, normalized size = 3.85

$$
\frac{-i\,d\mathrm{Li}_2\left(\cos(bx+a)+i\,\sin(bx+a)\right) + i\,d\mathrm{Li}_2\left(\cos(bx+a)-i\,\sin(bx+a)\right) + i\,d\mathrm{Li}_2\left(-\cos(bx+a)+i\,\sin(b\cdots\right.}{\cdots}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(cos(b*x + a) - I
*sin(b*x + a)) + I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) - I*d*dilog(-cos
(b*x + a) - I*sin(b*x + a)) + (b*d*x + b*c)*log(cos(b*x + a) + I*sin(b*x +
a) + 1) + (b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b*c - a*d
)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*log(-1/2*
cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x + a*d)*log(-cos(b*x + a)
+ I*sin(b*x + a) + 1) + (b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) +
1))/b^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$
\int (dx+c)\cos(bx+a)\csc(bx+a)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cos(b*x + a)*csc(b*x + a), x)
```

**maple [B]** time = 0.07, size = 215, normalized size = 3.31

$$-\frac{id\,x^2}{2}+icx+\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}-\frac{2c\ln\left(e^{i(bx+a)}\right)}{b}-\frac{2idax}{b}-\frac{id\,a^2}{b^2}+\frac{d\ln\left(e^{i(bx+a)}+1\right)x}{b}-\frac{id\,\mathrm{polylog}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] -1/2*I*d*x^2+I*c*x+1/b*c*ln(exp(I*(b*x+a))-1)+1/b*c*ln(exp(I*(b*x+a))+1)-2/
b*c*ln(exp(I*(b*x+a)))-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(exp(I*(b*x+a))+1)*x
-I*d*polylog(2,-exp(I*(b*x+a)))/b^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln
(1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(
b*x+a))-1)+2/b^2*d*a*ln(exp(I*(b*x+a)))
```

**maxima [B]** time = 0.46, size = 189, normalized size = 2.91

$$\frac{-i\,b^2dx^2-2i\,b^2cx-2i\,bdx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right)+2i\,bc\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)-1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*b^2*d*x^2 - 2*I*b^2*c*x - 2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x
+ a) + 1) + 2*I*b*c*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*I*b*d*x +
2*I*b*c)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*I*d*dilog(-e^(I*b*x +
I*a)) - 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(b*x + a)^2 + s
in(b*x + a)^2 + 2*cos(b*x + a) + 1) + (b*d*x + b*c)*log(cos(b*x + a)^2 + si
n(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x))/sin(a + b*x),x)
```

```
[Out] int((cos(a + b*x)*(c + d*x))/sin(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)\cos\left(a+bx\right)\csc\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*csc(a + b*x), x)
```

## 3.36 $\int \frac{\cot(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=17

$$\operatorname{Int}\left(\frac{\cot(a+bx)}{c+dx},x\right)$$

[Out] Unintegrable(cot(b*x+a)/(d*x+c),x)

**Rubi** [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + b*x]/(c + d*x),x]

[Out] Defer[Int][Cot[a + b*x]/(c + d*x), x]

Rubi steps

$$\int \frac{\cot(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)}{c+dx}\,dx$$

**Mathematica** [A] time = 3.66, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Cot[a + b*x]/(c + d*x),x]

[Out] Integrate[Cot[a + b*x]/(c + d*x), x]

**fricas** [A] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\cos(bx+a)\csc(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] integral(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)

**maple** [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\cos(a+b\,x)}{\sin(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)),x)
```

```
[Out] int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*csc(a + b*x)/(c + d*x), x)
```

## 3.37 $\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=17

$$\text{Int}\left(\frac{\cot(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)/(d*x+c)^2,x)
```

**Rubi** [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]/(c + d*x)^2,x]
[Out] Defer[Int][Cot[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx=\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$$

**Mathematica** [A] time = 6.98, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]/(c + d*x)^2,x]
[Out] Integrate[Cot[a + b*x]/(c + d*x)^2, x]
```

**fricas** [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cos(b*x + a)*csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c)^2, x)
```

**maple** [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
[Out] int(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c)^2, x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\cos(a+b\,x)}{\sin(a+b\,x)\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)^2),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)^2), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)**2,x)
[Out] Integral(cos(a + b*x)*csc(a + b*x)/(c + d*x)**2, x)
```

## 3.38 $\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\cot(a+bx)\csc(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x)
```

**Rubi [A]** time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$$

**Mathematica [A]** time = 2.93, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

**fricas [A]** time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m\cos(bx+a)\csc(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos(bx+a)\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

**maple [A]** time = 0.08, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos(bx+a)\left(\csc^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x)
[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\csc(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(a+b\,x)\,\left(c+d\,x\right)^m}{\sin(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^2,x)
[Out] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^2, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos(a+bx)\csc^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*cos(a + b*x)*csc(a + b*x)**2, x)
```

# 3.39 $\int (c + dx)^4 \cot(a + bx) \csc(a + bx) \, dx$

**Optimal**. Leaf size=208

$$-\frac{24id^4\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^5}+\frac{24id^4\text{Li}_4\left(e^{i(a+bx)}\right)}{b^5}-\frac{24d^3(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}+\frac{12id^2(c+d}{}$$

```
[Out] -8*d*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^4*csc(b*x+a)/b+12*I*d^2*
(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^3-12*I*d^2*(d*x+c)^2*polylog(2,exp(I
*(b*x+a)))/b^3-24*d^3*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^4+24*d^3*(d*x+c)
*polylog(3,exp(I*(b*x+a)))/b^4-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+24*I
*d^4*polylog(4,exp(I*(b*x+a)))/b^5
```

**Rubi [A]** time = 0.17, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4410, 4183, 2531, 6609, 2282, 6589}

$$-\frac{24d^3(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4}+\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-8*d*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^4*Csc[a + b*x]
)/b + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((12*I)*d
^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (24*d^3*(c + d*x)*PolyLog
[3, -E^(I*(a + b*x))])/b^4 + (24*d^3*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])
/b^4 - ((24*I)*d^4*PolyLog[4, -E^(I*(a + b*x))])/b^5 + ((24*I)*d^4*PolyLog[
4, E^(I*(a + b*x))])/b^5
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
```

```
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4\cot(a+bx)\csc(a+bx)\,dx = -\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{(4d)\int (c+dx)^3\csc(a+bx)\,dx}{b}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^4\csc(a+bx)}{b}-\frac{\left(12d^2\right)\int (c+dx \ldots}{}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{12id^2(c+dx)^2\mathrm{L}\ldots}{b^3}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{12id^2(c+dx)^2\mathrm{L}\ldots}{b^3}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{12id^2(c+dx)^2\mathrm{L}\ldots}{b^3}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{12id^2(c+dx)^2\mathrm{L}\ldots}{b^3}$$

**Mathematica [A]** time = 1.36, size = 308, normalized size = 1.48

$$8id\left(\frac{3d\left(b^2(c+dx)^2\mathrm{Li}_2(-\cos(a+bx)-i\sin(a+bx))+2ibd(c+dx)\mathrm{Li}_3(-\cos(a+bx)-i\sin(a+bx))-2d^2\mathrm{Li}_4(-\cos(a+bx)-i\sin(a+bx))\right)}{b^3}-\frac{3d\left(b^2(c+dx)^2\mathrm{Li}_2(\mathrm{co}\ldots\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-2*b*(c + d*x)^4*Csc[a] + (8*I)*d*((2*I)*(c + d*x)^3*ArcTanh[Cos[a + b*x]
+ I*Sin[a + b*x]] + (3*d*(b^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[
a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]]
- 2*d^2*PolyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]))/b^3 - (3*d*(b^2*(c + d
*x)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyL
og[3, Cos[a + b*x] + I*Sin[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Si
n[a + b*x]]))/b^3) + b*(c + d*x)^4*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] -
 b*(c + d*x)^4*Sec[a/2]*Sec[(a + b*x)/2]*Sin[(b*x)/2])/(2*b^2)
```

**fricas [C]** time = 0.55, size = 1021, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c
^4 - 12*I*d^4*polylog(4, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I
*d^4*polylog(4, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 12*I*d^4*poly
log(4, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4, -
cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*
c*d^3*x - 6*I*b^2*c^2*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a
) - (6*I*b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(cos(b*x +
a) - I*sin(b*x + a))*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*x -
6*I*b^2*c^2*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*
b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(-cos(b*x + a) - I*s
in(b*x + a))*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^
2*x + b^3*c^3*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 2*(b
^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x + a
) - I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a
^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin
(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*log(-
1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b^3*d^4*x^3
+ 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3
*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*d^4*x^3
 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^
3*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 12*(b*d^4*x +
 b*c*d^3)*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 12*(b*d^
4*x + b*c*d^3)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 12*
(b*d^4*x + b*c*d^3)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a)
 + 12*(b*d^4*x + b*c*d^3)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*
x + a))/(b^5*sin(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \cos(bx+a) \csc(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a)^2, x)
```

**maple [B]** time = 0.12, size = 716, normalized size = 3.44

$$\frac{8d^4a^3 \operatorname{arctanh}\left(e^{i(bx+a)}\right)}{b^5} - \frac{24d^3c \operatorname{polylog}\left(3, -e^{i(bx+a)}\right)}{b^4} + \frac{24d^3c \operatorname{polylog}\left(3, e^{i(bx+a)}\right)}{b^4} - \frac{8d\,c^3 \operatorname{arctanh}\left(e^{i(bx+a)}\right)}{b^2} - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x)
```

```
[Out] 8/b^5*d^4*a^3*arctanh(exp(I*(b*x+a)))-24/b^4*d^3*c*polylog(3,-exp(I*(b*x+a)
))+24/b^4*d^3*c*polylog(3,exp(I*(b*x+a)))-8/b^2*d*c^3*arctanh(exp(I*(b*x+a)
))-24/b^4*d^4*polylog(3,-exp(I*(b*x+a)))*x+24/b^4*d^4*polylog(3,exp(I*(b*x+
a)))*x-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+24*I/b^3*d^3*c*polylog(2,-ex
p(I*(b*x+a)))*x-24*I/b^3*d^3*c*polylog(2,exp(I*(b*x+a)))*x+24*I*d^4*polylog
(4,exp(I*(b*x+a)))/b^5-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4
)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)+12/b^2*d^2*c^2*ln(1-exp(I*(b*x+a)))
*x+12/b^3*d^2*c^2*ln(1-exp(I*(b*x+a)))*a-12/b^2*d^2*c^2*ln(exp(I*(b*x+a))+1
)*x-12/b^3*d^2*c^2*ln(exp(I*(b*x+a))+1)*a+12/b^2*d^3*c*ln(1-exp(I*(b*x+a)))
*x^2-12/b^4*d^3*c*ln(1-exp(I*(b*x+a)))*a^2-12/b^2*d^3*c*ln(exp(I*(b*x+a))+1
)*x^2+12/b^4*d^3*c*ln(exp(I*(b*x+a))+1)*a^2+4/b^2*d^4*ln(1-exp(I*(b*x+a)))*
x^3+4/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^3-4/b^2*d^4*ln(exp(I*(b*x+a))+1)*x^3-4
```

```
/b^5*d^4*ln(exp(I*(b*x+a))+1)*a^3-24/b^4*d^3*c*a^2*arctanh(exp(I*(b*x+a)))+
24/b^3*d^2*c^2*a*arctanh(exp(I*(b*x+a)))+12*I/b^3*d^4*polylog(2,-exp(I*(b*x
+a)))*x^2-12*I/b^3*d^4*polylog(2,exp(I*(b*x+a)))*x^2+12*I/b^3*d^2*c^2*polyl
og(2,-exp(I*(b*x+a)))-12*I/b^3*d^2*c^2*polylog(2,exp(I*(b*x+a)))
```

**maxima** [B] time = 0.84, size = 2944, normalized size = 14.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2*
a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*c^
3*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b)
- 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2*
a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*a*
c^2*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*b^2) + 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*
x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*
b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) -
 (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos
(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x +
a))*a^2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a
) + 1)*b^3) - 2*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*co
s(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a)
+ 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*l
og(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(
b*x + a))*a^3*d^4/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
 2*a) + 1)*b^4) + c^4/sin(b*x + a) - 4*a*c^3*d/(b*sin(b*x + a)) + 6*a^2*c^2
*d^2/(b^2*sin(b*x + a)) - 4*a^3*c*d^3/(b^3*sin(b*x + a)) + a^4*d^4/(b^4*sin
(b*x + a)) - ((4*(b*x + a)^3*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) - 4*((b*x + a)^3*d^4 + 3*(b*c
*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x +
a))*cos(2*b*x + 2*a) - (4*I*(b*x + a)^3*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(
b*x + a)^2 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a))*
sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (4*(b*x + a)^3*
d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*(b*x + a) - 4*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3
*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (4*I*(
b*x + a)^3*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^2 + (12*I*b^2*c^2*d^
2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin
(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b
*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2)*cos(b*x +
a) - (12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^2*d^4 + 12*a^2*d^4 + 24*
(b*c*d^3 - a*d^4)*(b*x + a) - 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d
^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (-12*I*b^2
*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*(b*x + a)^2*d^4 - 12*I*a^2*d^4 + (-24*I*b*
c*d^3 + 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) +
(12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^2*d^4 + 12*a^2*d^4 + 24*(b*c*
d^3 - a*d^4)*(b*x + a) - 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 +
a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (12*I*b^2*c^2*d
^2 - 24*I*a*b*c*d^3 + 12*I*(b*x + a)^2*d^4 + 12*I*a^2*d^4 + (24*I*b*c*d^3 -
```

```
24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (2*I*(b*
x + a)^3*d^4 + (6*I*b*c*d^3 - 6*I*a*d^4)*(b*x + a)^2 + (6*I*b^2*c^2*d^2 - 1
2*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a) + (-2*I*(b*x + a)^3*d^4 + (-6*I*b*c*
d^3 + 6*I*a*d^4)*(b*x + a)^2 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2
*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 2*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4
)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*
x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-2*I
*(b*x + a)^3*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a)^2 + (-6*I*b^2*c^2*d
^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^4)*(b*x + a) + (2*I*(b*x + a)^3*d^4 + (6*I*
b*c*d^3 - 6*I*a*d^4)*(b*x + a)^2 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 + 6*I*
a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 2*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*
d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 24
*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog(4, -e^(I*b*x
 + I*a)) + 24*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog
(4, e^(I*b*x + I*a)) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 24*I*a*d^4 + (-
24*I*b*c*d^3 - 24*I*(b*x + a)*d^4 + 24*I*a*d^4)*cos(2*b*x + 2*a) + 24*(b*c*
d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a))
 - (-24*I*b*c*d^3 - 24*I*(b*x + a)*d^4 + 24*I*a*d^4 + (24*I*b*c*d^3 + 24*I*
(b*x + a)*d^4 - 24*I*a*d^4)*cos(2*b*x + 2*a) - 24*(b*c*d^3 + (b*x + a)*d^4
- a*d^4)*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) - (2*I*(b*x + a)^4*d
^4 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c
*d^3 + 12*I*a^2*d^4)*(b*x + a)^2)*sin(b*x + a))/(-I*b^4*cos(2*b*x + 2*a) +
b^4*sin(2*b*x + 2*a) + I*b^4))/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)\,(c+d\,x)^4}{\sin(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^2,x)
[Out] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cos(a+bx)\csc^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a)**2,x)
[Out] Integral((c + d*x)**4*cos(a + b*x)*csc(a + b*x)**2, x)
```

# 3.40 $\int (c+dx)^3 \cot(a+bx) \csc(a+bx)\,dx$

**Optimal**. Leaf size=146

$$-\frac{6d^3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}+\frac{6id^2(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{6d(c+dx)^2\tanh^{-1}}{b^2}$$

```
[Out] -6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*(
d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^3-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+
a)))/b^3-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)
))/b^4
```

**Rubi [A]** time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4410, 4183, 2531, 2282, 6589}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{6d^3\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{PolyLog}(}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^3*Csc[a + b*x]
)/b + ((6*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(
c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, -E^(I*(a + b*
x))])/b^4 + (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\cot(a+bx)\csc(a+bx)\,dx = -\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{(3d)\int (c+dx)^2\csc(a+bx)\,dx}{b}$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}-\frac{\left(6d^2\right)\int (c+$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{6id^2(c+dx)}{}$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{6id^2(c+dx)}{}$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{6id^2(c+dx)}{}$$

**Mathematica [B]** time = 1.17, size = 311, normalized size = 2.13

$$-\frac{b^3c^3\csc(a+bx)+3b^3c^2dx\csc(a+bx)+3b^3cd^2x^2\csc(a+bx)+b^3d^3x^3\csc(a+bx)-3b^2c^2d\log\left(1-e^{i(a+bx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -((b^3*c^3*Csc[a + b*x] + 3*b^3*c^2*d*x*Csc[a + b*x] + 3*b^3*c*d^2*x^2*Csc[
a + b*x] + b^3*d^3*x^3*Csc[a + b*x] - 3*b^2*c^2*d*Log[1 - E^(I*(a + b*x))]
- 6*b^2*c*d^2*x*Log[1 - E^(I*(a + b*x))] - 3*b^2*d^3*x^2*Log[1 - E^(I*(a +
b*x))] + 3*b^2*c^2*d*Log[1 + E^(I*(a + b*x))] + 6*b^2*c*d^2*x*Log[1 + E^(I*
(a + b*x))] + 3*b^2*d^3*x^2*Log[1 + E^(I*(a + b*x))] - (6*I)*b*d^2*(c + d*x
)*PolyLog[2, -E^(I*(a + b*x))] + (6*I)*b*d^2*(c + d*x)*PolyLog[2, E^(I*(a +
 b*x))] + 6*d^3*PolyLog[3, -E^(I*(a + b*x))] - 6*d^3*PolyLog[3, E^(I*(a + b
*x))])/b^4)
```

**fricas [C]** time = 0.55, size = 669, normalized size = 4.58

$$-\frac{2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b^3c^3-6\,d^3\mathrm{polylog}\left(3,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\sin\left(bx+a\right)-6\,d^3\mathrm{p}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 6*d^3*p
olylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, co
s(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a)
+ I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) - I*sin(b*x
 + a))*sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(cos(b*x + a) + I*s
in(b*x + a))*sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(cos(b*x + a)
- I*sin(b*x + a))*sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(-cos(b*
x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(-
cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*
x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2
*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1
```

```
)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*
(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) + I
*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*
d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a))/(b^4*s
in(b*x + a))
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cos(bx+a) \csc(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a)^2, x)
```

**maple** [B] time = 0.09, size = 433, normalized size = 2.97

$$-\frac{6id^3 \operatorname{polylog}\left(2, e^{i(bx+a)}\right) x}{b^3}+\frac{6d^2c \ln\left(1-e^{i(bx+a)}\right) x}{b^2}+\frac{6d^2c \ln\left(1-e^{i(bx+a)}\right) a}{b^3}-\frac{6d^2c \ln\left(e^{i(bx+a)}+1\right) x}{b^2}-\frac{6d^2c \ln\left(e^{i(}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x)
```

```
[Out] -6*I/b^3*d^2*c*polylog(2,exp(I*(b*x+a)))+6/b^2*d^2*c*ln(1-exp(I*(b*x+a)))*x
+6/b^3*d^2*c*ln(1-exp(I*(b*x+a)))*a-6/b^2*d^2*c*ln(exp(I*(b*x+a))+1)*x-6/b^
3*d^2*c*ln(exp(I*(b*x+a))+1)*a-6/b^2*d*c^2*arctanh(exp(I*(b*x+a)))-6/b^4*d^
3*a^2*arctanh(exp(I*(b*x+a)))-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*po
lylog(3,exp(I*(b*x+a)))/b^4+12/b^3*d^2*c*a*arctanh(exp(I*(b*x+a)))+6*I/b^3*
d^2*c*polylog(2,-exp(I*(b*x+a)))-6*I/b^3*d^3*polylog(2,exp(I*(b*x+a)))*x-3/
b^2*d^3*ln(exp(I*(b*x+a))+1)*x^2+3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^2-2*I*(d^
3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)+3/b^
2*d^3*ln(1-exp(I*(b*x+a)))*x^2-3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^2+6*I/b^3*d
^3*polylog(2,-exp(I*(b*x+a)))*x
```

**maxima** [B] time = 0.56, size = 1770, normalized size = 12.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x
+ 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*
x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b
*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a)
)*c^2*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*b) - 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x
+ 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*
x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b
*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a)
)*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) +
1)*b^2) + 3*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*
b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(
2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
```

```
 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x
+ a))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a
) + 1)*b^3) + 2*c^3/sin(b*x + a) - 6*a*c^2*d/(b*sin(b*x + a)) + 6*a^2*c*d^2
/(b^2*sin(b*x + a)) - 2*a^3*d^3/(b^3*sin(b*x + a)) - 2*((6*(b*x + a)^2*d^3
+ 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)
*(b*x + a))*cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*
a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1)
 + (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^3 +
 (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x +
 a), -cos(b*x + a) + 1) - 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a
)^2)*cos(b*x + a) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2
 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a
)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (12*b*c*d^2
 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2
*b*x + 2*a) - (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(2*b*x +
2*a))*dilog(e^(I*b*x + I*a)) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*
d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x +
a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*
sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
 - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (3*I*(b*x
 + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b
*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*
x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-12*I*d^3*cos(2*b*x + 2*
a) + 12*d^3*sin(2*b*x + 2*a) + 12*I*d^3)*polylog(3, -e^(I*b*x + I*a)) - (12
*I*d^3*cos(2*b*x + 2*a) - 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*polylog(3, e^
(I*b*x + I*a)) - (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x +
a)^2)*sin(b*x + a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a) + 2
*I*b^3))/b
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^3}{{\sin\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^2,x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)^{3}\cos\left(a+bx\right)\csc^{2}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)*csc(a + b*x)**2, x)
```

# 3.41 $\int (c+dx)^2 \cot(a+bx)\csc(a+bx)\,dx$

**Optimal**. Leaf size=90

$$\frac{2id^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

```
[Out] -4*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^2*csc(b*x+a)/b+2*I*d^2*pol
ylog(2,-exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3
```

**Rubi [A]** time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4410, 4183, 2279, 2391}

$$\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-4*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^2*Csc[a + b*x])/
b + ((2*I)*d^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((2*I)*d^2*PolyLog[2, E^
(I*(a + b*x))])/b^3
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2\cot(a+bx)\csc(a+bx)\,dx = -\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{(2d)\int (c+dx)\csc(a+bx)\,dx}{b}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}-\frac{\left(2d^2\right)\int \log\left(\right.}{b}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{\left(2id^2\right)\text{Subst}}{}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{2id^2\text{Li}_2\left(-e^{i(a}\right.}{b^3}$$

**Mathematica [B]** time = 2.04, size = 234, normalized size = 2.60

$$-2b^2\csc(a)(c+dx)^2+b^2\csc\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)(c+dx)^2\csc\left(\frac{1}{2}(a+bx)\right)-b^2\sec\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)(c+dx)^2\sec\left(\frac{1}{2}(a+b\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-8*b*c*d*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] - 2*b^2*(c + d*x)^2*Csc[a]
+ 4*d^2*(2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + (((b*x +
ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b*x
+ ArcTan[Tan[a]]))]) + I*PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - I*Poly
Log[2, E^(I*(b*x + ArcTan[Tan[a]]))])*Sec[a])/Sqrt[Sec[a]^2]) + b^2*(c + d*
x)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] - b^2*(c + d*x)^2*Sec[a/2]*Sec[
(a + b*x)/2]*Sin[(b*x)/2])/(2*b^3)
```

**fricas [B]** time = 0.53, size = 375, normalized size = 4.17

$$-\frac{b^2d^2x^2+2\,b^2cdx+b^2c^2+i\,d^2\text{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\sin\left(bx+a\right)-i\,d^2\text{Li}_2\left(\cos\left(bx+a\right)-i\,\sin\left(b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*dilog(cos(b*x + a) + I*sin(b*
x + a))*sin(b*x + a) - I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x +
 a) + I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d^2*dilo
g(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(
b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x
 + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x
 + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*c
os(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*d^2*x + a*d^2)*lo
g(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(
-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a))/(b^3*sin(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\cos\left(bx+a\right)\csc\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*csc(b*x + a)^2, x)
```

**maple [B]** time = 0.03, size = 233, normalized size = 2.59

$$-\frac{d^2x^2}{b\sin(bx+a)}+\frac{2d^2\ln\left(1-e^{i(bx+a)}\right)x}{b^2}+\frac{2d^2\ln\left(1-e^{i(bx+a)}\right)a}{b^3}-\frac{2d^2\ln\left(e^{i(bx+a)}+1\right)x}{b^2}-\frac{2d^2\ln\left(e^{i(bx+a)}+1\right)a}{b^3}-\frac{2id^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x)
```

```
[Out] -1/b*d^2/sin(b*x+a)*x^2+2/b^2*d^2*ln(1-exp(I*(b*x+a)))*x+2/b^3*d^2*ln(1-exp
(I*(b*x+a)))*a-2/b^2*d^2*ln(exp(I*(b*x+a))+1)*x-2/b^3*d^2*ln(exp(I*(b*x+a))
+1)*a-2*I/b^3*d^2*dilog(1-exp(I*(b*x+a)))+2*I/b^3*d^2*dilog(exp(I*(b*x+a))+
1)-2/b^3*a*d^2*ln(csc(b*x+a)-cot(b*x+a))-2/b*c*d/sin(b*x+a)*x+2/b^2*c*d*ln(
csc(b*x+a)-cot(b*x+a))-1/b*c^2/sin(b*x+a)
```

**maxima [B]** time = 0.52, size = 556, normalized size = 6.18

$$\frac{\left(2\,bd^2x+2\,bcd-2\left(bd^2x+bcd\right)\cos\left(2\,bx+2\,a\right)-\left(2i\,bd^2x+2i\,bcd\right)\sin\left(2\,bx+2\,a\right)\right)\arctan\left(\sin\left(bx+a\right),\cos\left(b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] ((2*b*d^2*x + 2*b*c*d - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a) - (2*I*b*d^2*x
 + 2*I*b*c*d)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (
2*b*c*d*cos(2*b*x + 2*a) + 2*I*b*c*d*sin(2*b*x + 2*a) - 2*b*c*d)*arctan2(si
n(b*x + a), cos(b*x + a) - 1) - (2*b*d^2*x*cos(2*b*x + 2*a) + 2*I*b*d^2*x*s
in(2*b*x + 2*a) - 2*b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) + 2*(d^2*cos(2*b*x + 2*a)
 + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) - 2*(d^2*cos(2*b*x
 + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x + I*a)) - (I*b*d^2*x
 + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2*x + b*c*d)*si
n(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) -
 (-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*b*x + 2*a) - (b*d^2*x
+ b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x
+ a) + 1) - (2*I*b^2*d^2*x^2 + 4*I*b^2*c*d*x + 2*I*b^2*c^2)*sin(b*x + a))/(
-I*b^3*cos(2*b*x + 2*a) + b^3*sin(2*b*x + 2*a) + I*b^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}{{\sin\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x)^2,x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2\cos(a+bx)\csc^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*csc(a + b*x)**2, x)
```

# 3.42 $\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx$

**Optimal**. Leaf size=30

$$-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}$$

```
[Out] -d*arctanh(cos(b*x+a))/b^2-(d*x+c)*csc(b*x+a)/b
```

**Rubi [A]** time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4410, 3770}

$$-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -((d*ArcTanh[Cos[a + b*x]])/b^2) - ((c + d*x)*Csc[a + b*x])/b
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx &= -\frac{(c+dx)\csc(a+bx)}{b}+\frac{d\int \csc(a+bx)\,dx}{b}\\ &= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}\end{aligned}$$

**Mathematica [B]** time = 0.06, size = 131, normalized size = 4.37

$$\frac{d\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{d\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{c\csc(a+bx)}{b}-\frac{dx\csc(a)}{b}+\frac{dx\csc\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)}{2b}-\frac{dx\sec}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -((d*x*Csc[a])/b) - (c*Csc[a + b*x])/b - (d*Log[Cos[a/2 + (b*x)/2]])/b^2 +
(d*Log[Sin[a/2 + (b*x)/2]])/b^2 + (d*x*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x
)/2])/(2*b) - (d*x*Sec[a/2]*Sec[a/2 + (b*x)/2]*Sin[(b*x)/2])/(2*b)
```

**fricas [B]** time = 0.47, size = 62, normalized size = 2.07

$$-\frac{2\,bdx+d\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)\sin\left(bx+a\right)-d\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)\sin\left(bx+a\right)+2\,bc}{2\,b^2\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - d*log(-1/2*cos
(b*x + a) + 1/2)*sin(b*x + a) + 2*b*c)/(b^2*sin(b*x + a))
```

**giac [B]** time = 0.75, size = 801, normalized size = 26.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 +
b*d*x*tan(1/2*b*x)^2 - d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x
)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b
*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) + d*log(4
*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^
2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^
2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) + b*d*x*tan(1/2*a)^2 - d*log(4*(tan(1/2*b
*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a
)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*t
an(1/2*b*x)*tan(1/2*a)^2 + d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1
/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1
/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^2 + b*c*t
an(1/2*b*x)^2 + b*c*tan(1/2*a)^2 + b*d*x + d*log(4*(tan(1/2*b*x)^4*tan(1/2*
a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*
b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x) -
d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(
1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(
1/2*a)^2 + 1))*tan(1/2*b*x) + d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(
1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*ta
n(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a) - d*log(4*(tan(1/
2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(
1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*
tan(1/2*a) + b*c)/(b^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)*tan(1/2
*a)^2 - b^2*tan(1/2*b*x) - b^2*tan(1/2*a))
```

**maple [A]** time = 0.02, size = 52, normalized size = 1.73

$$-\frac{dx}{b\sin(bx+a)}+\frac{d\ln(\csc(bx+a)-\cot(bx+a))}{b^2}-\frac{c}{b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x)
```

```
[Out] -1/b*d/sin(b*x+a)*x+1/b^2*d*ln(csc(b*x+a)-cot(b*x+a))-1/b*c/sin(b*x+a)
```

**maxima [B]** time = 0.38, size = 259, normalized size = 8.63

$$-\frac{\left(4(bx+a)\cos(bx+a)\sin(2bx+2a)-4(bx+a)\cos(2bx+2a)\sin(bx+a)+\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1\right)\log\left(\cos(bx+a)^2+\sin(bx+a\right.\right.}{\left(\cos(2bx+2a)^2+\sin(2bx+2a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x +
2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x
```

```
+ 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (co
s(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x
 + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*
d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) +
2*c/sin(b*x + a) - 2*a*d/(b*sin(b*x + a)))/b
```

**mupad [B]** time = 2.28, size = 88, normalized size = 2.93

$$-\frac{d\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{b^2}+\frac{d\ln\left(d\,2\mathrm{i}-d\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,2\mathrm{i}\right)}{b^2}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(c+d\,x\right)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x))/sin(a + b*x)^2,x)
```

```
[Out] (d*log(d*2i - d*exp(a*1i)*exp(b*x*1i)*2i))/b^2 - (d*log(exp(a*1i + b*x*1i)*
1i + 1i))/b^2 - (exp(a*1i + b*x*1i)*(c + d*x)*2i)/(b*(exp(a*2i + b*x*2i) -
1))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int(c+dx)\cos(a+bx)\csc^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*csc(a + b*x)**2, x)
```

## 3.43 $\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=23

$$\operatorname{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x),x]
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

**Mathematica [A]** time = 16.76, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x),x]
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]
```

**fricas [A]** time = 0.49, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\cos(bx+a)\csc(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(cos(b*x + a)*csc(b*x + a)^2/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*csc(b*x + a)^2/(d*x + c), x)
```

**maple [A]** time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\left(\csc^2(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\cos(bx+a)\sin(2bx+2a)-2\cos(2bx+2a)\sin(bx+a)+\frac{\left(bd^2x+bcd+\left(bd^2x+bcd\right)\cos(2bx+2a)^2+\left(bd^2x+bcd\right)\sin(2\ldots\right.}{\ldots}}{bdx+(\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*
d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(sin
(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*
cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*(b*d^2*
x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) + (b*d^2*x + b*c*d + (b*d^2*x +
b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2
*x + b*c*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x
 + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*
b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x
 + a)), x) + 2*cos(b*x + a)*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a)*sin(b*x +
 a) + 2*sin(b*x + a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x +
b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a))
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(a+b\,x)}{\sin(a+b\,x)^2\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*csc(a + b*x)**2/(c + d*x), x)
```

# 3.44 $\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=23

$$\mathrm{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

**Rubi** [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2,x]
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

**Mathematica** [A] time = 20.30, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

**fricas** [A] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos(bx+a)\csc(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cos(b*x + a)*csc(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(cos(b*x + a)*csc(b*x + a)^2/(d*x + c)^2, x)
```

**maple** [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\left(\csc^2(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
[Out] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(a+b\,x)}{\sin(a+b\,x)^2\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)^2),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)^2), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(cos(a + b*x)*csc(a + b*x)**2/(c + d*x)**2, x)
```

# 3.45 $\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\cot(a+bx)\csc^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a)^2,x)
```

**Rubi [A]** time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$$

**Mathematica [A]** time = 6.31, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]
```

**fricas [A]** time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \cos(bx+a)\csc(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

**maple [A]** time = 0.10, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\left(\csc^3(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\csc(bx+a)^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(a+b\,x)\,\left(c+d\,x\right)^m}{\sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^3,x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^3, x)
```

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

## 3.46 $\int (c+dx)^4 \cot(a+bx)\csc^2(a+bx)\,dx$

**Optimal**. Leaf size=137

$$\frac{3d^4\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^5}-\frac{6id^3(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^4}+\frac{6d^2(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4}{\ldots}$$

```
[Out] -2*I*d*(d*x+c)^3/b^2-2*d*(d*x+c)^3*cot(b*x+a)/b^2-1/2*(d*x+c)^4*csc(b*x+a)^
2/b+6*d^2*(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b^3-6*I*d^3*(d*x+c)*polylog(2,ex
p(2*I*(b*x+a)))/b^4+3*d^4*polylog(3,exp(2*I*(b*x+a)))/b^5
```

**Rubi [A]** time = 0.26, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4410, 4184, 3717, 2190, 2531, 2282, 6589}

$$-\frac{6id^3(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^4}+\frac{3d^4\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^5}+\frac{6d^2(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)^3\cot(\ldots)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] ((-2*I)*d*(c + d*x)^3)/b^2 - (2*d*(c + d*x)^3*Cot[a + b*x])/b^2 - ((c + d*x
)^4*Csc[a + b*x]^2)/(2*b) + (6*d^2*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))]
)/b^3 - ((6*I)*d^3*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*
PolyLog[3, E^((2*I)*(a + b*x))])/b^5
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4 \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)^4\csc^2(a+bx)}{2b} + \frac{(2d)\int (c+dx)^3\csc^2(a+bx)\,dx}{b}$$
$$= -\frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\csc^2(a+bx)}{2b} + \frac{\left(6d^2\right)\int (c+dx}{b}$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\csc^2(a+bx)}{2b} -$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\csc^2(a+bx)}{2b} +$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\csc^2(a+bx)}{2b} +$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\csc^2(a+bx)}{2b} +$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\csc^2(a+bx)}{2b} +$$

**Mathematica [B]** time = 6.61, size = 504, normalized size = 3.68

$$\frac{6c^2d^2\csc(a)(\sin(a)\log(\sin(a)\cos(bx)+\cos(a)\sin(bx))-bx\cos(a))}{b^3\left(\sin^2(a)+\cos^2(a)\right)} + \frac{2\csc(a)\csc(a+bx)\left(c^3d\sin(bx)+3c^2d\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -1/2*((c + d*x)^4*Csc[a + b*x]^2)/b - (d^4*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((
2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3
*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2
*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b
*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*
x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^5 + (6*c^2*d^2*Cs
c[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*
```

```
(Cos[a]^2 + Sin[a]^2)) + (2*Csc[a]*Csc[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2
*x*Sin[b*x] + 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2 - (6*c*d^3*Csc[
a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]])
- Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b
*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x +
ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/
Sqrt[1 + Tan[a]^2]))/(b^4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

**fricas [C]** time = 0.70, size = 1071, normalized size = 7.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4 + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*cos
(b*x + a)*sin(b*x + a) + (12*I*b*d^4*x + 12*I*b*c*d^3 + (-12*I*b*d^4*x - 12
*I*b*c*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (-12*I*b
*d^4*x - 12*I*b*c*d^3 + (12*I*b*d^4*x + 12*I*b*c*d^3)*cos(b*x + a)^2)*dilog
(cos(b*x + a) - I*sin(b*x + a)) + (-12*I*b*d^4*x - 12*I*b*c*d^3 + (12*I*b*d
^4*x + 12*I*b*c*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))
+ (12*I*b*d^4*x + 12*I*b*c*d^3 + (-12*I*b*d^4*x - 12*I*b*c*d^3)*cos(b*x + a
)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x
 + b^2*c^2*d^2 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2
)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x +
 b^2*c^2*d^2 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2)*
log(cos(b*x + a) - I*sin(b*x + a) + 1) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2
*d^4 - (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b
*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^
4 - (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b*x
+ a) - 1/2*I*sin(b*x + a) + 1/2) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c
*d^3 - a^2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(
b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^
2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*
d^3 - a^2*d^4)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 12
*(d^4*cos(b*x + a)^2 - d^4)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 12*
(d^4*cos(b*x + a)^2 - d^4)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 12*(
d^4*cos(b*x + a)^2 - d^4)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 12*(
d^4*cos(b*x + a)^2 - d^4)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^5*
cos(b*x + a)^2 - b^5)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \cos(bx+a) \csc(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a)^3, x)
```

**maple [B]** time = 0.13, size = 716, normalized size = 5.23

$$-\frac{24id^3cax}{b^3}+\frac{12d^4 \operatorname{polylog}\left(3,-e^{i(bx+a)}\right)}{b^5}+\frac{12d^4 \operatorname{polylog}\left(3,e^{i(bx+a)}\right)}{b^5}+\frac{6d^4a^2 \ln\left(e^{i(bx+a)}-1\right)}{b^5}-\frac{12d^4a^2 \ln\left(e^{i(bx+a)}\right)}{b^5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] -24*I/b^3*d^3*c*a*x+12*d^4*polylog(3,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(3,
exp(I*(b*x+a)))/b^5+6/b^5*d^4*a^2*ln(exp(I*(b*x+a))-1)-12/b^5*d^4*a^2*ln(ex
p(I*(b*x+a)))+6/b^3*d^2*c^2*ln(exp(I*(b*x+a))-1)+6/b^3*d^2*c^2*ln(exp(I*(b*
x+a))+1)-12/b^3*d^2*c^2*ln(exp(I*(b*x+a)))-6/b^5*d^4*a^2*ln(1-exp(I*(b*x+a)
))+6/b^3*d^4*ln(1-exp(I*(b*x+a)))*x^2+6/b^3*d^4*ln(exp(I*(b*x+a))+1)*x^2-4*
I/b^2*d^4*x^3+8*I/b^5*d^4*a^3+2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*e
xp(2*I*(b*x+a))+6*b*c^2*d^2*x^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a
))-2*I*d^4*x^3*exp(2*I*(b*x+a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*
I*(b*x+a))-6*I*c^2*d^2*x*exp(2*I*(b*x+a))+2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*
x+a))+6*I*c*d^3*x^2+6*I*c^2*d^2*x+2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))-1)^2+12/
b^3*d^3*c*ln(exp(I*(b*x+a))+1)*x+12/b^3*d^3*c*ln(1-exp(I*(b*x+a)))*x+12/b^4
*d^3*c*ln(1-exp(I*(b*x+a)))*a+24/b^4*d^3*c*a*ln(exp(I*(b*x+a)))-12/b^4*d^3*
c*a*ln(exp(I*(b*x+a))-1)-12*I/b^4*d^3*c*polylog(2,-exp(I*(b*x+a)))-12*I/b^4
*d^3*c*polylog(2,exp(I*(b*x+a)))+12*I/b^4*d^4*a^2*x-12*I/b^2*d^3*c*x^2-12*I
/b^4*d^3*c*a^2-12*I/b^4*d^4*polylog(2,-exp(I*(b*x+a)))*x-12*I/b^4*d^4*polyl
og(2,exp(I*(b*x+a)))*x
```

**maxima [B]** time = 0.89, size = 4540, normalized size = 33.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (
2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x
+ a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) +
1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*c^3*d/((2*(2*cos(2*b*x + 2*a) - 1)*
cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x +
4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos
(2*b*x + 2*a) - 1)*b) - 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*si
n(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*
b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) -
 cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*a*c^2*d^2/((2*(
2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x
 + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*si
n(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^2) + 24*(4*(b*x + a)*cos(2*b*x
 + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a)
+ sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b
*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b
*x + 2*a))*a^2*c*d^3/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*
b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*
a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^3) -
 8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b
*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)
*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*s
in(4*b*x + 4*a) + sin(2*b*x + 2*a))*a^3*d^4/((2*(2*cos(2*b*x + 2*a) - 1)*co
s(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*
a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2
*b*x + 2*a) - 1)*b^4) + 6*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2
*sin(2*b*x + 2*a)^2 - 4*(b*x + a)^2*cos(2*b*x + 2*a) - 4*((b*x + a)^2*cos(2
*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*(2*cos(2*b*
x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2
- sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x +
2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*co
s(b*x + a) + 1) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x
+ 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*s
in(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*
x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*((b*x + a)^2*sin(2*b*x
+ 2*a) + b*x - (b*x + a)*cos(2*b*x + 2*a) + a)*sin(4*b*x + 4*a) + 4*(b*x +
```

```
a)*sin(2*b*x + 2*a))*c^2*d^2/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a)
- cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*
b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1
)*b^2) - 12*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2
*a)^2 - 4*(b*x + a)^2*cos(2*b*x + 2*a) - 4*((b*x + a)^2*cos(2*b*x + 2*a) +
(b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*(2*cos(2*b*x + 2*a) - 1)*
cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x +
4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos
(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1
) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*c
os(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin
(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*((b*x + a)^2*sin(2*b*x + 2*a) + b*x -
 (b*x + a)*cos(2*b*x + 2*a) + a)*sin(4*b*x + 4*a) + 4*(b*x + a)*sin(2*b*x +
 2*a))*a*c*d^3/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x +
4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^3) + 6*(8*
(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 - 4*(b*x
+ a)^2*cos(2*b*x + 2*a) - 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2
*b*x + 2*a))*cos(4*b*x + 4*a) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a
) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) -
 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*(2*cos(2
*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)
^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x
 + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2
*cos(b*x + a) + 1) - 4*((b*x + a)^2*sin(2*b*x + 2*a) + b*x - (b*x + a)*cos(
2*b*x + 2*a) + a)*sin(4*b*x + 4*a) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^4/
((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(
2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^4) - c^4/sin(b*x + a)^2 +
 4*a*c^3*d/(b*sin(b*x + a)^2) - 6*a^2*c^2*d^2/(b^2*sin(b*x + a)^2) + 4*a^3*
c*d^3/(b^3*sin(b*x + a)^2) - a^4*d^4/(b^4*sin(b*x + a)^2) + 2*((6*(b*x + a)
^2*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a) + 6*((b*x + a)^2*d^4 + 2*(b*c*d^3 -
 a*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 12*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*
d^4)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^4 + (12*I*b*c*d^3 - 1
2*I*a*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (-12*I*(b*x + a)^2*d^4 + (-24*I*b*
c*d^3 + 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(
b*x + a) + 1) - (6*(b*x + a)^2*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a) + 6*((b
*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 12*((b*x
+ a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x +
 a)^2*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - (12*
I*(b*x + a)^2*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a)
)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*((b*x + a)^3*d^4 + 3*(b*c*d^
3 - a*d^4)*(b*x + a)^2)*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^4*d^4 + (-8*I*b*
c*d^3 - 4*(-2*I*a - 1)*d^4)*(b*x + a)^3 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2)
*cos(2*b*x + 2*a) - (12*b*c*d^3 + 12*(b*x + a)*d^4 - 12*a*d^4 + 12*(b*c*d^3
 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) - 24*(b*c*d^3 + (b*x + a)*d^4 -
a*d^4)*cos(2*b*x + 2*a) - (-12*I*b*c*d^3 - 12*I*(b*x + a)*d^4 + 12*I*a*d^4)
*sin(4*b*x + 4*a) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 24*I*a*d^4)*sin(2*
b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - (12*b*c*d^3 + 12*(b*x + a)*d^4 - 12*a
*d^4 + 12*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) - 24*(b*c*d^3
+ (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) - (-12*I*b*c*d^3 - 12*I*(b*x + a)
*d^4 + 12*I*a*d^4)*sin(4*b*x + 4*a) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 -
24*I*a*d^4)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-3*I*(b*x + a)^2*d^
4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*
c*d^3 + 6*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (6*I*(b*x + a)^2*d^4 + (12
*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^4 +
 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 6*((b*x + a)^2*d^4 + 2*(
```

```
b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 + 2*cos(b*x + a) + 1) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*
a*d^4)*(b*x + a) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x
+ a))*cos(4*b*x + 4*a) + (6*I*(b*x + a)^2*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)
*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*
x + a))*sin(4*b*x + 4*a) - 6*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x +
a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a)
+ 1) + (-12*I*d^4*cos(4*b*x + 4*a) + 24*I*d^4*cos(2*b*x + 2*a) + 12*d^4*sin
(4*b*x + 4*a) - 24*d^4*sin(2*b*x + 2*a) - 12*I*d^4)*polylog(3, -e^(I*b*x +
I*a)) + (-12*I*d^4*cos(4*b*x + 4*a) + 24*I*d^4*cos(2*b*x + 2*a) + 12*d^4*si
n(4*b*x + 4*a) - 24*d^4*sin(2*b*x + 2*a) - 12*I*d^4)*polylog(3, e^(I*b*x +
I*a)) + (-4*I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2)*s
in(4*b*x + 4*a) + (2*(b*x + a)^4*d^4 + (8*b*c*d^3 - (8*a - 4*I)*d^4)*(b*x +
 a)^3 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-I*b^4*
cos(4*b*x + 4*a) + 2*I*b^4*cos(2*b*x + 2*a) + b^4*sin(4*b*x + 4*a) - 2*b^4*
sin(2*b*x + 2*a) - I*b^4))/b
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+bx)\,(c+dx)^4}{\sin(a+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^3,x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^3, x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.47 $\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx) \, dx$

**Optimal**. Leaf size=115

$$-\frac{3id^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^4}+\frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}-\frac{3id(c+dx)^2}{2b^2}$$

```
[Out] -3/2*I*d*(d*x+c)^2/b^2-3/2*d*(d*x+c)^2*cot(b*x+a)/b^2-1/2*(d*x+c)^3*csc(b*x
+a)^2/b+3*d^2*(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b^3-3/2*I*d^3*polylog(2,exp(2*
I*(b*x+a)))/b^4
```

**Rubi [A]** time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = $\{4410, 4184, 3717, 2190, 2279, 2391\}$

$$-\frac{3id^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^4}+\frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}-\frac{3id(c+dx)^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - (
(c + d*x)^3*Csc[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a +
b*x))])/b^3 - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)^3\csc^2(a+bx)}{2b}+\frac{(3d)\int (c+dx)^2\csc^2(a+bx)\,dx}{2b}$$
$$= -\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}+\frac{\left(3d^2\right)\int (c+dx$$
$$= -\frac{3id(c+dx)^2}{2b^2}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}-$$
$$= -\frac{3id(c+dx)^2}{2b^2}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}+$$
$$= -\frac{3id(c+dx)^2}{2b^2}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}+$$
$$= -\frac{3id(c+dx)^2}{2b^2}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}+$$

**Mathematica [B]** time = 6.41, size = 277, normalized size = 2.41

$$\frac{3cd^2\csc(a)(\sin(a)\log(\sin(a)\cos(bx)+\cos(a)\sin(bx))-bx\cos(a))}{b^3\left(\sin^2(a)+\cos^2(a)\right)}+\frac{3\csc(a)\csc(a+bx)\left(c^2d\sin(bx)+2cd^2x\right.}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -1/2*((c + d*x)^3*Csc[a + b*x]^2)/b + (3*c*d^2*Csc[a]*(-(b*x*Cos[a]) + Log[
Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (
3*Csc[a]*Csc[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*
x]))/(2*b^2) - (3*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x
*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Ta
n[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*Arc
Tan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + A
rcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^4*Sqrt[Sec[a]^2*(Cos[a]
^2 + Sin[a]^2)])
```

**fricas [B]** time = 0.75, size = 587, normalized size = 5.10

$$b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3+3\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d\right)\cos(bx+a)\sin(bx+a)+\left(-3i\,d^3\cos(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 3*(b^2*d^3*x
^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) + (-3*I*d^3*cos(b
*x + a)^2 + 3*I*d^3)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (3*I*d^3*cos(b*
```

```
x + a)^2 - 3*I*d^3)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (3*I*d^3*cos(b*x
 + a)^2 - 3*I*d^3)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-3*I*d^3*cos(b*
x + a)^2 + 3*I*d^3)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*d^3*x + b*
c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x +
a) + 1) - 3*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*log(co
s(b*x + a) - I*sin(b*x + a) + 1) - 3*(b*c*d^2 - a*d^3 - (b*c*d^2 - a*d^3)*c
os(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*(b*c*d
^2 - a*d^3 - (b*c*d^2 - a*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*
I*sin(b*x + a) + 1/2) - 3*(b*d^3*x + a*d^3 - (b*d^3*x + a*d^3)*cos(b*x + a)
^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*d^3*x + a*d^3 - (b*d^3*x
 + a*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1))/(b^4*cos
(b*x + a)^2 - b^4)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \cos (bx + a) \csc (bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a)^3, x)
```

**maple [B]** time = 0.10, size = 409, normalized size = 3.56

$$\frac{2b\,d^3x^3e^{2i(bx+a)} - 3id^3x^2e^{2i(bx+a)} + 6bc\,d^2x^2e^{2i(bx+a)} - 6ic\,d^2x\,e^{2i(bx+a)} + 6b\,c^2dx\,e^{2i(bx+a)} - 3ic^2d\,e^{2i(bx+a)} + 3id^3x^2}{b^2\left(e^{2i(bx+a)} - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] (2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*ex
p(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*
I*c^2*d*exp(2*I*(b*x+a))+3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*x+3
*I*c^2*d)/b^2/(exp(2*I*(b*x+a))-1)^2+3/b^3*d^2*c*ln(exp(I*(b*x+a))-1)+3/b^3
*d^2*c*ln(exp(I*(b*x+a))+1)-6/b^3*d^2*c*ln(exp(I*(b*x+a)))-3*I/b^2*d^3*x^2-
6*I/b^3*d^3*a*x-3*I/b^4*d^3*a^2+3/b^3*d^3*ln(exp(I*(b*x+a))+1)*x-3*I/b^4*d^
3*polylog(2,-exp(I*(b*x+a)))+3/b^3*d^3*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln(
1-exp(I*(b*x+a)))*a-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-3/b^4*d^3*a*ln(ex
p(I*(b*x+a))-1)+6/b^4*d^3*a*ln(exp(I*(b*x+a)))
```

**maxima [B]** time = 0.70, size = 1044, normalized size = 9.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (6*b^2*c^2*d + (6*b*d^3*x + 6*b*c*d^2 + 6*(b*d^3*x + b*c*d^2)*cos(4*b*x + 4
*a) - 12*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a) + (6*I*b*d^3*x + 6*I*b*c*d^2)
*sin(4*b*x + 4*a) + (-12*I*b*d^3*x - 12*I*b*c*d^2)*sin(2*b*x + 2*a))*arctan
2(sin(b*x + a), cos(b*x + a) + 1) + (6*b*c*d^2*cos(4*b*x + 4*a) - 12*b*c*d^
2*cos(2*b*x + 2*a) + 6*I*b*c*d^2*sin(4*b*x + 4*a) - 12*I*b*c*d^2*sin(2*b*x
+ 2*a) + 6*b*c*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) - (6*b*d^3*x*co
s(4*b*x + 4*a) - 12*b*d^3*x*cos(2*b*x + 2*a) + 6*I*b*d^3*x*sin(4*b*x + 4*a)
 - 12*I*b*d^3*x*sin(2*b*x + 2*a) + 6*b*d^3*x)*arctan2(sin(b*x + a), -cos(b*
x + a) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x)*cos(4*b*x + 4*a) + (-4*I*b^3*
d^3*x^3 - 4*I*b^3*c^3 - 6*b^2*c^2*d + (-12*I*b^3*c*d^2 + 6*b^2*d^3)*x^2 + (
-12*I*b^3*c^2*d + 12*b^2*c*d^2)*x)*cos(2*b*x + 2*a) - (6*d^3*cos(4*b*x + 4*
```

```
a) - 12*d^3*cos(2*b*x + 2*a) + 6*I*d^3*sin(4*b*x + 4*a) - 12*I*d^3*sin(2*b*
x + 2*a) + 6*d^3)*dilog(-e^(I*b*x + I*a)) - (6*d^3*cos(4*b*x + 4*a) - 12*d^
3*cos(2*b*x + 2*a) + 6*I*d^3*sin(4*b*x + 4*a) - 12*I*d^3*sin(2*b*x + 2*a) +
 6*d^3)*dilog(e^(I*b*x + I*a)) + (-3*I*b*d^3*x - 3*I*b*c*d^2 + (-3*I*b*d^3*
x - 3*I*b*c*d^2)*cos(4*b*x + 4*a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(2*b*x +
 2*a) + 3*(b*d^3*x + b*c*d^2)*sin(4*b*x + 4*a) - 6*(b*d^3*x + b*c*d^2)*sin(
2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (
-3*I*b*d^3*x - 3*I*b*c*d^2 + (-3*I*b*d^3*x - 3*I*b*c*d^2)*cos(4*b*x + 4*a)
+ (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(2*b*x + 2*a) + 3*(b*d^3*x + b*c*d^2)*sin(
4*b*x + 4*a) - 6*(b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*
x)*sin(4*b*x + 4*a) + (4*b^3*d^3*x^3 + 4*b^3*c^3 - 6*I*b^2*c^2*d + 6*(2*b^3
*c*d^2 + I*b^2*d^3)*x^2 + 12*(b^3*c^2*d + I*b^2*c*d^2)*x)*sin(2*b*x + 2*a))
/(-2*I*b^4*cos(4*b*x + 4*a) + 4*I*b^4*cos(2*b*x + 2*a) + 2*b^4*sin(4*b*x +
4*a) - 4*b^4*sin(2*b*x + 2*a) - 2*I*b^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,\left(c+d\,x\right)^3}{\sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^3,x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cos(a+bx) \csc^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)*csc(a + b*x)**3, x)
```

## 3.48 $\int (c+dx)^2 \cot(a+bx) \csc^2(a+bx)\, dx$

**Optimal**. Leaf size=54

$$\frac{d^2\log(\sin(a+bx))}{b^3}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\csc^2(a+bx)}{2b}$$

```
[Out] -d*(d*x+c)*cot(b*x+a)/b^2-1/2*(d*x+c)^2*csc(b*x+a)^2/b+d^2*ln(sin(b*x+a))/b
^3
```

**Rubi [A]** time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4410, 4184, 3475}

$$-\frac{d(c+dx)\cot(a+bx)}{b^2}+\frac{d^2\log(\sin(a+bx))}{b^3}-\frac{(c+dx)^2\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(c + d*x)*Cot[a + b*x])/b^2) - ((c + d*x)^2*Csc[a + b*x]^2)/(2*b) + (d
^2*Log[Sin[a + b*x]])/b^3
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2\cot(a+bx)\csc^2(a+bx)\,dx &= -\frac{(c+dx)^2\csc^2(a+bx)}{2b}+\frac{d\int (c+dx)\csc^2(a+bx)\,dx}{b}\\
&= -\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\csc^2(a+bx)}{2b}+\frac{d^2\int \cot(a+bx)\,dx}{b^2}\\
&= -\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\csc^2(a+bx)}{2b}+\frac{d^2\log(\sin(a+bx))}{b^3}
\end{aligned}$$

**Mathematica [C]** time = 0.89, size = 94, normalized size = 1.74

$$\frac{-b^2(c+dx)^2\csc^2(a+bx)+2bd\csc(a)\sin(bx)(c+dx)\csc(a+bx)-2id^2\tan^{-1}(\tan(a+bx))-2bd^2x\cot(a)+d^2 \ldots}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] ((2*I)*b*d^2*x - (2*I)*d^2*ArcTan[Tan[a + b*x]] - 2*b*d^2*x*Cot[a] - b^2*(c
 + d*x)^2*Csc[a + b*x]^2 + d^2*Log[Sin[a + b*x]^2] + 2*b*d*(c + d*x)*Csc[a]
*Csc[a + b*x]*Sin[b*x])/(2*b^3)
```

**fricas** [A] time = 0.72, size = 102, normalized size = 1.89

$$\frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + 2\left(bd^2x + bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right) + 2\left(d^2\cos\left(bx+a\right)^2 - d^2\right)\log\left(\frac{1}{2}\,\sin\left(bx+\right.\right.}{2\left(b^3\cos\left(bx+a\right)^2 - b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)
*sin(b*x + a) + 2*(d^2*cos(b*x + a)^2 - d^2)*log(1/2*sin(b*x + a)))/(b^3*co
s(b*x + a)^2 - b^3)
```

**giac** [B] time = 2.88, size = 3482, normalized size = 64.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4*
tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d^2*x^2*ta
n(1/2*b*x)^2*tan(1/2*a)^4 + b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^2*c*d
*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*
b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 4*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)
^4 + b^2*d^2*x^2*tan(1/2*b*x)^4 + 4*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2
 + 2*b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a
)^3 + b^2*d^2*x^2*tan(1/2*a)^4 + 2*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 4*
b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4 + 4*b*d^2*x*
tan(1/2*b*x)^4*tan(1/2*a) + 8*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 24*b*
d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^2 - 4*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)
^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/
2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(
1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^
5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^
3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1
/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1
/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan
(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^2 + 24*b*d^2*x*tan(1/2*b*x)^2*tan
(1/2*a)^3 - 8*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*ta
n(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*
tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x
)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2
*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1
/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2
*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*
x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^3*tan(1/2*a)^3 + 2*b^2*c*d*x*tan(1/2*a)^4 + 4*b*d^2*x*tan(1/2*b*x)*tan
(1/2*a)^4 - 4*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*ta
n(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*
tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x
)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2
*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1
/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2
```

```
*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*
x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^2*tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^2 + b^2*c^2*tan(1/2*b*x)^4
 + 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^2*d^2*x^2*tan(1/2*a)^2 + 4*b^2*c
^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 24*b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 24*
b*c*d*tan(1/2*b*x)^2*tan(1/2*a)^3 + b^2*c^2*tan(1/2*a)^4 + 4*b*c*d*tan(1/2*
b*x)*tan(1/2*a)^4 + 4*b^2*c*d*x*tan(1/2*b*x)^2 - 4*b*d^2*x*tan(1/2*b*x)^3 -
 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a) + 8*d^2*log(16*(tan(1/2*b*x)^8*tan(1/
2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*ta
n(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*
tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b
*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2
*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*t
an(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + t
an(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2
*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a) + 4*b^2*c*d*x*tan(1/2*a)^2 -
24*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^2 + 16*d^2*log(16*(tan(1/2*b*x)^8*tan(1/
2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*ta
n(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*
tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b
*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2
*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*t
an(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + t
an(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2
*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 4*b*d^2*x*tan(1/2*a)^3 +
8*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 +
 tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)
^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*
a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(
1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*
tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^
2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a
) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2
*a)^3 + b^2*d^2*x^2 + 2*b^2*c^2*tan(1/2*b*x)^2 - 4*b*c*d*tan(1/2*b*x)^3 - 2
4*b*c*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c^2*tan(1/2*a)^2 - 24*b*c*d*tan(1
/2*b*x)*tan(1/2*a)^2 - 4*b*c*d*tan(1/2*a)^3 + 2*b^2*c*d*x + 4*b*d^2*x*tan(1
/2*b*x) - 4*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(
1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*ta
n(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^
4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b
*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2
*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*t
an(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)
*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*
x)^2 + 4*b*d^2*x*tan(1/2*a) - 8*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2
*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)
^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)
^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(
1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + ta
n(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a)
- 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x
)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a
)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) - 4*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)
^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/
2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(
1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^
5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^
3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1
/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1
```

```
/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan
(1/2*a)^2 + 1))*tan(1/2*a)^2 + b^2*c^2 + 4*b*c*d*tan(1/2*b*x) + 4*b*c*d*tan
(1/2*a))/(b^3*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^3*tan(1/2*b*x)^3*tan(1/2*a)
^3 + b^3*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*b^3*tan(1/2*b*x)^3*tan(1/2*a) - 4*
b^3*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^3*tan(1/2*b*x)*tan(1/2*a)^3 + b^3*tan
(1/2*b*x)^2 + 2*b^3*tan(1/2*b*x)*tan(1/2*a) + b^3*tan(1/2*a)^2)
```

**maple [A]** time = 0.03, size = 95, normalized size = 1.76

$$-\frac{d^2x^2}{2b\sin(bx+a)^2}-\frac{d^2\cot(bx+a)x}{b^2}+\frac{d^2\ln(\sin(bx+a))}{b^3}-\frac{cdx}{b\sin(bx+a)^2}-\frac{cd\cot(bx+a)}{b^2}-\frac{c^2}{2b\sin(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] -1/2/b*d^2/sin(b*x+a)^2*x^2-1/b^2*d^2*cot(b*x+a)*x+d^2*ln(sin(b*x+a))/b^3-1
/b*c*d/sin(b*x+a)^2*x-1/b^2*c*d*cot(b*x+a)-1/2/b*c^2/sin(b*x+a)^2
```

**maxima [B]** time = 0.36, size = 1130, normalized size = 20.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (
2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x
+ a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) +
1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*c*d/((2*(2*cos(2*b*x + 2*a) - 1)*co
s(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*
a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2
*b*x + 2*a) - 1)*b) - 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2
*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x
 + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - co
s(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*a*d^2/((2*(2*cos(2
*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)
^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x
 + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^2) + (8*(b*x + a)^2*cos(2*b*x + 2*a)^
2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 - 4*(b*x + a)^2*cos(2*b*x + 2*a) - 4*(
(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*co
s(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
 - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(
b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x +
4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a
) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*((b*x
+ a)^2*sin(2*b*x + 2*a) + b*x - (b*x + a)*cos(2*b*x + 2*a) + a)*sin(4*b*x +
 4*a) + 4*(b*x + a)*sin(2*b*x + 2*a))*d^2/((2*(2*cos(2*b*x + 2*a) - 1)*cos(
4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)
^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b
*x + 2*a) - 1)*b^2) - c^2/sin(b*x + a)^2 + 2*a*c*d/(b*sin(b*x + a)^2) - a^2
*d^2/(b^2*sin(b*x + a)^2))/b
```

**mupad [B]** time = 2.56, size = 147, normalized size = 2.72

$$\frac{\frac{(c+d\,x)^2}{b}+\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,(c+d\,x)^2}{b}}{1+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}}-\frac{d^2\,x\,2\mathrm{i}}{b^2}+\frac{b\,c^2+2\,b\,c\,d\,x-c\,d\,2\mathrm{i}+b\,d^2\,x^2-d^2\,x\,2\mathrm{i}}{b^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}+\frac{d^2\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x)^3,x)
```

```
[Out] ((c + d*x)^2/b + (exp(a*2i + b*x*2i)*(c + d*x)^2)/b)/(exp(a*4i + b*x*4i) -
2*exp(a*2i + b*x*2i) + 1) - (d^2*x*2i)/b^2 + (b*c^2 - c*d*2i - d^2*x*2i + b
*d^2*x^2 + 2*b*c*d*x)/(b^2*(exp(a*2i + b*x*2i) - 1)) + (d^2*log(exp(a*2i)*e
xp(b*x*2i) - 1))/b^3
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \cos(a+bx) \csc^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*csc(a + b*x)**3, x)
```

## 3.49 $\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx$

**Optimal**. Leaf size=35

$$-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\csc^2(a+bx)}{2b}$$

```
[Out] -1/2*d*cot(b*x+a)/b^2-1/2*(d*x+c)*csc(b*x+a)^2/b
```

**Rubi [A]** time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4410, 3767, 8}

$$-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -(d*Cot[a + b*x])/(2*b^2) - ((c + d*x)*Csc[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx &= -\frac{(c+dx)\csc^2(a+bx)}{2b}+\frac{d\int \csc^2(a+bx)\,dx}{2b}\\ &= -\frac{(c+dx)\csc^2(a+bx)}{2b}-\frac{d\,\mathrm{Subst}\left(\int 1\,dx,x,\cot(a+bx)\right)}{2b^2}\\ &= -\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\csc^2(a+bx)}{2b}\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 48, normalized size = 1.37

$$-\frac{d\cot(a+bx)}{2b^2}-\frac{c\csc^2(a+bx)}{2b}-\frac{dx\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -1/2*(d*Cot[a + b*x])/b^2 - (c*Csc[a + b*x]^2)/(2*b) - (d*x*Csc[a + b*x]^2)
/(2*b)
```

**fricas [A]** time = 0.58, size = 44, normalized size = 1.26

$$\frac{bdx + d\cos(bx+a)\sin(bx+a) + bc}{2\left(b^2\cos(bx+a)^2 - b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x + d*cos(b*x + a)*sin(b*x + a) + b*c)/(b^2*cos(b*x + a)^2 - b^2)
```

**giac [B]** time = 0.36, size = 526, normalized size = 15.03

$$-\frac{bdx\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^4 + bc\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^4 + 2\,bdx\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^2 + 2\,bdx\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
 2*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4
+ 2*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 2*b
*c*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x*ta
n(1/2*b*x)^4 + 4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*tan(1/2*a)^4 + b
*c*tan(1/2*b*x)^4 + 2*d*tan(1/2*b*x)^4*tan(1/2*a) + 4*b*c*tan(1/2*b*x)^2*ta
n(1/2*a)^2 + 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)^2*tan(1/2
*a)^3 + b*c*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b*d*x*tan(1/2*
b*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 - 2*d*tan(1/2*b*x)^3 -
 12*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 - 12*d*tan(1/2*b*x)*ta
n(1/2*a)^2 - 2*d*tan(1/2*a)^3 + b*d*x + b*c + 2*d*tan(1/2*b*x) + 2*d*tan(1/
2*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3
+ b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*b^2*tan(1/2*b*x)^3*tan(1/2*a) - 4*b^2
*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*tan(1/2*b*x)*tan(1/2*a)^3 + b^2*tan(1/
2*b*x)^2 + 2*b^2*tan(1/2*b*x)*tan(1/2*a) + b^2*tan(1/2*a)^2)
```

**maple [A]** time = 0.03, size = 61, normalized size = 1.74

$$\frac{\frac{d\left(-\frac{bx+a}{2\sin(bx+a)^2}-\frac{\cot(bx+a)}{2}\right)}{b}+\frac{da}{2b\sin(bx+a)^2}-\frac{c}{2\sin(bx+a)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b*d*(-1/2*(b*x+a)/sin(b*x+a)^2-1/2*cot(b*x+a))+1/2/b*d*a/sin(b*x+a)^
2-1/2*c/sin(b*x+a)^2)
```

**maxima [B]** time = 0.39, size = 287, normalized size = 8.20

$$\frac{\frac{2\left(4\,(bx+a)\cos(2\,bx+2\,a)^2+4\,(bx+a)\sin(2\,bx+2\,a)^2-(2\,(bx+a)\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a))\cos(4\,bx+4\,a)-2\,(bx+a)\cos(2\,bx+2\,a)-(2\,(bx+a)\sin(2\,bx+2\right.}{\left(2\,(2\,\cos(2\,bx+2\,a)-1)\cos(4\,bx+4\,a)-\cos(4\,bx+4\,a)^2-4\,\cos(2\,bx+2\,a)^2-\sin(4\,bx+4\,a)^2+4\,\sin(4\,bx+4\,a)\sin(2\,bx+2\,a)-4\,\sin(2\,bx\right.}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (
2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x
+ a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) +
1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*d/((2*(2*cos(2*b*x + 2*a) - 1)*cos(
4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)
^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b
*x + 2*a) - 1)*b) - c/sin(b*x + a)^2 + a*d/(b*sin(b*x + a)^2))/b
```

**mupad** [B]   time = 1.72, size = 53, normalized size = 1.51

$$\frac{d\,1\mathrm{i} - \mathrm{e}^{a\,2\mathrm{i} + b\,x\,2\mathrm{i}}\,\left(-b\,\left(2\,c + 2\,d\,x\right) + d\,1\mathrm{i}\right)}{b^2\,\left(\mathrm{e}^{a\,2\mathrm{i} + b\,x\,2\mathrm{i}} - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x))/sin(a + b*x)^3,x)
```

```
[Out] (d*1i - exp(a*2i + b*x*2i)*(d*1i - b*(2*c + 2*d*x)))/(b^2*(exp(a*2i + b*x*2
i) - 1)^2)
```

**sympy** [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \cos(a + bx) \csc^3(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*csc(a + b*x)**3, x)
```

## 3.50 $\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\frac{\cot(a+bx)\csc^2(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

**Rubi** [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x),x]
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

**Mathematica** [A] time = 11.33, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x),x]
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]
```

**fricas** [A] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(cos(b*x + a)*csc(b*x + a)^3/(d*x + c), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*csc(b*x + a)^3/(d*x + c), x)
```

**maple** [**A**] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\left(\csc^3(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x)
[Out] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x)
```

**maxima** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

**mupad** [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(a+b\,x)}{\sin(a+b\,x)^3\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)), x)
```

**sympy** [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\csc^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**3/(d*x+c),x)
[Out] Integral(cos(a + b*x)*csc(a + b*x)**3/(c + d*x), x)
```

## 3.51 $\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2,x]
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx = \int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx$$

**Mathematica [A]** time = 10.64, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2,x]
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2, x]
```

**fricas [A]** time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cos(b*x + a)*csc(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
[Out] Timed out
```

**maple** [A] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\left(\csc^3(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x)
[Out] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(a+b\,x)}{\sin(a+b\,x)^3\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)^2),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)^2), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\csc^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**3/(d*x+c)**2,x)
[Out] Integral(cos(a + b*x)*csc(a + b*x)**3/(c + d*x)**2, x)
```

## 3.52 $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=196

$$-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3}+$$

```
[Out] -1/4*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+5/16*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2
-15/128*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/P
i^(1/2))*Pi^(1/2)/b^(7/2)+15/128*d^(5/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d
^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/64*d^2*cos(2*b*x+2*a)
*(d*x+c)^(1/2)/b^3
```

**Rubi [A]** time = 0.45, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(2a}{64b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b^3) - ((c + d*x)^(5/2)*Cos[2*a
 + 2*b*x])/(4*b) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^(5/2)*Sqrt[
Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(128*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(16*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^{5/2}\sin(2a+2bx)\,dx$$

$$= \frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{(5d)\int (c+dx)^{3/2}\cos(2a+2bx)\,dx}{8b}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{16b^2} - \frac{(15d^2)}{}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+d}{}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+d}{}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+d}{}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} - \frac{15d^{5/2}\sqrt{}}{}$$

**Mathematica** [A] time = 2.24, size = 179, normalized size = 0.91

$$\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(20bd(c+dx)\sin(2(a+bx))-\cos(2(a+bx))\left(16b^2(c+dx)^2-15d^2\right)\right)-15\sqrt{\pi}d^2\cos\left(2a-\frac{2bc}{d}\right)}{128b^3\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-15*d^2*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])
/Sqrt[Pi]] + 15*d^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(-((-15*d^2 + 16*b^2*(c +
 d*x)^2)*Cos[2*(a + b*x)]) + 20*b*d*(c + d*x)*Sin[2*(a + b*x)]))/(128*b^3*S
qrt[b/d])
```

**fricas [A]** time = 0.80, size = 222, normalized size = 1.13

$$-\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)C\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\left(16\,b^3d^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/128*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*
x + 16*b^3*c^2 - 15*b*d^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 -
 15*b*d^2)*cos(b*x + a)^2 + 40*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)*sin(b*x +
 a))*sqrt(d*x + c))/b^4
```

**giac [C]** time = 3.06, size = 1198, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(64*(I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I
*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 12*c*d
^2*((I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x +
c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d
)/b^2)/d^2 + (-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*
I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c
+ 2*I*a*d)/d)/b^2)/d^2) + d^3*((-I*sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c^2*d -
36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3)
 - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sq
rt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*b*c*d^2
 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3
)/d^3 + (I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d
*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c +
2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c
)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d
 - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)
*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 48*(-I*sqrt(p
i)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqr
t(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) +
 2*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d
*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.03, size = 234, normalized size = 1.19

$$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}}\right.\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}\right.}{4b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/8/b*d*(1/4/b
*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)
^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2
*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d
-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.55, size = 275, normalized size = 1.40

$$\frac{\sqrt{2}\left(160\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2d\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-8\left(16\sqrt{2}\,(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\,\sqrt{dx+c}\,bd^2\right)\cos\left(\frac{2\,((dx+c)b-bc+ad}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/1024*sqrt(2)*(160*sqrt(2)*(d*x + c)^(3/2)*b^2*d*sin(2*((d*x + c)*b - b*c
+ a*d)/d) - 8*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*b*
d^2)*cos(2*((d*x + c)*b - b*c + a*d)/d) + ((15*I - 15)*4^(1/4)*sqrt(pi)*d^3
*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (15*I + 15)*4^(1/4)*sqrt(pi)*d^3*(
b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + (-
(15*I + 15)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (1
5*I - 15)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(-2*I*b/d)))/b^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\sin(a+b\,x)\,{(c+d\,x)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Timed out
```

## 3.53 $\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=168

$$-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2}-\frac{(c+\ldots}{}$$

```
[Out] -1/4*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-3/32*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(
2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*Fre
snelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b
^(5/2)+3/16*d*sin(2*b*x+2*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.29, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\,\sin(2a+2bx\ldots}{16b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(4*b) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(32*b^(5/
2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])]*Sin[2*a - (2*b*c)/d])/(32*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b
*x])/(16*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{(3d)\int \sqrt{c+dx}\,\cos(2a+2bx)\,dx}{8b}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{\left(3d^2\right)\int \text{s}}{3}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{\left(3d^2\cos\right.}{}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{\left(3d\cos\right(}{}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} - \frac{3d^{3/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}$$

**Mathematica [A]** time = 0.84, size = 157, normalized size = 0.93

$$\frac{-3\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\,\sqrt{c+dx}}{\sqrt{\pi}}\right)-3\sqrt{\pi}\,d\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\,\sqrt{c+dx}}{\sqrt{\pi}}\right)-2\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\,(4b(c+dx)\cos(2(}{32d^2\left(\frac{b}{d}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-3*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sq
rt[Pi]] - 3*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2
*a - (2*b*c)/d] - 2*Sqrt[b/d]*Sqrt[c + d*x]*(4*b*(c + d*x)*Cos[2*(a + b*x)]
 - 3*d*Sin[2*(a + b*x)]))/(32*(b/d)^(5/2)*d^2)
```

**fricas [A]** time = 0.80, size = 167, normalized size = 0.99

$$-\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\left(2\,b^2dx+\right.}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/32*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x + a)*
sin(b*x + a) + 2*b^2*c - 4*(b^2*d*x + b^2*c)*cos(b*x + a)^2)*sqrt(d*x + c))
/b^3
```

**giac [C]** time = 0.51, size = 743, normalized size = 4.42

$$-\frac{16\left(\frac{i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2ibc+2iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}\right)c^2+d^2\left(\frac{i\sqrt{\pi}\left(16b^2c^2+8ibcd-3d^2\right)d\,\mathrm{erf}\left(-\cdots\right.}{\sqrt{bd}\left(\cdots\right.}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/64*(16*(I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*
sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((I
*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)
/d^2 + (-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt
(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*
a*d)/d)/b^2)/d^2) + 8*(-I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*
I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c -
2*I*a*d)/d)/b)*c)/d
```

**maple [A]** time = 0.02, size = 187, normalized size = 1.11

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)S\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{4b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/8/b*d*(1/4/b
*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(1
```

```
/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+
sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

**maxima [C]** time = 0.56, size = 256, normalized size = 1.52

$$-\frac{\sqrt{2}\left(32\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-24\sqrt{2}\,\sqrt{dx+c}\,bd\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-\left(-(3i+3)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/256*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2*cos(2*((d*x + c)*b - b*c + a
*d)/d) - 24*sqrt(2)*sqrt(d*x + c)*b*d*sin(2*((d*x + c)*b - b*c + a*d)/d) -
(-(3*I + 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (3
*I - 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(2*I*b/d)) - ((3*I - 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)
*cos(-2*(b*c - a*d)/d) - (3*I + 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin
(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\sin(a+b\,x)\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2), x)
```

**sympy [B]** time = 41.48, size = 665, normalized size = 3.96

$$-\frac{5\sqrt{\pi}\sqrt{\frac{d}{b}}\,(c+dx)^2\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)\Gamma\left(\frac{1}{4}\right)}{32d\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{\pi}\sqrt{\frac{d}{b}}\,(c+dx)^2\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}d\sqrt{\frac{b}{d}}}\right)}{2d}-\frac{21\sqrt{\pi}\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -5*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sq
rt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1/4)/(32*d*gamma(9/4)) + sqrt(pi)*sqr
t(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)
*d*sqrt(b/d)))/(2*d) - 21*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d
)*fresnels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(32*d*gam
ma(11/4)) + sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d)*fresnels(2*b
*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) - 15*sqrt(pi)*d*sqrt(d/b)*sin(
2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1
/4)/(512*b**2*gamma(9/4)) - 63*sqrt(pi)*d*sqrt(d/b)*cos(2*a - 2*b*c/d)*fres
nels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(512*b**2*gamma
(11/4)) + 5*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c/d)*sin(2*b*c/d + 2*b
*x)*gamma(1/4)/(64*sqrt(b)*sqrt(d)*gamma(9/4)) - 21*sqrt(d/b)*(c + d*x)**(3
/2)*cos(2*a - 2*b*c/d)*cos(2*b*c/d + 2*b*x)*gamma(3/4)/(64*sqrt(b)*sqrt(d)*
gamma(11/4)) + 15*sqrt(d)*sqrt(d/b)*sqrt(c + d*x)*sin(2*a - 2*b*c/d)*cos(2*
b*c/d + 2*b*x)*gamma(1/4)/(256*b**(3/2)*gamma(9/4)) + 63*sqrt(d)*sqrt(d/b)*
sqrt(c + d*x)*sin(2*b*c/d + 2*b*x)*cos(2*a - 2*b*c/d)*gamma(3/4)/(256*b**(3
/2)*gamma(11/4))
```

# 3.54 $\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=142

$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}$$

```
[Out] 1/8*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*d^(
1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2)
)*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/4*cos(2*b*x+2*a)*(d*x+c)^(1/2
)/b
```

**Rubi** [A] time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(4*b) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) -
 (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*S
in[2*a - (2*b*c)/d])/(8*b^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx &= \int \frac{1}{2}\sqrt{c+dx}\,\sin(2a+2bx)\,dx\\ &= \frac{1}{2}\int \sqrt{c+dx}\,\sin(2a+2bx)\,dx\\ &= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{4b}+\frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx}{8b}\\ &= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{4b}+\frac{\left(d\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{8b}-\frac{(d\,s\end{aligned}$$

$$\begin{aligned}&= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{4b}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)dx,x,\end{aligned}{4b}$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{4b}+\frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-$$

**Mathematica [A]** time = 0.26, size = 134, normalized size = 0.94

$$\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-2\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx))}{8b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + Sqrt[Pi]*Cos[2*a - (2*b*c)/d
]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[Pi]*FresnelS[(2*Sqr
t[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(8*b*Sqrt[b/d])
```

**fricas [A]** time = 0.75, size = 125, normalized size = 0.88

$$\frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)C\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\pi d\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-2\left(2b\cos(bx+a)\right.}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) - 2*(2*b*cos(b*x + a)^2 - b)*sqrt(d*x + c))/b^
2
```

**giac [C]** time = 0.56, size = 402, normalized size = 2.83

$$-\frac{4\left(\frac{i\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2ibc+2iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}\right)c-\frac{i\sqrt{\pi}(4bc+id)d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\ldots}{\sqrt{\ldots}}\right)}{d}\right)}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(4*(I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*s
qrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2
*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - I*sqrt(pi)
*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(
pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2
*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x
 + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)/d
```

**maple [A]** time = 0.02, size = 142, normalized size = 1.00

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\operatorname{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/16/b*d*Pi^(1
/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))
```

**maxima [C]** time = 0.45, size = 209, normalized size = 1.47

$$-\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{dx+c}\,b\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)+\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(\ldots\right.\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/64*sqrt(2)*(8*sqrt(2)*sqrt(d*x + c)*b*cos(2*((d*x + c)*b - b*c + a*d)/d)
 + ((I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I +
 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x
+ c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(
b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*
d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos{(a+b\,x)}\,\sin{(a+b\,x)}\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2), x)
```

**sympy [B]** time = 6.14, size = 389, normalized size = 2.74

$$-\frac{b^{\frac{3}{2}}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{5}{2}}\cos\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_{2}F_{3}\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\right|-\frac{b^{2}(c+dx)^{2}}{d^{2}}\right)}{4d^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{\sqrt{b}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{3}{2}}\sin\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right.}{8d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -b**(3/2)*sqrt(d/b)*(c + d*x)**(5/2)*cos(2*a - 2*b*c/d)*gamma(3/4)*gamma(5/
4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -b**2*(c + d*x)**2/d**2)/(4*d**(5/2)*
gamma(7/4)*gamma(9/4)) - sqrt(b)*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c
/d)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*(c + d*x
)**2/d**2)/(8*d**(3/2)*gamma(5/4)*gamma(7/4)) + sqrt(pi)*c*sqrt(d/b)*sin(2*
a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) + sqr
t(pi)*c*sqrt(d/b)*cos(2*a - 2*b*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d
*sqrt(b/d)))/(2*d) + sqrt(pi)*x*sqrt(d/b)*sin(2*a - 2*b*c/d)*fresnelc(2*b*s
qrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2 + sqrt(pi)*x*sqrt(d/b)*cos(2*a - 2*b
*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2
```

# 3.55 $\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=142

$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}$$

```
[Out] 1/8*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*d^(
1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2)
)*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/4*cos(2*b*x+2*a)*(d*x+c)^(1/2
)/b
```

**Rubi [A]** time = 0.22, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(4*b) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) -
 (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*S
in[2*a - (2*b*c)/d])/(8*b^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}\sqrt{c+dx}\sin(2a+2bx)\,dx$$

$$= \frac{1}{2}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx}{8b}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\left(d\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{8b} - \frac{(d\,\ldots}{}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)dx, x, \ldots\right.}{4b}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \ldots$$

**Mathematica [A]** time = 0.02, size = 134, normalized size = 0.94

$$\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-2\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx))}{8b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + Sqrt[Pi]*Cos[2*a - (2*b*c)/d
]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[Pi]*FresnelS[(2*Sqr
t[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(8*b*Sqrt[b/d])
```

**fricas [A]** time = 0.47, size = 125, normalized size = 0.88

$$\frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)C\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\pi d\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-2\left(2b\cos(bx+a)\ldots\right.}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) - 2*(2*b*cos(b*x + a)^2 - b)*sqrt(d*x + c))/b^
2
```

**giac [C]** time = 0.40, size = 402, normalized size = 2.83

$$-\frac{4\left(\frac{i\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2ibc+2iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}\right)c-\frac{i\sqrt{\pi}(4bc+id)d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i}{\sqrt{b}}\right.}{d}\right.}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(4*(I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*s
qrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2
*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - I*sqrt(pi)
*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(
pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2
*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x
 + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)/d
```

**maple [A]** time = 0.00, size = 142, normalized size = 1.00

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\operatorname{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/16/b*d*Pi^(1
/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))
```

**maxima [C]** time = 0.60, size = 209, normalized size = 1.47

$$-\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{dx+c}\,b\cos\left(\frac{2\left((dx+c)b-bc+ad\right)}{d}\right)+\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/64*sqrt(2)*(8*sqrt(2)*sqrt(d*x + c)*b*cos(2*((d*x + c)*b - b*c + a*d)/d)
 + ((I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I +
 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x
+ c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(
b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*
d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+bx)\sin(a+bx)\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2), x)
```

**sympy [B]** time = 6.21, size = 389, normalized size = 2.74

$$-\frac{b^{\frac{3}{2}}\sqrt{\frac{d}{b}}(c+dx)^{\frac{5}{2}}\cos\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{b^2(c+dx)^2}{d^2}\right)}{4d^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{\sqrt{b}\sqrt{\frac{d}{b}}(c+dx)^{\frac{3}{2}}\sin\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right.}{8d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -b**(3/2)*sqrt(d/b)*(c + d*x)**(5/2)*cos(2*a - 2*b*c/d)*gamma(3/4)*gamma(5/
4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -b**2*(c + d*x)**2/d**2)/(4*d**(5/2)*
gamma(7/4)*gamma(9/4)) - sqrt(b)*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c
/d)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*(c + d*x
)**2/d**2)/(8*d**(3/2)*gamma(5/4)*gamma(7/4)) + sqrt(pi)*c*sqrt(d/b)*sin(2*
a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) + sqr
t(pi)*c*sqrt(d/b)*cos(2*a - 2*b*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d
*sqrt(b/d)))/(2*d) + sqrt(pi)*x*sqrt(d/b)*sin(2*a - 2*b*c/d)*fresnelc(2*b*s
qrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2 + sqrt(pi)*x*sqrt(d/b)*cos(2*a - 2*b
*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2
```

# 3.56 $\int (c+dx)^{3/2} \cos(a+bx) \sin(a+bx)\, dx$

**Optimal**. Leaf size=168

$$-\frac{3\sqrt{\pi}\, d^{3/2} \sin\left(2a-\frac{2bc}{d}\right) C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}\, d^{3/2} \cos\left(2a-\frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2}-\frac{(c+\ldots}{}$$

```
[Out] -1/4*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-3/32*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(
2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*Fre
snelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b
^(5/2)+3/16*d*sin(2*b*x+2*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.28, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\pi}\, d^{3/2} \sin\left(2a-\frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}\, d^{3/2} \cos\left(2a-\frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(4*b) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(32*b^(5/
2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])]*Sin[2*a - (2*b*c)/d])/(32*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b
*x])/(16*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{(3d)\int \sqrt{c+dx}\,\cos(2a+2bx)\,dx}{8b}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{\left(3d^2\right)\int \ldots}{3\ldots}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{\left(3d^2\cos \ldots\right.}{\ldots}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{\left(3d\cos\left(\ldots\right.\right.}{\ldots}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} - \frac{3d^{3/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}}$$

**Mathematica [A]** time = 0.04, size = 157, normalized size = 0.93

$$\frac{-3\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-3\sqrt{\pi}\,d\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-2\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(4b(c+dx)\cos(2(\ldots}{32d^2\left(\frac{b}{d}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-3*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sq
rt[Pi]] - 3*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2
*a - (2*b*c)/d] - 2*Sqrt[b/d]*Sqrt[c + d*x]*(4*b*(c + d*x)*Cos[2*(a + b*x)]
 - 3*d*Sin[2*(a + b*x)]))/(32*(b/d)^(5/2)*d^2)
```

**fricas [A]** time = 0.50, size = 167, normalized size = 0.99

$$-\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{2\,(bc-ad)}{d}\right)S\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,C\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\left(2\,b^2dx+\ldots\right.}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/32*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x + a)*
sin(b*x + a) + 2*b^2*c - 4*(b^2*d*x + b^2*c)*cos(b*x + a)^2)*sqrt(d*x + c))
/b^3
```

**giac [C]** time = 1.02, size = 743, normalized size = 4.42

$$-\frac{16\left(\frac{i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2ibc+2iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}\right)c^2+d^2\left(\frac{i\sqrt{\pi}\left(16b^2c^2+8ibcd-3d^2\right)d\,\mathrm{erf}\left(-\cdots\right)}{\sqrt{bd}\left(\cdots\right)}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/64*(16*(I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*
sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((I
*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)
/d^2 + (-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt
(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*
a*d)/d)/b^2)/d^2) + 8*(-I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*
I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c -
2*I*a*d)/d)/b)*c)/d
```

**maple [A]** time = 0.00, size = 187, normalized size = 1.11

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{4b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/8/b*d*(1/4/b
*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(1
```

```
/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+
sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

**maxima [C]** time = 0.55, size = 256, normalized size = 1.52

$$-\frac{\sqrt{2}\left(32\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-24\sqrt{2}\sqrt{dx+c}\,bd\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-\left(-(3i+3)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/256*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2*cos(2*((d*x + c)*b - b*c + a
*d)/d) - 24*sqrt(2)*sqrt(d*x + c)*b*d*sin(2*((d*x + c)*b - b*c + a*d)/d) -
(-(3*I + 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (3
*I - 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(2*I*b/d)) - ((3*I - 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)
*cos(-2*(b*c - a*d)/d) - (3*I + 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin
(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2), x)
```

**sympy [B]** time = 42.22, size = 665, normalized size = 3.96

$$-\frac{5\sqrt{\pi}\sqrt{\frac{d}{b}}\,(c+dx)^2\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)\Gamma\left(\frac{1}{4}\right)}{32d\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{\pi}\sqrt{\frac{d}{b}}\,(c+dx)^2\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}d\sqrt{\frac{b}{d}}}\right)}{2d}-\frac{21\sqrt{\pi}\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -5*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sq
rt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1/4)/(32*d*gamma(9/4)) + sqrt(pi)*sqr
t(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)
*d*sqrt(b/d)))/(2*d) - 21*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d
)*fresnels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(32*d*gam
ma(11/4)) + sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d)*fresnels(2*b
*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) - 15*sqrt(pi)*d*sqrt(d/b)*sin(
2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1
/4)/(512*b**2*gamma(9/4)) - 63*sqrt(pi)*d*sqrt(d/b)*cos(2*a - 2*b*c/d)*fres
nels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(512*b**2*gamma
(11/4)) + 5*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c/d)*sin(2*b*c/d + 2*b
*x)*gamma(1/4)/(64*sqrt(b)*sqrt(d)*gamma(9/4)) - 21*sqrt(d/b)*(c + d*x)**(3
/2)*cos(2*a - 2*b*c/d)*cos(2*b*c/d + 2*b*x)*gamma(3/4)/(64*sqrt(b)*sqrt(d)*
gamma(11/4)) + 15*sqrt(d)*sqrt(d/b)*sqrt(c + d*x)*sin(2*a - 2*b*c/d)*cos(2*
b*c/d + 2*b*x)*gamma(1/4)/(256*b**(3/2)*gamma(9/4)) + 63*sqrt(d)*sqrt(d/b)*
sqrt(c + d*x)*sin(2*b*c/d + 2*b*x)*cos(2*a - 2*b*c/d)*gamma(3/4)/(256*b**(3
/2)*gamma(11/4))
```

# 3.57 $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=196

$$-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3}+$$

```
[Out] -1/4*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+5/16*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2
-15/128*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/P
i^(1/2))*Pi^(1/2)/b^(7/2)+15/128*d^(5/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d
^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/64*d^2*cos(2*b*x+2*a)
*(d*x+c)^(1/2)/b^3
```

**Rubi [A]** time = 0.33, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(2a}{64b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b^3) - ((c + d*x)^(5/2)*Cos[2*a
 + 2*b*x])/(4*b) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^(5/2)*Sqrt[
Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(128*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(16*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^{5/2}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{(5d)\int (c+dx)^{3/2}\cos(2a+2bx)\,dx}{8b}$$
$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{16b^2} - \frac{\left(15d^2\right)}{}$$
$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+d}{}$$
$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+d}{}$$
$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+d}{}$$
$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} - \frac{15d^{5/2}\sqrt{}}{}$$

**Mathematica [A]** time = 1.06, size = 179, normalized size = 0.91

$$\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(20bd(c+dx)\sin(2(a+bx)) - \cos(2(a+bx))\left(16b^2(c+dx)^2 - 15d^2\right)\right) - 15\sqrt{\pi}\,d^2\cos\left(2a - \frac{2bc}{d}\right)}{128b^3\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-15*d^2*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])
/Sqrt[Pi]] + 15*d^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(-((-15*d^2 + 16*b^2*(c +
 d*x)^2)*Cos[2*(a + b*x)]) + 20*b*d*(c + d*x)*Sin[2*(a + b*x)]))/(128*b^3*S
qrt[b/d])
```

**fricas [A]** time = 0.68, size = 222, normalized size = 1.13

$$-\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\left(16\,b^3d^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/128*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*
x + 16*b^3*c^2 - 15*b*d^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 -
 15*b*d^2)*cos(b*x + a)^2 + 40*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)*sin(b*x +
 a))*sqrt(d*x + c))/b^4
```

**giac [C]** time = 0.68, size = 1198, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(64*(I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I
*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 12*c*d
^2*((I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x +
c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d
)/b^2)/d^2 + (-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*
I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c
+ 2*I*a*d)/d)/b^2)/d^2) + d^3*((-I*sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c^2*d -
36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3)
 - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sq
rt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*b*c*d^2
 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3
)/d^3 + (I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d
*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c +
2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c
)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d
 - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)
*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 48*(-I*sqrt(p
i)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqr
t(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) +
 2*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d
*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.00, size = 234, normalized size = 1.19

$$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}}\right.\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}\right.}{4b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/8/b*d*(1/4/b
*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)
^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2
*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d
-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.47, size = 275, normalized size = 1.40

$$\sqrt{2}\left(160\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2d\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-8\left(16\sqrt{2}\,(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\sqrt{dx+c}\,bd^2\right)\cos\left(\frac{2\,((dx+c)b-bc+ad}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/1024*sqrt(2)*(160*sqrt(2)*(d*x + c)^(3/2)*b^2*d*sin(2*((d*x + c)*b - b*c
+ a*d)/d) - 8*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*b*
d^2)*cos(2*((d*x + c)*b - b*c + a*d)/d) + ((15*I - 15)*4^(1/4)*sqrt(pi)*d^3
*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (15*I + 15)*4^(1/4)*sqrt(pi)*d^3*(
b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + (-
(15*I + 15)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (1
5*I - 15)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(-2*I*b/d)))/b^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\sin(a+b\,x)\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Timed out
```

## 3.58 $\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=406

$$-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\cdots\right)}{16b^{7/2}}$$

```
[Out] 5/8*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/72*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^2+
1/4*(d*x+c)^(5/2)*sin(b*x+a)/b-1/12*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-5/864*d^
(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/864*d^(5/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d
^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-15/16*d^2*sin(
b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

**Rubi [A]** time = 1.14, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a +
 3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]
*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(
16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c + d*x)^(5/
2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3) -
 ((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\cos(a+bx)-\frac{1}{4}(c+dx)^{5/2}\cos(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{5/2}\cos(a+bx)\,dx-\frac{1}{4}\int (c+dx)^{5/2}\cos(3a+3bx)\,dx$$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b}+\frac{(5d)\int(c+dx \cdots}{\cdots}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{(c+dx \cdots}{\cdots}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}-\frac{15d^2\sqrt{c \cdots}}{\cdots}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}-\frac{15d^2\sqrt{c \cdots}}{\cdots}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}-\frac{15d^2\sqrt{c \cdots}}{\cdots}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{15d^{5/2}\cdots}{\cdots}$$

**Mathematica [C]** time = 15.07, size = 1171, normalized size = 2.88

$$-\frac{ie^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)c^2}{8b}-\frac{\left(-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-\sqrt{2\pi}\,C \cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/8*I)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])
/Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))
/d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]*
Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)
/d] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqr
t[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*
b*Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(8*b^3) + ((b/d)^(3
/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*
c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]*F
resnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (
4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b*(
c - 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(32*b^5) - (c
^2*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c
+ d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a
- (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqr
t[3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi
]*Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) +
 Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*C
os[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(
Cos[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) - ((b/d)^(3/2
)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c
^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b
*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d
*Sqrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*Sin
[3*(a + b*x)])))/(288*Sqrt[3]*b^5)
```

**fricas [A]** time = 0.59, size = 370, normalized size = 0.91

$$-\frac{5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*d^2*x
+ b^2*c*d)*cos(b*x + a)^3 - 30*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (12*b^3
*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2 - (12*b^3*d^2*x^2 + 24*b^3*
c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/
b^4
```

**giac [C]** time = 3.34, size = 2453, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/1728*(72*(sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
```

```
rt(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*((sqrt(6)*sqrt(pi)*(12*b^2*
c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b^2) - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(
d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 9*(sq
rt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d
*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)
/b^2)/d^2 - 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/
2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b -
 I*b*c + I*a*d)/d)/b^2)/d^2 + (sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d
^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6
*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e
^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2) - d^3*((sqrt(6)*sqrt(p
i)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(12*I*(d*x + c)^(5/2)*b^2
*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x
+ c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3
*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^3 - 27*(sqrt(2)*sqrt(pi)*(8*b
^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(
d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*
b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)
*b + I*b*c - I*a*d)/d)/b^3)/d^3 - 27*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^
2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^3) - 2*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)
*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqr
t(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*
a*d)/d)/b^3)/d^3 + (sqrt(6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*
d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b^3) + 6*(-12*I*(d*x + c)^(5/2)*b^2*d + 36*I*(d*x + c)^(3/2)*b^2*c*d - 3
6*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b
*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)
/b^3)/d^3) - 36*(sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b
*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)
*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b
) + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b
*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d
)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*
I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.05, size = 474, normalized size = 1.17

$$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{Fresne}\cdots\right.}{4b\sqrt{\frac{b}{d}}}\cdots\right.}{2b}\cdots\right.}{4b}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/8/b*d*(-1/2/b*d
*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2
)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1/2
4/b*d*(d*x+c)^(5/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/24/b*d*(-1/6/b*d*(d*
x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)*
sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+
c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.60, size = 543, normalized size = 1.34

$$-\frac{\left(240\,(dx+c)^{\frac{3}{2}}b^{3}\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)-2160\,(dx+c)^{\frac{3}{2}}b^{3}\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)+\left((5i+5)\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,bd^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\mathrm{co}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3456*(240*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d) - 2160*
(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d) + ((5*I + 5)*9^(1/4)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (5*I - 5)*9^(
1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(3*I*b/d)) + (-(405*I + 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-(b*c - a*d)/d) + (405*I - 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + ((405*I - 405)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (405*I + 405)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(-I*b/d)) + (-(5*I - 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*cos(-3*(b*c - a*d)/d) + (5*I + 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d
^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 24*(12
*(d*x + c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*sin(3*((d*x + c)*b - b*c +
a*d)/d) - 216*(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*sin(((d*x
+ c)*b - b*c + a*d)/d))*d/b^5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.59 $\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=353

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\ \cdots\right.}{24b^{5/2}}$$

```
[Out] 1/4*(d*x+c)^(3/2)*sin(b*x+a)/b-1/12*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b+1/144*d^
(3/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d^
(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))
*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*cos(b*x+a)
*(d*x+c)^(1/2)/b^2-1/24*d*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.68, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*x
])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt
[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a -
 (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/
2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + ((
c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(12
*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\cos(a+bx)-\frac{1}{4}(c+dx)^{3/2}\cos(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{3/2}\cos(a+bx)\,dx-\frac{1}{4}\int (c+dx)^{3/2}\cos(3a+3bx)\,dx$$

$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b}+\frac{d\int\sqrt{c+dx}\,s}{}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\,\mathrm{si}}{4b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\,\mathrm{si}}{4b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\,\mathrm{si}}{4b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\mathrm{co}}{}$$

**Mathematica [C]** time = 9.14, size = 677, normalized size = 1.92

$$\frac{d\left(\sqrt{2\pi}\sqrt{\frac{b}{d}}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(2bc\sin\left(a-\frac{bc}{d}\right)-3d\cos\left(a-\frac{bc}{d}\right)\right)+\sqrt{2\pi}\sqrt{\frac{b}{d}}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(3d\sin\right.}{16b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/8*I)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/S
qrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d
])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqrt
[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d]
+ 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/
Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*Sq
rt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) - (c*(-(Sqrt[2
*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - S
qrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]
 + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqrt[3]*b*Sqrt[
b/d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x
]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sqr
t[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*
c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*x
)] + 2*b*x*Sin[3*(a + b*x)]))))/(48*Sqrt[3]*b^3)
```

**fricas [A]** time = 0.63, size = 298, normalized size = 0.84

$$\frac{\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-27\,\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(
b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt
(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a)^3 -
3*b*d*cos(b*x + a) - 2*(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2)
*sin(b*x + a))*sqrt(d*x + c))/b^3
```

**giac [C]** time = 2.77, size = 1529, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/288*(12*(sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((sqrt(6)*sqrt(pi)*(12*b^2*c^2 +
4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)*b^2) - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x +
c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 9*(sqrt(2)*
sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c
)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/
d^2 - 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)
```

```
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d
 - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c
 + I*a*d)/d)/b^2)/d^2 + (sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*
erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(-2*I
*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((3*I
*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2) - 4*(sqrt(6)*sqrt(pi)*(6*b*c
 + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*
sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*
I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x
 + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*
x + c)*b + I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b +
 3*I*b*c - 3*I*a*d)/d)/b)*c)/d
```

**maple** [A] time = 0.04, size = 386, normalized size = 1.09

$$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{4b}-\frac{d(dx+\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/8/b*d*(-1/2/b*d
*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))-1/24/b*d*(d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(-
1/6/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima** [C] time = 0.59, size = 495, normalized size = 1.40

$$-\frac{\left(\frac{48\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{144\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+24\sqrt{dx+c}\,b^{2}\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)-216\sqrt{dx+c\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/576*(48*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 144*(
d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 24*sqrt(d*x + c)*b^
2*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*cos(((d*x + c)
*b - b*c + a*d)/d) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*
cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
```

```
)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (-(27*I - 27)*s
qrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (27*I + 27)*sqrt(
2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(I*b/d)) + ((27*I + 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*
d)/d) + (27*I - 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b
*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d))
)*d/b^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + d x\right)^{\frac{3}{2}} \sin^{2}{\left(a + b x \right)} \cos{\left(a + b x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x), x)
```

## 3.60 $\int \sqrt{c+dx}\,\cos(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt{}}\right.}{4b^{3/2}}$$

```
[Out] 1/72*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/72*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1
/8*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^
(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/4*sin(b*x+
a)*(d*x+c)^(1/2)/b-1/12*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

**Rubi [A]** time = 0.47, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (Sqrt[c + d*x]*Sin[
a + b*x])/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\cos(a+bx) - \frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int \sqrt{c+dx}\cos(a+bx)\,dx - \frac{1}{4}\int \sqrt{c+dx}\cos(3a+3bx)\,dx$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} + \frac{d\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}dx}{24b} - \ldots$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} + \frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)\int \ldots}{24b} \ldots$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} + \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Subst}\ldots}{\ldots}$$

$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\ldots}{\ldots}\right)}{12b^{3/2}} \ldots$$

**Mathematica [C]** time = 5.36, size = 264, normalized size = 0.87

$$\frac{\frac{\sqrt{6\pi}\sin\left(3a-\frac{3bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-6\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin(3(a+bx))}{\sqrt{\frac{b}{d}}} - 9i\sqrt{c+dx}\,e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c\ldots}{\ldots}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}\ldots\right.}{72b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (((-9*I)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt
[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/
Sqrt[(I*b*(c + d*x))/d]))/E^((I*(b*c + a*d))/d) + (Sqrt[6*Pi]*Cos[3*a - (3*
b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[6*Pi]*FresnelC[
Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] - 6*Sqrt[b/d]*Sqrt
[c + d*x]*Sin[3*(a + b*x)])/Sqrt[b/d])/(72*b)
```

**fricas** [A] time = 0.56, size = 245, normalized size = 0.81

$$\frac{\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{2}\,\pi d \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 - b)*sqrt(d*x +
 c)*sin(b*x + a))/b^2
```

**giac** [C] time = 2.85, size = 838, normalized size = 2.76

$$-\frac{\frac{\sqrt{6}\,\sqrt{\pi}\,(6\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{6}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{3i\,bc-3i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{9\,\sqrt{2}\,\sqrt{\pi}\,(2\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{9\,\sqrt{2}\,\sqrt{\pi}\,(2 \cdots}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/144*(sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c -
 I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(
6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b) - 6*(sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)
*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 6*I*sqrt(d*x + c)*d*e^((3*I*
(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c
 - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a
*d)/d)/b)/d
```

**maple** [A] time = 0.04, size = 294, normalized size = 0.97

$$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{12b}+\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d
)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresne
lS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/
d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima [C]** time = 0.57, size = 422, normalized size = 1.39

$$-\frac{\left(\frac{24\,\sqrt{dx+c}\,b^2\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{72\,\sqrt{dx+c}\,b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)+(i-1)\cdot\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/288*(24*sqrt(d*x + c)*b^2*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 72*sqrt
(d*x + c)*b^2*sin(((d*x + c)*b - b*c + a*d)/d)/d + (-(I + 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
3*I*b/d)) + ((9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/
d) - (9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(I*b/d)) + (-(9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)
*cos(-(b*c - a*d)/d) + (9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I - 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I
*b/d)))*d/b^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^2\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^2\left(a+bx\right)\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x), x)
```

## 3.61 $\int \sqrt{c+dx}\,\cos(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt{}}\right.}{4b^{3/2}}$$

```
[Out] 1/72*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/72*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1
/8*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^
(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/4*sin(b*x+
a)*(d*x+c)^(1/2)/b-1/12*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

**Rubi [A]** time = 0.47, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (Sqrt[c + d*x]*Sin[
a + b*x])/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\cos(a+bx)-\frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int \sqrt{c+dx}\cos(a+bx)\,dx-\frac{1}{4}\int \sqrt{c+dx}\cos(3a+3bx)\,dx$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}+\frac{d\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}dx}{24b}-\frac{d}{}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}+\frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)\int}{24b}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Subst}}{}$$

$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}}{}\right)}{12b^{3/2}}$$

**Mathematica [C]** time = 3.05, size = 264, normalized size = 0.87

$$\frac{\frac{\sqrt{6\pi}\sin\left(3a-\frac{3bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-6\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin(3(a+bx))}{\sqrt{\frac{b}{d}}}-9i\sqrt{c+dx}\,e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c}{}\right)}{\sqrt{-\frac{ib(c+dx}{d}}}\right.}{72b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (((-9*I)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt
[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/
Sqrt[(I*b*(c + d*x))/d]))/E^((I*(b*c + a*d))/d) + (Sqrt[6*Pi]*Cos[3*a - (3*
b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[6*Pi]*FresnelC[
Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] - 6*Sqrt[b/d]*Sqrt
[c + d*x]*Sin[3*(a + b*x)])/Sqrt[b/d])/(72*b)
```

**fricas** [A] time = 0.63, size = 245, normalized size = 0.81

$$\frac{\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right) - 9\,\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right) - 9\,\sqrt{2}\,\pi d \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 - b)*sqrt(d*x +
 c)*sin(b*x + a))/b^2
```

**giac** [C] time = 3.87, size = 838, normalized size = 2.76

$$-\frac{\frac{\sqrt{6}\,\sqrt{\pi}\,(6\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{6}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{3i\,bc-3i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b} - \frac{9\,\sqrt{2}\,\sqrt{\pi}\,(2\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b} - \frac{9\,\sqrt{2}\,\sqrt{\pi}\,(2 \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/144*(sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c -
 I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(
6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b) - 6*(sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)
*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 6*I*sqrt(d*x + c)*d*e^((3*I*
(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c
 - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a
*d)/d)/b)/d
```

**maple** [A] time = 0.00, size = 294, normalized size = 0.97

$$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b} - \frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}} - \frac{d\sqrt{dx+c}\,\sin\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{12b} + \cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d
)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresne
lS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/
d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima** [C] time = 0.58, size = 422, normalized size = 1.39

$$-\frac{\left(\frac{24\sqrt{dx+c}\,b^2\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{72\sqrt{dx+c}\,b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i-1)\cdot\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/288*(24*sqrt(d*x + c)*b^2*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 72*sqrt
(d*x + c)*b^2*sin(((d*x + c)*b - b*c + a*d)/d)/d + (-(I + 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
3*I*b/d)) + ((9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/
d) - (9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(I*b/d)) + (-(9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)
*cos(-(b*c - a*d)/d) + (9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I - 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I
*b/d)))*d/b^3
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^2\,\sqrt{c+d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx} \sin^{2}{\left(a + bx \right)} \cos{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x), x)
```

## 3.62 $\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=353

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\right.}{24b^{5/2}}$$

```
[Out] 1/4*(d*x+c)^(3/2)*sin(b*x+a)/b-1/12*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b+1/144*d^
(3/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d^
(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))
*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*cos(b*x+a)
*(d*x+c)^(1/2)/b^2-1/24*d*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.57, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\sqrt{\frac{\pi}{6}}\,d^{3/2}\,\text{si}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*x
])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt
[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a -
 (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/
2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + ((
c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(12
*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{4}(c+dx)^{3/2}\cos(a+bx)-\frac{1}{4}(c+dx)^{3/2}\cos(3a+3bx)\right)dx\\ &= \frac{1}{4}\int (c+dx)^{3/2}\cos(a+bx)\,dx-\frac{1}{4}\int (c+dx)^{3/2}\cos(3a+3bx)\,dx\\ &= \frac{(c+dx)^{3/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b}+\frac{d\int \sqrt{c+dx}\,\sin}{8b}\\ &= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\sin(}{4b}\\ &= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\sin(}{4b}\\ &= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\sin(}{4b}\\ &= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(\right.}{}\end{aligned}$$

**Mathematica [C]** time = 8.95, size = 677, normalized size = 1.92

$$\frac{d\left(\sqrt{2\pi}\sqrt{\frac{b}{d}}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(2bc\sin\left(a-\frac{bc}{d}\right)-3d\cos\left(a-\frac{bc}{d}\right)\right)+\sqrt{2\pi}\sqrt{\frac{b}{d}}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(3d\sin\left(a\right.\right.}{16b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/8*I)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/S
qrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d
])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqrt
[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d]
+ 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/
Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*Sq
rt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) - (c*(-(Sqrt[2
*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - S
qrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]
 + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqrt[3]*b*Sqrt[
b/d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x
]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sqr
t[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*
c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*x
)] + 2*b*x*Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)
```

**fricas [A]** time = 0.82, size = 298, normalized size = 0.84

$$\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-27\,\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(
b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt
(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a)^3 -
3*b*d*cos(b*x + a) - 2*(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2)
*sin(b*x + a))*sqrt(d*x + c))/b^3
```

**giac [C]** time = 6.94, size = 1529, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/288*(12*(sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((sqrt(6)*sqrt(pi)*(12*b^2*c^2 +
4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)*b^2) - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x +
c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 9*(sqrt(2)*
sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c
)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/
d^2 - 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)
```

```
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d
 - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c
 + I*a*d)/d)/b^2)/d^2 + (sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*
erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(-2*I
*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((3*I
*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2) - 4*(sqrt(6)*sqrt(pi)*(6*b*c
 + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*
sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*
I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x
 + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*
x + c)*b + I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b +
 3*I*b*c - 3*I*a*d)/d)/b)*c)/d
```

**maple [A]** time = 0.00, size = 386, normalized size = 1.09

$$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{4b}-\frac{d(dx+c)^{\frac{3}{2}}}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/8/b*d*(-1/2/b*d
*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))-1/24/b*d*(d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(-
1/6/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

**maxima [C]** time = 0.61, size = 495, normalized size = 1.40

$$-\frac{\left(\frac{48\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{144\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+24\sqrt{dx+c}\,b^2\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)-216\sqrt{dx+c}\,b^2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/576*(48*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 144*(
d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 24*sqrt(d*x + c)*b^
2*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*cos(((d*x + c)
*b - b*c + a*d)/d) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*
cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
```

```
)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (-(27*I - 27)*s
qrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (27*I + 27)*sqrt(
2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(I*b/d)) + ((27*I + 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*
d)/d) + (27*I - 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b
*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d))
)*d/b^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x), x)
```

# 3.63 $\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=406

$$-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\right.}{16b^{7/2}}$$

```
[Out] 5/8*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/72*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^2+
1/4*(d*x+c)^(5/2)*sin(b*x+a)/b-1/12*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-5/864*d^
(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/864*d^(5/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d
^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-15/16*d^2*sin(
b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

**Rubi [A]** time = 0.67, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a +
 3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]
*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(
16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c + d*x)^(5/
2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3) -
 ((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\cos(a+bx)-\frac{1}{4}(c+dx)^{5/2}\cos(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{5/2}\cos(a+bx)\,dx-\frac{1}{4}\int (c+dx)^{5/2}\cos(3a+3bx)\,dx$$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b}+\frac{(5d)\int (c+dx \ldots}{\ldots}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{(c+dx \ldots}{\ldots}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}-\frac{15d^2\sqrt{c \ldots}}{\ldots}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}-\frac{15d^2\sqrt{c \ldots}}{\ldots}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}-\frac{15d^2\sqrt{c \ldots}}{\ldots}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{15d^{5/2} \ldots}{\ldots}$$

**Mathematica [C]** time = 13.63, size = 1171, normalized size = 2.88

$$-\frac{ie^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)c^2}{8b}-\frac{\left(-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-\sqrt{2\pi}\,C \ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/8*I)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])
/Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))
/d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]*
Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)
/d] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqr
t[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*
b*Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(8*b^3) + ((b/d)^(3
/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*
c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]*F
resnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (
4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b*(
c - 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(32*b^5) - (c
^2*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c
+ d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a
- (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqr
t[3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi
]*Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) +
 Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*C
os[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(
Cos[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) - ((b/d)^(3/2
)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c
^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b
*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d
*Sqrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*Sin
[3*(a + b*x)])))/(288*Sqrt[3]*b^5)
```

**fricas [A]** time = 0.69, size = 370, normalized size = 0.91

$$-\frac{5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)S\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-405\,\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-405}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*d^2*x
+ b^2*c*d)*cos(b*x + a)^3 - 30*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (12*b^3
*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2 - (12*b^3*d^2*x^2 + 24*b^3*
c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/
b^4
```

**giac [C]** time = 2.46, size = 2453, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/1728*(72*(sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
```

```
rt(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*((sqrt(6)*sqrt(pi)*(12*b^2*
c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b^2) - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(
d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 9*(sq
rt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d
*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)
/b^2)/d^2 - 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/
2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b -
 I*b*c + I*a*d)/d)/b^2)/d^2 + (sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d
^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6
*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e
^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2) - d^3*((sqrt(6)*sqrt(p
i)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(12*I*(d*x + c)^(5/2)*b^2
*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x
+ c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3
*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^3 - 27*(sqrt(2)*sqrt(pi)*(8*b
^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(
d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*
b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)
*b + I*b*c - I*a*d)/d)/b^3)/d^3 - 27*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^
2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^3) - 2*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)
*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqr
t(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*
a*d)/d)/b^3)/d^3 + (sqrt(6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*
d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b^3) + 6*(-12*I*(d*x + c)^(5/2)*b^2*d + 36*I*(d*x + c)^(3/2)*b^2*c*d - 3
6*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b
*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)
/b^3)/d^3) - 36*(sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b
*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)
*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b
) + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b
*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d
)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*
I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.00, size = 474, normalized size = 1.17

$$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{Fresne}\right.}{4b\sqrt{\frac{b}{d}}}\right.}{2b}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/8/b*d*(-1/2/b*d
*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2
)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1/2
4/b*d*(d*x+c)^(5/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/24/b*d*(-1/6/b*d*(d*
x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)*
sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+
c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.60, size = 543, normalized size = 1.34

$$-\frac{\left(240\,(dx+c)^{\frac{3}{2}}b^{3}\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)-2160\,(dx+c)^{\frac{3}{2}}b^{3}\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)+\left((5i+5)\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,bd^{2}\left(\frac{b^{2}}{d^{2}}\right)^{\frac{1}{4}}\mathrm{co}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3456*(240*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d) - 2160*
(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d) + ((5*I + 5)*9^(1/4)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (5*I - 5)*9^(
1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(3*I*b/d)) + (-(405*I + 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-(b*c - a*d)/d) + (405*I - 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + ((405*I - 405)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (405*I + 405)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(-I*b/d)) + (-(5*I - 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*cos(-3*(b*c - a*d)/d) + (5*I + 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d
^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 24*(12
*(d*x + c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*sin(3*((d*x + c)*b - b*c +
a*d)/d) - 216*(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*sin(((d*x
+ c)*b - b*c + a*d)/d))*d/b^5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.64 $\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=407

$$\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)S}{4096b^{7/2}}$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/3
2*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b
^2+15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192*d^(5/2)*FresnelS(2*b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1
/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/2)*(d*x
+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/128*d^2*co
s(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b^3
```

**Rubi [A]** time = 1.05, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) - (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) + ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)-\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx\right)+\frac{1}{4}\int (c+dx)^{5/2}\sin(2a+2b$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}-\frac{(5d)\int(c}{}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{5d(c+d}{}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{15d^2\sqrt{c}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{15d^2\sqrt{c}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{15d^2\sqrt{c}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{15d^2\sqrt{c}}{}$$

**Mathematica [A]** time = 14.25, size = 550, normalized size = 1.35

$$-1024b^3c^2\sqrt{c+dx}\cos(2(a+bx))+256b^3c^2\sqrt{c+dx}\cos(4(a+bx))-1024b^3d^2x^2\sqrt{c+dx}\cos(2(a+bx))+2$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] - 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] + 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
rt[c + d*x]*Sin[2*(a + b*x)] - 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] -
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)
```

**fricas [A]** time = 0.88, size = 406, normalized size = 1.00

$$15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{4\,(bc-ad)}{d}\right)C\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-15\,\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) -
 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(320*b^3*d^2*x^2 + 640*b^3*c*d*x +
 320*b^3*c^2 + 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*c
os(b*x + a)^4 - 255*b*d^2 - 8*(128*b^3*d^2*x^2 + 256*b^3*c*d*x + 128*b^3*c^
2 - 75*b*d^2)*cos(b*x + a)^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 5*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

**giac [C]** time = 3.28, size = 2418, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/16384*(512*(-I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)))*c^3 + 24*c*d^2*((-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)
*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*
I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(
d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-4
*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (I*sqrt(2)*sqrt(pi)*(64*b
^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b
^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*
d + 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d
```

```
^2 + 16*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*
x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*
d)/d)/b^2)/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b
*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2
*I*b*c + 2*I*a*d)/d)/b^2)/d^2) + d^3*((I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 19
2*I*b^2*c^2*d - 72*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b^3) - 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x +
c)^(3/2)*b^2*c*d - 192*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2
 + 72*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(d*x + c)*b
+ 4*I*b*c - 4*I*a*d)/d)/b^3)/d^3 + (-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*
I*b^2*c^2*d - 72*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)*b^3) - 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x +
 c)^(3/2)*b^2*c*d - 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^
2 - 72*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((4*I*(d*x + c)*b
- 4*I*b*c + 4*I*a*d)/d)/b^3)/d^3 + 32*(-I*sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c
^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d +
48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*
b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)
/d)/b^3)/d^3 + 32*(I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 1
5*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*
I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)
*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqr
t(d*x + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 192
*(I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*erf
(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*d)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c +
 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 4*sqrt(d*x + c)*d*e
^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*I*(
d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x + c
)*b + 2*I*b*c - 2*I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*
I*b*c - 4*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.04, size = 470, normalized size = 1.15

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\ \cdots}{\sqrt{\pi}\sqrt{\frac{b}{d}}}\right.\right.}{8b\sqrt{\frac{b}{d}}}\cdots\right.}{4b}\cdots\right.}{8b}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-5/64/b*d*(1/8/b*d*(d*x+c)^(3/
2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.54, size = 547, normalized size = 1.34

$$-\frac{\left(1280\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-10240\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-32\left(\frac{64\,(dx+c)^{\frac{5}{2}}b^{4}}{d}-15\,\sqrt{dx+c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/65536*(1280*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) - 102
40*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 32*(64*(d*x + c
)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d)
+ 512*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)*
b - b*c + a*d)/d) + (-(480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-2*(b*c - a*d)/d) - (480*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^
2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) +
 ((30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d)
+ (30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))
*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b
^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b
^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((
480*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a
*d)/d) + (480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-
2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^3\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.65 $\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=351

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)S\Big(}{512b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b+3/1
024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/1024*d^(3/2)*FresnelC(2*b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5
/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^
(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2*a)*(d*
x+c)^(1/2)/b^2-3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.67, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\,\text{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) + ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right) dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^{3/2}\sin(2a+2bx)\,d$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} - \frac{(3d)\int \sqrt{c+}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\mathrm{co}}{}$$

**Mathematica [A]** time = 3.08, size = 393, normalized size = 1.12

$$\frac{3\sqrt{2\pi}\,d\sin\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) - 48\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + 3\sqrt{2\pi}\,d\cos\left(4a-\frac{4bc}{d}\right)S\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] + 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
+ 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

**fricas** [A] time = 0.58, size = 316, normalized size = 0.90

$$\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2
*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 4
8*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d))) - 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(
b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
+ 10*b^2*d*x + 10*b^2*c - 32*(b^2*d*x + b^2*c)*cos(b*x + a)^2 - 3*(2*b*d*co
s(b*x + a)^3 - 5*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

**giac** [C] time = 4.75, size = 1503, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2048*(64*(-I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^2 + d^2*((-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(
-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c -
 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x + c
)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x
 + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (I*sqrt(2)*sqrt(pi)*(64*b^2*c^2
- 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*s
qrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2 + 16
*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*
b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b
^2)/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*
I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c
```

```
+ 2*I*a*d)/d)/b^2)/d^2) + 16*(I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c
- I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*
sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) +
 8*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b) - 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b +
16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(
d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 4*sqrt(d*x + c)
*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c)/d
```

**maple** [A] time = 0.04, size = 376, normalized size = 1.07

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{8b}+\frac{d(dx+}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))+1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d))))
```

**maxima** [C] time = 0.51, size = 503, normalized size = 1.43

$$\frac{\left(\frac{256\,(dx+c)^{\frac{3}{2}}b^{3}\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{1024\,(dx+c)^{\frac{3}{2}}b^{3}\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d}-96\sqrt{dx+c}\,b^{2}\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)+768\sqrt{dx+c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8192*(256*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 1024
*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 96*sqrt(d*x + c
)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) + 768*sqrt(d*x + c)*b^2*sin(2*((d*
x + c)*b - b*c + a*d)/d) - ((48*I + 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d
^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (48*I - 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (
-(6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (6*
I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(I*b/d)) - ((6*I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*
cos(-4*(b*c - a*d)/d) - (6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(
-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - (-(48*I - 48)*4^(1/4
)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (48*I + 48)*
```

```
4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)\sin(a+bx)^3(c+dx)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.66 $\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=299

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\,\cdots}{\cdots}\right)}{64b^{3/2}}$$

```
[Out] -1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*FresnelS(2*b^(1/2)*2^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^
(3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/
2))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a)*(d*x
+c)^(1/2)/b+1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

**Rubi [A]** time = 0.50, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\,\sin(2a+2bx) - \frac{1}{8}\sqrt{c+dx}\,\sin(4a+4bx)\right) dx$$

$$= -\left(\frac{1}{8}\int \sqrt{c+dx}\,\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int \sqrt{c+dx}\,\sin(2a+2bx)\,dx$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b} - \frac{d\int \frac{\cos(4a+4bx)}{\sqrt{c+dx}}}{64b}$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b} - \frac{\left(d\cos\left(4a-\right.\right.}{}$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b} - \frac{\cos\left(4a-\frac{4bc}{d}\right)}{}$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(\right.}{}$$

**Mathematica [A]** time = 0.81, size = 264, normalized size = 0.88

$$\frac{-\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+\sqrt{2\pi}\sin\left(4a-\frac{4bc}{d}\right)S\left(2\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] - Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

**fricas [A]** time = 0.81, size = 244, normalized size = 0.82

$$-\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)-8\,\pi d\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_si
n(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 8*pi*d*sq
rt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d
))) + 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin
(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 16*b*cos(b*x + a)^2 + 5*b)*sqr
t(d*x + c))/b^2
```

**giac [C]** time = 2.83, size = 818, normalized size = 2.74

$$-\frac{\frac{i\,\sqrt{2}\,\sqrt{\pi}\,(8\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4i\,bc-4i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{i\,\sqrt{2}\,\sqrt{\pi}\,(8\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4i\,bc+4i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+8\left(-\frac{i\,\sqrt{2}\cdots}{\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/256*(I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*(-I*sqrt(2)*sqrt(pi)*d*e
rf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*
c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*
d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4
*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2
*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 8*I*sqrt(pi)*(4*b*c + I*d)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*
d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*
c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 4*sqrt(d*x + c)*
d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*
I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x
+ c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b +
 4*I*b*c - 4*I*a*d)/d)/b)/d
```

**maple [A]** time = 0.03, size = 286, normalized size = 0.96

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{dx+c}\,\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}-\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c
)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d))+1/64/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima** [C] time = 0.49, size = 425, normalized size = 1.42

$$\left(\frac{32\sqrt{dx+c}\,b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{128\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}+\left(-(8i-8)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(8\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(32*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 128*sqr
t(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d + (-(8*I - 8)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (8*I + 8)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(2*I*b/d)) + ((2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*
c - a*d)/d) + (2*I + 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*
d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(2*I + 2)*sqrt(2)*sqrt(pi)*b*(b
^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2
)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((8*I +
8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (8*I
- 8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^3\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\sin^3(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x), x)
```

## 3.67 $\int \sqrt{c+dx} \cos(a+bx) \sin^3(a+bx)\,dx$

**Optimal**. Leaf size=299

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\ldots}{\ldots}\right)}{64b^{3/2}}$$

```
[Out] -1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*FresnelS(2*b^(1/2)*2^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^
(3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/
2))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a)*(d*x
+c)^(1/2)/b+1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

**Rubi [A]** time = 0.46, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}+\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\Big(4\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx)-\frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$

$$= -\left(\frac{1}{8}\int \sqrt{c+dx}\sin(4a+4bx)\,dx\right)+\frac{1}{4}\int \sqrt{c+dx}\sin(2a+2bx)\,d$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}-\frac{d\int \frac{\cos(4a+4bx)}{\sqrt{c+dx}}}{64b}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}-\frac{\left(d\cos\left(4a-\right.\right.}{}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}-\frac{\cos\left(4a-\frac{4bc}{d}\right.}{}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(\right.}{}$$

**Mathematica [A]** time = 0.29, size = 264, normalized size = 0.88

$$\frac{-\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+\sqrt{2\pi}\sin\left(4a-\frac{4bc}{d}\right)S\left(2\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] - Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

**fricas** [A] time = 0.85, size = 244, normalized size = 0.82

$$-\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)-8\,\pi d\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_si
n(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 8*pi*d*sq
rt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d
))) + 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin
(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 16*b*cos(b*x + a)^2 + 5*b)*sqr
t(d*x + c))/b^2
```

**giac** [C] time = 1.07, size = 818, normalized size = 2.74

$$-\frac{\frac{i\,\sqrt{2}\,\sqrt{\pi}\,(8\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4i\,bc-4i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{i\,\sqrt{2}\,\sqrt{\pi}\,(8\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4i\,bc+4i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+8\left(-\frac{i\,\sqrt{2}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/256*(I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*(-I*sqrt(2)*sqrt(pi)*d*e
rf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*
c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*
d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4
*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2
*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 8*I*sqrt(pi)*(4*b*c + I*d)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*
d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*
c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 4*sqrt(d*x + c)*
d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*
I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x
+ c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b +
 4*I*b*c - 4*I*a*d)/d)/b)/d
```

**maple** [A] time = 0.00, size = 286, normalized size = 0.96

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{dx+c}\,\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}-\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c
)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d))+1/64/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima** [C] time = 0.48, size = 425, normalized size = 1.42

$$\left(\frac{32\,\sqrt{dx+c}\,b^2\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{128\,\sqrt{dx+c}\,b^2\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d}+\left(-(8i-8)\cdot 4^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)-(8\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(32*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 128*sqr
t(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d + (-(8*I - 8)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (8*I + 8)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(2*I*b/d)) + ((2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*
c - a*d)/d) + (2*I + 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*
d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(2*I + 2)*sqrt(2)*sqrt(pi)*b*(b
^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2
)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((8*I +
8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (8*I
- 8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^3\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^3{(a+bx)}\cos{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x), x)
```

# 3.68 $\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=351

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{}}{}\right.}{512b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b+3/1
024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/1024*d^(3/2)*FresnelC(2*b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5
/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^
(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2*a)*(d*
x+c)^(1/2)/b^2-3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.57, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) + ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^{3/2}\sin(2a+2b$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} - \frac{(3d)\int \sqrt{}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}}{}$$

**Mathematica [A]** time = 2.37, size = 393, normalized size = 1.12

$$\frac{3\sqrt{2\pi}\,d\sin\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) - 48\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + 3\sqrt{2\pi}\,d\cos\left(4a-\frac{4bc}{d}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] + 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
+ 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

**fricas [A]** time = 0.68, size = 316, normalized size = 0.90

$$\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2
*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 4
8*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d))) - 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(
b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
+ 10*b^2*d*x + 10*b^2*c - 32*(b^2*d*x + b^2*c)*cos(b*x + a)^2 - 3*(2*b*d*co
s(b*x + a)^3 - 5*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

**giac [C]** time = 6.77, size = 1503, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2048*(64*(-I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^2 + d^2*((-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(
-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c -
 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x + c
)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x
 + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (I*sqrt(2)*sqrt(pi)*(64*b^2*c^2
- 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*s
qrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2 + 16
*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*
b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b
^2)/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*
I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c
```

```
+ 2*I*a*d)/d)/b^2)/d^2) + 16*(I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c
- I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*
sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) +
 8*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b) - 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b +
16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(
d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 4*sqrt(d*x + c)
*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c)/d
```

**maple [A]** time = 0.00, size = 376, normalized size = 1.07

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{8b}+ \cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d))))
```

**maxima [C]** time = 0.50, size = 503, normalized size = 1.43

$$\frac{\left(\frac{256\,(dx+c)^{\frac{3}{2}}b^{3}\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{1024\,(dx+c)^{\frac{3}{2}}b^{3}\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d}-96\sqrt{dx+c}\,b^{2}\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)+768\sqrt{dx+\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8192*(256*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 1024
*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 96*sqrt(d*x + c
)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) + 768*sqrt(d*x + c)*b^2*sin(2*((d*
x + c)*b - b*c + a*d)/d) - ((48*I + 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d
^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (48*I - 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (
-(6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (6*
I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(I*b/d)) - ((6*I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*
cos(-4*(b*c - a*d)/d) - (6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(
-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - (-(48*I - 48)*4^(1/4
)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (48*I + 48)*
```

```
4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)\,\sin(a+bx)^3\,(c+dx)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**3,x)
[Out] Timed out
```

# 3.69 $\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=407

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right.}{4096b^{7/}}$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/3
2*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b
^2+15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192*d^(5/2)*FresnelS(2*b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1
/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/2)*(d*x
+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/128*d^2*co
s(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b^3
```

**Rubi [A]** time = 0.70, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) - (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) + ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^{5/2}\sin(2a+2bx)\,c$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} - \frac{(5d)\int (c+c}{}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} + \frac{5d(c+dx)^{3}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c+}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c+}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c+}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c+}}{}$$

**Mathematica [A]** time = 9.97, size = 550, normalized size = 1.35

$$\frac{-1024b^3c^2\sqrt{c+dx}\,\cos(2(a+bx)) + 256b^3c^2\sqrt{c+dx}\,\cos(4(a+bx)) - 1024b^3d^2x^2\sqrt{c+dx}\,\cos(2(a+bx)) + 256}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] - 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] + 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
rt[c + d*x]*Sin[2*(a + b*x)] - 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] -
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)
```

**fricas [A]** time = 0.91, size = 406, normalized size = 1.00

$$15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) -
 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(320*b^3*d^2*x^2 + 640*b^3*c*d*x +
 320*b^3*c^2 + 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*c
os(b*x + a)^4 - 255*b*d^2 - 8*(128*b^3*d^2*x^2 + 256*b^3*c*d*x + 128*b^3*c^
2 - 75*b*d^2)*cos(b*x + a)^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 5*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

**giac [C]** time = 4.04, size = 2418, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/16384*(512*(-I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)))*c^3 + 24*c*d^2*((-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)
*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*
I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(
d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-4
*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (I*sqrt(2)*sqrt(pi)*(64*b
^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b
^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*
d + 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d
```

```
^2 + 16*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*
x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*
d)/d)/b^2)/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b
*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2
*I*b*c + 2*I*a*d)/d)/b^2)/d^2) + d^3*((I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 19
2*I*b^2*c^2*d - 72*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b^3) - 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x +
c)^(3/2)*b^2*c*d - 192*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2
 + 72*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(d*x + c)*b
+ 4*I*b*c - 4*I*a*d)/d)/b^3)/d^3 + (-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*
I*b^2*c^2*d - 72*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)*b^3) - 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x +
 c)^(3/2)*b^2*c*d - 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^
2 - 72*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((4*I*(d*x + c)*b
- 4*I*b*c + 4*I*a*d)/d)/b^3)/d^3 + 32*(-I*sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c
^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d +
48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*
b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)
/d)/b^3)/d^3 + 32*(I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 1
5*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*
I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)
*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqr
t(d*x + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 192
*(I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*erf
(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*d)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c +
 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 4*sqrt(d*x + c)*d*e
^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*I*(
d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x + c
)*b + 2*I*b*c - 2*I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*
I*b*c - 4*I*a*d)/d)/b)*c^2)/d
```

**maple** [A] time = 0.00, size = 470, normalized size = 1.15

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\cdots\right)}{8b\sqrt{\frac{b}{d}}}\right)}{4b}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-5/64/b*d*(1/8/b*d*(d*x+c)^(3/
2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima** [C] time = 0.63, size = 547, normalized size = 1.34

$$-\frac{\left(1280\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-10240\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-32\left(\frac{64\,(dx+c)^{\frac{5}{2}}b^4}{d}-15\,\sqrt{dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/65536*(1280*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) - 102
40*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 32*(64*(d*x + c
)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d)
+ 512*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)*
b - b*c + a*d)/d) + (-(480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-2*(b*c - a*d)/d) - (480*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^
2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) +
 ((30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d)
+ (30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))
*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b
^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b
^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((
480*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a
*d)/d) + (480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-
2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.70 $\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=267

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}-$$

```
[Out] -1/8*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/
d)^m)-1/8*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+
c)/d)^m)-1/8*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(d*x+c)
/d)/b/((-I*b*(d*x+c)/d)^m)-1/8*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(d*x+c)/d
)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

**Rubi [A]** time = 0.28, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3308, 2181}

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(8*b*
(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(
8*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I)*(a -
(b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(8*b*(((-I)*b*(
c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/
d])/(8*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^m \sin(a+bx) + \frac{1}{4}(c+dx)^m \sin(3a+3bx)\right) dx$$
$$= \frac{1}{4}\int (c+dx)^m \sin(a+bx)\,dx + \frac{1}{4}\int (c+dx)^m \sin(3a+3bx)\,dx$$
$$= \frac{1}{8}i\int e^{-i(a+bx)}(c+dx)^m\,dx - \frac{1}{8}i\int e^{i(a+bx)}(c+dx)^m\,dx + \frac{1}{8}i\int e^{-i(3a-$$
$$= -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b} - \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+$$

**Mathematica [A]** time = 0.46, size = 250, normalized size = 0.94

$$\frac{e^{-\frac{3i(ad+bc)}{d}}(c+dx)^m\left(3e^{\frac{2i(ad+bc)}{d}}\left(-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)\right)-3\right.}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] ((c + d*x)^m*(3*E^(((2*I)*(b*c + a*d))/d)*(-((E^((2*I)*a)*Gamma[1 + m, ((-I
)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m) - (E^(((2*I)*b*c)/d)*Gamma[1 +
 m, (I*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^m) - (E^((6*I)*a)*((I*b*(c + d*
x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d)*(((-I)*b*
(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/(3^m*((b^2*(c + d*x)^2
)/d^2)^m)))/(24*b*E^(((3*I)*(b*c + a*d))/d))
```

**fricas [A]** time = 0.68, size = 184, normalized size = 0.69

$$-\frac{e^{\left(-\frac{dm\log\left(\frac{3ib}{d}\right)-3ibc+3iad}{d}\right)}\Gamma\left(m+1,\frac{3ibdx+3ibc}{d}\right)+3e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+1,\frac{ibdx+ibc}{d}\right)+3e^{\left(-\frac{dm\log\left(-\frac{ib}{d}\right)+ibc-iad}{d}\right)}\Gamma\left(m\right.}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/24*(e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gamma(m + 1, (3*I*b*d*
x + 3*I*b*c)/d) + 3*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I
*b*d*x + I*b*c)/d) + 3*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m + 1
, (-I*b*d*x - I*b*c)/d) + e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*ga
mma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d))/b
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a), x)
```

**maple [F]** time = 0.18, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^2(bx+a)\right)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2 \sin(a+b\,x)\,(c+d\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^m, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin(a+bx)\cos^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x)*cos(a + b*x)**2, x)
```

# 3.71 $\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=205

$$-\frac{8d^4\cos^3(a+bx)}{81b^5}-\frac{160d^4\cos(a+bx)}{27b^5}-\frac{160d^3(c+dx)\sin(a+bx)}{27b^4}-\frac{8d^3(c+dx)\sin(a+bx)\cos^2(a+bx)}{27b^4}+\frac{4d^2(}{}$$

```
[Out] -160/27*d^4*cos(b*x+a)/b^5+8/3*d^2*(d*x+c)^2*cos(b*x+a)/b^3-8/81*d^4*cos(b*
x+a)^3/b^5+4/9*d^2*(d*x+c)^2*cos(b*x+a)^3/b^3-1/3*(d*x+c)^4*cos(b*x+a)^3/b-
160/27*d^3*(d*x+c)*sin(b*x+a)/b^4+8/9*d*(d*x+c)^3*sin(b*x+a)/b^2-8/27*d^3*(
d*x+c)*cos(b*x+a)^2*sin(b*x+a)/b^4+4/9*d*(d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)/
b^2
```

**Rubi [A]** time = 0.20, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4405, 3311, 3296, 2638, 3310}

$$-\frac{160d^3(c+dx)\sin(a+bx)}{27b^4}+\frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3}+\frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3}-\frac{8d^3(c+dx)\sin(a+bx)\cos}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (-160*d^4*Cos[a + b*x])/(27*b^5) + (8*d^2*(c + d*x)^2*Cos[a + b*x])/(3*b^3)
 - (8*d^4*Cos[a + b*x]^3)/(81*b^5) + (4*d^2*(c + d*x)^2*Cos[a + b*x]^3)/(9*
b^3) - ((c + d*x)^4*Cos[a + b*x]^3)/(3*b) - (160*d^3*(c + d*x)*Sin[a + b*x]
)/(27*b^4) + (8*d*(c + d*x)^3*Sin[a + b*x])/(9*b^2) - (8*d^3*(c + d*x)*Cos[
a + b*x]^2*Sin[a + b*x])/(27*b^4) + (4*d*(c + d*x)^3*Cos[a + b*x]^2*Sin[a +
 b*x])/(9*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^4\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^4\cos^3(a+bx)}{3b}+\frac{(4d)\int (c+dx)^3\cos^3(a+bx)\,dx}{3b}$$
$$=\frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^4\cos^3(a+bx)}{3b}+\frac{4d(c+dx)^3\cos^2(}{9}$$
$$=-\frac{8d^4\cos^3(a+bx)}{81b^5}+\frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^4\cos^3(a+bx}{3b}$$
$$=\frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3}-\frac{8d^4\cos^3(a+bx)}{81b^5}+\frac{4d^2(c+dx)^2\cos^3(a+b}{9b^3}$$
$$=-\frac{16d^4\cos(a+bx)}{27b^5}+\frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3}-\frac{8d^4\cos^3(a+bx)}{81b^5}+\frac{4d}{}$$
$$=-\frac{160d^4\cos(a+bx)}{27b^5}+\frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3}-\frac{8d^4\cos^3(a+bx)}{81b^5}+\frac{4}{}$$

**Mathematica [A]** time = 1.55, size = 150, normalized size = 0.73

$$-\frac{-24bd(c+dx)\sin(a+bx)\left(\cos(2(a+bx))\left(3b^2(c+dx)^2-2d^2\right)+15b^2(c+dx)^2-82d^2\right)+81\cos(a+bx)\left(b^4(c\right.}{324b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/324*(81*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x]
 + (8*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)] -
 24*b*d*(c + d*x)*(-82*d^2 + 15*b^2*(c + d*x)^2 + (-2*d^2 + 3*b^2*(c + d*x)
^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/b^5
```

**fricas [A]** time = 0.49, size = 294, normalized size = 1.43

$$-\frac{\left(27\,b^4d^4x^4+108\,b^4cd^3x^3+27\,b^4c^4-36\,b^2c^2d^2+8\,d^4+18\left(9\,b^4c^2d^2-2\,b^2d^4\right)x^2+36\left(3\,b^4c^3d-2\,b^2cd^3\right)x\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/81*((27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 +
8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)
*x)*cos(b*x + a)^3 - 24*(9*b^2*d^4*x^2 + 18*b^2*c*d^3*x + 9*b^2*c^2*d^2 - 2
0*d^4)*cos(b*x + a) - 12*(6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d -
40*b*c*d^3 + (3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (
9*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + 2*(9*b^3*c^2*d^2 - 20*b*d^4)*x
)*sin(b*x + a))/b^5
```

**giac [A]** time = 0.25, size = 350, normalized size = 1.71

$$-\frac{\left(27\,b^4d^4x^4+108\,b^4cd^3x^3+162\,b^4c^2d^2x^2+108\,b^4c^3dx+27\,b^4c^4-36\,b^2d^4x^2-72\,b^2cd^3x-36\,b^2c^2d^2+8\,d^4\right)\cos}{324\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*
c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8
*d^4)*cos(3*b*x + 3*a)/b^5 - 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2
*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b
^2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^5 + 1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x
^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*sin(3*b*x + 3*a
)/b^5 + (b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*
d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

**maple [B]** time = 0.06, size = 835, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^4*d^4*(-1/3*(b*x+a)^4*cos(b*x+a)^3+4/9*(b*x+a)^3*(2+cos(b*x+a)^2)*
sin(b*x+a)+8/3*(b*x+a)^2*cos(b*x+a)-160/27*cos(b*x+a)-16/3*(b*x+a)*sin(b*x+
a)+4/9*(b*x+a)^2*cos(b*x+a)^3-8/27*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)-8/81
*cos(b*x+a)^3)-4/b^4*a*d^4*(-1/3*(b*x+a)^3*cos(b*x+a)^3+1/3*(b*x+a)^2*(2+co
s(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*co
s(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*sin(b*x+a))+4/b^3*c*d^3*(-1/3*(b*x+a)^3*co
s(b*x+a)^3+1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*
x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*sin(b*x+a))+
6/b^4*a^2*d^4*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin
(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))-12/b^3*a*c*d^3*(-1/3*(b*x+a)^2*co
s(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*co
s(b*x+a))+6/b^2*c^2*d^2*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x
+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))-4/b^4*a^3*d^4*(-1/3*(b*
x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+12/b^3*a^2*c*d^3*(-1/3*(
b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-12/b^2*a*c^2*d^2*(-1/3
*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+4/b*c^3*d*(-1/3*(b*x
+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-1/3/b^4*a^4*d^4*cos(b*x+a
)^3+4/3/b^3*a^3*c*d^3*cos(b*x+a)^3-2/b^2*a^2*c^2*d^2*cos(b*x+a)^3+4/3/b*a*c
^3*d*cos(b*x+a)^3-1/3*c^4*cos(b*x+a)^3)
```

**maxima [B]** time = 0.47, size = 889, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/324*(108*c^4*cos(b*x + a)^3 - 432*a*c^3*d*cos(b*x + a)^3/b + 648*a^2*c^2
*d^2*cos(b*x + a)^3/b^2 - 432*a^3*c*d^3*cos(b*x + a)^3/b^3 + 108*a^4*d^4*co
s(b*x + a)^3/b^4 + 36*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x +
 a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*c^3*d/b - 108*(3*(b*x + a)*cos(3*b
*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a
*c^2*d^2/b^2 + 108*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a)
 - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a^2*c*d^3/b^3 - 36*(3*(b*x + a)*cos(3
*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))
*a^3*d^4/b^4 + 18*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 -
 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a)
)*c^2*d^2/b^2 - 36*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2
- 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a
))*a*c*d^3/b^3 + 18*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2
```

```
 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x +
a))*a^2*d^4/b^4 + 12*(3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27
*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x +
 3*a) - 81*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^3/b^3 - 12*(3*(3*(b*x + a)^3
 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x +
 a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*sin(b*x +
 a))*a*d^4/b^4 + ((27*(b*x + a)^4 - 36*(b*x + a)^2 + 8)*cos(3*b*x + 3*a) +
81*((b*x + a)^4 - 12*(b*x + a)^2 + 24)*cos(b*x + a) - 12*(3*(b*x + a)^3 - 2
*b*x - 2*a)*sin(3*b*x + 3*a) - 324*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a)
)*d^4/b^4)/b
```

**mupad [B]** time = 1.90, size = 448, normalized size = 2.19

$$\frac{4\,x\cos\left(a+b\,x\right)^3\,\left(14\,c\,d^3-3\,b^2\,c^3\,d\right)}{9\,b^3}-\frac{\cos\left(a+b\,x\right)^3\,\left(27\,b^4\,c^4-252\,b^2\,c^2\,d^2+488\,d^4\right)}{81\,b^5}-\frac{8\,\cos\left(a+b\,x\right)\,\sin\left(a+b\right.}{27\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^4,x)
```

```
[Out] (4*x*cos(a + b*x)^3*(14*c*d^3 - 3*b^2*c^3*d))/(9*b^3) - (cos(a + b*x)^3*(48
8*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(81*b^5) - (8*cos(a + b*x)*sin(a + b
*x)^2*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^5) - (4*cos(a + b*x)^2*sin(a + b*x)*(
14*c*d^3 - 3*b^2*c^3*d))/(9*b^4) - (d^4*x^4*cos(a + b*x)^3)/(3*b) - (8*sin(
a + b*x)^3*(20*c*d^3 - 3*b^2*c^3*d))/(27*b^4) + (8*d^4*x^3*sin(a + b*x)^3)/
(9*b^2) - (8*x*sin(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^4) + (2*x^2*c
os(a + b*x)^3*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^3) - (4*c*d^3*x^3*cos(a + b*x)
^3)/(3*b) + (4*d^4*x^3*cos(a + b*x)^2*sin(a + b*x))/(3*b^2) + (8*d^4*x^2*co
s(a + b*x)*sin(a + b*x)^2)/(3*b^3) + (8*c*d^3*x^2*sin(a + b*x)^3)/(3*b^2) -
 (4*x*cos(a + b*x)^2*sin(a + b*x)*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^4) + (4*c*
d^3*x^2*cos(a + b*x)^2*sin(a + b*x))/b^2 + (16*c*d^3*x*cos(a + b*x)*sin(a +
 b*x)^2)/(3*b^3)
```

**sympy [A]** time = 7.19, size = 646, normalized size = 3.15

$$\begin{cases} -\frac{c^4\cos^3(a+bx)}{3b}-\frac{4c^3dx\cos^3(a+bx)}{3b}-\frac{2c^2d^2x^2\cos^3(a+bx)}{b}-\frac{4cd^3x^3\cos^3(a+bx)}{3b}-\frac{d^4x^4\cos^3(a+bx)}{3b}+\frac{8c^3d\sin^3(a+bx)}{9b^2}+\frac{4c^3d\sin(a+bx)\cos}{3b^2} \\ \left(c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}\right)\sin(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**4*cos(a + b*x)**3/(3*b) - 4*c**3*d*x*cos(a + b*x)**3/(3*b) -
 2*c**2*d**2*x**2*cos(a + b*x)**3/b - 4*c*d**3*x**3*cos(a + b*x)**3/(3*b) -
 d**4*x**4*cos(a + b*x)**3/(3*b) + 8*c**3*d*sin(a + b*x)**3/(9*b**2) + 4*c*
*3*d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 8*c**2*d**2*x*sin(a + b*x)**3/
(3*b**2) + 4*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**2 + 8*c*d**3*x**2*
sin(a + b*x)**3/(3*b**2) + 4*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**2/b**2
+ 8*d**4*x**3*sin(a + b*x)**3/(9*b**2) + 4*d**4*x**3*sin(a + b*x)*cos(a + b
*x)**2/(3*b**2) + 8*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 28*c*
*2*d**2*cos(a + b*x)**3/(9*b**3) + 16*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)
/(3*b**3) + 56*c*d**3*x*cos(a + b*x)**3/(9*b**3) + 8*d**4*x**2*sin(a + b*x)
**2*cos(a + b*x)/(3*b**3) + 28*d**4*x**2*cos(a + b*x)**3/(9*b**3) - 160*c*d
**3*sin(a + b*x)**3/(27*b**4) - 56*c*d**3*sin(a + b*x)*cos(a + b*x)**2/(9*b
**4) - 160*d**4*x*sin(a + b*x)**3/(27*b**4) - 56*d**4*x*sin(a + b*x)*cos(a
+ b*x)**2/(9*b**4) - 160*d**4*sin(a + b*x)**2*cos(a + b*x)/(27*b**5) - 488*
d**4*cos(a + b*x)**3/(81*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**
2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*cos(a)**2, True))
```

## 3.72 $\int (c+dx)^3 \cos^2(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=151

$$\frac{2d^3\sin^3(a+bx)}{27b^4}-\frac{14d^3\sin(a+bx)}{9b^4}+\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}+\frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d(c+dx)^2\sin(a+b}{3b^2}$$

```
[Out] 4/3*d^2*(d*x+c)*cos(b*x+a)/b^3+2/9*d^2*(d*x+c)*cos(b*x+a)^3/b^3-1/3*(d*x+c)
^3*cos(b*x+a)^3/b-14/9*d^3*sin(b*x+a)/b^4+2/3*d*(d*x+c)^2*sin(b*x+a)/b^2+1/
3*d*(d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)/b^2+2/27*d^3*sin(b*x+a)^3/b^4
```

**Rubi** [A] time = 0.13, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4405, 3311, 3296, 2637, 2633}

$$\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}+\frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d(c+dx)^2\sin(a+bx)}{3b^2}+\frac{d(c+dx)^2\sin(a+bx)\cos^2(a+b}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (4*d^2*(c + d*x)*Cos[a + b*x])/(3*b^3) + (2*d^2*(c + d*x)*Cos[a + b*x]^3)/(
9*b^3) - ((c + d*x)^3*Cos[a + b*x]^3)/(3*b) - (14*d^3*Sin[a + b*x])/(9*b^4)
 + (2*d*(c + d*x)^2*Sin[a + b*x])/(3*b^2) + (d*(c + d*x)^2*Cos[a + b*x]^2*S
in[a + b*x])/(3*b^2) + (2*d^3*Sin[a + b*x]^3)/(27*b^4)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^3\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^3\cos^3(a+bx)}{3b}+\frac{d\int (c+dx)^2\cos^3(a+bx)\,dx}{b}$$
$$=\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}+\frac{d(c+dx)^2\cos^2(a}{3b}$$
$$=\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}+\frac{2d(c+dx)^2\sin(a}{3b^2}$$
$$=\frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a}{3b}$$
$$=\frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a}{3b}$$

**Mathematica [A]** time = 0.89, size = 127, normalized size = 0.84

$$\frac{-27b(c+dx)\cos(a+bx)\left(b^2(c+dx)^2-6d^2\right)-3b(c+dx)\cos(3(a+bx))\left(3b^2(c+dx)^2-2d^2\right)+2d\sin(a+bx)\left(c\right.}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (-27*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 3*b*(c + d*x)*(-
2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 2*d*(-82*d^2 + 45*b^2*(c + d*
x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(108*b^
4)
```

**fricas [A]** time = 0.70, size = 183, normalized size = 1.21

$$-\frac{3\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3-2\,bcd^2+\left(9\,b^3c^2d-2\,bd^3\right)x\right)\cos\left(bx+a\right)^3-36\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)-\left(\right.}{27\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/27*(3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*
c^2*d - 2*b*d^3)*x)*cos(b*x + a)^3 - 36*(b*d^3*x + b*c*d^2)*cos(b*x + a) -
(18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - 40*d^3 + (9*b^2*d^3*x^2 +
 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/b^4
```

**giac [A]** time = 1.91, size = 231, normalized size = 1.53

$$-\frac{\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+9\,b^3c^2dx+3\,b^3c^3-2\,bd^3x-2\,bcd^2\right)\cos\left(3\,bx+3\,a\right)}{36\,b^4}-\frac{\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^
3*x - 2*b*c*d^2)*cos(3*b*x + 3*a)/b^4 - 1/4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
+ 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 + 1/108
*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)/b^
4 + 3/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/b^4
```

**maple [B]** time = 0.01, size = 447, normalized size = 2.96

$$\frac{d^3\left(-\frac{(bx+a)^3(\cos^3(bx+a))}{3}+\frac{(bx+a)^2(2+\cos^2(bx+a))\sin(bx+a)}{3}-\frac{4\sin(bx+a)}{3}+\frac{4(bx+a)\cos(bx+a)}{3}+\frac{2(bx+a)(\cos^3(bx+a))}{9}-\frac{2(2+\cos^2(bx+a))\sin(bx+a)}{27}\right)}{b^3}-\frac{3a\,d^3\left(-\frac{(bx+a}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^3*d^3*(-1/3*(b*x+a)^3*cos(b*x+a)^3+1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*
sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2
/27*(2+cos(b*x+a)^2)*sin(b*x+a))-3/b^3*a*d^3*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2
/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))+3/
b^2*c*d^2*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x
+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))+3/b^3*a^2*d^3*(-1/3*(b*x+a)*cos(b*x+a
)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-6/b^2*a*c*d^2*(-1/3*(b*x+a)*cos(b*x+a)
^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+3/b*c^2*d*(-1/3*(b*x+a)*cos(b*x+a)^3+1/
9*(2+cos(b*x+a)^2)*sin(b*x+a))+1/3/b^3*a^3*d^3*cos(b*x+a)^3-1/b^2*a^2*c*d^2
*cos(b*x+a)^3+1/b*a*c^2*d*cos(b*x+a)^3-1/3*c^3*cos(b*x+a)^3)
```

**maxima [B]** time = 0.37, size = 505, normalized size = 3.34

$$-\frac{36\,c^3\cos(bx+a)^3-\frac{108\,ac^2d\cos(bx+a)^3}{b}+\frac{108\,a^2cd^2\cos(bx+a)^3}{b^2}-\frac{36\,a^3d^3\cos(bx+a)^3}{b^3}+\frac{9\,(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/108*(36*c^3*cos(b*x + a)^3 - 108*a*c^2*d*cos(b*x + a)^3/b + 108*a^2*c*d^
2*cos(b*x + a)^3/b^2 - 36*a^3*d^3*cos(b*x + a)^3/b^3 + 9*(3*(b*x + a)*cos(3
*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))
*c^2*d/b - 18*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - si
n(3*b*x + 3*a) - 9*sin(b*x + a))*a*c*d^2/b^2 + 9*(3*(b*x + a)*cos(3*b*x + 3
*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a^2*d^3
/b^3 + 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*cos(b
*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))*c*d^2/b
^2 - 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*cos(b*x
 + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))*a*d^3/b^3
 + (3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27*((b*x + a)^3 - 6*
b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 81*((b*x +
 a)^2 - 2)*sin(b*x + a))*d^3/b^3)/b
```

**mupad [B]** time = 1.34, size = 290, normalized size = 1.92

$$\frac{\cos(a+b\,x)^3\,\left(14\,c\,d^2-3\,b^2\,c^3\right)}{9\,b^3}-\frac{2\sin(a+b\,x)^3\,\left(20\,d^3-9\,b^2\,c^2\,d\right)}{27\,b^4}-\frac{\cos(a+b\,x)^2\,\sin(a+b\,x)\,\left(14\,d^3-9\,b^2\right.}{9\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^3,x)
```

```
[Out] (cos(a + b*x)^3*(14*c*d^2 - 3*b^2*c^3))/(9*b^3) - (2*sin(a + b*x)^3*(20*d^3
 - 9*b^2*c^2*d))/(27*b^4) - (cos(a + b*x)^2*sin(a + b*x)*(14*d^3 - 9*b^2*c^
2*d))/(9*b^4) + (x*cos(a + b*x)^3*(14*d^3 - 9*b^2*c^2*d))/(9*b^3) - (d^3*x^
3*cos(a + b*x)^3)/(3*b) + (2*d^3*x^2*sin(a + b*x)^3)/(3*b^2) + (4*c*d^2*cos
(a + b*x)*sin(a + b*x)^2)/(3*b^3) + (4*d^3*x*cos(a + b*x)*sin(a + b*x)^2)/(
3*b^3) + (4*c*d^2*x*sin(a + b*x)^3)/(3*b^2) - (c*d^2*x^2*cos(a + b*x)^3)/b
+ (d^3*x^2*cos(a + b*x)^2*sin(a + b*x))/b^2 + (2*c*d^2*x*cos(a + b*x)^2*sin
(a + b*x))/b^2
```

**sympy [A]** time = 3.95, size = 391, normalized size = 2.59

$$\begin{cases} -\frac{c^3\cos^3(a+bx)}{3b} - \frac{c^2dx\cos^3(a+bx)}{b} - \frac{cd^2x^2\cos^3(a+bx)}{b} - \frac{d^3x^3\cos^3(a+bx)}{3b} + \frac{2c^2d\sin^3(a+bx)}{3b^2} + \frac{c^2d\sin(a+bx)\cos^2(a+bx)}{b^2} + \frac{4cd^2x\sin^3(a\cdot}{3b^2} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**3*cos(a + b*x)**3/(3*b) - c**2*d*x*cos(a + b*x)**3/b - c*d**
2*x**2*cos(a + b*x)**3/b - d**3*x**3*cos(a + b*x)**3/(3*b) + 2*c**2*d*sin(a
 + b*x)**3/(3*b**2) + c**2*d*sin(a + b*x)*cos(a + b*x)**2/b**2 + 4*c*d**2*x
*sin(a + b*x)**3/(3*b**2) + 2*c*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**2 +
2*d**3*x**2*sin(a + b*x)**3/(3*b**2) + d**3*x**2*sin(a + b*x)*cos(a + b*x)*
*2/b**2 + 4*c*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 14*c*d**2*cos(a
+ b*x)**3/(9*b**3) + 4*d**3*x*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 14*d*
*3*x*cos(a + b*x)**3/(9*b**3) - 40*d**3*sin(a + b*x)**3/(27*b**4) - 14*d**3
*sin(a + b*x)*cos(a + b*x)**2/(9*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2
/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)*cos(a)**2, True))
```

# 3.73 $\int (c+dx)^2 \cos^2(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=103

$$\frac{2d^2\cos^3(a+bx)}{27b^3}+\frac{4d^2\cos(a+bx)}{9b^3}+\frac{4d(c+dx)\sin(a+bx)}{9b^2}+\frac{2d(c+dx)\sin(a+bx)\cos^2(a+bx)}{9b^2}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

```
[Out] 4/9*d^2*cos(b*x+a)/b^3+2/27*d^2*cos(b*x+a)^3/b^3-1/3*(d*x+c)^2*cos(b*x+a)^3
/b+4/9*d*(d*x+c)*sin(b*x+a)/b^2+2/9*d*(d*x+c)*cos(b*x+a)^2*sin(b*x+a)/b^2
```

**Rubi** [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4405, 3310, 3296, 2638}

$$\frac{4d(c+dx)\sin(a+bx)}{9b^2}+\frac{2d(c+dx)\sin(a+bx)\cos^2(a+bx)}{9b^2}+\frac{2d^2\cos^3(a+bx)}{27b^3}+\frac{4d^2\cos(a+bx)}{9b^3}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (4*d^2*Cos[a + b*x])/(9*b^3) + (2*d^2*Cos[a + b*x]^3)/(27*b^3) - ((c + d*x)
^2*Cos[a + b*x]^3)/(3*b) + (4*d*(c + d*x)*Sin[a + b*x])/(9*b^2) + (2*d*(c +
 d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^2\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^2\cos^3(a+bx)}{3b} + \frac{(2d)\int (c+dx)\cos^3(a+bx)\,dx}{3b}$$
$$= \frac{2d^2\cos^3(a+bx)}{27b^3} - \frac{(c+dx)^2\cos^3(a+bx)}{3b} + \frac{2d(c+dx)\cos^2(a+bx)\,\mathrm{si}}{9b^2}$$
$$= \frac{2d^2\cos^3(a+bx)}{27b^3} - \frac{(c+dx)^2\cos^3(a+bx)}{3b} + \frac{4d(c+dx)\sin(a+bx)}{9b^2} +$$
$$= \frac{4d^2\cos(a+bx)}{9b^3} + \frac{2d^2\cos^3(a+bx)}{27b^3} - \frac{(c+dx)^2\cos^3(a+bx)}{3b} + \frac{4d(c+}{}$$

**Mathematica** [A] time = 0.49, size = 86, normalized size = 0.83

$$-\frac{27\cos(a+bx)\left(b^2(c+dx)^2-2d^2\right)+\cos(3(a+bx))\left(9b^2(c+dx)^2-2d^2\right)-6bd(c+dx)(9\sin(a+bx)+\sin(3(a}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/108*(27*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + (-2*d^2 + 9*b^2*(c + d
*x)^2)*Cos[3*(a + b*x)] - 6*b*d*(c + d*x)*(9*Sin[a + b*x] + Sin[3*(a + b*x)
]))/b^3
```

**fricas** [A] time = 0.72, size = 100, normalized size = 0.97

$$-\frac{\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2\right)\cos\left(bx+a\right)^3-12\,d^2\cos\left(bx+a\right)-6\left(2\,bd^2x+2\,bcd+\left(bd^2x+bcd\right)\cos\left(b\right.\right.}{27\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/27*((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^3 -
12*d^2*cos(b*x + a) - 6*(2*b*d^2*x + 2*b*c*d + (b*d^2*x + b*c*d)*cos(b*x +
a)^2)*sin(b*x + a))/b^3
```

**giac** [A] time = 4.95, size = 137, normalized size = 1.33

$$-\frac{\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2\right)\cos\left(3\,bx+3\,a\right)}{108\,b^3}-\frac{\left(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2\right)\cos\left(bx+a\right)}{4\,b^3}+\frac{\left(bd^2x+b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x + 3*a)/
b^3 - 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)/b^3 +
1/18*(b*d^2*x + b*c*d)*sin(3*b*x + 3*a)/b^3 + 1/2*(b*d^2*x + b*c*d)*sin(b*x
 + a)/b^3
```

**maple** [B] time = 0.01, size = 204, normalized size = 1.98

$$\frac{\frac{d^2\left(-\frac{(bx+a)^2\left(\cos^3(bx+a)\right)}{3}+\frac{2(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{9}+\frac{2\left(\cos^3(bx+a)\right)}{27}+\frac{4\cos(bx+a)}{9}\right)}{b^2}-\frac{2a\,d^2\left(-\frac{(bx+a)\left(\cos^3(bx+a)\right)}{3}+\frac{\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{9}\right)}{b^2}+2cd\left(-\frac{(bx+}{}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^2*d^2*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*si
n(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))-2/b^2*a*d^2*(-1/3*(b*x+a)*cos(b*
x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+2/b*c*d*(-1/3*(b*x+a)*cos(b*x+a)^3+
1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-1/3/b^2*a^2*d^2*cos(b*x+a)^3+2/3/b*a*c*d*c
os(b*x+a)^3-1/3*c^2*cos(b*x+a)^3)
```

**maxima [B]** time = 0.37, size = 243, normalized size = 2.36

$$-\frac{36\,c^2\cos\left(bx+a\right)^3-\frac{72\,acd\cos(bx+a)^3}{b}+\frac{36\,a^2d^2\cos(bx+a)^3}{b^2}+\frac{6\,(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)-9\,\sin(bx\cdots}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/108*(36*c^2*cos(b*x + a)^3 - 72*a*c*d*cos(b*x + a)^3/b + 36*a^2*d^2*cos(
b*x + a)^3/b^2 + 6*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a)
 - sin(3*b*x + 3*a) - 9*sin(b*x + a))*c*d/b - 6*(3*(b*x + a)*cos(3*b*x + 3*
a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a*d^2/b^
2 + ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*cos(b*x +
a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))*d^2/b^2)/b
```

**mupad [B]** time = 1.13, size = 145, normalized size = 1.41

$$\frac{12\,d^2\,\cos\left(a+b\,x\right)+2\,d^2\cos\left(a+b\,x\right)^3-9\,b^2\,c^2\cos\left(a+b\,x\right)^3+12\,b\,d^2\,x\,\sin\left(a+b\,x\right)-9\,b^2\,d^2\,x^2\cos\left(a+b\,x\right)\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2,x)
```

```
[Out] (12*d^2*cos(a + b*x) + 2*d^2*cos(a + b*x)^3 - 9*b^2*c^2*cos(a + b*x)^3 + 12
*b*d^2*x*sin(a + b*x) - 9*b^2*d^2*x^2*cos(a + b*x)^3 + 12*b*c*d*sin(a + b*x
) - 18*b^2*c*d*x*cos(a + b*x)^3 + 6*b*d^2*x*cos(a + b*x)^2*sin(a + b*x) + 6
*b*c*d*cos(a + b*x)^2*sin(a + b*x))/(27*b^3)
```

**sympy [A]** time = 2.03, size = 216, normalized size = 2.10

$$\begin{cases} -\frac{c^2\cos^3(a+bx)}{3b}-\frac{2cdx\cos^3(a+bx)}{3b}-\frac{d^2x^2\cos^3(a+bx)}{3b}+\frac{4cd\sin^3(a+bx)}{9b^2}+\frac{2cd\sin(a+bx)\cos^2(a+bx)}{3b^2}+\frac{4d^2x\sin^3(a+bx)}{9b^2}+\frac{2d^2x\sin(a\cdots}{} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**2*cos(a + b*x)**3/(3*b) - 2*c*d*x*cos(a + b*x)**3/(3*b) - d*
*2*x**2*cos(a + b*x)**3/(3*b) + 4*c*d*sin(a + b*x)**3/(9*b**2) + 2*c*d*sin(
a + b*x)*cos(a + b*x)**2/(3*b**2) + 4*d**2*x*sin(a + b*x)**3/(9*b**2) + 2*d
**2*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 4*d**2*sin(a + b*x)**2*cos(a
+ b*x)/(9*b**3) + 14*d**2*cos(a + b*x)**3/(27*b**3), Ne(b, 0)), ((c**2*x +
c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a)**2, True))
```

# 3.74 $\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=51

$$-\frac{d\sin^3(a+bx)}{9b^2}+\frac{d\sin(a+bx)}{3b^2}-\frac{(c+dx)\cos^3(a+bx)}{3b}$$

```
[Out] -1/3*(d*x+c)*cos(b*x+a)^3/b+1/3*d*sin(b*x+a)/b^2-1/9*d*sin(b*x+a)^3/b^2
```

**Rubi [A]** time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4405, 2633}

$$-\frac{d\sin^3(a+bx)}{9b^2}+\frac{d\sin(a+bx)}{3b^2}-\frac{(c+dx)\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)*Cos[a + b*x]^3)/(3*b) + (d*Sin[a + b*x])/(3*b^2) - (d*Sin[a + b
*x]^3)/(9*b^2)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx &= -\frac{(c+dx)\cos^3(a+bx)}{3b}+\frac{d\int \cos^3(a+bx)\,dx}{3b}\\ &= -\frac{(c+dx)\cos^3(a+bx)}{3b}-\frac{d\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(a+bx)\right)}{3b^2}\\ &= -\frac{(c+dx)\cos^3(a+bx)}{3b}+\frac{d\sin(a+bx)}{3b^2}-\frac{d\sin^3(a+bx)}{9b^2}\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 71, normalized size = 1.39

$$\frac{d(\sin(a+bx)-bx\cos(a+bx))}{4b^2}+\frac{d(\sin(3(a+bx))-3bx\cos(3(a+bx)))}{36b^2}-\frac{c\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/3*(c*Cos[a + b*x]^3)/b + (d*(-(b*x*Cos[a + b*x]) + Sin[a + b*x]))/(4*b^2
) + (d*(-3*b*x*Cos[3*(a + b*x)] + Sin[3*(a + b*x)]))/(36*b^2)
```

**fricas** [A] time = 0.68, size = 46, normalized size = 0.90

$$-\frac{3\,(bdx+bc)\cos\left(bx+a\right)^{3}-\left(d\cos\left(bx+a\right)^{2}+2\,d\right)\sin\left(bx+a\right)}{9\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/9*(3*(b*d*x + b*c)*cos(b*x + a)^3 - (d*cos(b*x + a)^2 + 2*d)*sin(b*x + a
))/b^2
```

**giac** [A] time = 0.17, size = 69, normalized size = 1.35

$$-\frac{(bdx+bc)\cos\left(3\,bx+3\,a\right)}{12\,b^{2}}-\frac{(bdx+bc)\cos\left(bx+a\right)}{4\,b^{2}}+\frac{d\sin\left(3\,bx+3\,a\right)}{36\,b^{2}}+\frac{d\sin\left(bx+a\right)}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/12*(b*d*x + b*c)*cos(3*b*x + 3*a)/b^2 - 1/4*(b*d*x + b*c)*cos(b*x + a)/b
^2 + 1/36*d*sin(3*b*x + 3*a)/b^2 + 1/4*d*sin(b*x + a)/b^2
```

**maple** [A] time = 0.01, size = 71, normalized size = 1.39

$$\frac{\frac{d\left(-\frac{(bx+a)\left(\cos^{3}(bx+a)\right)}{3}+\frac{\left(2+\cos^{2}(bx+a)\right)\sin(bx+a)}{9}\right)}{b}+\frac{da\left(\cos^{3}(bx+a)\right)}{3b}-\frac{c\left(\cos^{3}(bx+a)\right)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b*d*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+1/3/
b*d*a*cos(b*x+a)^3-1/3*c*cos(b*x+a)^3)
```

**maxima** [A] time = 0.39, size = 86, normalized size = 1.69

$$-\frac{12\,c\cos\left(bx+a\right)^{3}-\frac{12\,ad\cos(bx+a)^{3}}{b}+\frac{(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)-9\,\sin(bx+a))d}{b}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/36*(12*c*cos(b*x + a)^3 - 12*a*d*cos(b*x + a)^3/b + (3*(b*x + a)*cos(3*b
*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*d
/b)/b
```

**mupad** [B] time = 0.95, size = 58, normalized size = 1.14

$$\frac{\frac{2\,d\,\sin(a+b\,x)}{9}-b\,\left(\frac{c\cos(a+b\,x)^{3}}{3}+\frac{d\,x\cos(a+b\,x)^{3}}{3}\right)+\frac{d\cos(a+b\,x)^{2}\,\sin(a+b\,x)}{9}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x),x)
```

```
[Out] ((2*d*sin(a + b*x))/9 - b*((c*cos(a + b*x)^3)/3 + (d*x*cos(a + b*x)^3)/3) +
 (d*cos(a + b*x)^2*sin(a + b*x))/9)/b^2
```

**sympy [A]** time = 0.87, size = 85, normalized size = 1.67

$$\begin{cases} -\frac{c\cos^3(a+bx)}{3b} - \frac{dx\cos^3(a+bx)}{3b} + \frac{2d\sin^3(a+bx)}{9b^2} + \frac{d\sin(a+bx)\cos^2(a+bx)}{3b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sin(a)\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c*cos(a + b*x)**3/(3*b) - d*x*cos(a + b*x)**3/(3*b) + 2*d*sin(a
 + b*x)**3/(9*b**2) + d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2), Ne(b, 0)), (
(c*x + d*x**2/2)*sin(a)*cos(a)**2, True))
```

# 3.75 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=121

$$\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

```
[Out] 1/4*cos(a-b*c/d)*Si(b*c/d+b*x)/d+1/4*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d+1
/4*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d+1/4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d
```

**Rubi [A]** time = 0.22, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d) + (CosIntegral[
(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c
)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(
4*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx &= \int \left(\frac{\sin(a+bx)}{4(c+dx)}+\frac{\sin(3a+3bx)}{4(c+dx)}\right)dx\\ &= \frac{1}{4}\int \frac{\sin(a+bx)}{c+dx}\,dx+\frac{1}{4}\int \frac{\sin(3a+3bx)}{c+dx}\,dx\\ &= \frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx+\frac{1}{4}\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\\ &= \frac{\operatorname{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d}+\frac{\operatorname{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d}+\frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}\right.}{4d}\end{aligned}$$

**Mathematica [A]** time = 0.30, size = 100, normalized size = 0.83

$$\frac{\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Ci}\left(\frac{3b(c+dx)}{d}\right)+\sin\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3b(c+dx)}{d}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + CosIntegral[b*(c/d +
 x)]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Cos[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

**fricas [A]** time = 0.62, size = 152, normalized size = 1.26

$$\frac{\left(\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)+\operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+\left(\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)+2\,\cos\left(-\frac{3\,(bc-ad)}{d}\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*((cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*sin(-
(b*c - a*d)/d) + (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d*x
+ b*c)/d))*sin(-3*(b*c - a*d)/d) + 2*cos(-3*(b*c - a*d)/d)*sin_integral(3*(
b*d*x + b*c)/d) + 2*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

**giac [C]** time = 3.83, size = 6279, normalized size = 51.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/8*(imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_inte
gral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((b*
d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
 2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)
^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(
```

```
3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2
 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(b*x + b*c/d))*tan(
3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - i
mag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d
)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/
2*a)^2*tan(3/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 4*imag_part(cos_integral(b*x + b*c/d))*tan(
3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*imag_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
 + 8*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d) - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)
^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-b*x - b*c/d))*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*
d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((
b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2
*b*c/d)^2 - 4*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*
a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 8*sin_integral(3*(b*d*x + b*c)/d)*ta
n(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + imag_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag
_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral((b*d*x + b*c)/d)*
tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(3*b
*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(c
os_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(1/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(3*b*x +
3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*real_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*real_par
t(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2*r
eal_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)
^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan
(3/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(
1/2*a)^2*tan(3/2*b*c/d)^2 + 2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*
a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(b*x + b
*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*real_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
 + 2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d) - 2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a
)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*t
an(1/2*a)*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan
```

```
(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*real_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c
/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c
/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*
a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b
*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*rea
l_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 - 2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan
(1/2*a)^2 - imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2
 + imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 + imag_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 - 2*sin_integr
al(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2 - 2*sin_integral((b*d*x + b
*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2 + 4*imag_part(cos_integral(3*b*x + 3*b*c/d
))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) - 4*imag_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 8*sin_integral(3*(b*d
*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + imag_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + imag_part(cos_integral(
b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - imag_part(cos_integral(-b*x -
 b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*
b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*t
an(3/2*a)^2*tan(3/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2
*tan(3/2*b*c/d)^2 - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*t
an(3/2*b*c/d)^2 - imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2
*b*c/d)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/
d)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d
)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*s
in_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 4*imag_part(co
s_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*imag_pa
rt(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 8*s
in_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 4*ima
g_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
) - 4*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d) + 8*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d) - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1
/2*b*c/d)^2 - imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c
/d)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2
 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2
- 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 2*sin_i
ntegral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_inte
gral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integr
al(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-b*
x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d
)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a
)^2*tan(1/2*b*c/d)^2 + 4*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a
)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 4*imag_part(cos_integral(-3*b*x - 3*b*c
/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 8*sin_integral(3*(b*d*x +
 b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - imag_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integ
ral(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integra
l(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral
(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b
*d*x + b*c)/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral((b*d*x +
b*c)/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(b*x +
b*c/d))*tan(3/2*a)^2*tan(1/2*a) + 2*real_part(cos_integral(-b*x - b*c/d))*t
```

```
an(3/2*a)^2*tan(1/2*a) + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2
*a)*tan(1/2*a)^2 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*t
an(1/2*a)^2 + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3
/2*b*c/d) + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/
2*b*c/d) - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*
b*c/d) - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b
*c/d) - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d
)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)
^2 + 2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2
*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 - 2*real
_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d) - 2*real_part(
cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_integra
l(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(b*x
 + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(-b*x
- b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(3*b*x
+ 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(b*x + b*c
/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-b*x - b*c/d))*
tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))
*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - imag_part(cos_integral(3*b*x + 3*b*c/d))
*tan(3/2*a)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2 - imag_pa
rt(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2 + imag_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(3/2*a)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2
+ 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2 + imag_part(cos_integral(3*b
*x + 3*b*c/d))*tan(1/2*a)^2 - imag_part(cos_integral(b*x + b*c/d))*tan(1/2*
a)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - imag_part(cos_i
ntegral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)
*tan(1/2*a)^2 - 2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2 + 4*imag_part(
cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) - 4*imag_part(cos_
integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) + 8*sin_integral(3*(b
*d*x + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d) - imag_part(cos_integral(3*b*x + 3
*b*c/d))*tan(3/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*
c/d)^2 - imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2 + imag_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x
 + b*c)/d)*tan(3/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*c/d
)^2 + 4*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 4*
imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 8*sin_int
egral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d) + imag_part(cos_integral(3
*b*x + 3*b*c/d))*tan(1/2*b*c/d)^2 - imag_part(cos_integral(b*x + b*c/d))*ta
n(1/2*b*c/d)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - i
mag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*c/d)^2 + 2*sin_integral(
3*(b*d*x + b*c)/d)*tan(1/2*b*c/d)^2 - 2*sin_integral((b*d*x + b*c)/d)*tan(1
/2*b*c/d)^2 + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a) + 2*rea
l_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a) + 2*real_part(cos_integra
l(b*x + b*c/d))*tan(1/2*a) + 2*real_part(cos_integral(-b*x - b*c/d))*tan(1/
2*a) - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d) - 2*real_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d) - 2*real_part(cos_integr
al(b*x + b*c/d))*tan(1/2*b*c/d) - 2*real_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*c/d) + imag_part(cos_integral(3*b*x + 3*b*c/d)) + imag_part(cos_in
tegral(b*x + b*c/d)) - imag_part(cos_integral(-b*x - b*c/d)) - imag_part(co
s_integral(-3*b*x - 3*b*c/d)) + 2*sin_integral(3*(b*d*x + b*c)/d) + 2*sin_i
ntegral((b*d*x + b*c)/d))/(d*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(3/2*a)
^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^
2*tan(1/2*a)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2*b
*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(1/2*b*c/d)^2
```

```
 + d*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2 + d*tan(1/2*a)^2 +
d*tan(3/2*b*c/d)^2 + d*tan(1/2*b*c/d)^2 + d)
```

**maple [A]** time = 0.01, size = 167, normalized size = 1.38

$$\frac{\frac{b\left(\frac{3\,\mathrm{Si}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\sin\left(\frac{-3da+3cb}{d}\right)}{d}\right)}{12}+\frac{b\left(\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}\right)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x)
```

```
[Out] 1/b*(1/12*b*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*
x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)+1/4*b*(Si(b*x+a+(-a*d+b*c)/d)*
cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d))
```

**maxima [C]** time = 0.43, size = 273, normalized size = 2.26

$$-\frac{b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(i\,E_1\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)-i\,E_1\left(-\frac{3i\,bc+3i\,(bx+a)}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_int
egral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b*(I*
exp_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_integral
_e(1, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*
(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(1, -
(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(exp_integral_e
(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1, -(3*I*b*c
+ 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x), x)
```

# 3.76 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=168

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\right.}{4d^2}$$

```
[Out] 3/4*b*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^2+1/4*b*Ci(b*c/d+b*x)*cos(a-b*c/
d)/d^2-3/4*b*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2-1/4*b*Si(b*c/d+b*x)*sin
(a-b*c/d)/d^2-1/4*sin(b*x+a)/d/(d*x+c)-1/4*sin(3*b*x+3*a)/d/(d*x+c)
```

**Rubi [A]** time = 0.27, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sin[a + b*x]/(4*d*(c + d
*x)) - Sin[3*a + 3*b*x]/(4*d*(c + d*x)) - (b*Sin[a - (b*c)/d]*SinIntegral[(
b*c)/d + b*x])/(4*d^2) - (3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d +
3*b*x])/(4*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\sin(a+bx)}{4(c+dx)^2}+\frac{\sin(3a+3bx)}{4(c+dx)^2}\right)dx\\ &= \frac{1}{4}\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx+\frac{1}{4}\int \frac{\sin(3a+3bx)}{(c+dx)^2}\,dx\\ &= -\frac{\sin(a+bx)}{4d(c+dx)}-\frac{\sin(3a+3bx)}{4d(c+dx)}+\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{4d}+\frac{(3b)\int \frac{\cos(3a+3bx)}{c+dx}\,dx}{4d}\\ &= -\frac{\sin(a+bx)}{4d(c+dx)}-\frac{\sin(3a+3bx)}{4d(c+dx)}+\frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d}+\frac{\left(b\cos\left(\right.\right.}{}\\ &= \frac{b\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\sin(a+bx)}{4d(c+dx)}-\end{aligned}$$

**Mathematica [A]** time = 1.09, size = 139, normalized size = 0.83

$$-\frac{-b\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(b\left(\frac{c}{d}+x\right)\right)-3b\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3b(c+dx)}{d}\right)+b\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)+3b\sin\left(3a-\frac{3bc}{d}\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -1/4*(-(b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) - 3*b*Cos[3*a - (3*b*c
)/d]*CosIntegral[(3*b*(c + d*x))/d] + (d*Sin[a + b*x])/(c + d*x) + (d*Sin[3
*(a + b*x)])/(c + d*x) + b*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b*
Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^2
```

**fricas [A]** time = 0.83, size = 233, normalized size = 1.39

$$-\frac{8\,d\cos\left(bx+a\right)^2\sin\left(bx+a\right)+6\left(bdx+bc\right)\sin\left(-\frac{3\left(bc-ad\right)}{d}\right)\mathrm{Si}\left(\frac{3\left(bdx+bc\right)}{d}\right)+2\left(bdx+bc\right)\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*d*cos(b*x + a)^2*sin(b*x + a) + 6*(b*d*x + b*c)*sin(-3*(b*c - a*d)/
d)*sin_integral(3*(b*d*x + b*c)/d) + 2*(b*d*x + b*c)*sin(-(b*c - a*d)/d)*si
n_integral((b*d*x + b*c)/d) - ((b*d*x + b*c)*cos_integral((b*d*x + b*c)/d)
+ (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) - 3*((b
*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-3
*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 240, normalized size = 1.43

$$\frac{\frac{b^2\left(-\frac{3\sin(3bx+3a)}{((bx+a)d-da+cb)d}+\frac{\frac{9\,\mathrm{Si}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\sin\left(\frac{-3da+3cb}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}}{d}\right)}{12}+\frac{b^2\left(-\frac{\sin(bx+a)}{((bx+a)d-da+cb)d}+\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-d}{}\right.}{d}}{}\right.}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/12*b^2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d+3*(3*Si(3*b*x+3*a+3*
(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-
a*d+b*c)/d)/d)/d)+1/4*b^2*(-sin(b*x+a)/((b*x+a)*d-d*a+c*b)/d+(Si(b*x+a+(-a*
d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d
))
```

**maxima [C]** time = 0.52, size = 300, normalized size = 1.79

$$-\frac{b^2\left(i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(i\,E_2\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)-i\,E_2\left(-\frac{3i\,bc+3i}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*(I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^2
*(I*exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_inte
gral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d)
+ b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(exp_in
tegral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(2, -(
3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c*d + (
b*x + a)*d^2 - a*d^2)*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**2, x)
```

## 3.77 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=221

$$-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{S}}{8d^3}$$

```
[Out] -1/8*b*cos(b*x+a)/d^2/(d*x+c)-3/8*b*cos(3*b*x+3*a)/d^2/(d*x+c)-1/8*b^2*cos(
a-b*c/d)*Si(b*c/d+b*x)/d^3-9/8*b^2*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^3-9
/8*b^2*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3-1/8*b^2*Ci(b*c/d+b*x)*sin(a-b
*c/d)/d^3-1/8*sin(b*x+a)/d/(d*x+c)^2-1/8*sin(3*b*x+3*a)/d/(d*x+c)^2
```

**Rubi [A]** time = 0.32, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -(b*Cos[a + b*x])/(8*d^2*(c + d*x)) - (3*b*Cos[3*a + 3*b*x])/(8*d^2*(c + d*
x)) - (9*b^2*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^3) -
 (b^2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^3) - Sin[a + b*x]/(
8*d*(c + d*x)^2) - Sin[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/
d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) - (9*b^2*Cos[3*a - (3*b*c)/d]*SinInt
egral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\sin(a+bx)}{4(c+dx)^3}+\frac{\sin(3a+3bx)}{4(c+dx)^3}\right)dx$$
$$= \frac{1}{4}\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx+\frac{1}{4}\int \frac{\sin(3a+3bx)}{(c+dx)^3}\,dx$$
$$= -\frac{\sin(a+bx)}{8d(c+dx)^2}-\frac{\sin(3a+3bx)}{8d(c+dx)^2}+\frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{8d}+\frac{(3b)\int \frac{\cos(3a+3bx)}{(c+dx)^2}\,dx}{8d}$$
$$= -\frac{b\cos(a+bx)}{8d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{8d^2(c+dx)}-\frac{\sin(a+bx)}{8d(c+dx)^2}-\frac{\sin(3a+3bx)}{8d(c+dx)^2}-\frac{b^2\int \frac{\sin}{}}{8}$$
$$= -\frac{b\cos(a+bx)}{8d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{8d^2(c+dx)}-\frac{\sin(a+bx)}{8d(c+dx)^2}-\frac{\sin(3a+3bx)}{8d(c+dx)^2}-\frac{\left(9b^2\cos\right.}{}$$
$$= -\frac{b\cos(a+bx)}{8d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{8d^2(c+dx)}-\frac{9b^2\text{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{8d^3}-\frac{b^2\text{C}}{}$$

**Mathematica [A]** time = 2.56, size = 181, normalized size = 0.82

$$-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right)+b^2\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+9b^2\cos\left(3a\right.}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -1/8*(9*b^2*CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + b^2*CosIn
tegral[b*(c/d + x)]*Sin[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b*x] + d*Sin
[a + b*x]))/(c + d*x)^2 + (d*(3*b*(c + d*x)*Cos[3*(a + b*x)] + d*Sin[3*(a +
 b*x)]))/(c + d*x)^2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 9*b^
2*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^3
```

**fricas [A]** time = 0.53, size = 393, normalized size = 1.78

$$-\frac{8\,d^2\cos\left(bx+a\right)^2\sin\left(bx+a\right)+24\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3+18\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(8*d^2*cos(b*x + a)^2*sin(b*x + a) + 24*(b*d^2*x + b*c*d)*cos(b*x + a
)^3 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-3*(b*c - a*d)/d)*sin_in
tegral(3*(b*d*x + b*c)/d) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b
*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 16*(b*d^2*x + b*c*d)*cos(b*x +
 a) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x + b*c)/d)
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x + b*c)/d))*sin
(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(3*
(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-3*(b
*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 313, normalized size = 1.42

$$\frac{\frac{b^3\left(-\frac{3\sin(3bx+3a)}{2((bx+a)d-da+cb)^2 d}+\frac{-\frac{9\cos(3bx+3a)}{2((bx+a)d-da+cb)d}-\frac{9\left(\frac{3\,\mathrm{Si}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\sin\left(\frac{-3da+3cb}{d}\right)}{d}\right)}{2d}}{d}\right)}{12}+\frac{b^3\left(-\frac{\sin(bx+a)}{2((bx+a)d-da+cb)^2 d}+\cdots\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/12*b^3*(-3/2*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)^2/d+3/2*(-3*cos(3*b*
x+3*a)/((b*x+a)*d-d*a+c*b)/d-3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+
b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d)+1/4*b
^3*(-1/2*sin(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-d*a
+c*b)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*
sin((-a*d+b*c)/d)/d)/d)/d))
```

**maxima [C]** time = 0.65, size = 335, normalized size = 1.52

$$-\frac{b^3\left(i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(i\,E_3\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)-i\,E_3\left(-\frac{3i\,bc+3i\,(bx\cdots}{d}\right.\right.}{8\left(b^2c^2d\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^3*(I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^3
*(I*exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_inte
gral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d)
+ b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(exp_in
tegral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(3, -(
3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d
 - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**3,x)
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**3, x)
```

## 3.78 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

**Optimal**. Leaf size=270

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}+\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)}{8d^4}$$

```
[Out] -9/8*b^3*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^4-1/24*b^3*Ci(b*c/d+b*x)*cos(
a-b*c/d)/d^4-1/24*b*cos(b*x+a)/d^2/(d*x+c)^2-1/8*b*cos(3*b*x+3*a)/d^2/(d*x+
c)^2+9/8*b^3*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/24*b^3*Si(b*c/d+b*x)*
sin(a-b*c/d)/d^4-1/12*sin(b*x+a)/d/(d*x+c)^3+1/24*b^2*sin(b*x+a)/d^3/(d*x+c
)-1/12*sin(3*b*x+3*a)/d/(d*x+c)^3+3/8*b^2*sin(3*b*x+3*a)/d^3/(d*x+c)
```

**Rubi [A]** time = 0.38, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -(b*Cos[a + b*x])/(24*d^2*(c + d*x)^2) - (b*Cos[3*a + 3*b*x])/(8*d^2*(c + d
*x)^2) - (b^3*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(24*d^4) - (9*b^
3*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(8*d^4) - Sin[a + b*
x]/(12*d*(c + d*x)^3) + (b^2*Sin[a + b*x])/(24*d^3*(c + d*x)) - Sin[3*a + 3
*b*x]/(12*d*(c + d*x)^3) + (3*b^2*Sin[3*a + 3*b*x])/(8*d^3*(c + d*x)) + (b^
3*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(24*d^4) + (9*b^3*Sin[3*a -
(3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\sin(a+bx)}{4(c+dx)^4}+\frac{\sin(3a+3bx)}{4(c+dx)^4}\right)dx$$

$$= \frac{1}{4}\int \frac{\sin(a+bx)}{(c+dx)^4}\,dx+\frac{1}{4}\int \frac{\sin(3a+3bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\sin(a+bx)}{12d(c+dx)^3}-\frac{\sin(3a+3bx)}{12d(c+dx)^3}+\frac{b\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx}{12d}+\frac{b\int \frac{\cos(3a+3bx)}{(c+dx)^3}\,dx}{4d}$$

$$= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{8d^2(c+dx)^2}-\frac{\sin(a+bx)}{12d(c+dx)^3}-\frac{\sin(3a+3bx)}{12d(c+dx)^3}-\frac{b^2\int \frac{\mathrm{si}}{(}}{2}$$

$$= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{8d^2(c+dx)^2}-\frac{\sin(a+bx)}{12d(c+dx)^3}+\frac{b^2\sin(a+bx)}{24d^3(c+dx)}-\frac{\sin(3a}{12d(c}$$

$$= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{8d^2(c+dx)^2}-\frac{\sin(a+bx)}{12d(c+dx)^3}+\frac{b^2\sin(a+bx)}{24d^3(c+dx)}-\frac{\sin(3a}{12d(c}$$

$$= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{8d^2(c+dx)^2}-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos(}{}$$

**Mathematica [A]** time = 1.82, size = 300, normalized size = 1.11

$$-\frac{b^3(c+dx)^3\left(\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(b\left(\frac{c}{d}+x\right)\right)-\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right)+27b^3(c+dx)^3\left(\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3b(c}{d}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -1/24*(d*Cos[b*x]*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a]
) + d*Cos[3*b*x]*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*S
in[3*a]) - d*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a])*Sin
[b*x] - d*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin[3*a]
)*Sin[3*b*x] + b^3*(c + d*x)^3*(Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] -
 Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) + 27*b^3*(c + d*x)^3*(Cos[3*a -
 (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Sin[3*a - (3*b*c)/d]*SinIntegr
al[(3*b*(c + d*x))/d]))/(d^4*(c + d*x)^3)
```

**fricas [B]** time = 0.67, size = 558, normalized size = 2.07

$$-\frac{24\left(bd^3x+bcd^2\right)\cos(bx+a)^3-54\left(b^3d^3x^3+3b^3cd^2x^2+3b^3c^2dx+b^3c^3\right)\sin\left(-\frac{3(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3(bdx+bc)}{d}\right)-2(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(24*(b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - 54*(b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*
```

```
x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*s
in(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 16*(b*d^3*x + b*c*d^2)*c
os(b*x + a) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*co
s_integral((b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + b^3*c^3)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) + 27*((b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x
 + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_
integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) + 8*(2*b^2*d^3*x^2 + 4*
b^2*c*d^2*x + 2*b^2*c^2*d - (9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d -
 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x
+ c^3*d^4)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 381, normalized size = 1.41

$$\frac{\frac{b^4\left(-\frac{\sin(3bx+3a)}{((bx+a)d-da+cb)^3 d}+\frac{-\frac{3\cos(3bx+3a)}{2((bx+a)d-da+cb)^2 d}-\frac{3\left(-\frac{3\sin(3bx+3a)}{((bx+a)d-da+cb)d}+\frac{\frac{9\,\mathrm{Si}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\sin\left(\frac{-3da+3cb}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}}{d}\right)}{2d}}{d}\right)}{12}+\frac{b^4\left(-\cdots\right.}{\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x)
```

```
[Out] 1/b*(1/12*b^4*(-sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)^3/d+(-3/2*cos(3*b*x+3*a)
/((b*x+a)*d-d*a+c*b)^2/d-3/2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d+3*(3*
Si(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b
*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d)/d)+1/4*b^4*(-1/3*sin(b*x+a)/((b*x+a)*d-
d*a+c*b)^3/d+1/3*(-1/2*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d-1/2*(-sin(b*x+a)/
((b*x+a)*d-d*a+c*b)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+
(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)/d))
```

**maxima [C]** time = 0.88, size = 385, normalized size = 1.43

$$-\frac{b^4\left(iE_4\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-iE_4\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^4\left(iE_4\left(\frac{3ibc+3i(bx+a)d-3iad}{d}\right)-iE_4\left(-\frac{3ibc+3i(bx\cdots}{d}\right.\right.\cdots}{8\left(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3+(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/8*(b^4*(I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^4
*(I*exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_inte
gral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d)
+ b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
```

```
e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^4*(exp_in
tegral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(4, -(
3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^3*c^3*d
 - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3
 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*
b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^4,x)
[Out] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{\left(a + bx \right)} \cos^{2}{\left(a + bx \right)}}{\left(c + dx\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**4,x)
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**4, x)
```

# 3.79 $\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=162

$$\frac{i2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{i2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4ib(c+dx)}{d}\right)}{b}$$

```
[Out] 1/8*(d*x+c)^(1+m)/d/(1+m)+I*exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b*(
d*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)-I*(d*x+c)^m*GAMMA(1+m,4*I*b*(d
*x+c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

**Rubi [A]** time = 0.20, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3307, 2181}

$$\frac{i2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{i2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{4ib(c+dx)}{d}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(8*d*(1 + m)) + (I*E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Ga
mma[1 + m, ((-4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)
^m) - (I*(c + d*x)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*
E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^m - \frac{1}{8}(c+dx)^m\cos(4a+4bx)\right)dx$$
$$= \frac{(c+dx)^{1+m}}{8d(1+m)} - \frac{1}{8}\int (c+dx)^m \cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^{1+m}}{8d(1+m)} - \frac{1}{16}\int e^{-i(4a+4bx)}(c+dx)^m\,dx - \frac{1}{16}\int e^{i(4a+4bx)}(c+d$$
$$= \frac{(c+dx)^{1+m}}{8d(1+m)} + \frac{i4^{-3-m}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{4ib}{}\right.}{b}$$

**Mathematica [A]** time = 1.04, size = 213, normalized size = 1.31

$$\frac{4^{-m-3}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-id(m+1)\left(-\frac{ib(c+dx)}{d}\right)^m\left(\cos\left(4a-\frac{4bc}{d}\right)-i\sin\left(4a-\frac{4bc}{d}\right)\right)\Gamma\left(m+1,\frac{4ib(c+dx)}{d}\right)+}{bd(m+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (4^(-3 - m)*(c + d*x)^m*(2^(3 + 2*m)*b*(c + d*x)*((b^2*(c + d*x)^2)/d^2)^m
- I*d*(1 + m)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d]*
(Cos[4*a - (4*b*c)/d] - I*Sin[4*a - (4*b*c)/d]) + I*d*(1 + m)*((I*b*(c + d*
x))/d)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d]*(Cos[4*a - (4*b*c)/d] + I*Sin
[4*a - (4*b*c)/d])))/(b*d*(1 + m)*((b^2*(c + d*x)^2)/d^2)^m)
```

**fricas [A]** time = 0.59, size = 134, normalized size = 0.83

$$\frac{(-i\,dm-i\,d)e^{\left(-\frac{dm\log\left(\frac{4i\,b}{d}\right)-4i\,bc+4i\,ad}{d}\right)}\Gamma\left(m+1,\frac{4i\,bdx+4i\,bc}{d}\right)+(i\,dm+i\,d)e^{\left(-\frac{dm\log\left(-\frac{4i\,b}{d}\right)+4i\,bc-4i\,ad}{d}\right)}\Gamma\left(m+1,\frac{-4i\,bdx-4i\,bc}{d}\right)-}{64\,(bdm+bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/64*((-I*d*m - I*d)*e^(-(d*m*log(4*I*b/d) - 4*I*b*c + 4*I*a*d)/d)*gamma(m
+ 1, (4*I*b*d*x + 4*I*b*c)/d) + (I*d*m + I*d)*e^(-(d*m*log(-4*I*b/d) + 4*I*
b*c - 4*I*a*d)/d)*gamma(m + 1, (-4*I*b*d*x - 4*I*b*c)/d) + 8*(b*d*x + b*c)*
(d*x + c)^m)/(b*d*m + b*d)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^2, x)
```

**maple [F]** time = 0.15, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^2(bx+a)\right)\left(\sin^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(dm+d)\int (dx+c)^m \cos(4\,bx+4\,a)\,dx - e^{\left(m\log(dx+c)+\log(dx+c)\right)}}{8\,(dm+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((d*m + d)*integrate((d*x + c)^m*cos(4*b*x + 4*a), x) - e^(m*log(d*x +
 c) + log(d*x + c)))/(d*m + d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)^2\sin(a+b\,x)^2\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^m, x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.80 $\int (c+dx)^4 \cos^2(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=131

$$-\frac{3d^4\sin(4a+4bx)}{1024b^5}+\frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4}+\frac{3d^2(c+dx)^2\sin(4a+4bx)}{128b^3}-\frac{d(c+dx)^3\cos(4a+4bx)}{32b^2}-\frac{(c+}{}$$

```
[Out] 1/40*(d*x+c)^5/d+3/256*d^3*(d*x+c)*cos(4*b*x+4*a)/b^4-1/32*d*(d*x+c)^3*cos(
4*b*x+4*a)/b^2-3/1024*d^4*sin(4*b*x+4*a)/b^5+3/128*d^2*(d*x+c)^2*sin(4*b*x+
4*a)/b^3-1/32*(d*x+c)^4*sin(4*b*x+4*a)/b
```

**Rubi** [A] time = 0.16, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2637}

$$\frac{3d^2(c+dx)^2\sin(4a+4bx)}{128b^3}+\frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4}-\frac{d(c+dx)^3\cos(4a+4bx)}{32b^2}-\frac{3d^4\sin(4a+4bx)}{1024b^5}-\frac{(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^5/(40*d) + (3*d^3*(c + d*x)*Cos[4*a + 4*b*x])/(256*b^4) - (d*(c +
 d*x)^3*Cos[4*a + 4*b*x])/(32*b^2) - (3*d^4*Sin[4*a + 4*b*x])/(1024*b^5) +
(3*d^2*(c + d*x)^2*Sin[4*a + 4*b*x])/(128*b^3) - ((c + d*x)^4*Sin[4*a + 4*b
*x])/(32*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^4 \cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^4 - \frac{1}{8}(c+dx)^4\cos(4a+4bx)\right)dx$$
$$= \frac{(c+dx)^5}{40d} - \frac{1}{8}\int (c+dx)^4\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^5}{40d} - \frac{(c+dx)^4\sin(4a+4bx)}{32b} + \frac{d\int (c+dx)^3\sin(4a+4bx)\,dx}{8b}$$
$$= \frac{(c+dx)^5}{40d} - \frac{d(c+dx)^3\cos(4a+4bx)}{32b^2} - \frac{(c+dx)^4\sin(4a+4bx)}{32b} + \frac{(3d$$
$$= \frac{(c+dx)^5}{40d} - \frac{d(c+dx)^3\cos(4a+4bx)}{32b^2} + \frac{3d^2(c+dx)^2\sin(4a+4bx)}{128b^3} -$$
$$= \frac{(c+dx)^5}{40d} + \frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4} - \frac{d(c+dx)^3\cos(4a+4bx)}{32b^2} +$$
$$= \frac{(c+dx)^5}{40d} + \frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4} - \frac{d(c+dx)^3\cos(4a+4bx)}{32b^2} -$$

**Mathematica [A]** time = 1.27, size = 132, normalized size = 1.01

$$\frac{20bd(c+dx)\cos(4(a+bx))\left(3d^2-8b^2(c+dx)^2\right) - 5\sin(4(a+bx))\left(32b^4(c+dx)^4-24b^2d^2(c+dx)^2+3d^4\right)+12}{5120b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (128*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) +
20*b*d*(c + d*x)*(3*d^2 - 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 5*(3*d^4 -
24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Sin[4*(a + b*x)])/(5120*b^5)
```

**fricas [B]** time = 0.62, size = 466, normalized size = 3.56

$$32\,b^5d^4x^5 + 160\,b^5cd^3x^4 - 40\left(8\,b^3d^4x^3 + 24\,b^3cd^3x^2 + 8\,b^3c^3d - 3\,bcd^3 + 3\left(8\,b^3c^2d^2 - bd^4\right)x\right)\cos\left(bx+a\right)^4 + 40$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/1280*(32*b^5*d^4*x^5 + 160*b^5*c*d^3*x^4 - 40*(8*b^3*d^4*x^3 + 24*b^3*c*d
^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a
)^4 + 40*(8*b^5*c^2*d^2 - b^3*d^4)*x^3 + 40*(8*b^5*c^3*d - 3*b^3*c*d^3)*x^2
 + 40*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^
3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*(32*b^5*c^4 - 24*b^3*c^2*d^2 + 3*b
*d^4)*x - 5*(2*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 32*b^4*c^4 - 24*b^2*c^
2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^2 + 16*(8*b^4*c^3*d - 3*b^2*
c*d^3)*x)*cos(b*x + a)^3 - (32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 32*b^4*c^4
 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^2 + 16*(8*b^4*c^
3*d - 3*b^2*c*d^3)*x)*cos(b*x + a))*sin(b*x + a))/b^5
```

**giac [A]** time = 1.11, size = 224, normalized size = 1.71

$$\frac{1}{40}\,d^4x^5 + \frac{1}{8}\,cd^3x^4 + \frac{1}{4}\,c^2d^2x^3 + \frac{1}{4}\,c^3dx^2 + \frac{1}{8}\,c^4x - \frac{\left(8\,b^3d^4x^3 + 24\,b^3cd^3x^2 + 24\,b^3c^2d^2x + 8\,b^3c^3d - 3\,bd^4x - 3\,bcd^3\right)\mathrm{co}}{256\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/40*d^4*x^5 + 1/8*c*d^3*x^4 + 1/4*c^2*d^2*x^3 + 1/4*c^3*d*x^2 + 1/8*c^4*x
- 1/256*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d
- 3*b*d^4*x - 3*b*c*d^3)*cos(4*b*x + 4*a)/b^5 - 1/1024*(32*b^4*d^4*x^4 + 12
8*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4*c^3*d*x + 32*b^4*c^4 - 24*b
^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 + 3*d^4)*sin(4*b*x + 4*a)/b^5
```

**maple [B]** time = 0.09, size = 1915, normalized size = 14.62

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^4*d^4*((b*x+a)^4*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b
*x+a)^3*cos(b*x+a)^2+3/4*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a
)+3/32*(b*x+a)*cos(b*x+a)^2-3/64*cos(b*x+a)*sin(b*x+a)-21/256*b*x-21/256*a-
7/16*(b*x+a)^3-1/10*(b*x+a)^5-(b*x+a)^4*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))
*cos(b*x+a)+3/8*b*x+3/8*a)-1/4*(b*x+a)^3*sin(b*x+a)^4+3/4*(b*x+a)^2*(-1/4*(
sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*sin(b*x
+a)^4+3/128*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))-4/b^4*a*d^4*((b*x+a)^
3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/16*(b*x+a)^2*cos(b*x+a)^2+3/
8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-21/128*(b*x+a)^2-3/128*
sin(b*x+a)^2-3/32*(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*c
os(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^4+3/8*(b*x+a)*(-1/4*(sin
(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/128*sin(b*x+a)^4)+4/b
^3*c*d^3*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/16*(b*x+a)
^2*cos(b*x+a)^2+3/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-21/12
8*(b*x+a)^2-3/128*sin(b*x+a)^2-3/32*(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3
+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^4+3/8*
(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/128
*sin(b*x+a)^4)+6/b^4*a^2*d^4*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x
+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a
-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/
8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*co
s(b*x+a))-12/b^3*a*c*d^3*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2
*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/1
2*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*
x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a))+6/b^2*c^2*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1
/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/12*(b*
x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8
*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))
-4/b^4*a^3*d^4*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*
x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+12/b^3*a^2*c*d^3*((b*x+a)*(-1/2*cos(
b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(
-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a
)^4)-12/b^2*a*c^2*d^2*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1
/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))
*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+4/b*c^3*d*((b*x+a)*(-1/2*cos(
b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(
-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a
)^4)+1/b^4*a^4*d^4*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+
1/8*b*x+1/8*a)-4/b^3*a^3*c*d^3*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)
*sin(b*x+a)+1/8*b*x+1/8*a)+6/b^2*a^2*c^2*d^2*(-1/4*cos(b*x+a)^3*sin(b*x+a)+
1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)-4/b*a*c^3*d*(-1/4*cos(b*x+a)^3*sin
(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+c^4*(-1/4*cos(b*x+a)^3*sin
(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a))
```

**maxima [B]** time = 0.37, size = 735, normalized size = 5.61

$$\frac{160\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c^4 - \frac{640\,(4\,bx+4\,a-\sin(4\,bx+4\,a))ac^3d}{b} + \frac{960\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2c^2d^2}{b^2} - \frac{640\,(4\,bx+4\,a-\sin(4\,bx}{b^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5120*(160*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^4 - 640*(4*b*x + 4*a - sin(4
*b*x + 4*a))*a*c^3*d/b + 960*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*c^2*d^2/b
^2 - 640*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^3*c*d^3/b^3 + 160*(4*b*x + 4*a
- sin(4*b*x + 4*a))*a^4*d^4/b^4 + 160*(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*
x + 4*a) - cos(4*b*x + 4*a))*c^3*d/b - 480*(8*(b*x + a)^2 - 4*(b*x + a)*sin
(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*c^2*d^2/b^2 + 480*(8*(b*x + a)^2 - 4*(b
*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^2*c*d^3/b^3 - 160*(8*(b*x +
a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^3*d^4/b^4 + 40*(3
2*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4
*b*x + 4*a))*c^2*d^2/b^2 - 80*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*
a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a))*a*c*d^3/b^3 + 40*(32*(b*x + a)
^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a)
)*a^2*d^4/b^4 + 20*(32*(b*x + a)^4 - 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a)
 - 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a))*c*d^3/b^3 - 20*(32*(b*
x + a)^4 - 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 4*(8*(b*x + a)^3 - 3*b*
x - 3*a)*sin(4*b*x + 4*a))*a*d^4/b^4 + (128*(b*x + a)^5 - 20*(8*(b*x + a)^3
 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 5*(32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*
sin(4*b*x + 4*a))*d^4/b^4)/b
```

**mupad [B]** time = 1.68, size = 349, normalized size = 2.66

$$-\frac{15\,d^4\,\sin(4\,a+4\,b\,x) - 640\,b^5\,c^4\,x + 160\,b^4\,c^4\,\sin(4\,a+4\,b\,x) - 128\,b^5\,d^4\,x^5 + 160\,b^3\,c^3\,d\,\cos(4\,a+4\,b\,x) - 12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] -(15*d^4*sin(4*a + 4*b*x) - 640*b^5*c^4*x + 160*b^4*c^4*sin(4*a + 4*b*x) -
128*b^5*d^4*x^5 + 160*b^3*c^3*d*cos(4*a + 4*b*x) - 1280*b^5*c^3*d*x^2 - 640
*b^5*c*d^3*x^4 - 120*b^2*c^2*d^2*sin(4*a + 4*b*x) + 160*b^3*d^4*x^3*cos(4*a
 + 4*b*x) - 1280*b^5*c^2*d^2*x^3 - 120*b^2*d^4*x^2*sin(4*a + 4*b*x) + 160*b
^4*d^4*x^4*sin(4*a + 4*b*x) - 60*b*c*d^3*cos(4*a + 4*b*x) - 60*b*d^4*x*cos(
4*a + 4*b*x) + 960*b^4*c^2*d^2*x^2*sin(4*a + 4*b*x) - 240*b^2*c*d^3*x*sin(4
*a + 4*b*x) + 640*b^4*c^3*d*x*sin(4*a + 4*b*x) + 480*b^3*c^2*d^2*x*cos(4*a
+ 4*b*x) + 480*b^3*c*d^3*x^2*cos(4*a + 4*b*x) + 640*b^4*c*d^3*x^3*sin(4*a +
 4*b*x))/(5120*b^5)
```

**sympy [A]** time = 13.62, size = 1231, normalized size = 9.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*x*sin(a + b*x)**4/8 + c**4*x*sin(a + b*x)**2*cos(a + b*x)**
2/4 + c**4*x*cos(a + b*x)**4/8 + c**3*d*x**2*sin(a + b*x)**4/4 + c**3*d*x**
2*sin(a + b*x)**2*cos(a + b*x)**2/2 + c**3*d*x**2*cos(a + b*x)**4/4 + c**2*
d**2*x**3*sin(a + b*x)**4/4 + c**2*d**2*x**3*sin(a + b*x)**2*cos(a + b*x)**
2/2 + c**2*d**2*x**3*cos(a + b*x)**4/4 + c*d**3*x**4*sin(a + b*x)**4/8 + c*
d**3*x**4*sin(a + b*x)**2*cos(a + b*x)**2/4 + c*d**3*x**4*cos(a + b*x)**4/8
```

```
 + d**4*x**5*sin(a + b*x)**4/40 + d**4*x**5*sin(a + b*x)**2*cos(a + b*x)**2
/20 + d**4*x**5*cos(a + b*x)**4/40 + c**4*sin(a + b*x)**3*cos(a + b*x)/(8*b
) - c**4*sin(a + b*x)*cos(a + b*x)**3/(8*b) + c**3*d*x*sin(a + b*x)**3*cos(
a + b*x)/(2*b) - c**3*d*x*sin(a + b*x)*cos(a + b*x)**3/(2*b) + 3*c**2*d**2*
x**2*sin(a + b*x)**3*cos(a + b*x)/(4*b) - 3*c**2*d**2*x**2*sin(a + b*x)*cos
(a + b*x)**3/(4*b) + c*d**3*x**3*sin(a + b*x)**3*cos(a + b*x)/(2*b) - c*d**
3*x**3*sin(a + b*x)*cos(a + b*x)**3/(2*b) + d**4*x**4*sin(a + b*x)**3*cos(a
 + b*x)/(8*b) - d**4*x**4*sin(a + b*x)*cos(a + b*x)**3/(8*b) - c**3*d*sin(a
 + b*x)**4/(8*b**2) - c**3*d*cos(a + b*x)**4/(8*b**2) - 3*c**2*d**2*x*sin(a
 + b*x)**4/(32*b**2) + 9*c**2*d**2*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b*
*2) - 3*c**2*d**2*x*cos(a + b*x)**4/(32*b**2) - 3*c*d**3*x**2*sin(a + b*x)*
*4/(32*b**2) + 9*c*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**2) - 3*
c*d**3*x**2*cos(a + b*x)**4/(32*b**2) - d**4*x**3*sin(a + b*x)**4/(32*b**2)
 + 3*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**2) - d**4*x**3*cos(a
+ b*x)**4/(32*b**2) - 3*c**2*d**2*sin(a + b*x)**3*cos(a + b*x)/(32*b**3) +
3*c**2*d**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**3) - 3*c*d**3*x*sin(a + b*x
)**3*cos(a + b*x)/(16*b**3) + 3*c*d**3*x*sin(a + b*x)*cos(a + b*x)**3/(16*b
**3) - 3*d**4*x**2*sin(a + b*x)**3*cos(a + b*x)/(32*b**3) + 3*d**4*x**2*sin
(a + b*x)*cos(a + b*x)**3/(32*b**3) + 3*c*d**3*sin(a + b*x)**4/(64*b**4) +
3*c*d**3*cos(a + b*x)**4/(64*b**4) + 3*d**4*x*sin(a + b*x)**4/(256*b**4) -
9*d**4*x*sin(a + b*x)**2*cos(a + b*x)**2/(128*b**4) + 3*d**4*x*cos(a + b*x)
**4/(256*b**4) + 3*d**4*sin(a + b*x)**3*cos(a + b*x)/(256*b**5) - 3*d**4*si
n(a + b*x)*cos(a + b*x)**3/(256*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2
+ 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**2*cos(a)**2, True))
```

# 3.81 $\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=105

$$\frac{3d^3\cos(4a+4bx)}{1024b^4}+\frac{3d^2(c+dx)\sin(4a+4bx)}{256b^3}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}-\frac{(c+dx)^3\sin(4a+4bx)}{32b}+\frac{(c+dx)^4}{32d}$$

```
[Out] 1/32*(d*x+c)^4/d+3/1024*d^3*cos(4*b*x+4*a)/b^4-3/128*d*(d*x+c)^2*cos(4*b*x+
4*a)/b^2+3/256*d^2*(d*x+c)*sin(4*b*x+4*a)/b^3-1/32*(d*x+c)^3*sin(4*b*x+4*a)
/b
```

**Rubi [A]** time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2638}

$$\frac{3d^2(c+dx)\sin(4a+4bx)}{256b^3}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}+\frac{3d^3\cos(4a+4bx)}{1024b^4}-\frac{(c+dx)^3\sin(4a+4bx)}{32b}+\frac{(c+dx)^4}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^4/(32*d) + (3*d^3*Cos[4*a + 4*b*x])/(1024*b^4) - (3*d*(c + d*x)^2
*Cos[4*a + 4*b*x])/(128*b^2) + (3*d^2*(c + d*x)*Sin[4*a + 4*b*x])/(256*b^3)
 - ((c + d*x)^3*Sin[4*a + 4*b*x])/(32*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3\cos^2(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^3-\frac{1}{8}(c+dx)^3\cos(4a+4bx)\right) dx\\
&= \frac{(c+dx)^4}{32d}-\frac{1}{8}\int (c+dx)^3\cos(4a+4bx)\,dx\\
&= \frac{(c+dx)^4}{32d}-\frac{(c+dx)^3\sin(4a+4bx)}{32b}+\frac{(3d)\int (c+dx)^2\sin(4a+4bx)\,d}{32b}\\
&= \frac{(c+dx)^4}{32d}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}-\frac{(c+dx)^3\sin(4a+4bx)}{32b}+\frac{(3}{}\\
&= \frac{(c+dx)^4}{32d}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}+\frac{3d^2(c+dx)\sin(4a+4bx)}{256b^3}-\\
&= \frac{(c+dx)^4}{32d}+\frac{3d^3\cos(4a+4bx)}{1024b^4}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}+\frac{3d^2(c+}{}
\end{aligned}$$

**Mathematica [A]** time = 0.66, size = 106, normalized size = 1.01

$$\frac{-4b(c+dx)\sin(4(a+bx))\left(8b^2(c+dx)^2-3d^2\right)-3d\cos(4(a+bx))\left(8b^2(c+dx)^2-d^2\right)+32b^4x\left(4c^3+6c^2dx\right.}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (32*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(-d^2 + 8*b^2*(
c + d*x)^2)*Cos[4*(a + b*x)] - 4*b*(c + d*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*S
in[4*(a + b*x)])/(1024*b^4)
```

**fricas [B]** time = 0.50, size = 308, normalized size = 2.93

$$\frac{4\,b^4d^3x^4+16\,b^4cd^2x^3-3\left(8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3\right)\cos\left(bx+a\right)^4+3\left(8\,b^4c^2d-b^2d^3\right)x^2+3\left(8\,b^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/128*(4*b^4*d^3*x^4 + 16*b^4*c*d^2*x^3 - 3*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x
 + 8*b^2*c^2*d - d^3)*cos(b*x + a)^4 + 3*(8*b^4*c^2*d - b^2*d^3)*x^2 + 3*(8
*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 2*(8*b^
4*c^3 - 3*b^2*c*d^2)*x - 2*(2*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 8*b^3*c^3
 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^3 - (8*b^3*d^3*x^3 +
 24*b^3*c*d^2*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(
b*x + a))*sin(b*x + a))/b^4
```

**giac [A]** time = 0.23, size = 153, normalized size = 1.46

$$\frac{1}{32}\,d^3x^4+\frac{1}{8}\,cd^2x^3+\frac{3}{16}\,c^2dx^2+\frac{1}{8}\,c^3x-\frac{3\left(8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3\right)\cos\left(4\,bx+4\,a\right)}{1024\,b^4}-\frac{\left(8\,b^3d^3x^3+24\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/32*d^3*x^4 + 1/8*c*d^2*x^3 + 3/16*c^2*d*x^2 + 1/8*c^3*x - 3/1024*(8*b^2*d
^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(4*b*x + 4*a)/b^4 - 1/256*(
8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b*d^3*x -
 3*b*c*d^2)*sin(4*b*x + 4*a)/b^4
```

**maple [B]** time = 0.02, size = 1074, normalized size = 10.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^3*d^3*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/16*(
b*x+a)^2*cos(b*x+a)^2+3/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
-21/128*(b*x+a)^2-3/128*sin(b*x+a)^2-3/32*(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*
x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^
4+3/8*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)
+3/128*sin(b*x+a)^4)-3/b^3*a*d^3*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2
*b*x+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/
64*a-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a
)+3/8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a)
)*cos(b*x+a))+3/b^2*c*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/
```

```
2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/
12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b
*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b
*x+a))+3/b^3*a^2*d^3*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/
16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*
cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)-6/b^2*a*c*d^2*((b*x+a)*(-1/2*c
os(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a
)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*
x+a)^4)+3/b*c^2*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*
(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos
(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)-1/b^3*a^3*d^3*(-1/4*cos(b*x+a)^3*
sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+3/b^2*a^2*c*d^2*(-1/4*c
os(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)-3/b*a*c^2*d
*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+c^3
*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a))
```

**maxima** [B] time = 0.36, size = 442, normalized size = 4.21

$$\frac{32\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c^3-\frac{96\,(4\,bx+4\,a-\sin(4\,bx+4\,a))ac^2d}{b}+\frac{96\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2cd^2}{b^2}-\frac{32\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a}{b^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1024*(32*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^3 - 96*(4*b*x + 4*a - sin(4*b
*x + 4*a))*a*c^2*d/b + 96*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*c*d^2/b^2 -
32*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^3*d^3/b^3 + 24*(8*(b*x + a)^2 - 4*(b*
x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*c^2*d/b - 48*(8*(b*x + a)^2 - 4
*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*c*d^2/b^2 + 24*(8*(b*x +
a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^2*d^3/b^3 + 4*(32
*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*
b*x + 4*a))*c*d^2/b^2 - 4*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) -
 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a))*a*d^3/b^3 + (32*(b*x + a)^4 - 3*(8
*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*
b*x + 4*a))*d^3/b^3)/b
```

**mupad** [B] time = 1.69, size = 329, normalized size = 3.13

$$x^2\left(\frac{3\,c^2\,d}{64}+\frac{9\,d^3}{512\,b^2}\right)+x^2\left(\frac{9\,c^2\,d}{64}-\frac{9\,d^3}{512\,b^2}\right)+x\left(\frac{c^3}{32}+\frac{9\,c\,d^2}{256\,b^2}\right)+x\left(\frac{3\,c^3}{32}-\frac{9\,c\,d^2}{256\,b^2}\right)+\frac{d^3\,x^4}{32}-\frac{x\,\cos\left(4\,a+4\,b\,x\right)\,\left(\frac{c^3}{4}+\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] x^2*((3*c^2*d)/64 + (9*d^3)/(512*b^2)) + x^2*((9*c^2*d)/64 - (9*d^3)/(512*b
^2)) + x*(c^3/32 + (9*c*d^2)/(256*b^2)) + x*((3*c^3)/32 - (9*c*d^2)/(256*b^
2)) + (d^3*x^4)/32 - (x*cos(4*a + 4*b*x)*(c^3/4 + (9*c*d^2)/(32*b^2)))/8 +
(x*cos(4*a + 4*b*x)*(c^3/8 - (3*c*d^2)/(64*b^2)))/4 + (c*d^2*x^3)/8 + (cos(
4*a + 4*b*x)*((3*d^3)/128 - (3*b^2*c^2*d)/16))/(8*b^4) + (sin(4*a + 4*b*x)*
(3*c*d^2 - 8*b^2*c^3))/(256*b^3) - (x^2*cos(4*a + 4*b*x)*((3*c^2*d)/8 + (9*
d^3)/(64*b^2)))/8 + (x^2*cos(4*a + 4*b*x)*((3*c^2*d)/16 - (3*d^3)/(128*b^2)
))/4 - (d^3*x^3*sin(4*a + 4*b*x))/(32*b) + (3*x*sin(4*a + 4*b*x)*(d^3 - 8*b
^2*c^2*d))/(256*b^3) - (3*c*d^2*x^2*sin(4*a + 4*b*x))/(32*b)
```

**sympy** [A] time = 7.94, size = 835, normalized size = 7.95

$$\begin{cases} \frac{c^3x\sin^4(a+bx)}{8}+\frac{c^3x\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{c^3x\cos^4(a+bx)}{8}+\frac{3c^2dx^2\sin^4(a+bx)}{16}+\frac{3c^2dx^2\sin^2(a+bx)\cos^2(a+bx)}{8}+\frac{3c^2dx^2\cos^4(a+bx)}{16} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^2(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*x*sin(a + b*x)**4/8 + c**3*x*sin(a + b*x)**2*cos(a + b*x)**
2/4 + c**3*x*cos(a + b*x)**4/8 + 3*c**2*d*x**2*sin(a + b*x)**4/16 + 3*c**2*
d*x**2*sin(a + b*x)**2*cos(a + b*x)**2/8 + 3*c**2*d*x**2*cos(a + b*x)**4/16
 + c*d**2*x**3*sin(a + b*x)**4/8 + c*d**2*x**3*sin(a + b*x)**2*cos(a + b*x)
**2/4 + c*d**2*x**3*cos(a + b*x)**4/8 + d**3*x**4*sin(a + b*x)**4/32 + d**3
*x**4*sin(a + b*x)**2*cos(a + b*x)**2/16 + d**3*x**4*cos(a + b*x)**4/32 + c
**3*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**3*sin(a + b*x)*cos(a + b*x)**3/
(8*b) + 3*c**2*d*x*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*c**2*d*x*sin(a +
b*x)*cos(a + b*x)**3/(8*b) + 3*c*d**2*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*
b) - 3*c*d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d**3*x**3*sin(a + b
*x)**3*cos(a + b*x)/(8*b) - d**3*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b) -
3*c**2*d*sin(a + b*x)**4/(32*b**2) - 3*c**2*d*cos(a + b*x)**4/(32*b**2) - 3
*c*d**2*x*sin(a + b*x)**4/(64*b**2) + 9*c*d**2*x*sin(a + b*x)**2*cos(a + b*
x)**2/(32*b**2) - 3*c*d**2*x*cos(a + b*x)**4/(64*b**2) - 3*d**3*x**2*sin(a
+ b*x)**4/(128*b**2) + 9*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(64*b**2
) - 3*d**3*x**2*cos(a + b*x)**4/(128*b**2) - 3*c*d**2*sin(a + b*x)**3*cos(a
 + b*x)/(64*b**3) + 3*c*d**2*sin(a + b*x)*cos(a + b*x)**3/(64*b**3) - 3*d**
3*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) + 3*d**3*x*sin(a + b*x)*cos(a +
b*x)**3/(64*b**3) + 3*d**3*sin(a + b*x)**4/(256*b**4) + 3*d**3*cos(a + b*x)
**4/(256*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*
x**4/4)*sin(a)**2*cos(a)**2, True))
```

# 3.82 $\int (c+dx)^2 \cos^2(a+bx) \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=79

$$\frac{d^2 \sin(4a+4bx)}{256b^3}-\frac{d(c+dx)\cos(4a+4bx)}{64b^2}-\frac{(c+dx)^2\sin(4a+4bx)}{32b}+\frac{(c+dx)^3}{24d}$$

```
[Out] 1/24*(d*x+c)^3/d-1/64*d*(d*x+c)*cos(4*b*x+4*a)/b^2+1/256*d^2*sin(4*b*x+4*a)
/b^3-1/32*(d*x+c)^2*sin(4*b*x+4*a)/b
```

**Rubi [A]** time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2637}

$$-\frac{d(c+dx)\cos(4a+4bx)}{64b^2}+\frac{d^2\sin(4a+4bx)}{256b^3}-\frac{(c+dx)^2\sin(4a+4bx)}{32b}+\frac{(c+dx)^3}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^3/(24*d) - (d*(c + d*x)*Cos[4*a + 4*b*x])/(64*b^2) + (d^2*Sin[4*a
 + 4*b*x])/(256*b^3) - ((c + d*x)^2*Sin[4*a + 4*b*x])/(32*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2\cos^2(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^2-\frac{1}{8}(c+dx)^2\cos(4a+4bx)\right)dx\\
&= \frac{(c+dx)^3}{24d}-\frac{1}{8}\int (c+dx)^2\cos(4a+4bx)\,dx\\
&= \frac{(c+dx)^3}{24d}-\frac{(c+dx)^2\sin(4a+4bx)}{32b}+\frac{d\int (c+dx)\sin(4a+4bx)\,dx}{16b}\\
&= \frac{(c+dx)^3}{24d}-\frac{d(c+dx)\cos(4a+4bx)}{64b^2}-\frac{(c+dx)^2\sin(4a+4bx)}{32b}+d^2\int\\
&= \frac{(c+dx)^3}{24d}-\frac{d(c+dx)\cos(4a+4bx)}{64b^2}+\frac{d^2\sin(4a+4bx)}{256b^3}-\frac{(c+dx)^2\,\text{si}}{3}
\end{aligned}$$

**Mathematica [A]** time = 0.42, size = 77, normalized size = 0.97

$$\frac{-3\sin(4(a+bx))\left(8b^2(c+dx)^2-d^2\right)-12bd(c+dx)\cos(4(a+bx))+32b^3x\left(3c^2+3cdx+d^2x^2\right)}{768b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (32*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 12*b*d*(c + d*x)*Cos[4*(a + b*x)] -
 3*(-d^2 + 8*b^2*(c + d*x)^2)*Sin[4*(a + b*x)])/(768*b^3)
```

**fricas [B]** time = 0.44, size = 180, normalized size = 2.28

$$\frac{8\,b^3d^2x^3+24\,b^3cdx^2-24\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4+24\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2+3\left(8\,b^3c^2-bd^2\right)x-3\,(\ldots}{1\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(8*b^3*d^2*x^3 + 24*b^3*c*d*x^2 - 24*(b*d^2*x + b*c*d)*cos(b*x + a)^4
 + 24*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 3*(8*b^3*c^2 - b*d^2)*x - 3*(2*(8*
b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(b*x + a)^3 - (8*b^2*d^2*x
^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(b*x + a))*sin(b*x + a))/b^3
```

**giac [A]** time = 0.20, size = 94, normalized size = 1.19

$$\frac{1}{24}\,d^2x^3+\frac{1}{8}\,cdx^2+\frac{1}{8}\,c^2x-\frac{\left(bd^2x+bcd\right)\cos\left(4\,bx+4\,a\right)}{64\,b^3}-\frac{\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\sin\left(4\,bx+4\,a\right)}{256\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/24*d^2*x^3 + 1/8*c*d*x^2 + 1/8*c^2*x - 1/64*(b*d^2*x + b*c*d)*cos(4*b*x +
 4*a)/b^3 - 1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*sin(4*b*
x + 4*a)/b^3
```

**maple [B]** time = 0.02, size = 519, normalized size = 6.57

$$\frac{\frac{d^2\left((bx+a)^2\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)\left(\cos^2(bx+a)\right)}{8}+\frac{\cos(bx+a)\sin(bx+a)}{16}+\frac{7bx}{64}+\frac{7a}{64}-\frac{(bx+a)^3}{12}-(bx+a)^2\left(-\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\ldots}{b^2}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^2*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/8*(b
*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/12*(b*x+a)^
3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1
/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))-2/b^
2*a*d^2*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+
1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/
8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+2/b*c*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a
)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)
^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+1/b^2*a^2*d
^2*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)-2
/b*a*c*d*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/
8*a)+c^2*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/
8*a))
```

**maxima [B]** time = 0.34, size = 232, normalized size = 2.94

$$\frac{24\left(4\,bx+4\,a-\sin\left(4\,bx+4\,a\right)\right)c^2-\frac{48\,(4\,bx+4\,a-\sin(4\,bx+4\,a))acd}{b}+\frac{24\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2d^2}{b^2}+\frac{12\left(8\,(bx+a)^2-4\,(bx+a)\sin\ldots\right.}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/768*(24*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^2 - 48*(4*b*x + 4*a - sin(4*b*
x + 4*a))*a*c*d/b + 24*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*d^2/b^2 + 12*(8
*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*c*d/b - 12*
(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*d^2/b^2
 + (32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*
sin(4*b*x + 4*a))*d^2/b^2)/b
```

**mupad [B]** time = 1.31, size = 179, normalized size = 2.27

$$x\left(\frac{c^2}{32}+\frac{3\,d^2}{256\,b^2}\right)+x\left(\frac{3\,c^2}{32}-\frac{3\,d^2}{256\,b^2}\right)+\frac{d^2\,x^3}{24}+\frac{\sin\left(4\,a+4\,b\,x\right)\,\left(d^2-8\,b^2\,c^2\right)}{256\,b^3}-\frac{x\,\cos\left(4\,a+4\,b\,x\right)\,\left(\frac{c^2}{4}+\frac{3\,d^2}{32\,b^2}\right)}{8}+\frac{x\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] x*(c^2/32 + (3*d^2)/(256*b^2)) + x*((3*c^2)/32 - (3*d^2)/(256*b^2)) + (d^2*
x^3)/24 + (sin(4*a + 4*b*x)*(d^2 - 8*b^2*c^2))/(256*b^3) - (x*cos(4*a + 4*b
*x)*(c^2/4 + (3*d^2)/(32*b^2)))/8 + (x*cos(4*a + 4*b*x)*(c^2/8 - d^2/(64*b^
2)))/4 + (c*d*x^2)/8 - (d^2*x^2*sin(4*a + 4*b*x))/(32*b) - (c*d*cos(4*a + 4
*b*x))/(64*b^2) - (c*d*x*sin(4*a + 4*b*x))/(16*b)
```

**sympy [A]** time = 3.98, size = 493, normalized size = 6.24

$$\begin{cases}\frac{c^2x\sin^4(a+bx)}{8}+\frac{c^2x\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{c^2x\cos^4(a+bx)}{8}+\frac{cdx^2\sin^4(a+bx)}{8}+\frac{cdx^2\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{cdx^2\cos^4(a+bx)}{8}+\frac{d^2x}{} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin^2(a)\cos^2(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*x*sin(a + b*x)**4/8 + c**2*x*sin(a + b*x)**2*cos(a + b*x)**
2/4 + c**2*x*cos(a + b*x)**4/8 + c*d*x**2*sin(a + b*x)**4/8 + c*d*x**2*sin(
a + b*x)**2*cos(a + b*x)**2/4 + c*d*x**2*cos(a + b*x)**4/8 + d**2*x**3*sin(
a + b*x)**4/24 + d**2*x**3*sin(a + b*x)**2*cos(a + b*x)**2/12 + d**2*x**3*c
os(a + b*x)**4/24 + c**2*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**2*sin(a +
b*x)*cos(a + b*x)**3/(8*b) + c*d*x*sin(a + b*x)**3*cos(a + b*x)/(4*b) - c*d
*x*sin(a + b*x)*cos(a + b*x)**3/(4*b) + d**2*x**2*sin(a + b*x)**3*cos(a + b
*x)/(8*b) - d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) - c*d*sin(a + b*x)
**4/(16*b**2) - c*d*cos(a + b*x)**4/(16*b**2) - d**2*x*sin(a + b*x)**4/(64*
b**2) + 3*d**2*x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**2) - d**2*x*cos(a +
 b*x)**4/(64*b**2) - d**2*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) + d**2*sin
(a + b*x)*cos(a + b*x)**3/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*
x**3/3)*sin(a)**2*cos(a)**2, True))
```

# 3.83 $\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=53

$$-\frac{d\cos(4a+4bx)}{128b^2}-\frac{(c+dx)\sin(4a+4bx)}{32b}+\frac{(c+dx)^2}{16d}$$

```
[Out] 1/16*(d*x+c)^2/d-1/128*d*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)*sin(4*b*x+4*a)/b
```

**Rubi** [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4406, 3296, 2638}

$$-\frac{d\cos(4a+4bx)}{128b^2}-\frac{(c+dx)\sin(4a+4bx)}{32b}+\frac{(c+dx)^2}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^2/(16*d) - (d*Cos[4*a + 4*b*x])/(128*b^2) - ((c + d*x)*Sin[4*a + 4*b*x])/(32*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)-\frac{1}{8}(c+dx)\cos(4a+4bx)\right)dx\\
&= \frac{(c+dx)^2}{16d}-\frac{1}{8}\int (c+dx)\cos(4a+4bx)\,dx\\
&= \frac{(c+dx)^2}{16d}-\frac{(c+dx)\sin(4a+4bx)}{32b}+\frac{d\int \sin(4a+4bx)\,dx}{32b}\\
&= \frac{(c+dx)^2}{16d}-\frac{d\cos(4a+4bx)}{128b^2}-\frac{(c+dx)\sin(4a+4bx)}{32b}
\end{aligned}$$

**Mathematica** [A] time = 0.29, size = 54, normalized size = 1.02

$$-\frac{8(a+bx)(ad-2bc-bdx)+4b(c+dx)\sin(4(a+bx))+d\cos(4(a+bx))}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] -1/128*(8*(a + b*x)*(-2*b*c + a*d - b*d*x) + d*Cos[4*(a + b*x)] + 4*b*(c + 
d*x)*Sin[4*(a + b*x)])/b^2
```

**fricas [A]** time = 0.59, size = 85, normalized size = 1.60

$$\frac{b^2dx^2 - d\cos(bx+a)^4 + 2\,b^2cx + d\cos(bx+a)^2 - 2\left(2\,(bdx+bc)\cos(bx+a)^3 - (bdx+bc)\cos(bx+a)\right)\sin(b}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(b^2*d*x^2 - d*cos(b*x + a)^4 + 2*b^2*c*x + d*cos(b*x + a)^2 - 2*(2*(b
*d*x + b*c)*cos(b*x + a)^3 - (b*d*x + b*c)*cos(b*x + a))*sin(b*x + a))/b^2
```

**giac [A]** time = 1.36, size = 48, normalized size = 0.91

$$\frac{1}{16}\,dx^2 + \frac{1}{8}\,cx - \frac{d\cos(4\,bx+4\,a)}{128\,b^2} - \frac{(bdx+bc)\sin(4\,bx+4\,a)}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*d*x^2 + 1/8*c*x - 1/128*d*cos(4*b*x + 4*a)/b^2 - 1/32*(b*d*x + b*c)*si
n(4*b*x + 4*a)/b^2
```

**maple [B]** time = 0.02, size = 194, normalized size = 3.66

$$\frac{\frac{d\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{16}+\frac{\left(\sin^2(bx+a)\right)}{16}-(bx+a)\left(-\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-\frac{\left(\sin^4(bx+a)\right)}{16}\right)}{b} - \frac{da\left(-\frac{\left(\cos^3(bx+a)\right)\sin}{4}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)
^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)
+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)-1/b*d*a*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/
8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+c*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*
cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a))
```

**maxima [B]** time = 0.33, size = 96, normalized size = 1.81

$$\frac{4\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c - \frac{4\,(4\,bx+4\,a-\sin(4\,bx+4\,a))ad}{b} + \frac{\left(8\,(bx+a)^2-4\,(bx+a)\sin(4\,bx+4\,a)-\cos(4\,bx+4\,a)\right)d}{b}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/128*(4*(4*b*x + 4*a - sin(4*b*x + 4*a))*c - 4*(4*b*x + 4*a - sin(4*b*x + 
4*a))*a*d/b + (8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4
*a))*d/b)/b
```

**mupad [B]** time = 1.04, size = 57, normalized size = 1.08

$$\frac{c\,x}{8} + \frac{d\,x^2}{16} - \frac{d\,\cos(4\,a+4\,b\,x)}{128\,b^2} - \frac{c\,\sin(4\,a+4\,b\,x)}{32\,b} - \frac{d\,x\,\sin(4\,a+4\,b\,x)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x),x)
```

```
[Out] (c*x)/8 + (d*x^2)/16 - (d*cos(4*a + 4*b*x))/(128*b^2) - (c*sin(4*a + 4*b*x)
)/(32*b) - (d*x*sin(4*a + 4*b*x))/(32*b)
```

**sympy [A]** time = 2.00, size = 238, normalized size = 4.49

$$\begin{cases} \frac{cx\sin^4(a+bx)}{8} + \frac{cx\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{cx\cos^4(a+bx)}{8} + \frac{dx^2\sin^4(a+bx)}{16} + \frac{dx^2\sin^2(a+bx)\cos^2(a+bx)}{8} + \frac{dx^2\cos^4(a+bx)}{16} + \frac{c\sin^3 \\ \left(cx + \frac{dx^2}{2}\right)\sin^2(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c*x*sin(a + b*x)**4/8 + c*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
c*x*cos(a + b*x)**4/8 + d*x**2*sin(a + b*x)**4/16 + d*x**2*sin(a + b*x)**2*
cos(a + b*x)**2/8 + d*x**2*cos(a + b*x)**4/16 + c*sin(a + b*x)**3*cos(a + b
*x)/(8*b) - c*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d*x*sin(a + b*x)**3*cos(
a + b*x)/(8*b) - d*x*sin(a + b*x)*cos(a + b*x)**3/(8*b) - d*sin(a + b*x)**4
/(32*b**2) - d*cos(a + b*x)**4/(32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(
a)**2*cos(a)**2, True))
```

# 3.84 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=78

$$-\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\log(c+dx)}{8d}$$

```
[Out] -1/8*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d+1/8*ln(d*x+c)/d+1/8*Si(4*b*c/d+4*
b*x)*sin(4*a-4*b*c/d)/d
```

**Rubi [A]** time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4406, 3303, 3299, 3302}

$$-\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\log(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(8*d) + Log[c + d*x]
/(8*d) + (Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(8*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \left(\frac{1}{8(c+dx)} - \frac{\cos(4a+4bx)}{8(c+dx)}\right) dx$$
$$= \frac{\log(c+dx)}{8d} - \frac{1}{8}\int \frac{\cos(4a+4bx)}{c+dx}\,dx$$
$$= \frac{\log(c+dx)}{8d} - \frac{1}{8}\cos\left(4a-\frac{4bc}{d}\right)\int \frac{\cos\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx + \frac{1}{8}\sin\left(4a-\frac{4bc}{d}\right)\int$$
$$= -\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{8d} + \frac{\log(c+dx)}{8d} + \frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}$$

**Mathematica [A]** time = 0.16, size = 65, normalized size = 0.83

$$\frac{-\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4b(c+dx)}{d}\right)+\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)+\log(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (-(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d]) + Log[c + d*x] + Si
n[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d)
```

**fricas [A]** time = 0.49, size = 88, normalized size = 1.13

$$-\frac{\left(\text{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+\text{Ci}\left(-\frac{4\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-2\,\sin\left(-\frac{4\,(bc-ad)}{d}\right)\text{Si}\left(\frac{4\,(bdx+bc)}{d}\right)-2\,\log\left(dx+c\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/16*((cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-4*(b*d*x + b*c)/d))
*cos(-4*(b*c - a*d)/d) - 2*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*
c)/d) - 2*log(d*x + c))/d
```

**giac [C]** time = 0.22, size = 669, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/16*(2*log(abs(d*x + c))*tan(2*a)^2*tan(2*b*c/d)^2 - real_part(cos_integra
l(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - real_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(4*b*x +
 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 2*imag_part(cos_integral(-4*b*x - 4*b*
c/d))*tan(2*a)^2*tan(2*b*c/d) + 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^
2*tan(2*b*c/d) - 2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*
b*c/d)^2 + 2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d
)^2 - 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)^2 + 2*log(abs
(d*x + c))*tan(2*a)^2 + real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2
 + real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 4*real_part(cos_i
ntegral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*real_part(cos_integral(
-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 2*log(abs(d*x + c))*tan(2*b*c/d)
^2 + real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 + real_part(co
```

```
s_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*a) - 2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2
*a) + 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) - 2*imag_part(cos_integral
(4*b*x + 4*b*c/d))*tan(2*b*c/d) + 2*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*b*c/d) - 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d) + 2*log(ab
s(d*x + c)) - real_part(cos_integral(4*b*x + 4*b*c/d)) - real_part(cos_inte
gral(-4*b*x - 4*b*c/d)))/(d*tan(2*a)^2*tan(2*b*c/d)^2 + d*tan(2*a)^2 + d*ta
n(2*b*c/d)^2 + d)
```

**maple [A]** time = 0.03, size = 105, normalized size = 1.35

$$-\frac{\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{8d}-\frac{\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{8d}+\frac{\ln\left(\left(bx+a\right)d-da+cb\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] -1/8*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d-1/8*Ci(4*b*x+4*a+4*
(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d+1/8*ln((b*x+a)*d-d*a+c*b)/d
```

**maxima [C]** time = 0.41, size = 160, normalized size = 2.05

$$\frac{b\left(E_1\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+E_1\left(-\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)+b\left(-i\,E_1\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+i\,E_1\left(-\frac{4i\,bc+4}{}\right.\right.}{16\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/16*(b*(exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + exp_i
ntegral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c - a*d)/
d) + b*(-I*exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + I*e
xp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*(b*c - a
*d)/d) + 2*b*log(b*c + (b*x + a)*d - a*d))/(b*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}{\left(a+bx\right)}\cos^{2}{\left(a+bx\right)}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x), x)
```

## 3.85 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=104

$$\frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{\cos(4a+4bx)}{8d(c+dx)}-\frac{1}{8d(c+dx)}$$

```
[Out] -1/8/d/(d*x+c)+1/8*cos(4*b*x+4*a)/d/(d*x+c)+1/2*b*cos(4*a-4*b*c/d)*Si(4*b*c
/d+4*b*x)/d^2+1/2*b*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^2
```

**Rubi [A]** time = 0.17, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{\cos(4a+4bx)}{8d(c+dx)}-\frac{1}{8d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -1/(8*d*(c + d*x)) + Cos[4*a + 4*b*x]/(8*d*(c + d*x)) + (b*CosIntegral[(4*b
*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(2*d^2) + (b*Cos[4*a - (4*b*c)/d]*SinI
ntegral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \left(\frac{1}{8(c+dx)^2} - \frac{\cos(4a+4bx)}{8(c+dx)^2}\right) dx$$
$$= -\frac{1}{8d(c+dx)} - \frac{1}{8}\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx$$
$$= -\frac{1}{8d(c+dx)} + \frac{\cos(4a+4bx)}{8d(c+dx)} + \frac{b\int \frac{\sin(4a+4bx)}{c+dx}\,dx}{2d}$$
$$= -\frac{1}{8d(c+dx)} + \frac{\cos(4a+4bx)}{8d(c+dx)} + \frac{\left(b\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\sin\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx}{2d} + \frac{\left(b\sin\left(4\right.\right.}{}$$
$$= -\frac{1}{8d(c+dx)} + \frac{\cos(4a+4bx)}{8d(c+dx)} + \frac{b\operatorname{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{2d^2} + \frac{b\cos\left(4a-\right.}{}$$

**Mathematica [A]** time = 0.43, size = 81, normalized size = 0.78

$$\frac{4b\sin\left(4a-\frac{4bc}{d}\right)\operatorname{Ci}\left(\frac{4b(c+dx)}{d}\right)+4b\cos\left(4a-\frac{4bc}{d}\right)\operatorname{Si}\left(\frac{4b(c+dx)}{d}\right)+\frac{d(\cos(4(a+bx))-1)}{c+dx}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] ((d*(-1 + Cos[4*(a + b*x)]))/(c + d*x) + 4*b*CosIntegral[(4*b*(c + d*x))/d]
*Sin[4*a - (4*b*c)/d] + 4*b*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x)
)/d])/(8*d^2)
```

**fricas [A]** time = 0.55, size = 138, normalized size = 1.33

$$\frac{4\,d\cos\left(bx+a\right)^4-4\,d\cos\left(bx+a\right)^2+2\left(bdx+bc\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{4\,(bdx+bc)}{d}\right)+\left(\left(bdx+bc\right)\operatorname{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+(bd\right.}{4\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*cos(b*x + a)^4 - 4*d*cos(b*x + a)^2 + 2*(b*d*x + b*c)*cos(-4*(b*c
- a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + ((b*d*x + b*c)*cos_integral(4*(
b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(b
*c - a*d)/d))/(d^3*x + c*d^2)
```

**giac [C]** time = 0.97, size = 3218, normalized size = 30.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2
*tan(2*b*c/d)^2 - b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x
)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*d*x*real_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 2*b*d*x*real_part(cos_in
tegral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 2*b*d*x*re
al_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2
```

```
 - 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*
tan(2*b*c/d)^2 + b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(2*a)^2*tan(2*b*c/d)^2 - b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*t
an(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*c*sin_integral(4*(b*d*x + b*c)/
d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - b*d*x*imag_part(cos_integral(4*
b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + b*d*x*imag_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 2*b*d*x*sin_integral(4*(b*d*x + b*
c)/d)*tan(2*b*x)^2*tan(2*a)^2 + 4*b*d*x*imag_part(cos_integral(4*b*x + 4*b*
c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) - 4*b*d*x*imag_part(cos_integral(-
4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 8*b*d*x*sin_integral
(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 2*b*c*real_part(co
s_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 2*b*c*r
eal_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/
d) - b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/
d)^2 + b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b
*c/d)^2 - 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d)
^2 - 2*b*c*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*t
an(2*b*c/d)^2 - 2*b*c*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^
2*tan(2*a)*tan(2*b*c/d)^2 + b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*
tan(2*a)^2*tan(2*b*c/d)^2 - b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))
*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*
a)^2*tan(2*b*c/d)^2 + 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*b*x)^2*tan(2*a) + 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2
*b*x)^2*tan(2*a) - b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^
2*tan(2*a)^2 + b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*t
an(2*a)^2 - 2*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 -
 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)
 - 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c
/d) + 4*b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*
tan(2*b*c/d) - 4*b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2
*tan(2*a)*tan(2*b*c/d) + 8*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2
*tan(2*a)*tan(2*b*c/d) + 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*t
an(2*a)^2*tan(2*b*c/d) + 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*
tan(2*a)^2*tan(2*b*c/d) - b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*b*x)^2*tan(2*b*c/d)^2 + b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*b*x)^2*tan(2*b*c/d)^2 - 2*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)
^2*tan(2*b*c/d)^2 - 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*
a)*tan(2*b*c/d)^2 - 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2
*a)*tan(2*b*c/d)^2 + b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^
2*tan(2*b*c/d)^2 - b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2
*tan(2*b*c/d)^2 + 2*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*
c/d)^2 + b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 - b*d*
x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 + 2*b*d*x*sin_inte
gral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 + 2*b*c*real_part(cos_integral(4*b*x +
 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 2*b*c*real_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*b*x)^2*tan(2*a) - b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/
d))*tan(2*a)^2 + b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2
 - 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 - 2*b*c*real_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 2*b*c*real_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 4*b*d*x*imag_part(
cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*b*d*x*imag_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 8*b*d*x*sin_integral(
4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d) + 2*b*c*real_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 2*b*c*real_part(cos_integral(-4*b*
x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - b*d*x*imag_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*b*c/d)^2 + b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*b*c/d)^2 - 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 -
 2*b*c*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 2
*b*c*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 + b*
```

```
c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 - b*c*imag_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 + 2*b*c*sin_integral(4*(b*d*x + b
*c)/d)*tan(2*b*x)^2 + 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*a) + 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) - b*c*ima
g_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + b*c*imag_part(cos_integr
al(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 2*b*c*sin_integral(4*(b*d*x + b*c)/d)*ta
n(2*a)^2 - 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d) -
2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d) + 4*b*c*imag
_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*b*c*imag_par
t(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 8*b*c*sin_integra
l(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d) - b*c*imag_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*b*c/d)^2 + b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*b*c/d)^2 - 2*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 -
d*tan(2*b*x)^2*tan(2*b*c/d)^2 - 2*d*tan(2*b*x)*tan(2*a)*tan(2*b*c/d)^2 - d*
tan(2*a)^2*tan(2*b*c/d)^2 + b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))
- b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 2*b*d*x*sin_integral(4*
(b*d*x + b*c)/d) + 2*b*c*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)
+ 2*b*c*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) - 2*b*c*real_par
t(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d) - 2*b*c*real_part(cos_integra
l(-4*b*x - 4*b*c/d))*tan(2*b*c/d) + b*c*imag_part(cos_integral(4*b*x + 4*b*
c/d)) - b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 2*b*c*sin_integral(
4*(b*d*x + b*c)/d) - d*tan(2*b*x)^2 - 2*d*tan(2*b*x)*tan(2*a) - d*tan(2*a)^
2)/(d^3*x*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + c*d^2*tan(2*b*x)^2*tan(2
*a)^2*tan(2*b*c/d)^2 + d^3*x*tan(2*b*x)^2*tan(2*a)^2 + d^3*x*tan(2*b*x)^2*t
an(2*b*c/d)^2 + d^3*x*tan(2*a)^2*tan(2*b*c/d)^2 + c*d^2*tan(2*b*x)^2*tan(2*
a)^2 + c*d^2*tan(2*b*x)^2*tan(2*b*c/d)^2 + c*d^2*tan(2*a)^2*tan(2*b*c/d)^2
+ d^3*x*tan(2*b*x)^2 + d^3*x*tan(2*a)^2 + d^3*x*tan(2*b*c/d)^2 + c*d^2*tan(
2*b*x)^2 + c*d^2*tan(2*a)^2 + c*d^2*tan(2*b*c/d)^2 + d^3*x + c*d^2)
```

**maple** [A] time = 0.03, size = 156, normalized size = 1.50

$$\frac{-\frac{b^2\left(-\frac{4\cos(4bx+4a)}{((bx+a)d-da+cb)d}-\frac{4\left(\frac{4\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}-\frac{4\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}\right)}{d}\right)}{32}-\frac{b^2}{8((bx+a)d-da+cb)d}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] 1/b*(-1/32*b^2*(-4*cos(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)/d-4*(4*Si(4*b*x+4*a+4
*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(
-a*d+b*c)/d)/d)/d)-1/8*b^2/((b*x+a)*d-d*a+c*b)/d)
```

**maxima** [C] time = 0.44, size = 171, normalized size = 1.64

$$\frac{64\,b^2\left(E_2\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+E_2\left(-\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-b^2\left(64i\,E_2\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-64i\,E_2\right.}{1024\left(bcd+(bx+a)d^2-ad^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/1024*(64*b^2*(exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d)
+ exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c
- a*d)/d) - b^2*(64*I*exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*
d)/d) - 64*I*exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*s
in(-4*(b*c - a*d)/d) - 128*b^2)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^2\sin(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**2, x)
```

# 3.86 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=127

$$\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}-\frac{1}{16d(c+dx)^2}$$

```
[Out] -1/16/d/(d*x+c)^2+b^2*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^3+1/16*cos(4*b*x
+4*a)/d/(d*x+c)^2-b^2*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^3-1/4*b*sin(4*b*
x+4*a)/d^2/(d*x+c)
```

**Rubi [A]** time = 0.20, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}-\frac{1}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] -1/(16*d*(c + d*x)^2) + Cos[4*a + 4*b*x]/(16*d*(c + d*x)^2) + (b^2*Cos[4*a
- (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/d^3 - (b*Sin[4*a + 4*b*x])/(4*
d^2*(c + d*x)) - (b^2*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/
d^3
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{1}{8(c+dx)^3} - \frac{\cos(4a+4bx)}{8(c+dx)^3}\right)dx$$

$$= -\frac{1}{16d(c+dx)^2} - \frac{1}{8}\int \frac{\cos(4a+4bx)}{(c+dx)^3}\,dx$$

$$= -\frac{1}{16d(c+dx)^2} + \frac{\cos(4a+4bx)}{16d(c+dx)^2} + \frac{b\int \frac{\sin(4a+4bx)}{(c+dx)^2}\,dx}{4d}$$

$$= -\frac{1}{16d(c+dx)^2} + \frac{\cos(4a+4bx)}{16d(c+dx)^2} - \frac{b\sin(4a+4bx)}{4d^2(c+dx)} + \frac{b^2\int \frac{\cos(4a+4bx)}{c+dx}\,dx}{d^2}$$

$$= -\frac{1}{16d(c+dx)^2} + \frac{\cos(4a+4bx)}{16d(c+dx)^2} - \frac{b\sin(4a+4bx)}{4d^2(c+dx)} + \frac{\left(b^2\cos\left(4a-\frac{4bc}{d}\right)\right)\int}{d^2}$$

$$= -\frac{1}{16d(c+dx)^2} + \frac{\cos(4a+4bx)}{16d(c+dx)^2} + \frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{d^3} - \frac{b\sin(}{4d^2}$$

**Mathematica [A]** time = 0.85, size = 105, normalized size = 0.83

$$\frac{16b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4b(c+dx)}{d}\right) - 16b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right) + \frac{d(-4b(c+dx)\sin(4(a+bx))+d\cos(4(a+bx))-d)}{(c+dx)^2}}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] (16*b^2*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] + (d*(-d + d*Co
s[4*(a + b*x)] - 4*b*(c + d*x)*Sin[4*(a + b*x)]))/(c + d*x)^2 - 16*b^2*Sin[
4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(16*d^3)
```

**fricas [B]** time = 0.47, size = 255, normalized size = 2.01

$$\frac{d^2\cos(bx+a)^4 - d^2\cos(bx+a)^2 - 2\left(b^2d^2x^2+2b^2cdx+b^2c^2\right)\sin\left(-\frac{4(bc-ad)}{d}\right)\text{Si}\left(\frac{4(bdx+bc)}{d}\right) + \left(\left(b^2d^2x^2+2b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + ((b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(4*(b*d*x + b*c)/d) + (b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c
- a*d)/d) - 2*(2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - (b*d^2*x + b*c*d)*cos(b
*x + a))*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**giac [C]** time = 0.55, size = 5600, normalized size = 44.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*ta
n(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 8*b^2*d^
2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan
(2*b*c/d) - 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan
(2*a)^2*tan(2*b*c/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d
))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_integ
ral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 16*b^2*d^2*x^
2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 8*
b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*
tan(2*b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(4
*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 4*b^2*d^2*x^2*real_part(cos_inte
gral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 16*b^2*d^2*x^2*real_part(
cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 16*b^2*
d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan
(2*b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x
)^2*tan(2*a)^2*tan(2*b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 32*b^2*c*d*x*sin_integral(
4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 4*b^2*d^2*x^2*rea
l_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 4*b^2*d
^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^
2 + 16*b^2*c*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(
2*a)*tan(2*b*c/d)^2 - 16*b^2*c*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 32*b^2*c*d*x*sin_integral(4*(b*d*x
 + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*real_part(c
os_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*rea
l_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*c^
2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*
c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*t
an(2*a)^2*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b
*c/d))*tan(2*b*x)^2*tan(2*a) + 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x
- 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 16*b^2*d^2*x^2*sin_integral(4*(b*d*x +
b*c)/d)*tan(2*b*x)^2*tan(2*a) - 8*b^2*c*d*x*real_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 8*b^2*c*d*x*real_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 8*b^2*d^2*x^2*imag_part(cos_integr
al(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 8*b^2*d^2*x^2*imag_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 16*b^2*d^2*x^2*si
n_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d) + 32*b^2*c*d*x*real
_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 3
2*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)
*tan(2*b*c/d) - 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*a)^2*tan(2*b*c/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*a)^2*tan(2*b*c/d) - 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)
*tan(2*a)^2*tan(2*b*c/d) - 8*b^2*c^2*imag_part(cos_integral(4*b*x + 4*b*c/d
))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 8*b^2*c^2*imag_part(cos_integral(
-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 16*b^2*c^2*sin_in
tegral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 8*b^2*c*d*
x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 8*
b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/
d)^2 + 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(
2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*
a)*tan(2*b*c/d)^2 + 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)
*tan(2*b*c/d)^2 + 8*b^2*c^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*
b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 8*b^2*c^2*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 16*b^2*c^2*sin_integral(4*
(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 8*b^2*c*d*x*real_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^2*c*d*x*r
```

```
eal_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*
d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 + 4*b^2*d^2*x
^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 - 16*b^2*c*d*x*im
ag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 16*b^2*c*d*x
*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 32*b^2*c
*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a) - 4*b^2*d^2*x^2*
real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 - 4*b^2*d^2*x^2*real_pa
rt(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 4*b^2*c^2*real_part(cos_int
egral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 4*b^2*c^2*real_part(cos_i
ntegral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 16*b^2*c*d*x*imag_part
(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 16*b^2*c*d*x*im
ag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 32*b^2*
c*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d) + 16*b^2*d^
2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 16*b
^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)
 + 16*b^2*c^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)
*tan(2*b*c/d) + 16*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*
b*x)^2*tan(2*a)*tan(2*b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 32*b^2*c*d*x*sin_integral(4*(b*d*
x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d) - 4*b^2*d^2*x^2*real_part(cos_integral(
4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*b*c/d)^2 - 4*b^2*c^2*real_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 4*b^2*c^2*real_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 + 16*b^2*c*d*x*imag_part(cos_in
tegral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 16*b^2*c*d*x*imag_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 + 32*b^2*c*d*x*sin_i
ntegral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)^2 + 4*b*d^2*x*tan(2*b*x)^2
*tan(2*a)*tan(2*b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(4*b*x + 4*b*c/d
))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b*d^2*x*tan(2*b*x)*tan(2*a)^2*tan(2*b
*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2
 + 8*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 - 8*b
^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a) + 8*b^2*d^2*x^
2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) - 16*b^2*d^2*x^2*sin_i
ntegral(4*(b*d*x + b*c)/d)*tan(2*a) - 8*b^2*c^2*imag_part(cos_integral(4*b*
x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 8*b^2*c^2*imag_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 16*b^2*c^2*sin_integral(4*(b*d*x + b
*c)/d)*tan(2*b*x)^2*tan(2*a) - 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*a)^2 - 8*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))
*tan(2*a)^2 + 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*
b*c/d) - 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/
d) + 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d) + 8*b^2*c^
2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 8*b^
2*c^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) +
 16*b^2*c^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d) + 32*
b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) +
32*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d
) - 8*b^2*c^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c
/d) + 8*b^2*c^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*
b*c/d) - 16*b^2*c^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)
 - 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 - 8*
b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2 + 8*b^2*
c^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 8*b^
2*c^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 + 1
6*b^2*c^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)^2 + 4*b*c*d
*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 4*b*c*d*tan(2*b*x)*tan(2*a)^2*tan(2
*b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d)) + 4*b^2*
d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d)) + 4*b^2*c^2*real_part(cos
```

```
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 + 4*b^2*c^2*real_part(cos_integral
(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 - 16*b^2*c*d*x*imag_part(cos_integral(4*b*
x + 4*b*c/d))*tan(2*a) + 16*b^2*c*d*x*imag_part(cos_integral(-4*b*x - 4*b*c
/d))*tan(2*a) - 32*b^2*c*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) + 4*b
*d^2*x*tan(2*b*x)^2*tan(2*a) - 4*b^2*c^2*real_part(cos_integral(4*b*x + 4*b
*c/d))*tan(2*a)^2 - 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*a)^2 + 4*b*d^2*x*tan(2*b*x)*tan(2*a)^2 + 16*b^2*c*d*x*imag_part(cos_inte
gral(4*b*x + 4*b*c/d))*tan(2*b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(-
4*b*x - 4*b*c/d))*tan(2*b*c/d) + 32*b^2*c*d*x*sin_integral(4*(b*d*x + b*c)/
d)*tan(2*b*c/d) + 16*b^2*c^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*a)*tan(2*b*c/d) + 16*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*a)*tan(2*b*c/d) - 4*b^2*c^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan
(2*b*c/d)^2 - 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c
/d)^2 - 4*b*d^2*x*tan(2*b*x)*tan(2*b*c/d)^2 - 4*b*d^2*x*tan(2*a)*tan(2*b*c/
d)^2 + 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d)) + 8*b^2*c*d*x*r
eal_part(cos_integral(-4*b*x - 4*b*c/d)) - 8*b^2*c^2*imag_part(cos_integral
(4*b*x + 4*b*c/d))*tan(2*a) + 8*b^2*c^2*imag_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*a) - 16*b^2*c^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) + 4*b
*c*d*tan(2*b*x)^2*tan(2*a) + 4*b*c*d*tan(2*b*x)*tan(2*a)^2 + 8*b^2*c^2*imag
_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d) - 8*b^2*c^2*imag_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d) + 16*b^2*c^2*sin_integral(4*(b*d*
x + b*c)/d)*tan(2*b*c/d) - 4*b*c*d*tan(2*b*x)*tan(2*b*c/d)^2 - d^2*tan(2*b*
x)^2*tan(2*b*c/d)^2 - 4*b*c*d*tan(2*a)*tan(2*b*c/d)^2 - 2*d^2*tan(2*b*x)*ta
n(2*a)*tan(2*b*c/d)^2 - d^2*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*c^2*real_part
(cos_integral(4*b*x + 4*b*c/d)) + 4*b^2*c^2*real_part(cos_integral(-4*b*x -
 4*b*c/d)) - 4*b*d^2*x*tan(2*b*x) - 4*b*d^2*x*tan(2*a) - 4*b*c*d*tan(2*b*x)
 - d^2*tan(2*b*x)^2 - 4*b*c*d*tan(2*a) - 2*d^2*tan(2*b*x)*tan(2*a) - d^2*ta
n(2*a)^2)/(d^5*x^2*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2
*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + d^5*x^2*tan(2*b*x)^2*tan(2*a)^2 + d^5*x
^2*tan(2*b*x)^2*tan(2*b*c/d)^2 + d^5*x^2*tan(2*a)^2*tan(2*b*c/d)^2 + c^2*d^
3*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2*b*x)^2*tan(2*a)^
2 + 2*c*d^4*x*tan(2*b*x)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2*a)^2*tan(2*b*c/
d)^2 + d^5*x^2*tan(2*b*x)^2 + d^5*x^2*tan(2*a)^2 + c^2*d^3*tan(2*b*x)^2*tan
(2*a)^2 + d^5*x^2*tan(2*b*c/d)^2 + c^2*d^3*tan(2*b*x)^2*tan(2*b*c/d)^2 + c^
2*d^3*tan(2*a)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2*b*x)^2 + 2*c*d^4*x*tan(2*
a)^2 + 2*c*d^4*x*tan(2*b*c/d)^2 + d^5*x^2 + c^2*d^3*tan(2*b*x)^2 + c^2*d^3*
tan(2*a)^2 + c^2*d^3*tan(2*b*c/d)^2 + 2*c*d^4*x + c^2*d^3)
```

**maple [A]** time = 0.03, size = 193, normalized size = 1.52

$$\frac{-\frac{b^3\left(-\frac{2\cos(4bx+4a)}{((bx+a)d-da+cb)^2 d}-\frac{2\left(-\frac{4\sin(4bx+4a)}{((bx+a)d-da+cb)d}+\frac{\frac{16\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}+\frac{16\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}}{d}\right)}{d}\right)}{32}-\frac{b^3}{16((bx+a)d-da+cb)^2 d}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] 1/b*(-1/32*b^3*(-2*cos(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)^2/d-2*(-4*sin(4*b*x+4
*a)/((b*x+a)*d-d*a+c*b)/d+4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c
)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)/d)-1/16*b^3
/((b*x+a)*d-d*a+c*b)^2/d)
```

**maxima [C]** time = 0.49, size = 206, normalized size = 1.62

$$\frac{64\,b^3\left(E_3\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+E_3\left(-\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-b^3\left(64i\,E_3\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-64i\right.}{1024\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\left(bcd^2-ad^3\right)(bx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(64*b^3*(exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d)
+ exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c
- a*d)/d) - b^3*(64*I*exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*
d)/d) - 64*I*exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*s
in(-4*(b*c - a*d)/d) - 64*b^3)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3
+ a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**3, x)
```

## 3.87 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4} dx$

**Optimal**. Leaf size=158

$$-\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{b^2\cos(4a+4bx)}{3d^3(c+dx)}-\frac{b\sin(4a+4bx)}{12d^2(c+dx)^2}+\frac{\cos(4a}{24d(c}$$

```
[Out] -1/24/d/(d*x+c)^3+1/24*cos(4*b*x+4*a)/d/(d*x+c)^3-1/3*b^2*cos(4*b*x+4*a)/d^
3/(d*x+c)-4/3*b^3*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*b*x)/d^4-4/3*b^3*Ci(4*b*c/d
+4*b*x)*sin(4*a-4*b*c/d)/d^4-1/12*b*sin(4*b*x+4*a)/d^2/(d*x+c)^2
```

**Rubi [A]** time = 0.23, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{b^2\cos(4a+4bx)}{3d^3(c+dx)}-\frac{b\sin(4a+4b}{12d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -1/(24*d*(c + d*x)^3) + Cos[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*Cos[4*a
+ 4*b*x])/(3*d^3*(c + d*x)) - (4*b^3*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a
 - (4*b*c)/d])/(3*d^4) - (b*Sin[4*a + 4*b*x])/(12*d^2*(c + d*x)^2) - (4*b^3
*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{1}{8(c+dx)^4} - \frac{\cos(4a+4bx)}{8(c+dx)^4}\right)dx$$
$$= -\frac{1}{24d(c+dx)^3} - \frac{1}{8}\int \frac{\cos(4a+4bx)}{(c+dx)^4}\,dx$$
$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} + \frac{b\int \frac{\sin(4a+4bx)}{(c+dx)^3}\,dx}{6d}$$
$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} - \frac{b\sin(4a+4bx)}{12d^2(c+dx)^2} + \frac{b^2\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx}{3d^2}$$
$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} - \frac{b^2\cos(4a+4bx)}{3d^3(c+dx)} - \frac{b\sin(4a+4bx)}{12d^2(c+dx)^2} - \frac{(4}{}$$
$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} - \frac{b^2\cos(4a+4bx)}{3d^3(c+dx)} - \frac{b\sin(4a+4bx)}{12d^2(c+dx)^2} - \frac{(4}{}$$
$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} - \frac{b^2\cos(4a+4bx)}{3d^3(c+dx)} - \frac{4b^3\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)\mathrm{si}}{3d^4}$$

**Mathematica [A]** time = 1.65, size = 123, normalized size = 0.78

$$-\frac{32b^3\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4b(c+dx)}{d}\right) + 32b^3\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\frac{4b(c+dx)}{d}\right) + \frac{d\left(\cos(4(a+bx))\left(8b^2(c+dx)^2-d^2\right)+d(2b(c+dx)\sin(4(a}{(c+dx)^3}}{24d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -1/24*(32*b^3*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + (d*((-d
^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + d*(d + 2*b*(c + d*x)*Sin[4*(a +
b*x)])))/(c + d*x)^3 + 32*b^3*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*
x))/d])/d^4
```

**fricas [B]** time = 0.54, size = 406, normalized size = 2.57

$$-\frac{b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d + \left(8\,b^2d^3x^2 + 16\,b^2cd^2x + 8\,b^2c^2d - d^3\right)\cos\left(bx+a\right)^4 - \left(8\,b^2d^3x^2 + 16\,b^2cd^2x + 8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + (8*b^2*d^3*x^2 + 16*b^2*c*d
^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^4 - (8*b^2*d^3*x^2 + 16*b^2*c*d^2*x
+ 8*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
b^3*c^2*d*x + b^3*c^3)*cos(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d
) + (2*(b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cos(b*x + a
))*sin(b*x + a) + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c
^3)*cos_integral(4*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + b^3*c^3)*cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d
))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

**giac [C]** time = 0.64, size = 8508, normalized size = 53.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/12*(8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4
*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x^3*sin_integr
al(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x
^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b
*c/d) + 16*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)
^2*tan(2*a)^2*tan(2*b*c/d) - 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*real_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 24*b^
3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^
2*tan(2*b*c/d)^2 - 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c*d^2*x^2*sin_integral(4
*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*d^3*x^3*im
ag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 8*b^3*d^3*
x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 16*
b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 + 32*b^
3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*ta
n(2*b*c/d) - 32*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2
*b*x)^2*tan(2*a)*tan(2*b*c/d) + 64*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)
/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 48*b^3*c*d^2*x^2*real_part(cos_int
egral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c*d^2
*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(
2*b*c/d) - 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x
)^2*tan(2*b*c/d)^2 + 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)
/d)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*real_part(cos_integral(4
*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*re
al_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^
2 + 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2
*b*c/d)^2 - 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a
)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a
)^2*tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d)
)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 24*b^3*c^2*d*x*imag_part(cos_int
egral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c^
2*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^
2 + 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*ta
n(2*a) + 16*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x
)^2*tan(2*a) - 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*b*x)^2*tan(2*a)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 48*b^3*c*d^2*x^2*sin_integral(4*(b*d*x +
b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 - 16*b^3*d^3*x^3*real_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 16*b^3*d^3*x^3*real_part(cos_int
egral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 96*b^3*c*d^2*x^2*imag_
part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) - 96
*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2
*a)*tan(2*b*c/d) + 192*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*
b*x)^2*tan(2*a)*tan(2*b*c/d) + 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 16*b^3*d^3*x^3*real_part(cos_integral
(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c^2*d*x*real_part(cos_
integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c^
2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan
(2*b*c/d) - 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*b*x)^2*tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4
*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*sin_integral(4*(b*d
*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*real_part(cos_int
egral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*real_part(
```

```
cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 48*b^3*c^2*d*x*re
al_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2
 - 48*b^3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*ta
n(2*a)*tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b
*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - 24*b^3*c*d^2*x^2*imag_part(cos_integral(
-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c*d^2*x^2*sin_integra
l(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*d^3*x^2*tan(2*b*x)^2
*tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c
/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*c^3*imag_part(cos_integ
ral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*c^3*
sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 8*
b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 - 8*b^3*d
^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 + 16*b^3*d^3*
x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 + 48*b^3*c*d^2*x^2*real_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 48*b^3*c*d^2*x^2*
real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 8*b^3*d^3
*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + 8*b^3*d^3*x^3*im
ag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 16*b^3*d^3*x^3*sin_int
egral(4*(b*d*x + b*c)/d)*tan(2*a)^2 - 24*b^3*c^2*d*x*imag_part(cos_integral
(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 24*b^3*c^2*d*x*imag_part(cos_i
ntegral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 48*b^3*c^2*d*x*sin_int
egral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 - 48*b^3*c*d^2*x^2*real_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 48*b^3*c*d^2*
x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 3
2*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d
) - 32*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2
*b*c/d) + 64*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c
/d) + 96*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(2*a)*tan(2*b*c/d) - 96*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b*
c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 192*b^3*c^2*d*x*sin_integral(4*(
b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 48*b^3*c*d^2*x^2*real_
part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c*d^2*
x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 16*
b^3*c^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*ta
n(2*b*c/d) + 16*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x
)^2*tan(2*a)^2*tan(2*b*c/d) - 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*b*c/d)^2 + 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*b*c/d)^2 - 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*b*x)^2*tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c^2*d*x*sin_integral(4*(b*d*x +
 b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*real_part(cos_integ
ral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*real_part(
cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*c^3*real_p
art(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 1
6*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*t
an(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan
(2*a)^2*tan(2*b*c/d)^2 - 24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c^2*d*x*sin_integral(4*(b*d*x + b
*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^2*c*d^2*x*tan(2*b*x)^2*tan(2*a)^2*ta
n(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*b*x)^2 - 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*b*x)^2 + 48*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 +
 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a) + 16*b^3*
d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) + 48*b^3*c^2*d*x
*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 48*b^3*c^
2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 24*
b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + 24*b^3*
c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 48*b^3*c*d
```

```
^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 + 4*b^2*d^3*x^2*tan(2*b*x
)^2*tan(2*a)^2 - 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b
*x)^2*tan(2*a)^2 + 8*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
2*b*x)^2*tan(2*a)^2 - 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)
^2*tan(2*a)^2 - 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan
(2*b*c/d) - 16*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*
b*c/d) - 48*b^3*c^2*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)
^2*tan(2*b*c/d) - 48*b^3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*
tan(2*b*x)^2*tan(2*b*c/d) + 96*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x +
 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 96*b^3*c*d^2*x^2*imag_part(cos_integral(
-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 192*b^3*c*d^2*x^2*sin_integral(4
*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d) + 32*b^3*c^3*imag_part(cos_integral
(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) - 32*b^3*c^3*imag_par
t(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 64*b
^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) +
 48*b^3*c^2*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b
*c/d) + 48*b^3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2
*tan(2*b*c/d) - 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*t
an(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*
tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c
/d)^2 - 4*b^2*d^3*x^2*tan(2*b*x)^2*tan(2*b*c/d)^2 - 8*b^3*c^3*imag_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 + 8*b^3*c^3*imag_pa
rt(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^3*c^3
*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c^2*d
*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 48*b^
3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2
 - 16*b^2*d^3*x^2*tan(2*b*x)*tan(2*a)*tan(2*b*c/d)^2 - 4*b^2*d^3*x^2*tan(2*
a)^2*tan(2*b*c/d)^2 + 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*t
an(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*c^2*d*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)
^2 + 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d)) - 8*b^3*d^3*x^3
*imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 16*b^3*d^3*x^3*sin_integral(4*
(b*d*x + b*c)/d) + 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*
tan(2*b*x)^2 - 24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*b*x)^2 + 48*b^3*c^2*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 + 4
8*b^3*c*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a) + 48*b^3*
c*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) + 16*b^3*c^3*r
eal_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 16*b^3*c^3*
real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 24*b^3*c^
2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + 24*b^3*c^2*d*x*
imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 48*b^3*c^2*d*x*sin_i
ntegral(4*(b*d*x + b*c)/d)*tan(2*a)^2 + 8*b^2*c*d^2*x*tan(2*b*x)^2*tan(2*a)
^2 - 48*b^3*c*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)
 - 48*b^3*c*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)
- 16*b^3*c^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*
c/d) - 16*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*ta
n(2*b*c/d) + 96*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*
a)*tan(2*b*c/d) - 96*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*
tan(2*a)*tan(2*b*c/d) + 192*b^3*c^2*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*a)*tan(2*b*c/d) + 16*b^3*c^3*real_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*a)^2*tan(2*b*c/d) + 16*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*a)^2*tan(2*b*c/d) - 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*b*c/d)^2 - 48*b^3*c^2*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*b*c/d)^2 - 8*b^2*c*d^2*x*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^3*c^3*real_p
art(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*c^3*rea
l_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 32*b^2*c*d
^2*x*tan(2*b*x)*tan(2*a)*tan(2*b*c/d)^2 - 2*b*d^3*x*tan(2*b*x)^2*tan(2*a)*t
```

```
an(2*b*c/d)^2 - 8*b^2*c*d^2*x*tan(2*a)^2*tan(2*b*c/d)^2 - 2*b*d^3*x*tan(2*b
*x)*tan(2*a)^2*tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b
*x + 4*b*c/d)) - 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))
 + 48*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d) - 4*b^2*d^3*x^2*tan(2*b
*x)^2 + 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 - 8
*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 + 16*b^3*c^
3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 + 48*b^3*c^2*d*x*real_part(c
os_integral(4*b*x + 4*b*c/d))*tan(2*a) + 48*b^3*c^2*d*x*real_part(cos_integ
ral(-4*b*x - 4*b*c/d))*tan(2*a) - 16*b^2*d^3*x^2*tan(2*b*x)*tan(2*a) - 4*b^
2*d^3*x^2*tan(2*a)^2 - 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*t
an(2*a)^2 + 8*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2
- 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 + 4*b^2*c^2*d*tan(2
*b*x)^2*tan(2*a)^2 - 48*b^3*c^2*d*x*real_part(cos_integral(4*b*x + 4*b*c/d)
)*tan(2*b*c/d) - 48*b^3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*t
an(2*b*c/d) + 32*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*
tan(2*b*c/d) - 32*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a
)*tan(2*b*c/d) + 64*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*
b*c/d) + 4*b^2*d^3*x^2*tan(2*b*c/d)^2 - 8*b^3*c^3*imag_part(cos_integral(4*
b*x + 4*b*c/d))*tan(2*b*c/d)^2 + 8*b^3*c^3*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*b*c/d)^2 - 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2
*b*c/d)^2 - 4*b^2*c^2*d*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^2*c^2*d*tan(2*b*
x)*tan(2*a)*tan(2*b*c/d)^2 - 2*b*c*d^2*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2
 - 4*b^2*c^2*d*tan(2*a)^2*tan(2*b*c/d)^2 - 2*b*c*d^2*tan(2*b*x)*tan(2*a)^2*
tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d)) -
24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 48*b^3*c^2*d*x*s
in_integral(4*(b*d*x + b*c)/d) - 8*b^2*c*d^2*x*tan(2*b*x)^2 + 16*b^3*c^3*re
al_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a) + 16*b^3*c^3*real_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*a) - 32*b^2*c*d^2*x*tan(2*b*x)*tan(2*a) -
 2*b*d^3*x*tan(2*b*x)^2*tan(2*a) - 8*b^2*c*d^2*x*tan(2*a)^2 - 2*b*d^3*x*tan
(2*b*x)*tan(2*a)^2 - 16*b^3*c^3*real_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*b*c/d) - 16*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c
/d) + 8*b^2*c*d^2*x*tan(2*b*c/d)^2 + 2*b*d^3*x*tan(2*b*x)*tan(2*b*c/d)^2 +
2*b*d^3*x*tan(2*a)*tan(2*b*c/d)^2 + 4*b^2*d^3*x^2 + 8*b^3*c^3*imag_part(cos
_integral(4*b*x + 4*b*c/d)) - 8*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b
*c/d)) + 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d) - 4*b^2*c^2*d*tan(2*b*x
)^2 - 16*b^2*c^2*d*tan(2*b*x)*tan(2*a) - 2*b*c*d^2*tan(2*b*x)^2*tan(2*a) -
4*b^2*c^2*d*tan(2*a)^2 - 2*b*c*d^2*tan(2*b*x)*tan(2*a)^2 + 4*b^2*c^2*d*tan(
2*b*c/d)^2 + 2*b*c*d^2*tan(2*b*x)*tan(2*b*c/d)^2 + d^3*tan(2*b*x)^2*tan(2*b
*c/d)^2 + 2*b*c*d^2*tan(2*a)*tan(2*b*c/d)^2 + 2*d^3*tan(2*b*x)*tan(2*a)*tan
(2*b*c/d)^2 + d^3*tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^2*c*d^2*x + 2*b*d^3*x*tan
(2*b*x) + 2*b*d^3*x*tan(2*a) + 4*b^2*c^2*d + 2*b*c*d^2*tan(2*b*x) + d^3*tan
(2*b*x)^2 + 2*b*c*d^2*tan(2*a) + 2*d^3*tan(2*b*x)*tan(2*a) + d^3*tan(2*a)^2
)/(d^7*x^3*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 3*c*d^6*x^2*tan(2*b*x)^
2*tan(2*a)^2*tan(2*b*c/d)^2 + d^7*x^3*tan(2*b*x)^2*tan(2*a)^2 + d^7*x^3*tan
(2*b*x)^2*tan(2*b*c/d)^2 + d^7*x^3*tan(2*a)^2*tan(2*b*c/d)^2 + 3*c^2*d^5*x*
tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 3*c*d^6*x^2*tan(2*b*x)^2*tan(2*a)^
2 + 3*c*d^6*x^2*tan(2*b*x)^2*tan(2*b*c/d)^2 + 3*c*d^6*x^2*tan(2*a)^2*tan(2*
b*c/d)^2 + c^3*d^4*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + d^7*x^3*tan(2*b
*x)^2 + d^7*x^3*tan(2*a)^2 + 3*c^2*d^5*x*tan(2*b*x)^2*tan(2*a)^2 + d^7*x^3*
tan(2*b*c/d)^2 + 3*c^2*d^5*x*tan(2*b*x)^2*tan(2*b*c/d)^2 + 3*c^2*d^5*x*tan(
2*a)^2*tan(2*b*c/d)^2 + 3*c*d^6*x^2*tan(2*b*x)^2 + 3*c*d^6*x^2*tan(2*a)^2 +
 c^3*d^4*tan(2*b*x)^2*tan(2*a)^2 + 3*c*d^6*x^2*tan(2*b*c/d)^2 + c^3*d^4*tan
(2*b*x)^2*tan(2*b*c/d)^2 + c^3*d^4*tan(2*a)^2*tan(2*b*c/d)^2 + d^7*x^3 + 3*
c^2*d^5*x*tan(2*b*x)^2 + 3*c^2*d^5*x*tan(2*a)^2 + 3*c^2*d^5*x*tan(2*b*c/d)^
2 + 3*c*d^6*x^2 + c^3*d^4*tan(2*b*x)^2 + c^3*d^4*tan(2*a)^2 + c^3*d^4*tan(2
*b*c/d)^2 + 3*c^2*d^5*x + c^3*d^4)
```

**maple [A]** time = 0.03, size = 230, normalized size = 1.46

$$\frac{-\frac{b^4\left(-\frac{4\cos(4bx+4a)}{3((bx+a)d-da+cb)^3d}-\frac{4\left(-\frac{2\sin(4bx+4a)}{((bx+a)d-da+cb)^2d}+\frac{-\frac{8\cos(4bx+4a)}{((bx+a)d-da+cb)d}-\frac{8\left(\frac{4\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}-\frac{4\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}\right)}{d}}{d}\right)}{3d}\right)}{32}-\ \cdots}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x)
```

```
[Out] 1/b*(-1/32*b^4*(-4/3*cos(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)^3/d-4/3*(-2*sin(4*b
*x+4*a)/((b*x+a)*d-d*a+c*b)^2/d+2*(-4*cos(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)/d-
4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-
a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d)/d)-1/24*b^4/((b*x+a)*d-d*a+c*b)^3/
d)
```

**maxima [C]** time = 0.61, size = 256, normalized size = 1.62

$$\frac{3\,b^4\left(E_4\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+E_4\left(-\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-b^4\left(3i\,E_4\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-3i\,E_4\left(-\cdots\right.\right.}{48\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\left(bcd^3-ad^4\right)(bx+a)^2+3\left(b^2c^2d^2-2\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^4*(exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + e
xp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c - a
*d)/d) - b^4*(3*I*exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d
) - 3*I*exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4
*(b*c - a*d)/d) - 2*b^4)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b
*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2
- 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2}{{\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**4, x)
```

# 3.88 $\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=407

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{32b}$$

```
[Out] -1/16*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)
/d)^m)-1/16*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*
x+c)/d)^m)-1/32*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(d*x
+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/32*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(d*x+
c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)+1/32*5^(-1-m)*exp(5*I*(a-b*c
/d))*(d*x+c)^m*GAMMA(1+m,-5*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+1/32*5^(-
1-m)*(d*x+c)^m*GAMMA(1+m,5*I*b*(d*x+c)/d)/b/exp(5*I*(a-b*c/d))/((I*b*(d*x+c
)/d)^m)
```

**Rubi [A]** time = 0.40, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3308, 2181}

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(16*b
*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/
(16*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I)*(a
- (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(32*b*(((-I)*
b*(c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d*x
))/d])/(32*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (5^(-1 - m)*E
^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d])/(3
2*b*(((-I)*b*(c + d*x))/d)^m) + (5^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((5*I)
*b*(c + d*x))/d])/(32*b*E^((5*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^m \sin(a+bx) + \frac{1}{16}(c+dx)^m \sin(3a+3bx) - \frac{1}{16}(c+dx\right.$$
$$= \frac{1}{16}\int (c+dx)^m \sin(3a+3bx)\,dx - \frac{1}{16}\int (c+dx)^m \sin(5a+5bx)\,dx +$$
$$= \frac{1}{32}i\int e^{-i(3a+3bx)}(c+dx)^m\,dx - \frac{1}{32}i\int e^{i(3a+3bx)}(c+dx)^m\,dx - \frac{1}{32}i\int$$
$$= -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{16b} - \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx}{}$$

**Mathematica** [A] time = 0.63, size = 376, normalized size = 0.92

$$\frac{e^{-\frac{5i(ad+bc)}{d}}(c+dx)^m\left(-5\,3^{-m}e^{\frac{2i(ad+bc)}{d}}\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{6ia}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)+e^{\frac{6ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] ((c + d*x)^m*(30*E^(((4*I)*(b*c + a*d))/d)*(-((E^((2*I)*a)*Gamma[1 + m, ((-
I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m) - (E^(((2*I)*b*c)/d)*Gamma[1
+ m, (I*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^m) - (5*E^(((2*I)*(b*c + a*d))
/d)*(E^((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d]
 + E^(((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*
x))/d]))/(3^m*((b^2*(c + d*x)^2)/d^2)^m) + (3*(E^((10*I)*a)*((I*b*(c + d*x)
)/d)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d] + E^(((10*I)*b*c)/d)*(((-I)*b*(
c + d*x))/d)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d]))/(5^m*((b^2*(c + d*x)^2
)/d^2)^m)))/(480*b*E^(((5*I)*(b*c + a*d))/d))
```

**fricas** [A] time = 0.72, size = 276, normalized size = 0.68

$$\frac{3\,e^{\left(-\frac{dm\log\left(\frac{5\,i\,b}{d}\right)-5\,i\,bc+5\,i\,ad}{d}\right)}\Gamma\left(m+1,\frac{5\,i\,bdx+5\,i\,bc}{d}\right)-5\,e^{\left(-\frac{dm\log\left(\frac{3\,i\,b}{d}\right)-3\,i\,bc+3\,i\,ad}{d}\right)}\Gamma\left(m+1,\frac{3\,i\,bdx+3\,i\,bc}{d}\right)-30\,e^{\left(-\frac{dm\log\left(\frac{i\,b}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*e^(-(d*m*log(5*I*b/d) - 5*I*b*c + 5*I*a*d)/d)*gamma(m + 1, (5*I*b*
d*x + 5*I*b*c)/d) - 5*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gamma(m
 + 1, (3*I*b*d*x + 3*I*b*c)/d) - 30*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)
*gamma(m + 1, (I*b*d*x + I*b*c)/d) - 30*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*
d)/d)*gamma(m + 1, (-I*b*d*x - I*b*c)/d) - 5*e^(-(d*m*log(-3*I*b/d) + 3*I*b
*c - 3*I*a*d)/d)*gamma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d) + 3*e^(-(d*m*log(-5
*I*b/d) + 5*I*b*c - 5*I*a*d)/d)*gamma(m + 1, (-5*I*b*d*x - 5*I*b*c)/d))/b
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^3, x)
```

**maple** [F] time = 0.21, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^2(bx+a)\right)\left(\sin^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)
[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^3, x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2 \sin(a+b\,x)^3 (c+d\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^m,x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^m, x)
```

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a)**3,x)
[Out] Exception raised: HeuristicGCDFailed
```

# 3.89 $\int (c+dx)^4 \cos^2(a+bx) \sin^3(a+bx)\,dx$

**Optimal**. Leaf size=330

$$-\frac{3d^4\cos(a+bx)}{b^5}-\frac{d^4\cos(3a+3bx)}{162b^5}+\frac{3d^4\cos(5a+5bx)}{6250b^5}-\frac{3d^3(c+dx)\sin(a+bx)}{b^4}-\frac{d^3(c+dx)\sin(3a+3bx)}{54b^4}+\frac{3d^3}{}$$

```
[Out] -3*d^4*cos(b*x+a)/b^5+3/2*d^2*(d*x+c)^2*cos(b*x+a)/b^3-1/8*(d*x+c)^4*cos(b*
x+a)/b-1/162*d^4*cos(3*b*x+3*a)/b^5+1/36*d^2*(d*x+c)^2*cos(3*b*x+3*a)/b^3-1
/48*(d*x+c)^4*cos(3*b*x+3*a)/b+3/6250*d^4*cos(5*b*x+5*a)/b^5-3/500*d^2*(d*x
+c)^2*cos(5*b*x+5*a)/b^3+1/80*(d*x+c)^4*cos(5*b*x+5*a)/b-3*d^3*(d*x+c)*sin(
b*x+a)/b^4+1/2*d*(d*x+c)^3*sin(b*x+a)/b^2-1/54*d^3*(d*x+c)*sin(3*b*x+3*a)/b
^4+1/36*d*(d*x+c)^3*sin(3*b*x+3*a)/b^2+3/1250*d^3*(d*x+c)*sin(5*b*x+5*a)/b^
4-1/100*d*(d*x+c)^3*sin(5*b*x+5*a)/b^2
```

**Rubi [A]** time = 0.39, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$-\frac{3d^3(c+dx)\sin(a+bx)}{b^4}-\frac{d^3(c+dx)\sin(3a+3bx)}{54b^4}+\frac{3d^3(c+dx)\sin(5a+5bx)}{1250b^4}+\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}+\frac{d^2(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-3*d^4*Cos[a + b*x])/b^5 + (3*d^2*(c + d*x)^2*Cos[a + b*x])/(2*b^3) - ((c
+ d*x)^4*Cos[a + b*x])/(8*b) - (d^4*Cos[3*a + 3*b*x])/(162*b^5) + (d^2*(c +
 d*x)^2*Cos[3*a + 3*b*x])/(36*b^3) - ((c + d*x)^4*Cos[3*a + 3*b*x])/(48*b)
+ (3*d^4*Cos[5*a + 5*b*x])/(6250*b^5) - (3*d^2*(c + d*x)^2*Cos[5*a + 5*b*x]
)/(500*b^3) + ((c + d*x)^4*Cos[5*a + 5*b*x])/(80*b) - (3*d^3*(c + d*x)*Sin[
a + b*x])/b^4 + (d*(c + d*x)^3*Sin[a + b*x])/(2*b^2) - (d^3*(c + d*x)*Sin[3
*a + 3*b*x])/(54*b^4) + (d*(c + d*x)^3*Sin[3*a + 3*b*x])/(36*b^2) + (3*d^3*
(c + d*x)*Sin[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)^3*Sin[5*a + 5*b*x])/(
100*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^4\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^4\sin(a+bx)+\frac{1}{16}(c+dx)^4\sin(3a+3bx)-\frac{1}{16}(c+d\right.$$
$$=\frac{1}{16}\int (c+dx)^4\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^4\sin(5a+5bx)\,dx$$
$$=-\frac{(c+dx)^4\cos(a+bx)}{8b}-\frac{(c+dx)^4\cos(3a+3bx)}{48b}+\frac{(c+dx)^4\cos(}{80b}$$
$$=-\frac{(c+dx)^4\cos(a+bx)}{8b}-\frac{(c+dx)^4\cos(3a+3bx)}{48b}+\frac{(c+dx)^4\cos(}{80b}$$
$$=\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}-\frac{(c+dx)^4\cos(a+bx)}{8b}+\frac{d^2(c+dx)^2\cos}{36b^3}$$
$$=\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}-\frac{(c+dx)^4\cos(a+bx)}{8b}+\frac{d^2(c+dx)^2\cos}{36b^3}$$
$$=-\frac{3d^4\cos(a+bx)}{b^5}+\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}-\frac{(c+dx)^4\cos(a+b}{8b}$$

**Mathematica [A]** time = 3.17, size = 238, normalized size = 0.72

$$\frac{120bd(c+dx)\sin(a+bx)\left(16\cos(2(a+bx))\left(75b^2(c+dx)^2-68d^2\right)-27\cos(4(a+bx))\left(25b^2(c+dx)^2-6d^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-506250*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x] -
 3125*(8*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)
] + 81*(24*d^4 - 300*b^2*d^2*(c + d*x)^2 + 625*b^4*(c + d*x)^4)*Cos[5*(a +
b*x)] + 120*b*d*(c + d*x)*(17475*b^2*c^2 - 101794*d^2 + 34950*b^2*c*d*x + 1
7475*b^2*d^2*x^2 + 16*(-68*d^2 + 75*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 27*
(-6*d^2 + 25*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[a + b*x])/(4050000*b^5)
```

**fricas [A]** time = 0.81, size = 471, normalized size = 1.43

$$\frac{81\left(625\,b^4d^4x^4+2500\,b^4cd^3x^3+625\,b^4c^4-300\,b^2c^2d^2+24\,d^4+150\left(25\,b^4c^2d^2-2\,b^2d^4\right)x^2+100\left(25\,b^4c^3d-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/253125*(81*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 625*b^4*c^4 - 300*b^2*
c^2*d^2 + 24*d^4 + 150*(25*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 100*(25*b^4*c^3*d
 - 6*b^2*c*d^3)*x)*cos(b*x + a)^5 - 5*(16875*b^4*d^4*x^4 + 67500*b^4*c*d^3*
x^3 + 16875*b^4*c^4 - 11700*b^2*c^2*d^2 + 1736*d^4 + 450*(225*b^4*c^2*d^2 -
 26*b^2*d^4)*x^2 + 900*(75*b^4*c^3*d - 26*b^2*c*d^3)*x)*cos(b*x + a)^3 + 12
0*(2925*b^2*d^4*x^2 + 5850*b^2*c*d^3*x + 2925*b^2*c^2*d^2 - 6284*d^4)*cos(b
*x + a) + 60*(1950*b^3*d^4*x^3 + 5850*b^3*c*d^3*x^2 + 1950*b^3*c^3*d - 1256
8*b*c*d^3 - 27*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 25*b^3*c^3*d - 6*b*c*d^
3 + 3*(25*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^4 + (975*b^3*d^4*x^3 + 292
5*b^3*c*d^3*x^2 + 975*b^3*c^3*d - 434*b*c*d^3 + (2925*b^3*c^2*d^2 - 434*b*d
^4)*x)*cos(b*x + a)^2 + 2*(2925*b^3*c^2*d^2 - 6284*b*d^4)*x)*sin(b*x + a))/
b^5
```

**giac [A]** time = 0.87, size = 531, normalized size = 1.61

$$\frac{\left(625\,b^4d^4x^4+2500\,b^4cd^3x^3+3750\,b^4c^2d^2x^2+2500\,b^4c^3dx+625\,b^4c^4-300\,b^2d^4x^2-600\,b^2cd^3x-300\,b^2c^2d\right.}{50000\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/50000*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 3750*b^4*c^2*d^2*x^2 + 2500
*b^4*c^3*d*x + 625*b^4*c^4 - 300*b^2*d^4*x^2 - 600*b^2*c*d^3*x - 300*b^2*c^
2*d^2 + 24*d^4)*cos(5*b*x + 5*a)/b^5 - 1/1296*(27*b^4*d^4*x^4 + 108*b^4*c*d
^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^
2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*d^4)*cos(3*b*x + 3*a)/b^5 - 1/8*(b^
4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 -
 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^
5 - 1/2500*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 75*b^3*c^2*d^2*x + 25*b^3*c
^3*d - 6*b*d^4*x - 6*b*c*d^3)*sin(5*b*x + 5*a)/b^5 + 1/108*(3*b^3*d^4*x^3 +
 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*s
in(3*b*x + 3*a)/b^5 + 1/2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x
+ b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

**maple [B]** time = 0.12, size = 1812, normalized size = 5.49

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^4*d^4*(-1/3*(b*x+a)^4*(2+sin(b*x+a)^2)*cos(b*x+a)+8/15*(b*x+a)^3*s
in(b*x+a)+8/5*(b*x+a)^2*cos(b*x+a)-3424/1125*cos(b*x+a)-3424/1125*(b*x+a)*s
in(b*x+a)+4/45*(b*x+a)^3*sin(b*x+a)^3+4/45*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b
*x+a)+88/3375*(b*x+a)*sin(b*x+a)^3+88/10125*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5
*(b*x+a)^4*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-4/25*(b*x+a)^3*si
n(b*x+a)^5-12/125*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+
24/625*(b*x+a)*sin(b*x+a)^5+24/3125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos
(b*x+a))-4/b^4*a*d^4*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2/5*(b*x+a
)^2*sin(b*x+a)-856/1125*sin(b*x+a)+4/5*(b*x+a)*cos(b*x+a)+1/15*(b*x+a)^2*si
n(b*x+a)^3+2/45*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+22/3375*sin(b*x+a)^3+1/
5*(b*x+a)^3*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-3/25*(b*x+a)^2*s
in(b*x+a)^5-6/125*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+6/
625*sin(b*x+a)^5)+4/b^3*c*d^3*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2
/5*(b*x+a)^2*sin(b*x+a)-856/1125*sin(b*x+a)+4/5*(b*x+a)*cos(b*x+a)+1/15*(b*
x+a)^2*sin(b*x+a)^3+2/45*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+22/3375*sin(b*
x+a)^3+1/5*(b*x+a)^3*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-3/25*(b
*x+a)^2*sin(b*x+a)^5-6/125*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(
b*x+a)+6/625*sin(b*x+a)^5)+6/b^4*a^2*d^4*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*c
os(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3
+2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(
b*x+a)^2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*
sin(b*x+a)^2)*cos(b*x+a))-12/b^3*a*c*d^3*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*c
os(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3
+2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(
b*x+a)^2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*
sin(b*x+a)^2)*cos(b*x+a))+6/b^2*c^2*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*co
s(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+
2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b
*x+a)^2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*s
in(b*x+a)^2)*cos(b*x+a))-4/b^4*a^3*d^4*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b
*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*s
in(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)+12/b^3*a^2*c*d^3*(-1/3*(b*x+a)*(
2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8
/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)-12/b^2*a*c^
2*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(
b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(
```

```
b*x+a)^5)+4/b*c^3*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+
a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*
x+a)-1/25*sin(b*x+a)^5)+1/b^4*a^4*d^4*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*
cos(b*x+a)^3)-4/b^3*a^3*c*d^3*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+
a)^3)+6/b^2*a^2*c^2*d^2*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)-
4/b*a*c^3*d*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+c^4*(-1/5*si
n(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3))
```

**maxima** [**B**] time = 0.42, size = 1339, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4050000*(270000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^4 - 1080000*(3*co
s(b*x + a)^5 - 5*cos(b*x + a)^3)*a*c^3*d/b + 1620000*(3*cos(b*x + a)^5 - 5*
cos(b*x + a)^3)*a^2*c^2*d^2/b^2 - 1080000*(3*cos(b*x + a)^5 - 5*cos(b*x + a
)^3)*a^3*c*d^3/b^3 + 270000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a^4*d^4/b
^4 + 4500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) -
450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450
*sin(b*x + a))*c^3*d/b - 13500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a
)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*s
in(3*b*x + 3*a) + 450*sin(b*x + a))*a*c^2*d^2/b^2 + 13500*(45*(b*x + a)*cos
(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a)
- 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a^2*c*d^3/b^
3 - 4500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 4
50*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*
sin(b*x + a))*a^3*d^4/b^4 + 450*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) -
 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x
+ a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13
500*(b*x + a)*sin(b*x + a))*c^2*d^2/b^2 - 900*(27*(25*(b*x + a)^2 - 2)*cos(
5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2
 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b
*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*a*c*d^3/b^3 + 450*(27*(25*(b*x +
a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 675
0*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*
x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*a^2*d^4/b^4 + 60*(1
35*(25*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) - 1875*(3*(b*x + a)^3 -
2*b*x - 2*a)*cos(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x +
 a) - 81*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*si
n(3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^3/b^3 - 60*(135
*(25*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) - 1875*(3*(b*x + a)^3 - 2*
b*x - 2*a)*cos(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a
) - 81*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*sin(
3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*sin(b*x + a))*a*d^4/b^4 + (81*(625*
(b*x + a)^4 - 300*(b*x + a)^2 + 24)*cos(5*b*x + 5*a) - 3125*(27*(b*x + a)^4
 - 36*(b*x + a)^2 + 8)*cos(3*b*x + 3*a) - 506250*((b*x + a)^4 - 12*(b*x + a
)^2 + 24)*cos(b*x + a) - 1620*(25*(b*x + a)^3 - 6*b*x - 6*a)*sin(5*b*x + 5*
a) + 37500*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) + 2025000*((b*x +
 a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^4/b^4)/b
```

**mupad** [**B**] time = 4.61, size = 816, normalized size = 2.47

$$-\frac{3\,d^4\,\cos\left(a+b\,x\right)+\frac{d^4\,\cos(3\,a+3\,b\,x)}{162}-\frac{3\,d^4\,\cos(5\,a+5\,b\,x)}{6250}+\frac{b^4\,c^4\,\cos(a+b\,x)}{8}+\frac{b^4\,c^4\,\cos(3\,a+3\,b\,x)}{48}-\frac{b^4\,c^4\,\cos(5\,a+5\,b\,x)}{80}-\frac{3\,b^2\,c^2 \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] -(3*d^4*cos(a + b*x) + (d^4*cos(3*a + 3*b*x))/162 - (3*d^4*cos(5*a + 5*b*x)
)/6250 + (b^4*c^4*cos(a + b*x))/8 + (b^4*c^4*cos(3*a + 3*b*x))/48 - (b^4*c^
4*cos(5*a + 5*b*x))/80 - (3*b^2*c^2*d^2*cos(a + b*x))/2 - (b^3*c^3*d*sin(3*
a + 3*b*x))/36 + (b^3*c^3*d*sin(5*a + 5*b*x))/100 - (3*b^2*d^4*x^2*cos(a +
b*x))/2 + (b^4*d^4*x^4*cos(a + b*x))/8 - (b^3*d^4*x^3*sin(a + b*x))/2 + 3*b
*c*d^3*sin(a + b*x) - (b^2*c^2*d^2*cos(3*a + 3*b*x))/36 + (3*b^2*c^2*d^2*co
s(5*a + 5*b*x))/500 + 3*b*d^4*x*sin(a + b*x) - (b^2*d^4*x^2*cos(3*a + 3*b*x
))/36 + (3*b^2*d^4*x^2*cos(5*a + 5*b*x))/500 + (b^4*d^4*x^4*cos(3*a + 3*b*x
))/48 - (b^4*d^4*x^4*cos(5*a + 5*b*x))/80 - (b^3*d^4*x^3*sin(3*a + 3*b*x))/
36 + (b^3*d^4*x^3*sin(5*a + 5*b*x))/100 + (b*c*d^3*sin(3*a + 3*b*x))/54 - (
3*b*c*d^3*sin(5*a + 5*b*x))/1250 - (b^3*c^3*d*sin(a + b*x))/2 + (b*d^4*x*si
n(3*a + 3*b*x))/54 - (3*b*d^4*x*sin(5*a + 5*b*x))/1250 - 3*b^2*c*d^3*x*cos(
a + b*x) + (b^4*c^3*d*x*cos(a + b*x))/2 + (b^4*c^2*d^2*x^2*cos(3*a + 3*b*x)
)/8 - (3*b^4*c^2*d^2*x^2*cos(5*a + 5*b*x))/40 - (b^2*c*d^3*x*cos(3*a + 3*b*
x))/18 + (b^4*c^3*d*x*cos(3*a + 3*b*x))/12 + (3*b^2*c*d^3*x*cos(5*a + 5*b*x
))/250 - (b^4*c^3*d*x*cos(5*a + 5*b*x))/20 + (b^4*c*d^3*x^3*cos(a + b*x))/2
 - (3*b^3*c^2*d^2*x*sin(a + b*x))/2 - (3*b^3*c*d^3*x^2*sin(a + b*x))/2 + (b
^4*c*d^3*x^3*cos(3*a + 3*b*x))/12 - (b^4*c*d^3*x^3*cos(5*a + 5*b*x))/20 + (
3*b^4*c^2*d^2*x^2*cos(a + b*x))/4 - (b^3*c^2*d^2*x*sin(3*a + 3*b*x))/12 - (
b^3*c*d^3*x^2*sin(3*a + 3*b*x))/12 + (3*b^3*c^2*d^2*x*sin(5*a + 5*b*x))/100
 + (3*b^3*c*d^3*x^2*sin(5*a + 5*b*x))/100)/b^5
```

**sympy [A]** time = 20.16, size = 1098, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**4*cos(a + b*x
)**5/(15*b) - 4*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 8*c**3*d*x
*cos(a + b*x)**5/(15*b) - 2*c**2*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/
b - 4*c**2*d**2*x**2*cos(a + b*x)**5/(5*b) - 4*c*d**3*x**3*sin(a + b*x)**2*
cos(a + b*x)**3/(3*b) - 8*c*d**3*x**3*cos(a + b*x)**5/(15*b) - d**4*x**4*si
n(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d**4*x**4*cos(a + b*x)**5/(15*b) +
104*c**3*d*sin(a + b*x)**5/(225*b**2) + 52*c**3*d*sin(a + b*x)**3*cos(a + b
*x)**2/(45*b**2) + 8*c**3*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2) + 104*c*
*2*d**2*x*sin(a + b*x)**5/(75*b**2) + 52*c**2*d**2*x*sin(a + b*x)**3*cos(a
+ b*x)**2/(15*b**2) + 8*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**4/(5*b**2) +
 104*c*d**3*x**2*sin(a + b*x)**5/(75*b**2) + 52*c*d**3*x**2*sin(a + b*x)**3
*cos(a + b*x)**2/(15*b**2) + 8*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**4/(5*
b**2) + 104*d**4*x**3*sin(a + b*x)**5/(225*b**2) + 52*d**4*x**3*sin(a + b*x
)**3*cos(a + b*x)**2/(45*b**2) + 8*d**4*x**3*sin(a + b*x)*cos(a + b*x)**4/(
15*b**2) + 104*c**2*d**2*sin(a + b*x)**4*cos(a + b*x)/(75*b**3) + 676*c**2*
d**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 1712*c**2*d**2*cos(a + b*
x)**5/(1125*b**3) + 208*c*d**3*x*sin(a + b*x)**4*cos(a + b*x)/(75*b**3) + 1
352*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 3424*c*d**3*x*cos
(a + b*x)**5/(1125*b**3) + 104*d**4*x**2*sin(a + b*x)**4*cos(a + b*x)/(75*b
**3) + 676*d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 1712*d**4
*x**2*cos(a + b*x)**5/(1125*b**3) - 50272*c*d**3*sin(a + b*x)**5/(16875*b**
4) - 20456*c*d**3*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 3424*c*d**3
*sin(a + b*x)*cos(a + b*x)**4/(1125*b**4) - 50272*d**4*x*sin(a + b*x)**5/(1
6875*b**4) - 20456*d**4*x*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 342
4*d**4*x*sin(a + b*x)*cos(a + b*x)**4/(1125*b**4) - 50272*d**4*sin(a + b*x)
**4*cos(a + b*x)/(16875*b**5) - 303368*d**4*sin(a + b*x)**2*cos(a + b*x)**3
/(50625*b**5) - 760816*d**4*cos(a + b*x)**5/(253125*b**5), Ne(b, 0)), ((c**
4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*
*3*cos(a)**2, True))
```

# 3.90 $\int (c+dx)^3 \cos^2(a+bx) \sin^3(a+bx) \, dx$

**Optimal**. Leaf size=259

$$-\frac{3d^3\sin(a+bx)}{4b^4}-\frac{d^3\sin(3a+3bx)}{216b^4}+\frac{3d^3\sin(5a+5bx)}{5000b^4}+\frac{3d^2(c+dx)\cos(a+bx)}{4b^3}+\frac{d^2(c+dx)\cos(3a+3bx)}{72b^3}-$$

```
[Out] 3/4*d^2*(d*x+c)*cos(b*x+a)/b^3-1/8*(d*x+c)^3*cos(b*x+a)/b+1/72*d^2*(d*x+c)*
cos(3*b*x+3*a)/b^3-1/48*(d*x+c)^3*cos(3*b*x+3*a)/b-3/1000*d^2*(d*x+c)*cos(5
*b*x+5*a)/b^3+1/80*(d*x+c)^3*cos(5*b*x+5*a)/b-3/4*d^3*sin(b*x+a)/b^4+3/8*d*
(d*x+c)^2*sin(b*x+a)/b^2-1/216*d^3*sin(3*b*x+3*a)/b^4+1/48*d*(d*x+c)^2*sin(
3*b*x+3*a)/b^2+3/5000*d^3*sin(5*b*x+5*a)/b^4-3/400*d*(d*x+c)^2*sin(5*b*x+5*
a)/b^2
```

**Rubi** [A] time = 0.28, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2637}

$$\frac{3d^2(c+dx)\cos(a+bx)}{4b^3}+\frac{d^2(c+dx)\cos(3a+3bx)}{72b^3}-\frac{3d^2(c+dx)\cos(5a+5bx)}{1000b^3}+\frac{3d(c+dx)^2\sin(a+bx)}{8b^2}+\frac{d(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (3*d^2*(c + d*x)*Cos[a + b*x])/(4*b^3) - ((c + d*x)^3*Cos[a + b*x])/(8*b) +
 (d^2*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*Cos[3*a + 3*b*x])
/(48*b) - (3*d^2*(c + d*x)*Cos[5*a + 5*b*x])/(1000*b^3) + ((c + d*x)^3*Cos[
5*a + 5*b*x])/(80*b) - (3*d^3*Sin[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Sin[
a + b*x])/(8*b^2) - (d^3*Sin[3*a + 3*b*x])/(216*b^4) + (d*(c + d*x)^2*Sin[3
*a + 3*b*x])/(48*b^2) + (3*d^3*Sin[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x
)^2*Sin[5*a + 5*b*x])/(400*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^3 \cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^3\sin(a+bx) + \frac{1}{16}(c+dx)^3\sin(3a+3bx) - \frac{1}{16}(c+dx)^3\right.$$
$$= \frac{1}{16}\int (c+dx)^3\sin(3a+3bx)\,dx - \frac{1}{16}\int (c+dx)^3\sin(5a+5bx)\,dx + \frac{1}{8}$$
$$= -\frac{(c+dx)^3\cos(a+bx)}{8b} - \frac{(c+dx)^3\cos(3a+3bx)}{48b} + \frac{(c+dx)^3\cos(5a}{80b}$$
$$= -\frac{(c+dx)^3\cos(a+bx)}{8b} - \frac{(c+dx)^3\cos(3a+3bx)}{48b} + \frac{(c+dx)^3\cos(5a}{80b}$$
$$= \frac{3d^2(c+dx)\cos(a+bx)}{4b^3} - \frac{(c+dx)^3\cos(a+bx)}{8b} + \frac{d^2(c+dx)\cos(3a}{72b^3}$$
$$= \frac{3d^2(c+dx)\cos(a+bx)}{4b^3} - \frac{(c+dx)^3\cos(a+bx)}{8b} + \frac{d^2(c+dx)\cos(3a}{72b^3}$$

**Mathematica [A]** time = 1.45, size = 369, normalized size = 1.42

$$\frac{3375b^3c^3\cos(5(a+bx)) + 10125b^3c^2dx\cos(5(a+bx)) + 10125b^3cd^2x^2\cos(5(a+bx)) + 3375b^3d^3x^3\cos(5(a+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-33750*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 1875*b*(c + d
*x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 3375*b^3*c^3*Cos[5*(a +
 b*x)] - 810*b*c*d^2*Cos[5*(a + b*x)] + 10125*b^3*c^2*d*x*Cos[5*(a + b*x)]
- 810*b*d^3*x*Cos[5*(a + b*x)] + 10125*b^3*c*d^2*x^2*Cos[5*(a + b*x)] + 337
5*b^3*d^3*x^3*Cos[5*(a + b*x)] + 101250*b^2*c^2*d*Sin[a + b*x] - 202500*d^3
*Sin[a + b*x] + 202500*b^2*c*d^2*x*Sin[a + b*x] + 101250*b^2*d^3*x^2*Sin[a
+ b*x] + 5625*b^2*c^2*d*Sin[3*(a + b*x)] - 1250*d^3*Sin[3*(a + b*x)] + 1125
0*b^2*c*d^2*x*Sin[3*(a + b*x)] + 5625*b^2*d^3*x^2*Sin[3*(a + b*x)] - 2025*b
^2*c^2*d*Sin[5*(a + b*x)] + 162*d^3*Sin[5*(a + b*x)] - 4050*b^2*c*d^2*x*Sin
[5*(a + b*x)] - 2025*b^2*d^3*x^2*Sin[5*(a + b*x)])/(270000*b^4)
```

**fricas [A]** time = 0.53, size = 296, normalized size = 1.14

$$\frac{135\left(25\,b^3d^3x^3 + 75\,b^3cd^2x^2 + 25\,b^3c^3 - 6\,bcd^2 + 3\left(25\,b^3c^2d - 2\,bd^3\right)x\right)\cos(bx+a)^5 - 75\left(75\,b^3d^3x^3 + 225\,b^3cd^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16875*(135*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 25*b^3*c^3 - 6*b*c*d^2 +
3*(25*b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^5 - 75*(75*b^3*d^3*x^3 + 225*b^3
*c*d^2*x^2 + 75*b^3*c^3 - 26*b*c*d^2 + (225*b^3*c^2*d - 26*b*d^3)*x)*cos(b*
x + a)^3 + 11700*(b*d^3*x + b*c*d^2)*cos(b*x + a) + (5850*b^2*d^3*x^2 + 117
00*b^2*c*d^2*x + 5850*b^2*c^2*d - 81*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*
b^2*c^2*d - 2*d^3)*cos(b*x + a)^4 - 12568*d^3 + (2925*b^2*d^3*x^2 + 5850*b^
2*c*d^2*x + 2925*b^2*c^2*d - 434*d^3)*cos(b*x + a)^2)*sin(b*x + a))/b^4
```

**giac [A]** time = 0.28, size = 351, normalized size = 1.36

$$\frac{\left(25\,b^3d^3x^3 + 75\,b^3cd^2x^2 + 75\,b^3c^2dx + 25\,b^3c^3 - 6\,bd^3x - 6\,bcd^2\right)\cos(5\,bx+5\,a)}{2000\,b^4} - \frac{\left(3\,b^3d^3x^3 + 9\,b^3cd^2x^2 + 9\,b^3c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2000*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 75*b^3*c^2*d*x + 25*b^3*c^3 - 6
*b*d^3*x - 6*b*c*d^2)*cos(5*b*x + 5*a)/b^4 - 1/144*(3*b^3*d^3*x^3 + 9*b^3*c
*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x - 2*b*c*d^2)*cos(3*b*x + 3
*a)/b^4 - 1/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*
b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 - 3/10000*(25*b^2*d^3*x^2 + 50*b^2*c*
d^2*x + 25*b^2*c^2*d - 2*d^3)*sin(5*b*x + 5*a)/b^4 + 1/432*(9*b^2*d^3*x^2 +
 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)/b^4 + 3/8*(b^2*d^3*
x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/b^4
```

**maple [B]** time = 0.02, size = 992, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^3*d^3*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2/5*(b*x+a)^2*si
n(b*x+a)-856/1125*sin(b*x+a)+4/5*(b*x+a)*cos(b*x+a)+1/15*(b*x+a)^2*sin(b*x+
a)^3+2/45*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+22/3375*sin(b*x+a)^3+1/5*(b*x
+a)^3*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-3/25*(b*x+a)^2*sin(b*x
+a)^5-6/125*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+6/625*si
n(b*x+a)^5)-3/b^3*a*d^3*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*co
s(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x
+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x
+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos
(b*x+a))+3/b^2*c*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*cos(b
*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x+a)
^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)
-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*
x+a))+3/b^3*a^2*d^3*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+
a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*
x+a)-1/25*sin(b*x+a)^5)-6/b^2*a*c*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*
x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*si
n(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)+3/b*c^2*d*(-1/3*(b*x+a)*(2+sin(b*
x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b
*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)-1/b^3*a^3*d^3*(-1/5
*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+3/b^2*a^2*c*d^2*(-1/5*sin(b*x
+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)-3/b*a*c^2*d*(-1/5*sin(b*x+a)^2*cos(b*
x+a)^3-2/15*cos(b*x+a)^3)+c^3*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+
a)^3))
```

**maxima [B]** time = 0.38, size = 766, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/270000*(18000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^3 - 54000*(3*cos(b*
x + a)^5 - 5*cos(b*x + a)^3)*a*c^2*d/b + 54000*(3*cos(b*x + a)^5 - 5*cos(b*
x + a)^3)*a^2*c*d^2/b^2 - 18000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a^3*d
^3/b^3 + 225*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a)
 - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) +
450*sin(b*x + a))*c^2*d/b - 450*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x +
a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*
sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*c*d^2/b^2 + 225*(45*(b*x + a)*cos(5*
b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9
*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a^2*d^3/b^3 + 1
```

```
5*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3
*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x
 + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*c*
d^2/b^2 - 15*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2
 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a
)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b
*x + a))*a*d^3/b^3 + (135*(25*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) -
 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 33750*((b*x + a)^3 -
 6*b*x - 6*a)*cos(b*x + a) - 81*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 625
*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*sin(b*x +
a))*d^3/b^3)/b
```

**mupad [B]** time = 2.57, size = 516, normalized size = 1.99

$$-\frac{\frac{3\,d^3\,\sin(a+b\,x)}{4}+\frac{d^3\,\sin(3\,a+3\,b\,x)}{216}-\frac{3\,d^3\,\sin(5\,a+5\,b\,x)}{5000}+\frac{b^3\,c^3\,\cos(a+b\,x)}{8}+\frac{b^3\,c^3\,\cos(3\,a+3\,b\,x)}{48}-\frac{b^3\,c^3\,\cos(5\,a+5\,b\,x)}{80}-\frac{b^2\,c^2\,d\,\sin(3\,a+3\,b}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] -((3*d^3*sin(a + b*x))/4 + (d^3*sin(3*a + 3*b*x))/216 - (3*d^3*sin(5*a + 5*
b*x))/5000 + (b^3*c^3*cos(a + b*x))/8 + (b^3*c^3*cos(3*a + 3*b*x))/48 - (b^
3*c^3*cos(5*a + 5*b*x))/80 - (b^2*c^2*d*sin(3*a + 3*b*x))/48 + (3*b^2*c^2*d
*sin(5*a + 5*b*x))/400 + (b^3*d^3*x^3*cos(a + b*x))/8 - (3*b^2*d^3*x^2*sin(
a + b*x))/8 - (3*b*c*d^2*cos(a + b*x))/4 - (3*b*d^3*x*cos(a + b*x))/4 + (b^
3*d^3*x^3*cos(3*a + 3*b*x))/48 - (b^3*d^3*x^3*cos(5*a + 5*b*x))/80 - (b^2*d
^3*x^2*sin(3*a + 3*b*x))/48 + (3*b^2*d^3*x^2*sin(5*a + 5*b*x))/400 - (b*c*d
^2*cos(3*a + 3*b*x))/72 + (3*b*c*d^2*cos(5*a + 5*b*x))/1000 - (3*b^2*c^2*d*
sin(a + b*x))/8 - (b*d^3*x*cos(3*a + 3*b*x))/72 + (3*b*d^3*x*cos(5*a + 5*b*
x))/1000 + (3*b^3*c^2*d*x*cos(a + b*x))/8 - (3*b^2*c*d^2*x*sin(a + b*x))/4
+ (b^3*c^2*d*x*cos(3*a + 3*b*x))/16 - (3*b^3*c^2*d*x*cos(5*a + 5*b*x))/80 +
 (3*b^3*c*d^2*x^2*cos(a + b*x))/8 - (b^2*c*d^2*x*sin(3*a + 3*b*x))/24 + (3*
b^2*c*d^2*x*sin(5*a + 5*b*x))/200 + (b^3*c*d^2*x^2*cos(3*a + 3*b*x))/16 - (
3*b^3*c*d^2*x^2*cos(5*a + 5*b*x))/80)/b^4
```

**sympy [A]** time = 11.14, size = 690, normalized size = 2.66

$$\begin{cases} -\frac{c^3\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2c^3\cos^5(a+bx)}{15b}-\frac{c^2dx\sin^2(a+bx)\cos^3(a+bx)}{b}-\frac{2c^2dx\cos^5(a+bx)}{5b}-\frac{cd^2x^2\sin^2(a+bx)\cos^3(a+bx)}{b}-\frac{2cd^2x^2\,c}{} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^3(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**3*cos(a + b*x
)**5/(15*b) - c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**3/b - 2*c**2*d*x*cos(a
 + b*x)**5/(5*b) - c*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/b - 2*c*d**2
*x**2*cos(a + b*x)**5/(5*b) - d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**3/(3*
b) - 2*d**3*x**3*cos(a + b*x)**5/(15*b) + 26*c**2*d*sin(a + b*x)**5/(75*b**
2) + 13*c**2*d*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 2*c**2*d*sin(a +
 b*x)*cos(a + b*x)**4/(5*b**2) + 52*c*d**2*x*sin(a + b*x)**5/(75*b**2) + 26
*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 4*c*d**2*x*sin(a + b*
x)*cos(a + b*x)**4/(5*b**2) + 26*d**3*x**2*sin(a + b*x)**5/(75*b**2) + 13*d
**3*x**2*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 2*d**3*x**2*sin(a + b*
x)*cos(a + b*x)**4/(5*b**2) + 52*c*d**2*sin(a + b*x)**4*cos(a + b*x)/(75*b*
*3) + 338*c*d**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 856*c*d**2*co
s(a + b*x)**5/(1125*b**3) + 52*d**3*x*sin(a + b*x)**4*cos(a + b*x)/(75*b**3
```

```
) + 338*d**3*x*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 856*d**3*x*cos(
a + b*x)**5/(1125*b**3) - 12568*d**3*sin(a + b*x)**5/(16875*b**4) - 5114*d*
*3*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 856*d**3*sin(a + b*x)*cos(
a + b*x)**4/(1125*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**
3 + d**3*x**4/4)*sin(a)**3*cos(a)**2, True))
```

# 3.91 $\int (c+dx)^2 \cos^2(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=184

$$\frac{d^2\cos(a+bx)}{4b^3}+\frac{d^2\cos(3a+3bx)}{216b^3}-\frac{d^2\cos(5a+5bx)}{1000b^3}+\frac{d(c+dx)\sin(a+bx)}{4b^2}+\frac{d(c+dx)\sin(3a+3bx)}{72b^2}-\frac{d(c+dx)\mathrm{s}}{2}$$

```
[Out] 1/4*d^2*cos(b*x+a)/b^3-1/8*(d*x+c)^2*cos(b*x+a)/b+1/216*d^2*cos(3*b*x+3*a)/
b^3-1/48*(d*x+c)^2*cos(3*b*x+3*a)/b-1/1000*d^2*cos(5*b*x+5*a)/b^3+1/80*(d*x
+c)^2*cos(5*b*x+5*a)/b+1/4*d*(d*x+c)*sin(b*x+a)/b^2+1/72*d*(d*x+c)*sin(3*b*
x+3*a)/b^2-1/200*d*(d*x+c)*sin(5*b*x+5*a)/b^2
```

**Rubi [A]** time = 0.20, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$\frac{d(c+dx)\sin(a+bx)}{4b^2}+\frac{d(c+dx)\sin(3a+3bx)}{72b^2}-\frac{d(c+dx)\sin(5a+5bx)}{200b^2}+\frac{d^2\cos(a+bx)}{4b^3}+\frac{d^2\cos(3a+3bx)}{216b^3}-\frac{d^2\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (d^2*Cos[a + b*x])/(4*b^3) - ((c + d*x)^2*Cos[a + b*x])/(8*b) + (d^2*Cos[3*
a + 3*b*x])/(216*b^3) - ((c + d*x)^2*Cos[3*a + 3*b*x])/(48*b) - (d^2*Cos[5*
a + 5*b*x])/(1000*b^3) + ((c + d*x)^2*Cos[5*a + 5*b*x])/(80*b) + (d*(c + d*
x)*Sin[a + b*x])/(4*b^2) + (d*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^2) - (d*(c
+ d*x)*Sin[5*a + 5*b*x])/(200*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2\cos^2(a+bx)\sin^3(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^2\sin(a+bx)+\frac{1}{16}(c+dx)^2\sin(3a+3bx)-\frac{1}{16}(c+dx)^2\right.\\
&= \frac{1}{16}\int (c+dx)^2\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^2\sin(5a+5bx)\,dx+\\
&= -\frac{(c+dx)^2\cos(a+bx)}{8b}-\frac{(c+dx)^2\cos(3a+3bx)}{48b}+\frac{(c+dx)^2\cos(5a}{80b}\\
&= -\frac{(c+dx)^2\cos(a+bx)}{8b}-\frac{(c+dx)^2\cos(3a+3bx)}{48b}+\frac{(c+dx)^2\cos(5a}{80b}\\
&= \frac{d^2\cos(a+bx)}{4b^3}-\frac{(c+dx)^2\cos(a+bx)}{8b}+\frac{d^2\cos(3a+3bx)}{216b^3}-\frac{(c+dx)}{}
\end{aligned}$$

**Mathematica** [A] time = 0.88, size = 127, normalized size = 0.69

$$\frac{-6750\cos(a+bx)\left(b^2(c+dx)^2-2d^2\right)-125\cos(3(a+bx))\left(9b^2(c+dx)^2-2d^2\right)+27\cos(5(a+bx))\left(25b^2(c+\right.}{54000b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-6750*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 125*(-2*d^2 + 9*b^2*(c + d
*x)^2)*Cos[3*(a + b*x)] + 27*(-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*(a + b*x)]
 + 30*b*d*(c + d*x)*(450*Sin[a + b*x] + 25*Sin[3*(a + b*x)] - 9*Sin[5*(a +
b*x)]))/(54000*b^3)
```

**fricas** [A] time = 0.47, size = 166, normalized size = 0.90

$$\frac{27\left(25\,b^2d^2x^2+50\,b^2cdx+25\,b^2c^2-2\,d^2\right)\cos\left(bx+a\right)^5-5\left(225\,b^2d^2x^2+450\,b^2cdx+225\,b^2c^2-26\,d^2\right)\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(b*x + a
)^5 - 5*(225*b^2*d^2*x^2 + 450*b^2*c*d*x + 225*b^2*c^2 - 26*d^2)*cos(b*x +
a)^3 + 780*d^2*cos(b*x + a) - 30*(9*(b*d^2*x + b*c*d)*cos(b*x + a)^4 - 26*b
*d^2*x - 26*b*c*d - 13*(b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/b^3
```

**giac** [A] time = 0.25, size = 209, normalized size = 1.14

$$\frac{\left(25\,b^2d^2x^2+50\,b^2cdx+25\,b^2c^2-2\,d^2\right)\cos\left(5\,bx+5\,a\right)}{2000\,b^3}-\frac{\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2\right)\cos\left(3\,bx+3\,a\right)}{432\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2000*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(5*b*x + 5*a
)/b^3 - 1/432*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x
+ 3*a)/b^3 - 1/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)
/b^3 - 1/200*(b*d^2*x + b*c*d)*sin(5*b*x + 5*a)/b^3 + 1/72*(b*d^2*x + b*c*d
)*sin(3*b*x + 3*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*sin(b*x + a)/b^3
```

**maple** [B] time = 0.02, size = 466, normalized size = 2.53

$$\frac{d^2\left(-\frac{(bx+a)^2\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{3}+\frac{4\cos(bx+a)}{15}+\frac{4(bx+a)\sin(bx+a)}{15}+\frac{2(bx+a)\left(\sin^3(bx+a)\right)}{45}+\frac{2\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{135}+\frac{(bx+a)^2\left(\frac{8}{3}+\sin^4(bx+a)+\frac{4\left(\sin^2(bx+a)\right)}{3}\right)c}{5}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^2*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*cos(b*x+a)+
4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x+a)^2)*co
s(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-2/25*
(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))-
2/b^2*a*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/1
5*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/2
```

```
5*sin(b*x+a)^5)+2/b*c*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(
b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*co
s(b*x+a)-1/25*sin(b*x+a)^5)+1/b^2*a^2*d^2*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2
/15*cos(b*x+a)^3)-2/b*a*c*d*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)
^3)+c^2*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3))
```

**maxima [B]** time = 0.35, size = 375, normalized size = 2.04

$$\frac{3600\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)c^2-\frac{7200\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)acd}{b}+\frac{3600\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)a^2d^2}{b^2}+\frac{30}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/54000*(3600*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^2 - 7200*(3*cos(b*x +
 a)^5 - 5*cos(b*x + a)^3)*a*c*d/b + 3600*(3*cos(b*x + a)^5 - 5*cos(b*x + a)
^3)*a^2*d^2/b^2 + 30*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*
x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x +
 3*a) + 450*sin(b*x + a))*c*d/b - 30*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b
*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a)
+ 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*d^2/b^2 + (27*(25*(b*x + a)^2 -
 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*
x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)
*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*d^2/b^2)/b
```

**mupad [B]** time = 0.81, size = 249, normalized size = 1.35

$$\frac{780\,d^2\,\cos(a+b\,x)+130\,d^2\,\cos(a+b\,x)^3-54\,d^2\,\cos(a+b\,x)^5-1125\,b^2\,c^2\,\cos(a+b\,x)^3+675\,b^2\,c^2\,\cos(a+b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] (780*d^2*cos(a + b*x) + 130*d^2*cos(a + b*x)^3 - 54*d^2*cos(a + b*x)^5 - 11
25*b^2*c^2*cos(a + b*x)^3 + 675*b^2*c^2*cos(a + b*x)^5 + 780*b*d^2*x*sin(a
+ b*x) - 1125*b^2*d^2*x^2*cos(a + b*x)^3 + 675*b^2*d^2*x^2*cos(a + b*x)^5 +
 780*b*c*d*sin(a + b*x) - 2250*b^2*c*d*x*cos(a + b*x)^3 + 1350*b^2*c*d*x*co
s(a + b*x)^5 + 390*b*d^2*x*cos(a + b*x)^2*sin(a + b*x) - 270*b*d^2*x*cos(a
+ b*x)^4*sin(a + b*x) + 390*b*c*d*cos(a + b*x)^2*sin(a + b*x) - 270*b*c*d*c
os(a + b*x)^4*sin(a + b*x))/(3375*b^3)
```

**sympy [A]** time = 5.96, size = 382, normalized size = 2.08

$$\begin{cases} -\frac{c^2\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2c^2\cos^5(a+bx)}{15b}-\frac{2cdx\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{4cdx\cos^5(a+bx)}{15b}-\frac{d^2x^2\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2d^2x^2\cos^5}{15b} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin^3(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**2*cos(a + b*x
)**5/(15*b) - 2*c*d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 4*c*d*x*cos(a
 + b*x)**5/(15*b) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d**
2*x**2*cos(a + b*x)**5/(15*b) + 52*c*d*sin(a + b*x)**5/(225*b**2) + 26*c*d*
sin(a + b*x)**3*cos(a + b*x)**2/(45*b**2) + 4*c*d*sin(a + b*x)*cos(a + b*x)
**4/(15*b**2) + 52*d**2*x*sin(a + b*x)**5/(225*b**2) + 26*d**2*x*sin(a + b*
x)**3*cos(a + b*x)**2/(45*b**2) + 4*d**2*x*sin(a + b*x)*cos(a + b*x)**4/(15
```

```
*b**2) + 52*d**2*sin(a + b*x)**4*cos(a + b*x)/(225*b**3) + 338*d**2*sin(a +
 b*x)**2*cos(a + b*x)**3/(675*b**3) + 856*d**2*cos(a + b*x)**5/(3375*b**3),
 Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a)**2, True))
```

## 3.92 $\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=109

$$\frac{d\sin(a+bx)}{8b^2}+\frac{d\sin(3a+3bx)}{144b^2}-\frac{d\sin(5a+5bx)}{400b^2}-\frac{(c+dx)\cos(a+bx)}{8b}-\frac{(c+dx)\cos(3a+3bx)}{48b}+\frac{(c+dx)\cos(5a}{80b}$$

```
[Out] -1/8*(d*x+c)*cos(b*x+a)/b-1/48*(d*x+c)*cos(3*b*x+3*a)/b+1/80*(d*x+c)*cos(5*
b*x+5*a)/b+1/8*d*sin(b*x+a)/b^2+1/144*d*sin(3*b*x+3*a)/b^2-1/400*d*sin(5*b*
x+5*a)/b^2
```

**Rubi [A]** time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3296, 2637}

$$\frac{d\sin(a+bx)}{8b^2}+\frac{d\sin(3a+3bx)}{144b^2}-\frac{d\sin(5a+5bx)}{400b^2}-\frac{(c+dx)\cos(a+bx)}{8b}-\frac{(c+dx)\cos(3a+3bx)}{48b}+\frac{(c+dx)\cos(5a}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -((c + d*x)*Cos[a + b*x])/(8*b) - ((c + d*x)*Cos[3*a + 3*b*x])/(48*b) + ((c
 + d*x)*Cos[5*a + 5*b*x])/(80*b) + (d*Sin[a + b*x])/(8*b^2) + (d*Sin[3*a +
3*b*x])/(144*b^2) - (d*Sin[5*a + 5*b*x])/(400*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)\sin(a+bx)+\frac{1}{16}(c+dx)\sin(3a+3bx)-\frac{1}{16}(c+dx)\sin(\right.\\
&= \frac{1}{16}\int (c+dx)\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)\sin(5a+5bx)\,dx+\frac{1}{8}\int\\
&= -\frac{(c+dx)\cos(a+bx)}{8b}-\frac{(c+dx)\cos(3a+3bx)}{48b}+\frac{(c+dx)\cos(5a+5b}{80b}\\
&= -\frac{(c+dx)\cos(a+bx)}{8b}-\frac{(c+dx)\cos(3a+3bx)}{48b}+\frac{(c+dx)\cos(5a+5b}{80b}
\end{aligned}$$

**Mathematica [A]** time = 0.29, size = 94, normalized size = 0.86

$$\frac{-450b(c+dx)\cos(a+bx)-75b(c+dx)\cos(3(a+bx))+45bc\cos(5(a+bx))+450d\sin(a+bx)+25d\sin(3(a}{3600b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-450*b*(c + d*x)*Cos[a + b*x] - 75*b*(c + d*x)*Cos[3*(a + b*x)] + 45*b*c*C
os[5*(a + b*x)] + 45*b*d*x*Cos[5*(a + b*x)] + 450*d*Sin[a + b*x] + 25*d*Sin
[3*(a + b*x)] - 9*d*Sin[5*(a + b*x)])/(3600*b^2)
```

**fricas [A]** time = 0.93, size = 76, normalized size = 0.70

$$\frac{45\,(bdx+bc)\cos(bx+a)^5-75\,(bdx+bc)\cos(bx+a)^3-\left(9\,d\cos(bx+a)^4-13\,d\cos(bx+a)^2-26\,d\right)\sin(b}{225\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/225*(45*(b*d*x + b*c)*cos(b*x + a)^5 - 75*(b*d*x + b*c)*cos(b*x + a)^3 -
(9*d*cos(b*x + a)^4 - 13*d*cos(b*x + a)^2 - 26*d)*sin(b*x + a))/b^2
```

**giac [A]** time = 1.17, size = 106, normalized size = 0.97

$$\frac{(bdx+bc)\cos(5\,bx+5\,a)}{80\,b^2}-\frac{(bdx+bc)\cos(3\,bx+3\,a)}{48\,b^2}-\frac{(bdx+bc)\cos(bx+a)}{8\,b^2}-\frac{d\sin(5\,bx+5\,a)}{400\,b^2}+\frac{d\sin(3\,bx}{144\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/80*(b*d*x + b*c)*cos(5*b*x + 5*a)/b^2 - 1/48*(b*d*x + b*c)*cos(3*b*x + 3*
a)/b^2 - 1/8*(b*d*x + b*c)*cos(b*x + a)/b^2 - 1/400*d*sin(5*b*x + 5*a)/b^2
+ 1/144*d*sin(3*b*x + 3*a)/b^2 + 1/8*d*sin(b*x + a)/b^2
```

**maple [A]** time = 0.02, size = 163, normalized size = 1.50

$$\frac{\frac{d\left(-\frac{(bx+a)\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{3}+\frac{\left(\sin^3(bx+a)\right)}{45}+\frac{2\sin(bx+a)}{15}+\frac{(bx+a)\left(\frac{8}{3}+\sin^4(bx+a)+\frac{4\left(\sin^2(bx+a)\right)}{3}\right)\cos(bx+a)}{5}-\frac{\left(\sin^5(bx+a)\right)}{25}\right)}{b}-\frac{da\left(-\frac{\left(\sin^2(bx+a)\right)\left(\cos^3(bx+a)\right.}{5}\right.}{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15
*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25
*sin(b*x+a)^5)-1/b*d*a*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+c
*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3))
```

**maxima [A]** time = 0.34, size = 139, normalized size = 1.28

$$\frac{240\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)c-\frac{240\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)ad}{b}+\frac{(45\,(bx+a)\cos(5\,bx+5\,a)-75\,(bx+a)\cos(3\,bx+3\,a}{}}{3600\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3600*(240*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c - 240*(3*cos(b*x + a)^5
 - 5*cos(b*x + a)^3)*a*d/b + (45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*
```

```
cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin
(3*b*x + 3*a) + 450*sin(b*x + a))*d/b)/b
```

**mupad [B]** time = 1.24, size = 99, normalized size = 0.91

$$\frac{26\,d\,\sin\left(a+b\,x\right)-75\,b\,c\cos\left(a+b\,x\right)^3+45\,b\,c\cos\left(a+b\,x\right)^5+13\,d\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)-9\,d\cos\left(a+b\,x\right)}{225\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x),x)
```

```
[Out] (26*d*sin(a + b*x) - 75*b*c*cos(a + b*x)^3 + 45*b*c*cos(a + b*x)^5 + 13*d*c
os(a + b*x)^2*sin(a + b*x) - 9*d*cos(a + b*x)^4*sin(a + b*x) - 75*b*d*x*cos
(a + b*x)^3 + 45*b*d*x*cos(a + b*x)^5)/(225*b^2)
```

**sympy [A]** time = 3.05, size = 163, normalized size = 1.50

$$\begin{cases} -\frac{c\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2c\cos^5(a+bx)}{15b}-\frac{dx\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2dx\cos^5(a+bx)}{15b}+\frac{26d\sin^5(a+bx)}{225b^2}+\frac{13d\sin^3(a+bx)\cos^2(a+bx)}{45b^2} \\ \left(cx+\frac{dx^2}{2}\right)\sin^3(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c*cos(a + b*x)**5/(
15*b) - d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d*x*cos(a + b*x)**5/(
15*b) + 26*d*sin(a + b*x)**5/(225*b**2) + 13*d*sin(a + b*x)**3*cos(a + b*x)
**2/(45*b**2) + 2*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2), Ne(b, 0)), ((c*
x + d*x**2/2)*sin(a)**3*cos(a)**2, True))
```

# 3.93 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=185

$$-\frac{\sin\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5bc}{d}+5bx\right)}{16d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{8d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+\right.}{8d}$$

```
[Out] 1/8*cos(a-b*c/d)*Si(b*c/d+b*x)/d+1/16*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d-
1/16*cos(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d-1/16*Ci(5*b*c/d+5*b*x)*sin(5*a-5*
b*c/d)/d+1/16*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d+1/8*Ci(b*c/d+b*x)*sin(a-
b*c/d)/d
```

**Rubi** [A] time = 0.34, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4406, 3303, 3299, 3302}

$$-\frac{\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -(CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(16*d) + (CosIntegra
l[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(16*d) + (CosIntegral[(b*c)/d +
b*x]*Sin[a - (b*c)/d])/(8*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x]
)/(8*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (C
os[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \left(\frac{\sin(a+bx)}{8(c+dx)} + \frac{\sin(3a+3bx)}{16(c+dx)} - \frac{\sin(5a+5bx)}{16(c+dx)}\right)dx$$
$$= \frac{1}{16}\int \frac{\sin(3a+3bx)}{c+dx}\,dx - \frac{1}{16}\int \frac{\sin(5a+5bx)}{c+dx}\,dx + \frac{1}{8}\int \frac{\sin(a+bx)}{c+dx}\,dx$$
$$= -\left(\frac{1}{16}\cos\left(5a-\frac{5bc}{d}\right)\int \frac{\sin\left(\frac{5bc}{d}+5bx\right)}{c+dx}\,dx\right) + \frac{1}{16}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\sin\left(\frac{3bc}{d}+\right.}{c+d}$$
$$= -\frac{\operatorname{Ci}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{16d} + \frac{\operatorname{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{16d} + \frac{\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{8}$$

**Mathematica [A]** time = 0.49, size = 154, normalized size = 0.83

$$\frac{\sin\left(5a-\frac{5bc}{d}\right)\left(-\operatorname{Ci}\left(\frac{5b(c+dx)}{d}\right)\right) + \sin\left(3a-\frac{3bc}{d}\right)\operatorname{Ci}\left(\frac{3b(c+dx)}{d}\right) + 2\sin\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(b\left(\frac{c}{d}+x\right)\right) + 2\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(b\left(\right.\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] (-(CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d]) + CosIntegral[(3*b*
(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + 2*CosIntegral[b*(c/d + x)]*Sin[a - (b*
c)/d] + 2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Cos[3*a - (3*b*c)/d]*
SinIntegral[(3*b*(c + d*x))/d] - Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c +
 d*x))/d])/(16*d)
```

**fricas [A]** time = 0.65, size = 228, normalized size = 1.23

$$2\left(\operatorname{Ci}\left(\frac{bdx+bc}{d}\right) + \operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right) + \left(\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right) + \operatorname{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right) - \left(\operatorname{Ci}\left(\frac{5\,(bdx+bc)}{d}\right) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/32*(2*(cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*si
n(-(b*c - a*d)/d) + (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d
*x + b*c)/d))*sin(-3*(b*c - a*d)/d) - (cos_integral(5*(b*d*x + b*c)/d) + co
s_integral(-5*(b*d*x + b*c)/d))*sin(-5*(b*c - a*d)/d) - 2*cos(-5*(b*c - a*d
)/d)*sin_integral(5*(b*d*x + b*c)/d) + 2*cos(-3*(b*c - a*d)/d)*sin_integral
(3*(b*d*x + b*c)/d) + 4*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/
d
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.02, size = 253, normalized size = 1.37

$$\frac{-\frac{b\left(\frac{5\operatorname{Si}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\cos\left(\frac{-5da+5cb}{d}\right)}{d} - \frac{5\operatorname{Ci}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\sin\left(\frac{-5da+5cb}{d}\right)}{d}\right)}{80} + \frac{b\left(\frac{\operatorname{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d} - \frac{\operatorname{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}\right)}{8} + \frac{b}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/b*(-1/80*b*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d-5*Ci(5*b
*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)+1/8*b*(Si(b*x+a+(-a*d+b*c)/d)
*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)+1/48*b*(3*
Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b
*c)/d)*sin(3*(-a*d+b*c)/d)/d))
```

**maxima [C]** time = 0.47, size = 407, normalized size = 2.20

$$\frac{b\left(-2i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+2i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(-i\,E_1\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+i\,E_1\left(-\frac{3i\,bc-}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/32*(b*(-2*I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*ex
p_integral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) +
b*(-I*exp_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*exp_in
tegral_e(1, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d
) + b*(I*exp_integral_e(1, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) - I*exp
_integral_e(1, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c - a*d
)/d) - 2*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b*(exp
_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1,
 -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) + b*(exp_
integral_e(1, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_integral_e(1,
-(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d))/(b*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{3}{\left(a + bx \right)} \cos^{2}{\left(a + bx \right)}}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x), x)
```

## 3.94 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2} dx$

**Optimal**. Leaf size=257

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{8d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\mathrm{Ci}\left(\frac{5bc}{d}+5bx\right)}{16d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\right.}{8d^2}$$

```
[Out] -5/16*b*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d^2+3/16*b*Ci(3*b*c/d+3*b*x)*cos
(3*a-3*b*c/d)/d^2+1/8*b*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^2+5/16*b*Si(5*b*c/d+5*
b*x)*sin(5*a-5*b*c/d)/d^2-3/16*b*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2-1/8
*b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2-1/8*sin(b*x+a)/d/(d*x+c)-1/16*sin(3*b*x+3
*a)/d/(d*x+c)+1/16*sin(5*b*x+5*a)/d/(d*x+c)
```

**Rubi [A]** time = 0.42, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegra}}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) - (5*b*Cos[5*a - (5*b*c)/
d]*CosIntegral[(5*b*c)/d + 5*b*x])/(16*d^2) - Sin[a + b*x]/(8*d*(c + d*x))
- Sin[3*a + 3*b*x]/(16*d*(c + d*x)) + Sin[5*a + 5*b*x]/(16*d*(c + d*x)) - (
b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) - (3*b*Sin[3*a - (3*
b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Sin[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \left(\frac{\sin(a+bx)}{8(c+dx)^2}+\frac{\sin(3a+3bx)}{16(c+dx)^2}-\frac{\sin(5a+5bx)}{16(c+dx)^2}\right)dx$$

$$= \frac{1}{16}\int \frac{\sin(3a+3bx)}{(c+dx)^2}\,dx - \frac{1}{16}\int \frac{\sin(5a+5bx)}{(c+dx)^2}\,dx + \frac{1}{8}\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx$$

$$= -\frac{\sin(a+bx)}{8d(c+dx)} - \frac{\sin(3a+3bx)}{16d(c+dx)} + \frac{\sin(5a+5bx)}{16d(c+dx)} + \frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{8d} + \frac{(3b)\int}{}$$

$$= -\frac{\sin(a+bx)}{8d(c+dx)} - \frac{\sin(3a+3bx)}{16d(c+dx)} + \frac{\sin(5a+5bx)}{16d(c+dx)} - \frac{\left(5b\cos\left(5a-\frac{5bc}{d}\right)\right)\int \frac{\cos(}{}}{16d}$$

$$= \frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{8d^2} + \frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d^2} - \frac{5b\cos\left(5a\right.}{}$$

**Mathematica [A]** time = 1.47, size = 213, normalized size = 0.83

$$\frac{2b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+3b\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right)-5b\cos\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5b(c+dx)}{d}\right)-2b\sin\left(a-\frac{bc}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (2*b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 3*b*Cos[3*a - (3*b*c)/d]*C
osIntegral[(3*b*(c + d*x))/d] - 5*b*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*(
c + d*x))/d] - (2*d*Sin[a + b*x])/(c + d*x) - (d*Sin[3*(a + b*x)])/(c + d*x
) + (d*Sin[5*(a + b*x)])/(c + d*x) - 2*b*Sin[a - (b*c)/d]*SinIntegral[b*(c/
d + x)] - 3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 5*b*Sin
[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d^2)
```

**fricas [A]** time = 0.91, size = 347, normalized size = 1.35

$$\frac{10\,(bdx+bc)\sin\left(-\frac{5\,(bc-ad)}{d}\right)\text{Si}\left(\frac{5\,(bdx+bc)}{d}\right)-6\,(bdx+bc)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\text{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-4\,(bdx+bc)\sin\left(-\frac{bc-a}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(10*(b*d*x + b*c)*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d
) - 6*(b*d*x + b*c)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) -
 4*(b*d*x + b*c)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 2*((b*
d*x + b*c)*cos_integral((b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-(b*d
*x + b*c)/d))*cos(-(b*c - a*d)/d) + 3*((b*d*x + b*c)*cos_integral(3*(b*d*x
+ b*c)/d) + (b*d*x + b*c)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a
*d)/d) - 5*((b*d*x + b*c)*cos_integral(5*(b*d*x + b*c)/d) + (b*d*x + b*c)*c
os_integral(-5*(b*d*x + b*c)/d))*cos(-5*(b*c - a*d)/d) + 32*(d*cos(b*x + a)
^4 - d*cos(b*x + a)^2)*sin(b*x + a))/(d^3*x + c*d^2)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.03, size = 365, normalized size = 1.42

$$\frac{-\frac{b^2\left(-\frac{5\sin(5bx+5a)}{((bx+a)d-da+cb)d}+\frac{\frac{25\,\mathrm{Si}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\sin\left(\frac{-5da+5cb}{d}\right)}{d}+\frac{25\,\mathrm{Ci}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\cos\left(\frac{-5da+5cb}{d}\right)}{d}}{d}\right)}{80}+\frac{b^2\left(-\frac{\sin(bx+a)}{((bx+a)d-da+cb)d}+\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-d}{}\right.}{d}\right.}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/b*(-1/80*b^2*(-5*sin(5*b*x+5*a)/((b*x+a)*d-d*a+c*b)/d+5*(5*Si(5*b*x+5*a+5
*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(
-a*d+b*c)/d)/d)/d)+1/8*b^2*(-sin(b*x+a)/((b*x+a)*d-d*a+c*b)/d+(Si(b*x+a+(-a
*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/
d)+1/48*b^2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d+3*(3*Si(3*b*x+3*a+3*(-
a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*
d+b*c)/d)/d)/d))
```

**maxima [C]** time = 0.58, size = 438, normalized size = 1.70

$$b^2\left(-2i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+2i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(-i\,E_2\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+i\,E_2\left(-\frac{3i\,bc+3}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(b^2*(-2*I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*
exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
+ b^2*(-I*exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*ex
p_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^2*(I*exp_integral_e(2, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) -
 I*exp_integral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c
 - a*d)/d) - 2*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d)
- b^2*(exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_int
egral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d)
 + b^2*(exp_integral_e(2, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_in
tegral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d
))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**2, x)
```

# 3.95 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=338

$$\frac{25b^2\sin\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5bc}{d}+5bx\right)}{32d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{32d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{16d^3}-\frac{b^2\cos\left(a-\frac{bc}{d}\right.}{16}$$

```
[Out] -1/16*b*cos(b*x+a)/d^2/(d*x+c)-3/32*b*cos(3*b*x+3*a)/d^2/(d*x+c)+5/32*b*cos
(5*b*x+5*a)/d^2/(d*x+c)-1/16*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3-9/32*b^2*co
s(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^3+25/32*b^2*cos(5*a-5*b*c/d)*Si(5*b*c/d+
5*b*x)/d^3+25/32*b^2*Ci(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^3-9/32*b^2*Ci(3*b
*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3-1/16*b^2*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^3-1/
16*sin(b*x+a)/d/(d*x+c)^2-1/32*sin(3*b*x+3*a)/d/(d*x+c)^2+1/32*sin(5*b*x+5*
a)/d/(d*x+c)^2
```

**Rubi [A]** time = 0.50, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{25b^2\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{32d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosInt}}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] -(b*Cos[a + b*x])/(16*d^2*(c + d*x)) - (3*b*Cos[3*a + 3*b*x])/(32*d^2*(c +
d*x)) + (5*b*Cos[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (25*b^2*CosIntegral[(5*
b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(32*d^3) - (9*b^2*CosIntegral[(3*b*c)
/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(32*d^3) - (b^2*CosIntegral[(b*c)/d + b*x
]*Sin[a - (b*c)/d])/(16*d^3) - Sin[a + b*x]/(16*d*(c + d*x)^2) - Sin[3*a +
3*b*x]/(32*d*(c + d*x)^2) + Sin[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*Cos[
a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Cos[3*a - (3*b*c
)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) + (25*b^2*Cos[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\sin(a+bx)}{8(c+dx)^3}+\frac{\sin(3a+3bx)}{16(c+dx)^3}-\frac{\sin(5a+5bx)}{16(c+dx)^3}\right)dx$$

$$= \frac{1}{16}\int \frac{\sin(3a+3bx)}{(c+dx)^3}\,dx-\frac{1}{16}\int \frac{\sin(5a+5bx)}{(c+dx)^3}\,dx+\frac{1}{8}\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx$$

$$= -\frac{\sin(a+bx)}{16d(c+dx)^2}-\frac{\sin(3a+3bx)}{32d(c+dx)^2}+\frac{\sin(5a+5bx)}{32d(c+dx)^2}+\frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{16d}+\frac{(3b)\int$$

$$= -\frac{b\cos(a+bx)}{16d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{32d^2(c+dx)}+\frac{5b\cos(5a+5bx)}{32d^2(c+dx)}-\frac{\sin(a+bx)}{16d(c+dx)^2}-\frac{\mathrm{si}}{3}$$

$$= -\frac{b\cos(a+bx)}{16d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{32d^2(c+dx)}+\frac{5b\cos(5a+5bx)}{32d^2(c+dx)}-\frac{\sin(a+bx)}{16d(c+dx)^2}-\frac{\mathrm{si}}{3}$$

$$= -\frac{b\cos(a+bx)}{16d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{32d^2(c+dx)}+\frac{5b\cos(5a+5bx)}{32d^2(c+dx)}+\frac{25b^2\mathrm{Ci}\left(\frac{5bc}{d}+5b\right.}{32}$$

**Mathematica [A]** time = 3.90, size = 279, normalized size = 0.83

$$\frac{-2\left(b^2\sin\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+b^2\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\frac{d(b(c+dx)\cos(a+bx)+d\sin(a+bx))}{(c+dx)^2}\right)+25b^2\sin\left(5a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (25*b^2*CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] - 9*b^2*CosInte
gral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] - (d*(3*b*(c + d*x)*Cos[3*(a +
 b*x)] + d*Sin[3*(a + b*x)]))/(c + d*x)^2 + (d*(5*b*(c + d*x)*Cos[5*(a + b*
x)] + d*Sin[5*(a + b*x)]))/(c + d*x)^2 - 2*(b^2*CosIntegral[b*(c/d + x)]*Si
n[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b*x] + d*Sin[a + b*x]))/(c + d*x)^
2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) - 9*b^2*Cos[3*a - (3*b*c
)/d]*SinIntegral[(3*b*(c + d*x))/d] + 25*b^2*Cos[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*(c + d*x))/d])/(32*d^3)
```

**fricas [A]** time = 0.83, size = 585, normalized size = 1.73

$$\frac{160\left(bd^2x+bcd\right)\cos\left(bx+a\right)^5-224\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3+50\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{5\,(bc-ad)}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(160*(b*d^2*x + b*c*d)*cos(b*x + a)^5 - 224*(b*d^2*x + b*c*d)*cos(b*x
+ a)^3 + 50*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-5*(b*c - a*d)/d)*sin
_integral(5*(b*d*x + b*c)/d) - 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 4*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 64*(b
*d^2*x + b*c*d)*cos(b*x + a) + 32*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)
*sin(b*x + a) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*
x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x +
b*c)/d))*sin(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_in
tegral(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d) + 25*((b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2)*cos_integral(5*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*cos_integral(-5*(b*d*x + b*c)/d))*sin(-5*(b*c - a*d)/d))/(d
^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.03, size = 475, normalized size = 1.41

$$-\frac{b^3\left(-\frac{5\sin(5bx+5a)}{2((bx+a)d-da+cb)^2 d}+\frac{-\frac{25\cos(5bx+5a)}{2((bx+a)d-da+cb)d}-\frac{25\left(\frac{5\,\mathrm{Si}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\cos\left(\frac{-5da+5cb}{d}\right)}{d}-\frac{5\,\mathrm{Ci}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\sin\left(\frac{-5da+5cb}{d}\right)}{d}\right)}{2d}}{d}\right)}{80}+\frac{b^3\left(-\frac{\sin(bx+a)}{2((bx+a)d-da+cb)^2 d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] 1/b*(-1/80*b^3*(-5/2*sin(5*b*x+5*a)/((b*x+a)*d-d*a+c*b)^2/d+5/2*(-5*cos(5*b
*x+5*a)/((b*x+a)*d-d*a+c*b)/d-5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d
+b*c)/d)/d-5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d)/d)+1/8*
b^3*(-1/2*sin(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-d*
a+c*b)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)
*sin((-a*d+b*c)/d)/d)/d)/d)+1/48*b^3*(-3/2*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*
b)^2/d+3/2*(-3*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d-3*(3*Si(3*b*x+3*a+3*(-a
*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d
+b*c)/d)/d)/d)/d))
```

**maxima [C]** time = 0.78, size = 473, normalized size = 1.40

$$\frac{b^3\left(-2i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+2i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(-i\,E_3\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+i\,E_3\left(-\frac{3i\,bc+3}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(b^3*(-2*I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*
exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
+ b^3*(-I*exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*ex
```

```
p_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^3*(I*exp_integral_e(3, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) -
 I*exp_integral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c
 - a*d)/d) - 2*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d)
- b^3*(exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_int
egral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d)
 + b^3*(exp_integral_e(3, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_in
tegral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d
))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d
^3)*(b*x + a))*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**3, x)
```

# 3.96 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

**Optimal**. Leaf size=413

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{48d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5bc}{d}+5bx\right)}{96d^4}+\frac{b^3\sin\left(a\cdots\right.}{}$$

```
[Out] 125/96*b^3*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d^4-9/32*b^3*Ci(3*b*c/d+3*b*x
)*cos(3*a-3*b*c/d)/d^4-1/48*b^3*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^4-1/48*b*cos(b
*x+a)/d^2/(d*x+c)^2-1/32*b*cos(3*b*x+3*a)/d^2/(d*x+c)^2+5/96*b*cos(5*b*x+5*
a)/d^2/(d*x+c)^2-125/96*b^3*Si(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^4+9/32*b^3
*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/48*b^3*Si(b*c/d+b*x)*sin(a-b*c/d)
/d^4-1/24*sin(b*x+a)/d/(d*x+c)^3+1/48*b^2*sin(b*x+a)/d^3/(d*x+c)-1/48*sin(3
*b*x+3*a)/d/(d*x+c)^3+3/32*b^2*sin(3*b*x+3*a)/d^3/(d*x+c)+1/48*sin(5*b*x+5*
a)/d/(d*x+c)^3-25/96*b^2*sin(5*b*x+5*a)/d^3/(d*x+c)
```

**Rubi [A]** time = 0.59, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{48d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\text{CosI}\cdots}{96d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] -(b*Cos[a + b*x])/(48*d^2*(c + d*x)^2) - (b*Cos[3*a + 3*b*x])/(32*d^2*(c +
d*x)^2) + (5*b*Cos[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) - (b^3*Cos[a - (b*c)/
d]*CosIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*Cos[3*a - (3*b*c)/d]*CosIn
tegral[(3*b*c)/d + 3*b*x])/(32*d^4) + (125*b^3*Cos[5*a - (5*b*c)/d]*CosInte
gral[(5*b*c)/d + 5*b*x])/(96*d^4) - Sin[a + b*x]/(24*d*(c + d*x)^3) + (b^2*
Sin[a + b*x])/(48*d^3*(c + d*x)) - Sin[3*a + 3*b*x]/(48*d*(c + d*x)^3) + (3
*b^2*Sin[3*a + 3*b*x])/(32*d^3*(c + d*x)) + Sin[5*a + 5*b*x]/(48*d*(c + d*x
)^3) - (25*b^2*Sin[5*a + 5*b*x])/(96*d^3*(c + d*x)) + (b^3*Sin[a - (b*c)/d]
*SinIntegral[(b*c)/d + b*x])/(48*d^4) + (9*b^3*Sin[3*a - (3*b*c)/d]*SinInte
gral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*Sin[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*c)/d + 5*b*x])/(96*d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\sin(a+bx)}{8(c+dx)^4}+\frac{\sin(3a+3bx)}{16(c+dx)^4}-\frac{\sin(5a+5bx)}{16(c+dx)^4}\right)dx$$

$$= \frac{1}{16}\int \frac{\sin(3a+3bx)}{(c+dx)^4}\,dx - \frac{1}{16}\int \frac{\sin(5a+5bx)}{(c+dx)^4}\,dx + \frac{1}{8}\int \frac{\sin(a+bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\sin(a+bx)}{24d(c+dx)^3} - \frac{\sin(3a+3bx)}{48d(c+dx)^3} + \frac{\sin(5a+5bx)}{48d(c+dx)^3} + \frac{b\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx}{24d} + \frac{b\int \frac{\text{cc}}{}}{}$$

$$= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{\sin(a+bx)}{24d(c+dx)^3} - \frac{\text{si}}{4}$$

$$= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{\sin(a+bx)}{24d(c+dx)^3} + \frac{b^2}{4}$$

$$= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{\sin(a+bx)}{24d(c+dx)^3} + \frac{b^2}{4}$$

$$= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Ci}}{48d^4}$$

**Mathematica [A]** time = 2.90, size = 457, normalized size = 1.11

$$\frac{-27b^3(c+dx)^3\left(\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right)-\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)\right)+125b^3(c+dx)^3\left(\cos\left(5a-\frac{5bc}{d}\right)\text{Ci}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] (-(d*Cos[3*b*x]*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*Si
n[3*a])) + d*Cos[5*b*x]*(5*b*d*(c + d*x)*Cos[5*a] - (-2*d^2 + 25*b^2*(c + d
*x)^2)*Sin[5*a]) + d*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*
x)*Sin[3*a])*Sin[3*b*x] - d*((-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*a] + 5*b*d
*(c + d*x)*Sin[5*a])*Sin[5*b*x] - 2*(d*Cos[b*x]*(b*d*(c + d*x)*Cos[a] - (-2
*d^2 + b^2*(c + d*x)^2)*Sin[a]) - d*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*
d*(c + d*x)*Sin[a])*Sin[b*x] + b^3*(c + d*x)^3*(Cos[a - (b*c)/d]*CosIntegra
l[b*(c/d + x)] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)])) - 27*b^3*(c +
d*x)^3*(Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Sin[3*a - (3*
b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]) + 125*b^3*(c + d*x)^3*(Cos[5*a - (5
*b*c)/d]*CosIntegral[(5*b*(c + d*x))/d] - Sin[5*a - (5*b*c)/d]*SinIntegral[
(5*b*(c + d*x))/d]))/(96*d^4*(c + d*x)^3)
```

**fricas [B]** time = 0.62, size = 824, normalized size = 2.00

$$160\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^5-224\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^3-250\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/192*(160*(b*d^3*x + b*c*d^2)*cos(b*x + a)^5 - 224*(b*d^3*x + b*c*d^2)*cos
(b*x + a)^3 - 250*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)
*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 54*(b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-3*(b*c - a*d)/d)*sin_integr
al(3*(b*d*x + b*c)/d) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
b^3*c^3)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 64*(b*d^3*x +
b*c*d^2)*cos(b*x + a) - 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
 b^3*c^3)*cos_integral((b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d
) - 27*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integ
ral(3*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b
^3*c^3)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) + 125*((b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(5*(b*d*x
 + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_
integral(-5*(b*d*x + b*c)/d))*cos(-5*(b*c - a*d)/d) - 32*(2*b^2*d^3*x^2 + 4
*b^2*c*d^2*x + 2*b^2*c^2*d + (25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*
d - 2*d^3)*cos(b*x + a)^4 - (21*b^2*d^3*x^2 + 42*b^2*c*d^2*x + 21*b^2*c^2*d
 - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*
x + c^3*d^4)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.03, size = 580, normalized size = 1.40

$$-\frac{b^4\left(-\frac{5\sin(5bx+5a)}{3((bx+a)d-da+cb)^3d}+\frac{-\frac{25\cos(5bx+5a)}{6((bx+a)d-da+cb)^2d}-\frac{25\left(-\frac{5\sin(5bx+5a)}{((bx+a)d-da+cb)d}+\frac{\frac{25\,\mathrm{Si}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\sin\left(\frac{-5da+5cb}{d}\right)}{d}+\frac{25\,\mathrm{Ci}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\cos\left(\frac{-5da+5cb}{d}\right)}{d}}{d}\right)}{6d}}{d}\right)}{80}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x)
```

```
[Out] 1/b*(-1/80*b^4*(-5/3*sin(5*b*x+5*a)/((b*x+a)*d-d*a+c*b)^3/d+5/3*(-5/2*cos(5
*b*x+5*a)/((b*x+a)*d-d*a+c*b)^2/d-5/2*(-5*sin(5*b*x+5*a)/((b*x+a)*d-d*a+c*b
)/d+5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+
5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)/d)/d)/d)+1/8*b^4*(-1/3*sin(b*x+a)/((
b*x+a)*d-d*a+c*b)^3/d+1/3*(-1/2*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d-1/2*(-si
```

```
n(b*x+a)/((b*x+a)*d-d*a+c*b)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+
Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)/d)+1/48*b^4*(-sin(3*b*x+3
*a)/((b*x+a)*d-d*a+c*b)^3/d+(-3/2*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)^2/d-3/
2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d+3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d
)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/
d)/d)/d)/d))
```

**maxima [C]** time = 1.10, size = 523, normalized size = 1.27

$$b^4\left(-2i\,E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+2i\,E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^4\left(-i\,E_4\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+i\,E_4\left(-\frac{3i}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/32*(b^4*(-2*I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*
exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
+ b^4*(-I*exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*ex
p_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^4*(I*exp_integral_e(4, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) -
 I*exp_integral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c
 - a*d)/d) - 2*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d)
- b^4*(exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_int
egral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d)
 + b^4*(exp_integral_e(4, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_in
tegral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d
))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^
4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^
4)*(b*x + a))*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**4, x)
```

## 3.97 $\int (c+dx)^m \cos(a+bx) \cot(a+bx) \, dx$

**Optimal**. Leaf size=144

$$\text{Int}\left(\csc(a+bx)(c+dx)^m,x\right)+\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma}{2b}$$

```
[Out] 1/2*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d
)^m)+1/2*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+c
)/d)^m)+Unintegrable((d*x+c)^m*csc(b*x+a),x)
```

**Rubi [A]** time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cos(a+bx) \cot(a+bx) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*(
((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(2
*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x)^m*Csc[
a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx &= \int (c+dx)^m \csc(a+bx)\,dx - \int (c+dx)^m \sin(a+bx)\,dx\\ &= -\left(\frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^m\,dx\right)+\frac{1}{2}i\int e^{i(a+bx)}(c+dx)^m\,dx+\int (c+dx)^m\\ &= \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m}{}\end{aligned}$$

**Mathematica [A]** time = 6.38, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos(a+bx) \cot(a+bx) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x], x]
```

**fricas [A]** time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \cos(bx+a)\cot(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

**maple** [A] time = 0.24, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)
[Out] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\cot(a+b\,x)\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^m,x)
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^m, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*cot(b*x+a),x)
[Out] Integral((c + d*x)**m*cos(a + b*x)*cot(a + b*x), x)
```

## 3.98 $\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx$

**Optimal**. Leaf size=333

$$\frac{24d^4\text{Li}_5\left(-e^{i(a+bx)}\right)}{b^5}-\frac{24d^4\text{Li}_5\left(e^{i(a+bx)}\right)}{b^5}+\frac{24d^4\cos(a+bx)}{b^5}-\frac{24id^3(c+dx)\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{24id^3(c+dx)\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}$$

```
[Out] -2*(d*x+c)^4*arctanh(exp(I*(b*x+a)))/b+24*d^4*cos(b*x+a)/b^5-12*d^2*(d*x+c)
^2*cos(b*x+a)/b^3+(d*x+c)^4*cos(b*x+a)/b+4*I*d*(d*x+c)^3*polylog(2,-exp(I*(
b*x+a)))/b^2-4*I*d*(d*x+c)^3*polylog(2,exp(I*(b*x+a)))/b^2-12*d^2*(d*x+c)^2
*polylog(3,-exp(I*(b*x+a)))/b^3+12*d^2*(d*x+c)^2*polylog(3,exp(I*(b*x+a)))/
b^3-24*I*d^3*(d*x+c)*polylog(4,-exp(I*(b*x+a)))/b^4+24*I*d^3*(d*x+c)*polylo
g(4,exp(I*(b*x+a)))/b^4+24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*polylo
g(5,exp(I*(b*x+a)))/b^5+24*d^3*(d*x+c)*sin(b*x+a)/b^4-4*d*(d*x+c)^3*sin(b*x
+a)/b^2
```

**Rubi [A]** time = 0.28, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4408, 3296, 2638, 4183, 2531, 6609, 2282, 6589}

$$-\frac{24id^3(c+dx)\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}+\frac{24id^3(c+dx)\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4}-\frac{12d^2(c+dx)^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b + (24*d^4*Cos[a + b*x])/b^5 - (
12*d^2*(c + d*x)^2*Cos[a + b*x])/b^3 + ((c + d*x)^4*Cos[a + b*x])/b + ((4*I
)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((4*I)*d*(c + d*x)^3*Po
lyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a +
b*x))])/b^3 + (12*d^2*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((24*I
)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((24*I)*d^3*(c + d*x)*P
olyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^4*PolyLog[5, -E^(I*(a + b*x))])/b^5
 - (24*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5 + (24*d^3*(c + d*x)*Sin[a + b*x
])/b^4 - (4*d*(c + d*x)^3*Sin[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^4\csc(a+bx)\,dx - \int (c+dx)^4\sin(a+bx)\,dx$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b} - \frac{(4d)\int (c+dx}{}$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b} + \frac{4id(c+dx)^3\mathrm{I}}{b}$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3} + \frac{(c+dx)^4}{}$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3} + \frac{(c+dx)^4}{}$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{24d^4\cos(a+bx)}{b^5} - \frac{12d^2(c+dx)^2\cos}{b^3}$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{24d^4\cos(a+bx)}{b^5} - \frac{12d^2(c+dx)^2\cos}{b^3}$$

**Mathematica [B]** time = 1.32, size = 837, normalized size = 2.51

$$\frac{c^4\cos(a+bx)b^4 + d^4x^4\cos(a+bx)b^4 + 4cd^3x^3\cos(a+bx)b^4 + 6c^2d^2x^2\cos(a+bx)b^4 + 4c^3dx\cos(a+bx)b^4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (b^4*c^4*Cos[a + b*x] - 12*b^2*c^2*d^2*Cos[a + b*x] + 24*d^4*Cos[a + b*x] +
 4*b^4*c^3*d*x*Cos[a + b*x] - 24*b^2*c*d^3*x*Cos[a + b*x] + 6*b^4*c^2*d^2*x
^2*Cos[a + b*x] - 12*b^2*d^4*x^2*Cos[a + b*x] + 4*b^4*c*d^3*x^3*Cos[a + b*x
] + b^4*d^4*x^4*Cos[a + b*x] + b^4*c^4*Log[1 - E^(I*(a + b*x))] + 4*b^4*c^3
*d*x*Log[1 - E^(I*(a + b*x))] + 6*b^4*c^2*d^2*x^2*Log[1 - E^(I*(a + b*x))]
+ 4*b^4*c*d^3*x^3*Log[1 - E^(I*(a + b*x))] + b^4*d^4*x^4*Log[1 - E^(I*(a +
b*x))] - b^4*c^4*Log[1 + E^(I*(a + b*x))] - 4*b^4*c^3*d*x*Log[1 + E^(I*(a +
 b*x))] - 6*b^4*c^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] - 4*b^4*c*d^3*x^3*Log[
1 + E^(I*(a + b*x))] - b^4*d^4*x^4*Log[1 + E^(I*(a + b*x))] + (4*I)*b^3*d*(
c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))] - (4*I)*b^3*d*(c + d*x)^3*PolyLog[2
, E^(I*(a + b*x))] - 12*b^2*c^2*d^2*PolyLog[3, -E^(I*(a + b*x))] - 24*b^2*c
*d^3*x*PolyLog[3, -E^(I*(a + b*x))] - 12*b^2*d^4*x^2*PolyLog[3, -E^(I*(a +
b*x))] + 12*b^2*c^2*d^2*PolyLog[3, E^(I*(a + b*x))] + 24*b^2*c*d^3*x*PolyLo
g[3, E^(I*(a + b*x))] + 12*b^2*d^4*x^2*PolyLog[3, E^(I*(a + b*x))] - (24*I)
*b*c*d^3*PolyLog[4, -E^(I*(a + b*x))] - (24*I)*b*d^4*x*PolyLog[4, -E^(I*(a
+ b*x))] + (24*I)*b*c*d^3*PolyLog[4, E^(I*(a + b*x))] + (24*I)*b*d^4*x*Poly
Log[4, E^(I*(a + b*x))] + 24*d^4*PolyLog[5, -E^(I*(a + b*x))] - 24*d^4*Poly
Log[5, E^(I*(a + b*x))] - 4*b^3*c^3*d*Sin[a + b*x] + 24*b*c*d^3*Sin[a + b*x
] - 12*b^3*c^2*d^2*x*Sin[a + b*x] + 24*b*d^4*x*Sin[a + b*x] - 12*b^3*c*d^3*
x^2*Sin[a + b*x] - 4*b^3*d^4*x^3*Sin[a + b*x])/b^5
```

**fricas [C]** time = 0.75, size = 1367, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
cos(b*x + a) - I*sin(b*x + a)) - 24*d^4*polylog(5, -cos(b*x + a) + I*sin(b*
x + a)) - 24*d^4*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) - 2*(b^4*d^4*x^
4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 -
2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a) - (-4*I*b^3*d^
4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(cos(
b*x + a) + I*sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b
^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (-4*I*
b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilo
g(-cos(b*x + a) + I*sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 +
 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a))
+ (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*
c^4)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^
3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*sin(b
*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(b
*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^
4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b
*c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 +
4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2
*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a)
+ 1) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, cos(b*x + a) + I*sin(b*x +
a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, cos(b*x + a) - I*sin(b*x +
a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) + I*sin(b*x +
a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) - I*sin(b*x +
 a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(b*x +
a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polyl
```

```
og(3, cos(b*x + a) - I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^
2*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2
*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 8*
(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2 - 2
*b*d^4)*x)*sin(b*x + a))/b^5
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \cos(bx+a)\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*cot(b*x + a), x)
```

**maple [B]** time = 0.23, size = 1295, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] -12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,exp(I*(
b*x+a)))-1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))+12/b^3*d^4*polylog(3,exp(I*(b*x
+a)))*x^2-12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+24*d^4*polylog(5,-exp(I
*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5+1/2*(d^4*x^4*b^4+4*b^4*
c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x-4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^
4*x^2-12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x-12*I*b^3*c^2*d^2*x-12*c^2*d^2*b^2-4
*I*b^3*c^3*d+24*I*b*d^4*x+24*d^4+24*I*b*c*d^3)/b^5*exp(-I*(b*x+a))+1/2*(d^4
*x^4*b^4+4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x+4*I*b^3*d^4*x^3+b^
4*c^4-12*b^2*d^4*x^2+12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x+12*I*b^3*c^2*d^2*x-1
2*c^2*d^2*b^2+4*I*b^3*c^3*d-24*I*b*d^4*x+24*d^4-24*I*b*c*d^3)/b^5*exp(I*(b*
x+a))-2/b*c^4*arctanh(exp(I*(b*x+a)))-4/b^2*c^3*d*ln(exp(I*(b*x+a))+1)*a-24
*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x-24*I/b^4*c*d^3*polylog(4,-exp(I*(b*
x+a)))+4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+4*I/b^2*d^4*polylog(2,-exp(
I*(b*x+a)))*x^3+8/b^4*c*d^3*a^3*arctanh(exp(I*(b*x+a)))-12/b^3*c^2*d^2*a^2*
arctanh(exp(I*(b*x+a)))+8/b^2*c^3*d*a*arctanh(exp(I*(b*x+a)))+6/b^3*c^2*d^2
*ln(exp(I*(b*x+a))+1)*a^2+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))+1)-2/b^5*d^4*a^4*
arctanh(exp(I*(b*x+a)))-4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+4/b*c^3*d*ln(1-exp
(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a-6/b*c^2*d^2*ln(exp(I*(b*x
+a))+1)*x^2-24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x-6/b^3*c^2*d^2*a^2*ln(
1-exp(I*(b*x+a)))+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2+24/b^3*c*d^3*polylog
(3,exp(I*(b*x+a)))*x+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))-4*I/b^2*d^4*p
olylog(2,exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b
^2*c^3*d*polylog(2,exp(I*(b*x+a)))+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4-1/b*d^4
*ln(exp(I*(b*x+a))+1)*x^4-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I
/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3
+4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3-4/
b^4*c*d^3*ln(exp(I*(b*x+a))+1)*a^3+12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a))
)*x^2+12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x
```

**maxima [B]** time = 0.64, size = 1538, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^4*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 4*a*c^3*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 6*a^2*c^2*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x +
a) - 1))/b^2 - 4*a^3*c*d^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(co
s(b*x + a) - 1))/b^3 + a^4*d^4*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + lo
g(cos(b*x + a) - 1))/b^4 + (48*d^4*polylog(5, -e^(I*b*x + I*a)) - 48*d^4*po
lylog(5, e^(I*b*x + I*a)) - (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - 8*I*a*d^4
)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a
)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + 24*I*a^2*b*c*d^3 - 8*I*a^3*d^4)
*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*I*(b*x + a)^4*d^4
+ (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^
3 + 12*I*a^2*d^4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + 24*I*
a^2*b*c*d^3 - 8*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) +
 1) + 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*(a^2 - 2)*d^4
 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 -
2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3
- (a^3 - 6*a)*d^4)*(b*x + a))*cos(b*x + a) - (-8*I*b^3*c^3*d + 24*I*a*b^2*c
^2*d^2 - 24*I*a^2*b*c*d^3 - 8*I*(b*x + a)^3*d^4 + 8*I*a^3*d^4 + (-24*I*b*c*
d^3 + 24*I*a*d^4)*(b*x + a)^2 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*
a^2*d^4)*(b*x + a))*dilog(-e^(I*b*x + I*a)) - (8*I*b^3*c^3*d - 24*I*a*b^2*c
^2*d^2 + 24*I*a^2*b*c*d^3 + 8*I*(b*x + a)^3*d^4 - 8*I*a^3*d^4 + (24*I*b*c*d
^3 - 24*I*a*d^4)*(b*x + a)^2 + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^
2*d^4)*(b*x + a))*dilog(e^(I*b*x + I*a)) - ((b*x + a)^4*d^4 + 4*(b*c*d^3 -
a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 +
4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(co
s(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + ((b*x + a)^4*d^4 + 4*
(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*
x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x +
 a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (48*I*b*c*
d^3 + 48*I*(b*x + a)*d^4 - 48*I*a*d^4)*polylog(4, -e^(I*b*x + I*a)) - (-48*
I*b*c*d^3 - 48*I*(b*x + a)*d^4 + 48*I*a*d^4)*polylog(4, e^(I*b*x + I*a)) -
24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*
d^4)*(b*x + a))*polylog(3, -e^(I*b*x + I*a)) + 24*(b^2*c^2*d^2 - 2*a*b*c*d^
3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, e
^(I*b*x + I*a)) - 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2
 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*
c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*sin(b*x + a))/b^4)/b
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,\mathrm{cot}\left(a+b\,x\right)\,{\left(c+d\,x\right)}^4 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^4,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^4, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cos(a+bx) \cot(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*cos(a + b*x)*cot(a + b*x), x)
```

## 3.99 $\int (c+dx)^3 \cos(a+bx)\cot(a+bx)\,dx$

**Optimal**. Leaf size=254

$$-\frac{6id^3\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6id^3\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\sin(a+bx)}{b^4}-\frac{6d^2(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-$$

```
[Out] -2*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b-6*d^2*(d*x+c)*cos(b*x+a)/b^3+(d*x+c)
^3*cos(b*x+a)/b+3*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-3*I*d*(d*x+c
)^2*polylog(2,exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/
b^3+6*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-6*I*d^3*polylog(4,-exp(I*(b
*x+a)))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+6*d^3*sin(b*x+a)/b^4-3*d*
(d*x+c)^2*sin(b*x+a)/b^2
```

**Rubi** [A] time = 0.20, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4408, 3296, 2637, 4183, 2531, 6609, 2282, 6589}

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (6*d^2*(c + d*x)*Cos[a + b*x]
)/b^3 + ((c + d*x)^3*Cos[a + b*x])/b + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^(
I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2
- (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*(c + d*x)*Pol
yLog[3, E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^
4 + ((6*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (6*d^3*Sin[a + b*x])/b^4
- (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^3\csc(a+bx)\,dx - \int (c+dx)^3\sin(a+bx)\,dx$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^3\cos(a+bx)}{b} - \frac{(3d)\int (c+dx)^2}{b}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^3\cos(a+bx)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2}{b^2}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\cos(}{b}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\cos(}{b}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\cos(}{b}$$

**Mathematica** [A] time = 0.94, size = 330, normalized size = 1.30

$$\frac{-2b^3(c+dx)^3\tanh^{-1}(\cos(a+bx)+i\sin(a+bx)) + 3id\left(b^2(c+dx)^2\mathrm{Li}_2(-\cos(a+bx)-i\sin(a+bx)) + 2ibd(c+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] + (3*I)*d*(b^2*(
c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)
```

```
*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*PolyLog[4, -Cos[a + b*x
] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x] + I
*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*
x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]) + Cos[b*x]*(b*(c + d
*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a] - 3*d*(-2*d^2 + b^2*(c + d*x)^2)*Sin[
a]) - (3*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*(c + d*x)*(-6*d^2 + b^2*(c
 + d*x)^2)*Sin[a])*Sin[b*x])/b^4
```

**fricas [C]** time = 0.79, size = 921, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4,
 cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(
b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 2*(b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*c
os(b*x + a) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(co
s(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2
*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*
c*d^2*x - 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d
^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x +
 a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*
x + a) + I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d
*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
+ 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b
*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*
sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)
+ 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^
3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c
*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*po
lylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x +
 b^2*c^2*d - 2*d^3)*sin(b*x + a))/b^4
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cos(bx+a)\cot(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*cot(b*x + a), x)
```

**maple [B]** time = 0.15, size = 847, normalized size = 3.33

$$-\frac{6icd^2 \operatorname{polylog}\left(2, e^{i(bx+a)}\right) x}{b^2}+\frac{6icd^2 \operatorname{polylog}\left(2, -e^{i(bx+a)}\right) x}{b^2}+\frac{d^3 \ln\left(1-e^{i(bx+a)}\right) x^3}{b}+\frac{d^3 \ln\left(1-e^{i(bx+a)}\right) a^3}{b^4}-\frac{d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] -6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a))
)/b^4-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,exp(I*(b
```

```
*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-6/b^3*d^3*polylog(3,-exp(I*(b
*x+a)))*x-6*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4+6*I/b^2*c*d^2*polylog(2,-e
xp(I*(b*x+a)))*x+1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I
*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4
*exp(I*(b*x+a))+1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*
b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*
exp(-I*(b*x+a))-2/b*c^3*arctanh(exp(I*(b*x+a)))+2/b^4*d^3*a^3*arctanh(exp(I
*(b*x+a)))-3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a+3/b^3*c*d^2*a^2*ln(exp(I*(b*x
+a))+1)-1/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^3-6/b^3*c*d^2*a^2*arctanh(exp(I*(b
*x+a)))+6/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))+3*I/b^2*d^3*polylog(2,-exp(I*
(b*x+a)))*x^2+3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylo
g(2,exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-3/b*c^2*d*ln(
exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I
*(b*x+a)))*a-3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+3/b*c*d^2*ln(1-exp(I*(b*x
+a)))*x^2-3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/b*d^3*ln(1-exp(I*(b*x+a)))*x
^3+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3
```

**maxima** [B] time = 0.52, size = 919, normalized size = 3.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 3*a*c^2*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 3*a^2*c*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a)
 - 1))/b^2 - a^3*d^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x
+ a) - 1))/b^3 - (12*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 12*I*d^3*polylog(
4, e^(I*b*x + I*a)) + (2*I*(b*x + a)^3*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x
 + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*a^2*d^3)*(b*x + a))*arctan2
(sin(b*x + a), cos(b*x + a) + 1) + (2*I*(b*x + a)^3*d^3 + (6*I*b*c*d^2 - 6*
I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*a^2*d^3)*(b*x
+ a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^3*d^3 - 6*b*c
*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d
^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(b*x + a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*
d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x
 + a))*dilog(-e^(I*b*x + I*a)) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x
 + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*dilog(e^
(I*b*x + I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 + 2*cos(b*x + a) + 1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x +
a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2
+ sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^
3)*polylog(3, -e^(I*b*x + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*poly
log(3, e^(I*b*x + I*a)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a
^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))/b^3)/b
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\cot(a+b\,x)\,{(c+d\,x)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^3,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^3, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cos(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)*cot(a + b*x), x)
```

# 3.100 $\int (c+dx)^2 \cos(a+bx)\cot(a+bx)\,dx$

**Optimal**. Leaf size=171

$$-\frac{2d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{2d^2\cos(a+bx)}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{2d(c}{}$$

```
[Out] -2*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-2*d^2*cos(b*x+a)/b^3+(d*x+c)^2*cos(b
*x+a)/b+2*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(
2,exp(I*(b*x+a)))/b^2-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,
exp(I*(b*x+a)))/b^3-2*d*(d*x+c)*sin(b*x+a)/b^2
```

**Rubi [A]** time = 0.14, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {4408, 3296, 2638, 4183, 2531, 2282, 6589}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{PolyLog}\left(3,\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (2*d^2*Cos[a + b*x])/b^3 + ((
c + d*x)^2*Cos[a + b*x])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))
])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (2*d^2*PolyL
og[3, -E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 - (2
*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^2\csc(a+bx)\,dx - \int (c+dx)^2\sin(a+bx)\,dx$$

$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^2\cos(a+bx)}{b} - \frac{(2d)\int(c+dx}{}$$

$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{2id(c+dx)\mathrm{L}}{b}$$

$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a+}{b}$$

$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a+}{b}$$

**Mathematica [A]** time = 0.85, size = 221, normalized size = 1.29

$$\cos(bx)\left(\cos(a)\left(b^2(c+dx)^2-2d^2\right)-2bd\sin(a)(c+dx)\right)-\sin(bx)\left(\sin(a)\left(b^2(c+dx)^2-2d^2\right)+2bd\cos(a)(c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))] - b^2*(c + d*x)^2*Log[1 + E^(I*(a
 + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (2*I)*b*d*(c
 + d*x)*PolyLog[2, E^(I*(a + b*x))] - 2*d^2*PolyLog[3, -E^(I*(a + b*x))] +
2*d^2*PolyLog[3, E^(I*(a + b*x))] + Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Co
s[a] - 2*b*d*(c + d*x)*Sin[a]) - (2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c
 + d*x)^2)*Sin[a])*Sin[b*x])/b^3
```

**fricas [C]** time = 1.54, size = 558, normalized size = 3.26

$$2\,d^2\mathrm{polylog}\left(3,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+2\,d^2\mathrm{polylog}\left(3,\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)-2\,d^2\mathrm{polylog}\left(3,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a)) - 2*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 2*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) + (-2*I*b*d^2*x - 2*I*b*c*d)*dil
og(cos(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(cos(b*x
 + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-cos(b*x + a) +
I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x
 + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b*x
 + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I*sin
(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a) + 1/
2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x
+ a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d -
 a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*
d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(b*d
^2*x + b*c*d)*sin(b*x + a))/b^3
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \cos(bx+a)\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*cot(b*x + a), x)
```

**maple [B]** time = 0.13, size = 479, normalized size = 2.80

$$\frac{\left(d^2x^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)e^{i(bx+a)}}{2b^3}+\frac{\left(d^2x^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2\right)e^{-i(b}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] 1/2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I
*(b*x+a))+1/2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)
/b^3*exp(-I*(b*x+a))-2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*ln(exp(I*(b*x
+a))+1)*a+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-2
/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2
*d^2*polylog(3,exp(I*(b*x+a)))/b^3-2/b*c^2*arctanh(exp(I*(b*x+a)))-1/b*d^2*
ln(exp(I*(b*x+a))+1)*x^2+1/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2+2*I/b^2*d^2*pol
ylog(2,-exp(I*(b*x+a)))*x+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-e
xp(I*(b*x+a)))*a^2-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x+4/b^2*c*d*a*arct
anh(exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+2*I/b^2*c*d*polyl
og(2,-exp(I*(b*x+a)))
```

**maxima [B]** time = 0.47, size = 507, normalized size = 2.96

$$c^2\Big(2\cos(bx+a)-\log(\cos(bx+a)+1)+\log(\cos(bx+a)-1)\Big)-\frac{2\,acd\big(2\cos(bx+a)-\log(\cos(bx+a)+1)+\log(\cos(bx+a)-1)\big)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 2*a*c*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
 + a^2*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b^2 - (4*d^2*polylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x + I*
a)) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*arctan2(sin
```

```
(b*x + a), cos(b*x + a) + 1) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^
2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(b*x + a) + (-4*I*b*c*d - 4*I*(b
*x + a)*d^2 + 4*I*a*d^2)*dilog(-e^(I*b*x + I*a)) + (4*I*b*c*d + 4*I*(b*x +
a)*d^2 - 4*I*a*d^2)*dilog(e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d -
a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
 - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin
(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(b
*x + a))/b^2)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\cot(a+b\,x)\,{(c+d\,x)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^2,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \cos(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*cot(a + b*x), x)
```

# 3.101 $\int (c + dx) \cos(a + bx) \cot(a + bx) \, dx$

**Optimal**. Leaf size=94

$$\frac{id\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}-\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

```
[Out] -2*(d*x+c)*arctanh(exp(I*(b*x+a)))/b+(d*x+c)*cos(b*x+a)/b+I*d*polylog(2,-ex
p(I*(b*x+a)))/b^2-I*d*polylog(2,exp(I*(b*x+a)))/b^2-d*sin(b*x+a)/b^2
```

**Rubi [A]** time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4408, 3296, 2637, 4183, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}-\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)*Cos[a + b*x])/b + (I
*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^
2 - (d*Sin[a + b*x])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
```

```
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)\csc(a+bx)\,dx - \int (c+dx)\sin(a+bx)\,dx$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\cos(a+bx)}{b} - \frac{d\int \cos(a+bx)}{b}$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\cos(a+bx)}{b} - \frac{d\sin(a+bx)}{b^2} +$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\cos(a+bx)}{b} + \frac{id\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}$$

**Mathematica [A]** time = 0.17, size = 176, normalized size = 1.87

$$\frac{d\left(i\left(\text{Li}_2\left(-e^{i(a+bx)}\right) - \text{Li}_2\left(e^{i(a+bx)}\right)\right) + (a+bx)\left(\log\left(1-e^{i(a+bx)}\right) - \log\left(1+e^{i(a+bx)}\right)\right) - a\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (c*Cos[a + b*x])/b - (c*Log[Cos[(a + b*x)/2]])/b + (c*Log[Sin[(a + b*x)/2]]
)/b + (d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) -
 a*Log[Tan[(a + b*x)/2]] + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(
I*(a + b*x))])))/b^2 + (d*Cos[b*x]*(b*x*Cos[a] - Sin[a]))/b^2 - (d*(Cos[a]
+ b*x*Sin[a])*Sin[b*x])/b^2
```

**fricas [B]** time = 0.65, size = 277, normalized size = 2.95

$$\frac{2\,(bdx+bc)\cos\,(bx+a) - i\,d\text{Li}_2\,(\cos\,(bx+a) + i\,\sin\,(bx+a)) + i\,d\text{Li}_2\,(\cos\,(bx+a) - i\,\sin\,(bx+a)) - i\,d\text{Li}_2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b*d*x + b*c)*cos(b*x + a) - I*d*dilog(cos(b*x + a) + I*sin(b*x + a)
) + I*d*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*dilog(-cos(b*x + a) + I*
sin(b*x + a)) + I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b*d*x + b*c)*l
og(cos(b*x + a) + I*sin(b*x + a) + 1) - (b*d*x + b*c)*log(cos(b*x + a) - I*
sin(b*x + a) + 1) + (b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
+ 1/2) + (b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b
*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b*d*x + a*d)*log(-co
s(b*x + a) - I*sin(b*x + a) + 1) - 2*d*sin(b*x + a))/b^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\cos(bx+a)\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cos(b*x + a)*cot(b*x + a), x)
```

**maple [B]** time = 0.08, size = 199, normalized size = 2.12

$$\frac{d\cos(bx+a)x}{b}-\frac{d\sin(bx+a)}{b^2}+\frac{c\cos(bx+a)}{b}+\frac{d\ln\left(1-e^{i(bx+a)}\right)x}{b}-\frac{d\ln\left(e^{i(bx+a)}+1\right)x}{b}+\frac{id\,\mathrm{dilog}\left(e^{i(bx+a)}+1\right)}{b^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] 1/b*d*cos(b*x+a)*x-d*sin(b*x+a)/b^2+1/b*c*cos(b*x+a)+1/b*d*ln(1-exp(I*(b*x+
a)))*x-1/b*d*ln(exp(I*(b*x+a))+1)*x+I/b^2*d*dilog(exp(I*(b*x+a))+1)-I/b^2*d
*dilog(1-exp(I*(b*x+a)))+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-1/b^2*d*ln(exp(I*(b
*x+a))+1)*a-1/b^2*d*a*ln(csc(b*x+a)-cot(b*x+a))+1/b*c*ln(csc(b*x+a)-cot(b*x
+a))
```

**maxima [B]** time = 0.45, size = 199, normalized size = 2.12

$$-\frac{2i\,bdx\arctan\left(\sin(bx+a),-\cos(bx+a)+1\right)-2i\,bc\arctan\left(\sin(bx+a),\cos(bx+a)-1\right)+(2i\,bdx+2i\,bc)\,\mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2(
sin(b*x + a), cos(b*x + a) - 1) + (2*I*b*d*x + 2*I*b*c)*arctan2(sin(b*x + a
), cos(b*x + a) + 1) - 2*(b*d*x + b*c)*cos(b*x + a) - 2*I*d*dilog(-e^(I*b*x
 + I*a)) + 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (b*d*x + b*c)*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 2*d*sin(b*x + a))/b^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\cot(a+b\,x)\,(c+d\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x),x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*cot(a + b*x), x)
```

## 3.102 $\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=70

$$\text{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right)-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] -cos(a-b*c/d)*Si(b*c/d+b*x)/d-Ci(b*c/d+b*x)*sin(a-b*c/d)/d+Unintegrable(csc
(b*x+a)/(d*x+c),x)
```

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x),x]
```

```
[Out] -((CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d) - (Cos[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][Csc[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx &= \int \frac{\csc(a+bx)}{c+dx}\,dx-\int \frac{\sin(a+bx)}{c+dx}\,dx\\ &= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)-\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\int\\ &= -\frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\csc(a+bx)}{c+dx}\,dx\end{aligned}$$

**Mathematica [A]** time = 8.35, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x), x]
```

**fricas [A]** time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)\cot(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*cot(b*x + a)/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\cot(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(b*x + a)/(d*x + c), x)
```

**maple [A]** time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\cos (bx+a) \cot (bx+a)}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(i E_1\left(\frac{i bdx+i bc}{d}\right) - i E_1\left(-\frac{i bdx+i bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right) + 2d\int \frac{\sin(bx+a)}{(dx+c)\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(bx+a)+1\right)}\,dx + 2d\int \frac{}{(dx+c)\left(\cos(\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*((I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 2*d*integrate(sin(b*x + a)/((d*x + c
)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x + a
) + c), x) + 2*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x +
c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + (exp_integral
_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-
(b*c - a*d)/d))/d
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (a+b\,x)\,\cot (a+b\,x)}{c+d\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*cot(a + b*x))/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)*cot(a + b*x))/(c + d*x), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos (a+bx) \cot (a+bx)}{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*cot(a + b*x)/(c + d*x), x)
```

## 3.103 $\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=88

$$\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}$$

```
[Out] -b*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^2+b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2+sin(b*x+
a)/d/(d*x+c)+Unintegrable(csc(b*x+a)/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2) + Sin[a + b*x]/(d*(c
 + d*x)) + (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int]
[Csc[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx &= \int \frac{\csc(a+bx)}{(c+dx)^2}\,dx-\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}-\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}+\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}-\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,d}{d}\\
&= -\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int
\end{aligned}$$

**Mathematica [A]** time = 4.02, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2, x]
```

**fricas [A]** time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)\cot(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\cot(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(b*x + a)/(d*x + c)^2, x)
```

**maple** [A] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\cot(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(i E_2\left(\frac{i bdx+i bc}{d}\right) - i E_2\left(-\frac{i bdx+i bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right) + 2\left(d^2x+cd\right)\int \frac{\sin(bx+a)}{(dx+c)^2\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(bx+a)+1\right)}\,dx + 2\left(d^2\right.}{2\left(d^2x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 2*(d^2*x + c*d)*integrate(sin(b*x +
a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2
 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*
x + a)), x) + 2*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2 + 2*c*d*x + (
d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(b*x
 + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) + (exp_integr
al_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin
(-(b*c - a*d)/d))/(d^2*x + c*d)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,\cot(a+b\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*cot(a + b*x))/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)*cot(a + b*x))/(c + d*x)^2, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*cot(a + b*x)/(c + d*x)**2, x)
```

## 3.104 $\int (c+dx)^m \cot^2(a+bx)\,dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\cot^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable((d*x+c)^m*cot(b*x+a)^2,x)
```

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cot^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]^2,x]
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int (c+dx)^m \cot^2(a+bx)\,dx$$

**Mathematica [A]** time = 1.20, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^2, x]
```

**fricas [A]** time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \cot(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*cot(b*x + a)^2, x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cot(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cot(b*x + a)^2, x)
```

**maple [A]** time = 0.09, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cot^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cot(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cot(b*x+a)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cot (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cot(b*x + a)^2, x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.05

$$\int \cot (a + b\,x)^2\,(c + d\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(cot(a + b*x)^2*(c + d*x)^m, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \cot^2 (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*cot(a + b*x)**2, x)
```

# 3.105 $\int (c+dx)^4 \cot^2(a+bx)\,dx$

**Optimal**. Leaf size=155

$$\frac{3id^4\text{Li}_4\left(e^{2i(a+bx)}\right)}{b^5}+\frac{6d^3(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^4}-\frac{6id^2(c+dx)^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}+\frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+$$

```
[Out] -I*(d*x+c)^4/b-1/5*(d*x+c)^5/d-(d*x+c)^4*cot(b*x+a)/b+4*d*(d*x+c)^3*ln(1-ex
p(2*I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^3+6*d^3
*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^4+3*I*d^4*polylog(4,exp(2*I*(b*x+a))
)/b^5
```

**Rubi [A]** time = 0.23, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3720, 3717, 2190, 2531, 6609, 2282, 6589, 32}

$$-\frac{6id^2(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{6d^3(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^4}+\frac{3id^4\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^5}+\frac{4d(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^4)/b - (c + d*x)^5/(5*d) - ((c + d*x)^4*Cot[a + b*x])/b + (
4*d*(c + d*x)^3*Log[1 - E^((2*I)*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)^2*
PolyLog[2, E^((2*I)*(a + b*x))])/b^3 + (6*d^3*(c + d*x)*PolyLog[3, E^((2*I)
*(a + b*x))])/b^4 + ((3*I)*d^4*PolyLog[4, E^((2*I)*(a + b*x))])/b^5
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
```

```
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4\cot^2(a+bx)\,dx = -\frac{(c+dx)^4\cot(a+bx)}{b}+\frac{(4d)\int (c+dx)^3\cot(a+bx)\,dx}{b}-\int (c+dx)^4\,dx$$

$$= -\frac{i(c+dx)^4}{b}-\frac{(c+dx)^5}{5d}-\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(8id)\int \frac{e^{2i(a+bx)}(c+dx)^3}{1-e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^4}{b}-\frac{(c+dx)^5}{5d}-\frac{(c+dx)^4\cot(a+bx)}{b}+\frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2}-$$

$$= -\frac{i(c+dx)^4}{b}-\frac{(c+dx)^5}{5d}-\frac{(c+dx)^4\cot(a+bx)}{b}+\frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2}-$$

$$= -\frac{i(c+dx)^4}{b}-\frac{(c+dx)^5}{5d}-\frac{(c+dx)^4\cot(a+bx)}{b}+\frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2}-$$

$$= -\frac{i(c+dx)^4}{b}-\frac{(c+dx)^5}{5d}-\frac{(c+dx)^4\cot(a+bx)}{b}+\frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2}-$$

$$= -\frac{i(c+dx)^4}{b}-\frac{(c+dx)^5}{5d}-\frac{(c+dx)^4\cot(a+bx)}{b}+\frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2}-$$

**Mathematica [B]** time = 6.72, size = 795, normalized size = 5.13

$$-\frac{e^{ia}\csc(a)\left(b^4e^{-2ia}x^4+2ib^3\left(1-e^{-2ia}\right)\log\left(1-e^{-i(a+bx)}\right)x^3+2ib^3\left(1-e^{-2ia}\right)\log\left(1+e^{-i(a+bx)}\right)x^3-6e^{-2ia}\left(-1+e\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]^2,x]
```

```
[Out] -1/5*(x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)) - (2
*c*d^3*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a
))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1
 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a
+ b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2
*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*x
))]))/E^((2*I)*a)))/b^4 - (d^4*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I
)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E
^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b^2*x
^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a + b*x
))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a
))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^((-I)*(
a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^5 + (4*c^3*
d*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(
b^2*(Cos[a]^2 + Sin[a]^2)) + (Csc[a]*Csc[a + b*x]*(c^4*Sin[b*x] + 4*c^3*d*x
*Sin[b*x] + 6*c^2*d^2*x^2*Sin[b*x] + 4*c*d^3*x^3*Sin[b*x] + d^4*x^4*Sin[b*x
]))/b - (6*c^2*d^2*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-
Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a
]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan
[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcT
an[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^3*Sqrt[Sec[a]^2*(Cos[a]^2 +
Sin[a]^2)])
```

**fricas [C]** time = 0.95, size = 856, normalized size = 5.52

$$-\frac{10\,b^4d^4x^4+40\,b^4cd^3x^3+60\,b^4c^2d^2x^2+40\,b^4c^3dx+10\,b^4c^4-15i\,d^4\mathrm{polylog}\left(4,\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/10*(10*b^4*d^4*x^4 + 40*b^4*c*d^3*x^3 + 60*b^4*c^2*d^2*x^2 + 40*b^4*c^3*
d*x + 10*b^4*c^4 - 15*I*d^4*polylog(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a
))*sin(2*b*x + 2*a) + 15*I*d^4*polylog(4, cos(2*b*x + 2*a) - I*sin(2*b*x +
2*a))*sin(2*b*x + 2*a) - (-30*I*b^2*d^4*x^2 - 60*I*b^2*c*d^3*x - 30*I*b^2*c
^2*d^2)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - (30
*I*b^2*d^4*x^2 + 60*I*b^2*c*d^3*x + 30*I*b^2*c^2*d^2)*dilog(cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2
+ 3*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*
a) + 1/2)*sin(2*b*x + 2*a) - 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^
3 - a^3*d^4)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(
2*b*x + 2*a) - 20*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^
2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x +
2*a) + 1)*sin(2*b*x + 2*a) - 20*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*
d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(2*b*x + 2*a) -
I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) - 30*(b*d^4*x + b*c*d^3)*polylog(3
, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 30*(b*d^4*x + b
*c*d^3)*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a)
+ 10*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b
^4*c^4)*cos(2*b*x + 2*a) + 2*(b^5*d^4*x^5 + 5*b^5*c*d^3*x^4 + 10*b^5*c^2*d^
2*x^3 + 10*b^5*c^3*d*x^2 + 5*b^5*c^4*x)*sin(2*b*x + 2*a))/(b^5*sin(2*b*x +
2*a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \cot(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^4*cot(b*x + a)^2, x)
```

**maple [B]** time = 0.13, size = 913, normalized size = 5.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cot(b*x+a)^2,x)
[Out] 24/b^4*d^3*c*polylog(3,-exp(I*(b*x+a)))+24/b^4*d^3*c*polylog(3,exp(I*(b*x+a
)))+24/b^4*d^4*polylog(3,-exp(I*(b*x+a)))*x+24/b^4*d^4*polylog(3,exp(I*(b*x
+a)))*x-24*I/b^3*d^3*c*polylog(2,exp(I*(b*x+a)))*x-c*d^3*x^4+24*I*d^4*polyl
og(4,exp(I*(b*x+a)))/b^5-1/5*d^4*x^5-c^4*x-2*c^2*d^2*x^3-2*c^3*d*x^2+8/b^5*
d^4*a^3*ln(exp(I*(b*x+a)))-4/b^5*d^4*a^3*ln(exp(I*(b*x+a))-1)+4/b^2*d*c^3*l
n(exp(I*(b*x+a))-1)+4/b^2*d*c^3*ln(exp(I*(b*x+a))+1)-8/b^2*d*c^3*ln(exp(I*(
b*x+a)))+24*I/b^5*d^4*polylog(4,-exp(I*(b*x+a)))-6*I/b^5*d^4*a^4-2*I/b*d^4*
x^4-24*I/b^2*d^2*c^2*a*x+24*I/b^3*d^3*c*a^2*x-24*I/b^3*d^3*c*polylog(2,-exp
(I*(b*x+a)))*x-12*I/b^3*d^2*c^2*polylog(2,-exp(I*(b*x+a)))+16*I/b^4*d^3*c*a
^3-8*I/b^4*d^4*a^3*x-8*I/b*d^3*c*x^3-12*I/b*d^2*c^2*x^2-12*I/b^3*d^2*c^2*a^
2-12*I/b^3*d^4*polylog(2,-exp(I*(b*x+a)))*x^2+24/b^3*d^2*c^2*a*ln(exp(I*(b*
x+a)))-24/b^4*d^3*c*a^2*ln(exp(I*(b*x+a)))-12/b^3*d^2*c^2*a*ln(exp(I*(b*x+a
))-1)+12/b^4*d^3*c*a^2*ln(exp(I*(b*x+a))-1)-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*
d^2*x^2+4*c^3*d*x+c^4)/b/(exp(2*I*(b*x+a))-1)+12/b^2*d^2*c^2*ln(1-exp(I*(b*
x+a)))*x+12/b^3*d^2*c^2*ln(1-exp(I*(b*x+a)))*a+12/b^2*d^2*c^2*ln(exp(I*(b*x
+a))+1)*x+12/b^2*d^3*c*ln(1-exp(I*(b*x+a)))*x^2-12/b^4*d^3*c*ln(1-exp(I*(b*
x+a)))*a^2+12/b^2*d^3*c*ln(exp(I*(b*x+a))+1)*x^2+4/b^2*d^4*ln(1-exp(I*(b*x+
a)))*x^3+4/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^3+4/b^2*d^4*ln(exp(I*(b*x+a))+1)*
x^3-12*I/b^3*d^4*polylog(2,exp(I*(b*x+a)))*x^2-12*I/b^3*d^2*c^2*polylog(2,e
xp(I*(b*x+a)))
```

**maxima [B]** time = 1.21, size = 3229, normalized size = 20.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="maxima")
[Out] -((b*x + a + 1/tan(b*x + a))*c^4 - 4*(b*x + a + 1/tan(b*x + a))*a*c^3*d/b +
 6*(b*x + a + 1/tan(b*x + a))*a^2*c^2*d^2/b^2 - 4*(b*x + a + 1/tan(b*x + a)
)*a^3*c*d^3/b^3 + (b*x + a + 1/tan(b*x + a))*a^4*d^4/b^4 + 2*((b*x + a)^2*c
os(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*
x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
- (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(co
s(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x
 + 2*a))*c^3*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*
a) + 1)*b) - 6*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*
a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 +
 sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2
*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a)
 + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a*c^2*d^2/((cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^2) + 6*((b*x + a)^2*cos(2*b*x +
2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) +
(b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)
 + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*
x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^
```

```
2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*b^3) - 2*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2
- 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(
2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
 + 4*(b*x + a)*sin(2*b*x + 2*a))*a^3*d^4/((cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^4) - (-I*(b*x + a)^5*d^4 + (-5*I*b*c*d^
3 + 5*I*a*d^4)*(b*x + a)^4 + (-10*I*b^2*c^2*d^2 + 20*I*a*b*c*d^3 - 10*I*a^2
*d^4)*(b*x + a)^3 - (20*(b*x + a)^3*d^4 + 60*(b*c*d^3 - a*d^4)*(b*x + a)^2
+ 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) - 20*((b*x + a)^3*d^4
+ 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a))*cos(2*b*x + 2*a) - (20*I*(b*x + a)^3*d^4 + (60*I*b*c*d^3 - 60*I
*a*d^4)*(b*x + a)^2 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + 60*I*a^2*d^4)*(
b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (20*(
b*x + a)^3*d^4 + 60*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 60*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + a^2*d^4)*(b*x + a) - 20*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*
x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2
*a) + (-20*I*(b*x + a)^3*d^4 + (-60*I*b*c*d^3 + 60*I*a*d^4)*(b*x + a)^2 + (
-60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4)*(b*x + a))*sin(2*b*x +
2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*(b*x + a)^5*d^4 + (5*I*
b*c*d^3 - 5*(I*a + 2)*d^4)*(b*x + a)^4 + (10*I*b^2*c^2*d^2 - 20*(I*a + 2)*b
*c*d^3 + (10*I*a^2 + 40*a)*d^4)*(b*x + a)^3 - 60*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)*(b*x + a)^2)*cos(2*b*x + 2*a) + (60*b^2*c^2*d^2 - 120*a*b*c*d^3
 + 60*(b*x + a)^2*d^4 + 60*a^2*d^4 + 120*(b*c*d^3 - a*d^4)*(b*x + a) - 60*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)
*(b*x + a))*cos(2*b*x + 2*a) + (-60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*
(b*x + a)^2*d^4 - 60*I*a^2*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a))*
sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (60*b^2*c^2*d^2 - 120*a*b*c*d^3
 + 60*(b*x + a)^2*d^4 + 60*a^2*d^4 + 120*(b*c*d^3 - a*d^4)*(b*x + a) - 60*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)
*(b*x + a))*cos(2*b*x + 2*a) + (-60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*
(b*x + a)^2*d^4 - 60*I*a^2*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a))*
sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (10*I*(b*x + a)^3*d^4 + (30*I*b*
c*d^3 - 30*I*a*d^4)*(b*x + a)^2 + (30*I*b^2*c^2*d^2 - 60*I*a*b*c*d^3 + 30*I
*a^2*d^4)*(b*x + a) + (-10*I*(b*x + a)^3*d^4 + (-30*I*b*c*d^3 + 30*I*a*d^4)
*(b*x + a)^2 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 - 30*I*a^2*d^4)*(b*x + a
))*cos(2*b*x + 2*a) + 10*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2
 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log
(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (10*I*(b*x + a)^3*
d^4 + (30*I*b*c*d^3 - 30*I*a*d^4)*(b*x + a)^2 + (30*I*b^2*c^2*d^2 - 60*I*a*
b*c*d^3 + 30*I*a^2*d^4)*(b*x + a) + (-10*I*(b*x + a)^3*d^4 + (-30*I*b*c*d^3
 + 30*I*a*d^4)*(b*x + a)^2 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 - 30*I*a^2
*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 10*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^
4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 120*
(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog(4, -e^(I*b*x
+ I*a)) + 120*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog
(4, e^(I*b*x + I*a)) + (120*I*b*c*d^3 + 120*I*(b*x + a)*d^4 - 120*I*a*d^4 +
 (-120*I*b*c*d^3 - 120*I*(b*x + a)*d^4 + 120*I*a*d^4)*cos(2*b*x + 2*a) + 12
0*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x
+ I*a)) + (120*I*b*c*d^3 + 120*I*(b*x + a)*d^4 - 120*I*a*d^4 + (-120*I*b*c*
d^3 - 120*I*(b*x + a)*d^4 + 120*I*a*d^4)*cos(2*b*x + 2*a) + 120*(b*c*d^3 +
(b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) - ((b*
x + a)^5*d^4 + (5*b*c*d^3 - (5*a - 10*I)*d^4)*(b*x + a)^4 + (10*b^2*c^2*d^2
 - (20*a - 40*I)*b*c*d^3 + 10*(a^2 - 4*I*a)*d^4)*(b*x + a)^3 - (-60*I*b^2*c
^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-5
*I*b^4*cos(2*b*x + 2*a) + 5*b^4*sin(2*b*x + 2*a) + 5*I*b^4))/b
```

**mupad** [F]    time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(a+b\,x\right)^2\,\left(c+d\,x\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] int(cot(a + b*x)^2*(c + d*x)^4, x)
```

**sympy** [F]    time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**4*cot(a + b*x)**2, x)
```

# 3.106 $\int (c+dx)^3 \cot^2(a+bx)\,dx$

**Optimal**. Leaf size=127

$$\frac{3d^3\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^2(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{i(c+dx)^3}{b}$$

```
[Out] -I*(d*x+c)^3/b-1/4*(d*x+c)^4/d-(d*x+c)^3*cot(b*x+a)/b+3*d*(d*x+c)^2*ln(1-ex
p(2*I*(b*x+a)))/b^2-3*I*d^2*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^3+3/2*d^3
*polylog(3,exp(2*I*(b*x+a)))/b^4
```

**Rubi [A]** time = 0.20, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3720, 3717, 2190, 2531, 2282, 6589, 32}

$$-\frac{3id^2(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{3d^3\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^4}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\cot(a}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) - ((c + d*x)^3*Cot[a + b*x])/b + (
3*d*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*Po
lyLog[2, E^((2*I)*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((2*I)*(a + b*x))]
)/(2*b^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
```

```
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\cot^2(a+bx)\,dx = -\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{(3d)\int (c+dx)^2\cot(a+bx)\,dx}{b}-\int (c+dx)^3\,dx$$

$$= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^4}{4d}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(6id)\int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^4}{4d}-\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-$$

$$= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^4}{4d}-\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-$$

$$= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^4}{4d}-\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-$$

$$= -\frac{i(c+dx)^3}{b}-\frac{(c+dx)^4}{4d}-\frac{(c+dx)^3\cot(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-$$

**Mathematica [B]** time = 6.15, size = 374, normalized size = 2.94

$$-\frac{3c^2d(bx\cot(a)-\log(\sin(a+bx)))}{b^2}+\frac{3cd^2\left(-b^2x^2e^{i\tan^{-1}(\tan(a))}\cot(a)\sqrt{\sec^2(a)}-i\mathrm{Li}_2\left(e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right)+ibx\left(\pi\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]^2,x]
```

```
[Out] -1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)) - (3*c^2*d*(b*x*Cot[a]
 - Log[Sin[a + b*x]]))/b^2 + (3*c*d^2*(I*b*x*(Pi - 2*ArcTan[Tan[a]]) + Pi*L
og[1 + E^((-2*I)*b*x)] + 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + A
rcTan[Tan[a]]))] - Pi*Log[Cos[b*x]] - 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan
[Tan[a]]]] - I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))] - b^2*E^(I*ArcT
an[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2]))/b^3 + (d^3*(I + Cot[a])*(I*b^3*x^3
- b^3*x^3*Cot[a] + 3*b^2*x^2*Log[1 - E^((-I)*(a + b*x))] + 3*b^2*x^2*Log[1
+ E^((-I)*(a + b*x))] + (6*I)*b*x*PolyLog[2, -E^((-I)*(a + b*x))] + (6*I)*b
*x*PolyLog[2, E^((-I)*(a + b*x))] + 6*PolyLog[3, -E^((-I)*(a + b*x))] + 6*P
olyLog[3, E^((-I)*(a + b*x))])*Sin[a])/(b^4*E^(I*a)) + ((c + d*x)^3*Csc[a]*
Csc[a + b*x]*Sin[b*x])/b
```

**fricas** [C] time = 0.78, size = 599, normalized size = 4.72

$$-\frac{4\,b^3d^3x^3+12\,b^3cd^2x^2+12\,b^3c^2dx+4\,b^3c^3-3\,d^3\mathrm{polylog}\left(3,\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)\right)\sin\left(2\,bx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 4*b^3*c^3 - 3*d^3
*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 3*d^3
*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - (-6*I
*b*d^3*x - 6*I*b*c*d^2)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*
b*x + 2*a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(cos(2*b*x + 2*a) - I*sin(2*b
*x + 2*a))*sin(2*b*x + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/
2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*
x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c
*d^2 - a^2*d^3)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)*sin(2*b*x +
 2*a) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(2*
b*x + 2*a) - I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) + 4*(b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(2*b*x + 2*a) + (b^4*d^3*x^4 +
4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*x)*sin(2*b*x + 2*a))/(b^4*sin
(2*b*x + 2*a))
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cot (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cot(b*x + a)^2, x)
```

**maple** [B] time = 0.11, size = 573, normalized size = 4.51

$$\frac{3d^3a^2\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^4}-\frac{6d^3a^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^4}+\frac{3d\,c^2\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^2}+\frac{3d\,c^2\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b^2}-\frac{6d\,c^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}-\frac{2i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cot(b*x+a)^2,x)
```

```
[Out] 3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-1)-6/b^4*d^3*a^2*ln(exp(I*(b*x+a)))+3/b^2*d
*c^2*ln(exp(I*(b*x+a))-1)+3/b^2*d*c^2*ln(exp(I*(b*x+a))+1)-6/b^2*d*c^2*ln(e
xp(I*(b*x+a)))-2*I/b*d^3*x^3+4*I/b^4*d^3*a^3-12*I/b^2*d^2*c*a*x+6*d^3*polyl
og(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4-3/2*c^2*d*x^2
-c*d^2*x^3-1/4*d^3*x^4-c^3*x+3/b^2*d^3*ln(exp(I*(b*x+a))+1)*x^2+3/b^2*d^3*l
n(1-exp(I*(b*x+a)))*x^2-3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^2-2*I*(d^3*x^3+3*c
*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))-1)-6*I/b^3*d^3*polylog(2,exp(I*
(b*x+a)))*x+6/b^2*d^2*c*ln(exp(I*(b*x+a))+1)*x-6*I/b^3*d^2*c*polylog(2,exp(
I*(b*x+a)))+12/b^3*d^2*c*a*ln(exp(I*(b*x+a)))-6/b^3*d^2*c*a*ln(exp(I*(b*x+a
))-1)-6*I/b^3*d^2*c*polylog(2,-exp(I*(b*x+a)))+6*I/b^3*d^3*a^2*x-6*I/b*d^2*
c*x^2-6*I/b^3*d^2*c*a^2-6*I/b^3*d^3*polylog(2,-exp(I*(b*x+a)))*x+6/b^2*d^2*
c*ln(1-exp(I*(b*x+a)))*x+6/b^3*d^2*c*ln(1-exp(I*(b*x+a)))*a
```

**maxima** [B] time = 0.74, size = 1945, normalized size = 15.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a + 1/tan(b*x + a))*c^3 - 6*(b*x + a + 1/tan(b*x + a))*a*c^2
*d/b + 6*(b*x + a + 1/tan(b*x + a))*a^2*c*d^2/b^2 - 2*(b*x + a + 1/tan(b*x
+ a))*a^3*d^3/b^3 + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b
*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2
*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + s
in(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b
*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*c^2*d/((cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) - 6*((b*x + a)^2*cos(2*b*x +
 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) +
 (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a
) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a
*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*
b^2) + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 -
 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
+ 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^3) - 2*(-I*(b*x + a)^4*d^3 + (-4*I*b*c*d
^2 + 4*I*a*d^3)*(b*x + a)^3 - (12*(b*x + a)^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b
*x + a) - 12*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x +
2*a) - (12*I*(b*x + a)^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (12*(b*x + a)^2*d^3
+ 24*(b*c*d^2 - a*d^3)*(b*x + a) - 12*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3
)*(b*x + a))*cos(2*b*x + 2*a) + (-12*I*(b*x + a)^2*d^3 + (-24*I*b*c*d^2 + 2
4*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a)
 + 1) + (I*(b*x + a)^4*d^3 + (4*I*b*c*d^2 - 4*(I*a + 2)*d^3)*(b*x + a)^3 -
24*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(2*b*x + 2*a) + (24*b*c*d^2 + 24*(b*x
+ a)*d^3 - 24*a*d^3 - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a)
 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(2*b*x + 2*a))*dilo
g(-e^(I*b*x + I*a)) + (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 - 24*(b*c*d
^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (-24*I*b*c*d^2 - 24*I*(b*x +
 a)*d^3 + 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (6*I*(b*x
+ a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3
+ (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2
*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d
^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*
a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^
3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos
(b*x + a) + 1) + (-24*I*d^3*cos(2*b*x + 2*a) + 24*d^3*sin(2*b*x + 2*a) + 24
*I*d^3)*polylog(3, -e^(I*b*x + I*a)) + (-24*I*d^3*cos(2*b*x + 2*a) + 24*d^3
*sin(2*b*x + 2*a) + 24*I*d^3)*polylog(3, e^(I*b*x + I*a)) - ((b*x + a)^4*d^
3 + (4*b*c*d^2 - (4*a - 8*I)*d^3)*(b*x + a)^3 - (-24*I*b*c*d^2 + 24*I*a*d^3
)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-4*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(2*b
*x + 2*a) + 4*I*b^3))/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cot (a + b x)^2 (c + d x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] int(cot(a + b*x)^2*(c + d*x)^3, x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cot^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*cot(a + b*x)**2, x)
```

# 3.107 $\int (c+dx)^2 \cot^2(a+bx)\,dx$

**Optimal**. Leaf size=97

$$-\frac{id^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}$$

```
[Out] -I*(d*x+c)^2/b-1/3*(d*x+c)^3/d-(d*x+c)^2*cot(b*x+a)/b+2*d*(d*x+c)*ln(1-exp(
2*I*(b*x+a)))/b^2-I*d^2*polylog(2,exp(2*I*(b*x+a)))/b^3
```

**Rubi [A]** time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3720, 3717, 2190, 2279, 2391, 32}

$$-\frac{id^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) - ((c + d*x)^2*Cot[a + b*x])/b + (
2*d*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b^2 - (I*d^2*PolyLog[2, E^((2*I
)*(a + b*x))])/b^3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
```

```
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2\cot^2(a+bx)\,dx = -\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{(2d)\int (c+dx)\cot(a+bx)\,dx}{b}-\int (c+dx)^2\,dx$$
$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(4id)\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx}{b}$$
$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}$$
$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}$$
$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}$$

**Mathematica [B]** time = 6.03, size = 198, normalized size = 2.04

$$-\frac{2cd(bx\cot(a)-\log(\sin(a+bx)))}{b^2}+\frac{d^2\left(-b^2x^2e^{i\tan^{-1}(\tan(a))}\cot(a)\sqrt{\sec^2(a)}-i\mathrm{Li}_2\left(e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right)+ibx\left(\pi\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]^2,x]
```

```
[Out] -1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)) - (2*c*d*(b*x*Cot[a] - Log[Sin[a + b*x
]]))/b^2 + (d^2*(I*b*x*(Pi - 2*ArcTan[Tan[a]]) + Pi*Log[1 + E^((-2*I)*b*x)]
 + 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - Pi*
Log[Cos[b*x]] - 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] - I*PolyLog
[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))] - b^2*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]
*Sqrt[Sec[a]^2]))/b^3 + ((c + d*x)^2*Csc[a]*Csc[a + b*x]*Sin[b*x])/b
```

**fricas [B]** time = 0.72, size = 384, normalized size = 3.96

$$-\frac{6\,b^2d^2x^2+12\,b^2cdx+6\,b^2c^2+3i\,d^2\mathrm{Li}_2\left(\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)\right)\sin\left(2\,bx+2\,a\right)-3i\,d^2\mathrm{Li}_2\left(\cos\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^2*d^2*x^2 + 12*b^2*c*d*x + 6*b^2*c^2 + 3*I*d^2*dilog(cos(2*b*x +
2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 3*I*d^2*dilog(cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 6*(b*c*d - a*d^2)*log(-1/2*cos(2
*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b*c*d - a
*d^2)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x +
 2*a) - 6*(b*d^2*x + a*d^2)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)
*sin(2*b*x + 2*a) - 6*(b*d^2*x + a*d^2)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x
 + 2*a) + 1)*sin(2*b*x + 2*a) + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
(2*b*x + 2*a) + 2*(b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 3*b^3*c^2*x)*sin(2*b*x + 2
*a))/(b^3*sin(2*b*x + 2*a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\cot(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cot(b*x + a)^2, x)
```

**maple [B]** time = 0.09, size = 297, normalized size = 3.06

$$-\frac{d^2x^3}{3}-cd\,x^2-c^2x-\frac{4id^2ax}{b^2}+\frac{2dc\ln\left(e^{i(bx+a)}-1\right)}{b^2}+\frac{2dc\ln\left(e^{i(bx+a)}+1\right)}{b^2}-\frac{4dc\ln\left(e^{i(bx+a)}\right)}{b^2}-\frac{2id^2\operatorname{polylog}\left(2,-e^{i(bx+a)}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cot(b*x+a)^2,x)
```

```
[Out] -1/3*d^2*x^3-c*d*x^2-c^2*x-4*I/b^2*d^2*a*x+2/b^2*d*c*ln(exp(I*(b*x+a))-1)+2
/b^2*d*c*ln(exp(I*(b*x+a))+1)-4/b^2*d*c*ln(exp(I*(b*x+a)))-2*I/b^3*d^2*poly
log(2,-exp(I*(b*x+a)))-2*I/b^3*d^2*a^2-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^
3+2/b^2*d^2*ln(exp(I*(b*x+a))+1)*x-2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*
x+a))-1)+2/b^2*d^2*ln(1-exp(I*(b*x+a)))*x+2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a-
2*I/b*d^2*x^2-2/b^3*d^2*a*ln(exp(I*(b*x+a))-1)+4/b^3*d^2*a*ln(exp(I*(b*x+a)
))
```

**maxima [B]** time = 0.74, size = 646, normalized size = 6.66

$$\frac{-i\,b^3d^2x^3-3i\,b^3cdx^2-3i\,b^3c^2x-6\,b^2c^2-\left(6\,bd^2x+6\,bcd-6\left(bd^2x+bcd\right)\cos\left(2\,bx+2\,a\right)-\left(6i\,bd^2x+6i\,bcd\right)\sin\ldots\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (-I*b^3*d^2*x^3 - 3*I*b^3*c*d*x^2 - 3*I*b^3*c^2*x - 6*b^2*c^2 - (6*b*d^2*x
+ 6*b*c*d - 6*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a) - (6*I*b*d^2*x + 6*I*b*c*d
)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*b*c*d*cos(
2*b*x + 2*a) + 6*I*b*c*d*sin(2*b*x + 2*a) - 6*b*c*d)*arctan2(sin(b*x + a),
cos(b*x + a) - 1) - (6*b*d^2*x*cos(2*b*x + 2*a) + 6*I*b*d^2*x*sin(2*b*x + 2
*a) - 6*b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*b^3*d^2*x^3
+ (3*I*b^3*c*d - 6*b^2*d^2)*x^2 - 3*(-I*b^3*c^2 + 4*b^2*c*d)*x)*cos(2*b*x +
 2*a) - 6*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I
*b*x + I*a)) - 6*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilo
g(e^(I*b*x + I*a)) + (3*I*b*d^2*x + 3*I*b*c*d + (-3*I*b*d^2*x - 3*I*b*c*d)*
cos(2*b*x + 2*a) + 3*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (3*I*b*d^2*x + 3*I*b*c*d + (-3*I*
b*d^2*x - 3*I*b*c*d)*cos(2*b*x + 2*a) + 3*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a
))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (b^3*d^2*x^3
 + 3*(b^3*c*d + 2*I*b^2*d^2)*x^2 + (3*b^3*c^2 + 12*I*b^2*c*d)*x)*sin(2*b*x
+ 2*a))/(-3*I*b^3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) + 3*I*b^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] int(cot(a + b*x)^2*(c + d*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)^{2}\cot^{2}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*cot(a + b*x)**2, x)
```

# 3.108 $\int (c+dx)\cot^2(a+bx)\,dx$

**Optimal**. Leaf size=41

$$\frac{d\log(\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}-cx-\frac{dx^2}{2}$$

```
[Out] -c*x-1/2*d*x^2-(d*x+c)*cot(b*x+a)/b+d*ln(sin(b*x+a))/b^2
```

**Rubi [A]** time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3720, 3475}

$$\frac{d\log(\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}-cx-\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]^2,x]
```

```
[Out] -(c*x) - (d*x^2)/2 - ((c + d*x)*Cot[a + b*x])/b + (d*Log[Sin[a + b*x]])/b^2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cot^2(a+bx)\,dx &= -\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\int\cot(a+bx)\,dx}{b}-\int(c+dx)\,dx\\ &= -cx-\frac{dx^2}{2}-\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\log(\sin(a+bx))}{b^2}\end{aligned}$$

**Mathematica [C]** time = 0.48, size = 82, normalized size = 2.00

$$\frac{d\log(\sin(a+bx))}{b^2}-\frac{c\cot(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(a+bx)\right)}{b}+\frac{dx\csc(a)\sin(bx)\csc(a+bx)}{b}-\frac{dx\csc(a)(bx\sin(a$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]^2,x]
```

```
[Out] -((c*Cot[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a + b*x]^2])/b) + (d
*Log[Sin[a + b*x]])/b^2 - (d*x*Csc[a]*(2*Cos[a] + b*x*Sin[a]))/(2*b) + (d*x
*Csc[a]*Csc[a + b*x]*Sin[b*x])/b
```

**fricas [B]** time = 0.59, size = 97, normalized size = 2.37

$$-\frac{2\,bdx-d\log\left(-\frac{1}{2}\cos\left(2\,bx+2\,a\right)+\frac{1}{2}\right)\sin\left(2\,bx+2\,a\right)+2\,bc+2\left(bdx+bc\right)\cos\left(2\,bx+2\,a\right)+\left(b^2dx^2+2\,b^2cx\right)}{2\,b^2\sin\left(2\,bx+2\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x - d*log(-1/2*cos(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) + 2*b*c
 + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d*x^2 + 2*b^2*c*x)*sin(2*b*x + 2
*a))/(b^2*sin(2*b*x + 2*a))
```

**giac [B]** time = 2.53, size = 1375, normalized size = 33.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d*x^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*d*x^2*tan(1/2*b*x)*tan(1/2*
a)^2 + 2*b^2*c*x*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c*x*tan(1/2*b*x)*tan(1/2
*a)^2 - b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*d*x^2*tan(1/2*b*x) - b^2*d*
x^2*tan(1/2*a) - b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*c*x*tan(1/2*b*x) +
 b*d*x*tan(1/2*b*x)^2 - 2*b^2*c*x*tan(1/2*a) + 4*b*d*x*tan(1/2*b*x)*tan(1/2
*a) - d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3
 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*
x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/
2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*ta
n(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 +
2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a
)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2
*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan
(1/2*a) + b*d*x*tan(1/2*a)^2 - d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*
tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3
+ 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2
*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1
/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2
*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2
 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2
 + 1))*tan(1/2*b*x)*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2 + 4*b*c*tan(1/2*b*x)*
tan(1/2*a) + b*c*tan(1/2*a)^2 - b*d*x + d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)
^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/
2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(
1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^
5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^
3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1
/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1
/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan
(1/2*a)^2 + 1))*tan(1/2*b*x) + d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*
tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3
+ 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2
*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1
/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2
*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2
 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2
 + 1))*tan(1/2*a) - b*c)/(b^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)*
tan(1/2*a)^2 - b^2*tan(1/2*b*x) - b^2*tan(1/2*a))
```

**maple [A]** time = 0.05, size = 49, normalized size = 1.20

$$-\frac{d\,x^{2}}{2}-c x-\frac{d \cot \left(b x +a \right) x}{b}+\frac{d \ln \left(\sin \left(b x +a \right)\right)}{b^{2}}-\frac{c \cot \left(b x +a \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cot(b*x+a)^2,x)
```

```
[Out] -1/2*d*x^2-c*x-1/b*d*cot(b*x+a)*x+d*ln(sin(b*x+a))/b^2-1/b*c*cot(b*x+a)
```

**maxima [B]** time = 0.58, size = 292, normalized size = 7.12

$$-\frac{2\left(bx+a+\frac{1}{\tan(bx+a)}\right)c-\frac{2\left(bx+a+\frac{1}{\tan(bx+a)}\right)ad}{b}+\frac{\left((bx+a)^2\cos(2\,bx+2\,a)^2+(bx+a)^2\sin(2\,bx+2\,a)^2-2\,(bx+a)^2\cos(2\,bx+2\,a)+(bx+a)^2-\left(\cos(\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a + 1/tan(b*x + a))*c - 2*(b*x + a + 1/tan(b*x + a))*a*d/b +
 ((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x
+ a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos
(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2
*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x
 + a)*sin(2*b*x + 2*a))*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos
(2*b*x + 2*a) + 1)*b))/b
```

**mupad [B]** time = 1.57, size = 67, normalized size = 1.63

$$-\frac{d\,x^2}{2}+\frac{d\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-1\right)}{b^2}-\frac{(c+d\,x)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}-\frac{x\,(b\,c+d\,2\mathrm{i})}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x),x)
```

```
[Out] (d*log(exp(a*2i)*exp(b*x*2i) - 1))/b^2 - (d*x^2)/2 - ((c + d*x)*2i)/(b*(exp
(a*2i + b*x*2i) - 1)) - (x*(d*2i + b*c))/b
```

**sympy [A]** time = 0.48, size = 104, normalized size = 2.54

$$\begin{cases} \tilde{\infty}\left(cx+\frac{dx^2}{2}\right) & \text{for } a=0\wedge b=0 \\ \left(cx+\frac{dx^2}{2}\right)\cot^2(a) & \text{for } b=0 \\ \tilde{\infty}\left(cx+\frac{dx^2}{2}\right) & \text{for } a=-bx \\ -cx-\frac{dx^2}{2}-\frac{c}{b\tan(a+bx)}-\frac{dx}{b\tan(a+bx)}-\frac{d\log\left(\tan^2(a+bx)+1\right)}{2b^2}+\frac{d\log(\tan(a+bx))}{b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*(c*x + d*x**2/2), Eq(a, 0) & Eq(b, 0)), ((c*x + d*x**2/2)*co
t(a)**2, Eq(b, 0)), (zoo*(c*x + d*x**2/2), Eq(a, -b*x)), (-c*x - d*x**2/2 -
 c/(b*tan(a + b*x)) - d*x/(b*tan(a + b*x)) - d*log(tan(a + b*x)**2 + 1)/(2*
b**2) + d*log(tan(a + b*x))/b**2, True))
```

## 3.109 $\int \frac{\cot^2(a+bx)}{c+dx} \, dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\frac{\cot^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^2/(d*x+c),x)
```

**Rubi [A]** time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^2(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^2/(c + d*x),x]
[Out] Defer[Int][Cot[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)}{c+dx} \, dx = \int \frac{\cot^2(a+bx)}{c+dx} \, dx$$

**Mathematica [A]** time = 4.52, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^2/(c + d*x),x]
[Out] Integrate[Cot[a + b*x]^2/(c + d*x), x]
```

**fricas [A]** time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(cot(b*x + a)^2/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)^2}{dx+c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(cot(b*x + a)^2/(d*x + c), x)
```

**maple** [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(cot(b*x+a)^2/(d*x+c),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\left(bdx + (bdx+bc)\cos(2\,bx+2\,a)^2 + (bdx+bc)\sin(2\,bx+2\,a)^2 + bc - 2\,(bdx+bc)\cos(2\,bx+2\,a)\right)\log(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] ((b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x + b
*c*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*integr
ate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x +
b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*
(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - (b*d^3*x + b*c*d^2 + (b
*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a)
^2 - 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*
x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 +
(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x +
 b*c^2)*cos(b*x + a)), x) - (b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*
d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*log
(d*x + c) - 2*d*sin(2*b*x + 2*a))/(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(
2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)
*cos(2*b*x + 2*a))
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cot(a+b\,x)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/(c + d*x),x)
```

```
[Out] int(cot(a + b*x)^2/(c + d*x), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(cot(a + b*x)**2/(c + d*x), x)
```

# 3.110 $\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\frac{\cot^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^2/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^2/(c + d*x)^2,x]
[Out] Defer[Int][Cot[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx = \int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

**Mathematica [A]** time = 2.35, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^2/(c + d*x)^2,x]
[Out] Integrate[Cot[a + b*x]^2/(c + d*x)^2, x]
```

**fricas [A]** time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cot(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(cot(b*x + a)^2/(d*x + c)^2, x)
```

**maple** [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2/(d*x+c)^2,x)
[Out] int(cot(b*x+a)^2/(d*x+c)^2,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cot(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/(c + d*x)^2,x)
[Out] int(cot(a + b*x)^2/(c + d*x)^2, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(cot(a + b*x)**2/(c + d*x)**2, x)
```

# 3.111 $\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$

**Optimal**. Leaf size=39

$$\text{Int}\left(\csc^3(a+bx)(c+dx)^m,x\right)-\text{Int}\left(\csc(a+bx)(c+dx)^m,x\right)$$

```
[Out] -Unintegrable((d*x+c)^m*csc(b*x+a),x)+Unintegrable((d*x+c)^m*csc(b*x+a)^3,x
)
```

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -Defer[Int][(c + d*x)^m*Csc[a + b*x], x] + Defer[Int][(c + d*x)^m*Csc[a + b
*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx=-\int (c+dx)^m \csc(a+bx)\,dx+\int (c+dx)^m \csc^3(a+bx)\,dx$$

**Mathematica [A]** time = 38.57, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x], x]
```

**fricas [A]** time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \cot(bx+a)^2 \csc(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cot(bx+a)^2 \csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

**maple** [**A**] time = 0.10, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cot^2(bx+a)\right)\csc(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x)
[Out] int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x)
```

**maxima** [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cot(bx+a)^2 \csc(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

**mupad** [**A**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(a+b\,x)^2\,(c+d\,x)^m}{\sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x)^m)/sin(a + b*x),x)
[Out] int((cot(a + b*x)^2*(c + d*x)^m)/sin(a + b*x), x)
```

**sympy** [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cot(b*x+a)**2*csc(b*x+a),x)
[Out] Integral((c + d*x)**m*cot(a + b*x)**2*csc(a + b*x), x)
```

# 3.112 $\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx$

**Optimal**. Leaf size=416

$$-\frac{12d^4\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^5}+\frac{12d^4\text{Li}_3\left(e^{i(a+bx)}\right)}{b^5}-\frac{12d^4\text{Li}_5\left(-e^{i(a+bx)}\right)}{b^5}+\frac{12d^4\text{Li}_5\left(e^{i(a+bx)}\right)}{b^5}+\frac{12id^3(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^4}-$$

```
[Out] -12*d^2*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^3+(d*x+c)^4*arctanh(exp(I*(b*x+
a)))/b-2*d*(d*x+c)^3*csc(b*x+a)/b^2-1/2*(d*x+c)^4*cot(b*x+a)*csc(b*x+a)/b+1
2*I*d^3*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^4-12*I*d^3*(d*x+c)*polylog(2,e
xp(I*(b*x+a)))/b^4+12*I*d^3*(d*x+c)*polylog(4,-exp(I*(b*x+a)))/b^4-12*I*d^3
*(d*x+c)*polylog(4,exp(I*(b*x+a)))/b^4-12*d^4*polylog(3,-exp(I*(b*x+a)))/b^
5+6*d^2*(d*x+c)^2*polylog(3,-exp(I*(b*x+a)))/b^3+12*d^4*polylog(3,exp(I*(b*
x+a)))/b^5-6*d^2*(d*x+c)^2*polylog(3,exp(I*(b*x+a)))/b^3-2*I*d*(d*x+c)^3*po
lylog(2,-exp(I*(b*x+a)))/b^2+2*I*d*(d*x+c)^3*polylog(2,exp(I*(b*x+a)))/b^2-
12*d^4*polylog(5,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(5,exp(I*(b*x+a)))/b^5
```

**Rubi [A]** time = 0.50, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4415, 4183, 2531, 6609, 2282, 6589, 4186}

$$\frac{12id^3(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^4}-\frac{12id^3(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^4}+\frac{12id^3(c+dx)\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] (-12*d^2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^3 + ((c + d*x)^4*ArcTanh[E
^(I*(a + b*x))])/b - (2*d*(c + d*x)^3*Csc[a + b*x])/b^2 - ((c + d*x)^4*Cot[
a + b*x]*Csc[a + b*x])/(2*b) + ((12*I)*d^3*(c + d*x)*PolyLog[2, -E^(I*(a +
b*x))])/b^4 - ((2*I)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((12
*I)*d^3*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^4 + ((2*I)*d*(c + d*x)^3*P
olyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^4*PolyLog[3, -E^(I*(a + b*x))])/b^5
 + (6*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (12*d^4*PolyLog[3
, E^(I*(a + b*x))])/b^5 - (6*d^2*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b
^3 + ((12*I)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))])/b^4 - ((12*I)*d^3*
(c + d*x)*PolyLog[4, E^(I*(a + b*x))])/b^4 - (12*d^4*PolyLog[5, -E^(I*(a +
b*x))])/b^5 + (12*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4415

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)^4 \csc(a+bx)\,dx + \int (c+dx)^4 \csc^3(a+bx)\,dx$$

$$= \frac{2(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)^3 \csc(a+bx)}{b^2} - \frac{(c+dx)^4 \cot(a \ldots}{}$$

$$= -\frac{12d^2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+ \ldots}{}$$

$$= -\frac{12d^2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+ \ldots}{}$$

$$= -\frac{12d^2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+ \ldots}{}$$

$$= -\frac{12d^2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+ \ldots}{}$$

$$= -\frac{12d^2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+ \ldots}{}$$

$$= -\frac{12d^2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+ \ldots}{}$$

**Mathematica [B]** time = 8.30, size = 966, normalized size = 2.32

$$\frac{-c^4\log\left(1-e^{i(a+bx)}\right)b^4-d^4x^4\log\left(1-e^{i(a+bx)}\right)b^4-4cd^3x^3\log\left(1-e^{i(a+bx)}\right)b^4-6c^2d^2x^2\log\left(1-e^{i(a+bx)}\right)b^4-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] (-(b^4*c^4*Log[1 - E^(I*(a + b*x))]) + 12*b^2*c^2*d^2*Log[1 - E^(I*(a + b*x
))] - 4*b^4*c^3*d*x*Log[1 - E^(I*(a + b*x))] + 24*b^2*c*d^3*x*Log[1 - E^(I*
(a + b*x))] - 6*b^4*c^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] + 12*b^2*d^4*x^2*L
og[1 - E^(I*(a + b*x))] - 4*b^4*c*d^3*x^3*Log[1 - E^(I*(a + b*x))] - b^4*d^
4*x^4*Log[1 - E^(I*(a + b*x))] + b^4*c^4*Log[1 + E^(I*(a + b*x))] - 12*b^2*
c^2*d^2*Log[1 + E^(I*(a + b*x))] + 4*b^4*c^3*d*x*Log[1 + E^(I*(a + b*x))] -
 24*b^2*c*d^3*x*Log[1 + E^(I*(a + b*x))] + 6*b^4*c^2*d^2*x^2*Log[1 + E^(I*(
a + b*x))] - 12*b^2*d^4*x^2*Log[1 + E^(I*(a + b*x))] + 4*b^4*c*d^3*x^3*Log[
1 + E^(I*(a + b*x))] + b^4*d^4*x^4*Log[1 + E^(I*(a + b*x))] - (4*I)*b*d*(c
+ d*x)*(-6*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*x))] + (4*I)*b*d*
(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))] + 12*b^2*c
^2*d^2*PolyLog[3, -E^(I*(a + b*x))] - 24*d^4*PolyLog[3, -E^(I*(a + b*x))] +
 24*b^2*c*d^3*x*PolyLog[3, -E^(I*(a + b*x))] + 12*b^2*d^4*x^2*PolyLog[3, -E
^(I*(a + b*x))] - 12*b^2*c^2*d^2*PolyLog[3, E^(I*(a + b*x))] + 24*d^4*PolyL
og[3, E^(I*(a + b*x))] - 24*b^2*c*d^3*x*PolyLog[3, E^(I*(a + b*x))] - 12*b^
2*d^4*x^2*PolyLog[3, E^(I*(a + b*x))] + (24*I)*b*c*d^3*PolyLog[4, -E^(I*(a
+ b*x))] + (24*I)*b*d^4*x*PolyLog[4, -E^(I*(a + b*x))] - (24*I)*b*c*d^3*Pol
yLog[4, E^(I*(a + b*x))] - (24*I)*b*d^4*x*PolyLog[4, E^(I*(a + b*x))] - 24*
d^4*PolyLog[5, -E^(I*(a + b*x))] + 24*d^4*PolyLog[5, E^(I*(a + b*x))])/(2*b
^5) - (Csc[a + b*x]^2*(b*c^4*Cos[a + b*x] + 4*b*c^3*d*x*Cos[a + b*x] + 6*b*
c^2*d^2*x^2*Cos[a + b*x] + 4*b*c*d^3*x^3*Cos[a + b*x] + b*d^4*x^4*Cos[a + b
*x] + 4*c^3*d*Sin[a + b*x] + 12*c^2*d^2*x*Sin[a + b*x] + 12*c*d^3*x^2*Sin[a
 + b*x] + 4*d^4*x^3*Sin[a + b*x]))/(2*b^2)
```

**fricas [C]** time = 2.06, size = 2762, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x +
 b^4*c^4)*cos(b*x + a) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c
^3*d + 24*I*b*c*d^3 + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d
 - 24*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 - 12*I*(b^
3*c^2*d^2 - 2*b*d^4)*x)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (4*I*b^3*d^4
*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d - 24*I*b*c*d^3 + (-4*I*b^3*d^4*x^
3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 24*I*b*c*d^3 - 12*I*(b^3*c^2*d^2 -
 2*b*d^4)*x)*cos(b*x + a)^2 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*dilog(cos(b*x
 + a) - I*sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*
c^3*d + 24*I*b*c*d^3 + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*
d - 24*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 - 12*I*(b
^3*c^2*d^2 - 2*b*d^4)*x)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (4*I*b^3*d
^4*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d - 24*I*b*c*d^3 + (-4*I*b^3*d^4*
x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 24*I*b*c*d^3 - 12*I*(b^3*c^2*d^2
 - 2*b*d^4)*x)*cos(b*x + a)^2 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*dilog(-cos(
b*x + a) - I*sin(b*x + a)) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*
b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*
x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c
^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*log(
cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c
```

```
^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^
4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 +
4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)
*x)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*(
a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4 - 12*a^2)*d^4 - (b^4*c^
4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4
- 12*a^2)*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) +
 1/2) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*
b*c*d^3 + (a^4 - 12*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c
^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4 - 12*a^2)*d^4)*cos(b*x + a)^2)*log(-1
/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^
3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a
^2)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3
+ 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2
)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*co
s(b*x + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*log(-cos(b*x + a) + I*sin(b*x
 + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2
*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*
d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d
^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^
4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(b^4*c^3*d - 6*b
^2*c*d^3)*x)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 24*(d^4*cos(b*x + a)
^2 - d^4)*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*(d^4*cos(b*x + a)^
2 - d^4)*polylog(5, cos(b*x + a) - I*sin(b*x + a)) - 24*(d^4*cos(b*x + a)^2
 - d^4)*polylog(5, -cos(b*x + a) + I*sin(b*x + a)) - 24*(d^4*cos(b*x + a)^2
 - d^4)*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b
*c*d^3 + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x + a)^2)*polylog(4, cos(b*x
+ a) + I*sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3 + (24*I*b*d^4*x + 24
*I*b*c*d^3)*cos(b*x + a)^2)*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + (24
*I*b*d^4*x + 24*I*b*c*d^3 + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x + a)^2)*
polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3
+ (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)^2)*polylog(4, -cos(b*x + a) -
I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4 - (
b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4)*cos(b*x + a)^2)*polylog(
3, cos(b*x + a) + I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c
^2*d^2 - 2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4)*cos(b*
x + a)^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b
^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d
^2 - 2*d^4)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12
*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4 - (b^2*d^4*x^2 + 2*b^2*
c*d^3*x + b^2*c^2*d^2 - 2*d^4)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) - I
*sin(b*x + a)) + 8*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c
^3*d)*sin(b*x + a))/(b^5*cos(b*x + a)^2 - b^5)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \cot(bx+a)^2 \csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cot(b*x + a)^2*csc(b*x + a), x)
```

**maple [B]** time = 0.19, size = 1673, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x)
```

```
[Out] 6/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))-6/b^3*c^2*d^2*polylog(3,exp(I*(b*x
+a)))+1/2/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))-6/b^3*d^4*polylog(3,exp(I*(b*x+a
)))*x^2+6/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2-12*d^4*polylog(3,-exp(I*(b
*x+a)))/b^5+12*d^4*polylog(3,exp(I*(b*x+a)))/b^5-12*d^4*polylog(5,-exp(I*(b
*x+a)))/b^5+12*d^4*polylog(5,exp(I*(b*x+a)))/b^5+1/b*c^4*arctanh(exp(I*(b*x
+a)))-6/b^5*d^4*a^2*ln(1-exp(I*(b*x+a)))+6/b^3*d^4*ln(1-exp(I*(b*x+a)))*x^2
-6/b^3*d^4*ln(exp(I*(b*x+a))+1)*x^2+2/b^2*c^3*d*ln(exp(I*(b*x+a))+1)*a-4/b^
4*c*d^3*a^3*arctanh(exp(I*(b*x+a)))+6/b^3*c^2*d^2*a^2*arctanh(exp(I*(b*x+a)
))-4/b^2*c^3*d*a*arctanh(exp(I*(b*x+a)))-3/b^3*c^2*d^2*ln(exp(I*(b*x+a))+1)
*a^2+1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^4*x^4*b*exp(3*I*(b*x+a))+4*c*d^3*x^3*b
*exp(3*I*(b*x+a))+6*c^2*d^2*x^2*b*exp(3*I*(b*x+a))+d^4*x^4*b*exp(I*(b*x+a))
+4*c^3*d*x*b*exp(3*I*(b*x+a))+4*c*d^3*x^3*b*exp(I*(b*x+a))-4*I*d^4*x^3*exp(
3*I*(b*x+a))+c^4*b*exp(3*I*(b*x+a))+6*c^2*d^2*x^2*b*exp(I*(b*x+a))-12*I*c^2
*d^2*x*exp(3*I*(b*x+a))+4*c^3*d*x*b*exp(I*(b*x+a))+4*I*d^4*x^3*exp(I*(b*x+a
))+4*I*c^3*d*exp(I*(b*x+a))+c^4*b*exp(I*(b*x+a))-12*I*c*d^3*x^2*exp(3*I*(b*
x+a))+12*I*c*d^3*x^2*exp(I*(b*x+a))-4*I*c^3*d*exp(3*I*(b*x+a))+12*I*c^2*d^2
*x*exp(I*(b*x+a)))+24/b^4*c*d^3*a*arctanh(exp(I*(b*x+a)))-1/2/b^5*d^4*a^4*l
n(exp(I*(b*x+a))+1)+1/b^5*d^4*a^4*arctanh(exp(I*(b*x+a)))+2/b*c^3*d*ln(exp(
I*(b*x+a))+1)*x-2/b*c^3*d*ln(1-exp(I*(b*x+a)))*x-2/b^2*c^3*d*ln(1-exp(I*(b*
x+a)))*a+3/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+12/b^3*c*d^3*polylog(3,-exp(I
*(b*x+a)))*x+3/b^3*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))-3/b*c^2*d^2*ln(1-exp(I*
(b*x+a)))*x^2-12/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x-1/2/b*d^4*ln(1-exp(I
*(b*x+a)))*x^4+1/2/b*d^4*ln(exp(I*(b*x+a))+1)*x^4+2/b*c*d^3*ln(exp(I*(b*x+a
))+1)*x^3-2/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3-2/b^4*c*d^3*ln(1-exp(I*(b*x+a)
))*a^3+2/b^4*c*d^3*ln(exp(I*(b*x+a))+1)*a^3-12/b^3*d^3*c*ln(exp(I*(b*x+a))+
1)*x+12/b^3*d^3*c*ln(1-exp(I*(b*x+a)))*x+12/b^4*d^3*c*ln(1-exp(I*(b*x+a)))*
a-12/b^4*c*d^3*ln(exp(I*(b*x+a))+1)*a-2*I/b^2*d^4*polylog(2,-exp(I*(b*x+a))
)*x^3+12*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x+2*I/b^2*d^4*polylog(2,exp(I
*(b*x+a)))*x^3-12*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x+12*I/b^4*d^4*polylo
g(2,-exp(I*(b*x+a)))*x+12*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))-12*I/b^4*c
*d^3*polylog(4,exp(I*(b*x+a)))-2*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+2*I
/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))+12*I/b^4*c*d^3*polylog(2,-exp(I*(b*x+a
)))+6/b^5*d^4*a^2*ln(exp(I*(b*x+a))+1)-12/b^3*c^2*d^2*arctanh(exp(I*(b*x+a)
))-12/b^5*d^4*a^2*arctanh(exp(I*(b*x+a)))-12*I/b^4*d^3*c*polylog(2,exp(I*(b
*x+a)))-12*I/b^4*d^4*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^3*polylog(2,-e
xp(I*(b*x+a)))*x^2+6*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-6*I/b^2*poly
log(2,-exp(I*(b*x+a)))*c^2*d^2*x+6*I/b^2*polylog(2,exp(I*(b*x+a)))*c^2*d^2*
x
```

**maxima [B]** time = 5.49, size = 6952, normalized size = 16.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(c^4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1)) - 4*a*c^3*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(
cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + 6*a^2*c^2*d^2*(2*cos(b*x + a
)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^2
 - 4*a^3*c*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1)
 - log(cos(b*x + a) - 1))/b^3 + a^4*d^4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1
) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^4 + 4*((2*(b*x + a)^4*
d^4 - 24*b^2*c^2*d^2 + 48*a*b*c*d^3 - 24*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x
 + a)^3 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 8*(b
^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x +
a) + 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b
*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)
*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3
- 6*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 4*((b*x + a)^4*d^4 - 12*b^2*c^2*d
```

```
^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c
^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*
c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a
) + (2*I*(b*x + a)^4*d^4 - 24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a^2*d^4
 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d
^3 + (12*I*a^2 - 24*I)*d^4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d
^2 + (24*I*a^2 - 48*I)*b*c*d^3 + (-8*I*a^3 + 48*I*a)*d^4)*(b*x + a))*sin(4*
b*x + 4*a) + (-4*I*(b*x + a)^4*d^4 + 48*I*b^2*c^2*d^2 - 96*I*a*b*c*d^3 + 48
*I*a^2*d^4 + (-16*I*b*c*d^3 + 16*I*a*d^4)*(b*x + a)^3 + (-24*I*b^2*c^2*d^2
+ 48*I*a*b*c*d^3 + (-24*I*a^2 + 48*I)*d^4)*(b*x + a)^2 + (-16*I*b^3*c^3*d +
 48*I*a*b^2*c^2*d^2 + (-48*I*a^2 + 96*I)*b*c*d^3 + (16*I*a^3 - 96*I*a)*d^4)
*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (24
*b^2*c^2*d^2 - 48*a*b*c*d^3 + 24*a^2*d^4 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*cos(4*b*x + 4*a) - 48*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(2*
b*x + 2*a) + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^2*d^4)*sin(4*b*x +
 4*a) + (-48*I*b^2*c^2*d^2 + 96*I*a*b*c*d^3 - 48*I*a^2*d^4)*sin(2*b*x + 2*a
))*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*(b*x + a)^4*d^4 + 8*(b*c*d^
3 - a*d^4)*(b*x + a)^3 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*
x + a)^2 + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*
a)*d^4)*(b*x + a) + 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 +
6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d -
3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(4*b
*x + 4*a) - 4*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c
^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*
c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a
) + (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^
2*c^2*d^2 - 24*I*a*b*c*d^3 + (12*I*a^2 - 24*I)*d^4)*(b*x + a)^2 + (8*I*b^3*
c^3*d - 24*I*a*b^2*c^2*d^2 + (24*I*a^2 - 48*I)*b*c*d^3 + (-8*I*a^3 + 48*I*a
)*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^4*d^4 + (-16*I*b*c*d^3
 + 16*I*a*d^4)*(b*x + a)^3 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 + (-24*I*a
^2 + 48*I)*d^4)*(b*x + a)^2 + (-16*I*b^3*c^3*d + 48*I*a*b^2*c^2*d^2 + (-48*
I*a^2 + 96*I)*b*c*d^3 + (16*I*a^3 - 96*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a
))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (-4*I*(b*x + a)^4*d^4 - 16*b^
3*c^3*d + 48*a*b^2*c^2*d^2 - 48*a^2*b*c*d^3 + 16*a^3*d^4 - 16*(I*b*c*d^3 +
(-I*a + 1)*d^4)*(b*x + a)^3 + (-24*I*b^2*c^2*d^2 - 48*(-I*a + 1)*b*c*d^3 +
(-24*I*a^2 + 48*a)*d^4)*(b*x + a)^2 + (-16*I*b^3*c^3*d - 48*(-I*a + 1)*b^2*
c^2*d^2 + (-48*I*a^2 + 96*a)*b*c*d^3 + (16*I*a^3 - 48*a^2)*d^4)*(b*x + a))*
cos(3*b*x + 3*a) + (-4*I*(b*x + a)^4*d^4 + 16*b^3*c^3*d - 48*a*b^2*c^2*d^2
+ 48*a^2*b*c*d^3 - 16*a^3*d^4 + (-16*I*b*c*d^3 - 16*(-I*a - 1)*d^4)*(b*x +
a)^3 + (-24*I*b^2*c^2*d^2 - 48*(-I*a - 1)*b*c*d^3 + (-24*I*a^2 - 48*a)*d^4)
*(b*x + a)^2 + (-16*I*b^3*c^3*d - 48*(-I*a - 1)*b^2*c^2*d^2 + (-48*I*a^2 -
96*a)*b*c*d^3 + (16*I*a^3 + 48*a^2)*d^4)*(b*x + a))*cos(b*x + a) - (8*b^3*c
^3*d - 24*a*b^2*c^2*d^2 + 8*(b*x + a)^3*d^4 + 24*(a^2 - 2)*b*c*d^3 - 8*(a^3
 - 6*a)*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*
d^3 + (a^2 - 2)*d^4)*(b*x + a) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)
^3*d^4 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x +
 a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*cos(4*b*x
+ 4*a) - 16*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 2)*b*
c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2
- 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-8*I*b^3*c^3*
d + 24*I*a*b^2*c^2*d^2 - 8*I*(b*x + a)^3*d^4 + (-24*I*a^2 + 48*I)*b*c*d^3 +
 (8*I*a^3 - 48*I*a)*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a)^2 + (-24*I
*b^2*c^2*d^2 + 48*I*a*b*c*d^3 + (-24*I*a^2 + 48*I)*d^4)*(b*x + a))*sin(4*b*
x + 4*a) - (16*I*b^3*c^3*d - 48*I*a*b^2*c^2*d^2 + 16*I*(b*x + a)^3*d^4 + (4
8*I*a^2 - 96*I)*b*c*d^3 + (-16*I*a^3 + 96*I*a)*d^4 + (48*I*b*c*d^3 - 48*I*a
*d^4)*(b*x + a)^2 + (48*I*b^2*c^2*d^2 - 96*I*a*b*c*d^3 + (48*I*a^2 - 96*I)*
d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (8*b^3*c^3*d -
24*a*b^2*c^2*d^2 + 8*(b*x + a)^3*d^4 + 24*(a^2 - 2)*b*c*d^3 - 8*(a^3 - 6*a)
*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (
```

```
a^2 - 2)*d^4)*(b*x + a) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4
+ 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 +
 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*cos(4*b*x + 4*a)
- 16*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 2)*b*c*d^3 -
 (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (8*I*b^3*c^3*d - 24*I
*a*b^2*c^2*d^2 + 8*I*(b*x + a)^3*d^4 + (24*I*a^2 - 48*I)*b*c*d^3 + (-8*I*a^
3 + 48*I*a)*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^2 + (24*I*b^2*c^2*d
^2 - 48*I*a*b*c*d^3 + (24*I*a^2 - 48*I)*d^4)*(b*x + a))*sin(4*b*x + 4*a) +
(-16*I*b^3*c^3*d + 48*I*a*b^2*c^2*d^2 - 16*I*(b*x + a)^3*d^4 + (-48*I*a^2 +
 96*I)*b*c*d^3 + (16*I*a^3 - 96*I*a)*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*
x + a)^2 + (-48*I*b^2*c^2*d^2 + 96*I*a*b*c*d^3 + (-48*I*a^2 + 96*I)*d^4)*(b
*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)^4*d^4 + 1
2*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4 + (-4*I*b*c*d^3 + 4*I*a*d^4
)*(b*x + a)^3 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 + (-6*I*a^2 + 12*I)*d^4)
*(b*x + a)^2 + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 + (-12*I*a^2 + 24*I)*b*
c*d^3 + (4*I*a^3 - 24*I*a)*d^4)*(b*x + a) + (-I*(b*x + a)^4*d^4 + 12*I*b^2*
c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4 + (-4*I*b*c*d^3 + 4*I*a*d^4)*(b*x +
 a)^3 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 + (-6*I*a^2 + 12*I)*d^4)*(b*x +
a)^2 + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 + (-12*I*a^2 + 24*I)*b*c*d^3 +
(4*I*a^3 - 24*I*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (2*I*(b*x + a)^4*d^4
- 24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a^2*d^4 + (8*I*b*c*d^3 - 8*I*a*d
^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + (12*I*a^2 - 24*I)*d^
4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + (24*I*a^2 - 48*I)*b*
c*d^3 + (-8*I*a^3 + 48*I*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^4
*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*
x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b
^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x +
a))*sin(4*b*x + 4*a) - 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 -
 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^
3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 -
2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^4*d^4 - 12*I*b^2*c
^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4 + (4*I*b*c*d^3 - 4*I*a*d^4)*(b*x + a
)^3 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 + (6*I*a^2 - 12*I)*d^4)*(b*x + a)^2
 + (4*I*b^3*c^3*d - 12*I*a*b^2*c^2*d^2 + (12*I*a^2 - 24*I)*b*c*d^3 + (-4*I*
a^3 + 24*I*a)*d^4)*(b*x + a) + (I*(b*x + a)^4*d^4 - 12*I*b^2*c^2*d^2 + 24*I
*a*b*c*d^3 - 12*I*a^2*d^4 + (4*I*b*c*d^3 - 4*I*a*d^4)*(b*x + a)^3 + (6*I*b^
2*c^2*d^2 - 12*I*a*b*c*d^3 + (6*I*a^2 - 12*I)*d^4)*(b*x + a)^2 + (4*I*b^3*c
^3*d - 12*I*a*b^2*c^2*d^2 + (12*I*a^2 - 24*I)*b*c*d^3 + (-4*I*a^3 + 24*I*a)
*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^4*d^4 + 24*I*b^2*c^2*d^
2 - 48*I*a*b*c*d^3 + 24*I*a^2*d^4 + (-8*I*b*c*d^3 + 8*I*a*d^4)*(b*x + a)^3
+ (-12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 + (-12*I*a^2 + 24*I)*d^4)*(b*x + a)^2
 + (-8*I*b^3*c^3*d + 24*I*a*b^2*c^2*d^2 + (-24*I*a^2 + 48*I)*b*c*d^3 + (8*I
*a^3 - 48*I*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^4*d^4 - 12*b^2
*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*
(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*
a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*sin(4*b*x
 + 4*a) + 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 +
 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)
*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 -
(a^3 - 6*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 - 2*cos(b*x + a) + 1) + (48*I*d^4*cos(4*b*x + 4*a) - 96*I*d^4*cos(2*
b*x + 2*a) - 48*d^4*sin(4*b*x + 4*a) + 96*d^4*sin(2*b*x + 2*a) + 48*I*d^4)*
polylog(5, -e^(I*b*x + I*a)) + (-48*I*d^4*cos(4*b*x + 4*a) + 96*I*d^4*cos(2
*b*x + 2*a) + 48*d^4*sin(4*b*x + 4*a) - 96*d^4*sin(2*b*x + 2*a) - 48*I*d^4)
*polylog(5, e^(I*b*x + I*a)) + (48*b*c*d^3 + 48*(b*x + a)*d^4 - 48*a*d^4 +
48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) - 96*(b*c*d^3 + (b*x
+ a)*d^4 - a*d^4)*cos(2*b*x + 2*a) + (48*I*b*c*d^3 + 48*I*(b*x + a)*d^4 - 4
```

```
8*I*a*d^4)*sin(4*b*x + 4*a) + (-96*I*b*c*d^3 - 96*I*(b*x + a)*d^4 + 96*I*a*
d^4)*sin(2*b*x + 2*a))*polylog(4, -e^(I*b*x + I*a)) - (48*b*c*d^3 + 48*(b*x
 + a)*d^4 - 48*a*d^4 + 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a
) - 96*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) - (-48*I*b*c*d^3
- 48*I*(b*x + a)*d^4 + 48*I*a*d^4)*sin(4*b*x + 4*a) - (96*I*b*c*d^3 + 96*I*
(b*x + a)*d^4 - 96*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, e^(I*b*x + I*a)) +
 (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*(b*x + a)^2*d^4 + (-24*I*a^2 +
48*I)*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a) + (-24*I*b^2*c^2*d^2 + 4
8*I*a*b*c*d^3 - 24*I*(b*x + a)^2*d^4 + (-24*I*a^2 + 48*I)*d^4 + (-48*I*b*c*
d^3 + 48*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (48*I*b^2*c^2*d^2 - 96*I*a*
b*c*d^3 + 48*I*(b*x + a)^2*d^4 + (48*I*a^2 - 96*I)*d^4 + (96*I*b*c*d^3 - 96
*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*
x + a)^2*d^4 + (a^2 - 2)*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x + 4
*a) - 48*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + (a^2 - 2)*d^4 + 2*(
b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a))
+ (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*(b*x + a)^2*d^4 + (24*I*a^2 - 4
8*I)*d^4 + (48*I*b*c*d^3 - 48*I*a*d^4)*(b*x + a) + (24*I*b^2*c^2*d^2 - 48*I
*a*b*c*d^3 + 24*I*(b*x + a)^2*d^4 + (24*I*a^2 - 48*I)*d^4 + (48*I*b*c*d^3 -
 48*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (-48*I*b^2*c^2*d^2 + 96*I*a*b*c*
d^3 - 48*I*(b*x + a)^2*d^4 + (-48*I*a^2 + 96*I)*d^4 + (-96*I*b*c*d^3 + 96*I
*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x
+ a)^2*d^4 + (a^2 - 2)*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x + 4*a
) + 48*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + (a^2 - 2)*d^4 + 2*(b*
c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) + (
4*(b*x + a)^4*d^4 - 16*I*b^3*c^3*d + 48*I*a*b^2*c^2*d^2 - 48*I*a^2*b*c*d^3
+ 16*I*a^3*d^4 + (16*b*c*d^3 - (16*a + 16*I)*d^4)*(b*x + a)^3 + (24*b^2*c^2
*d^2 - (48*a + 48*I)*b*c*d^3 + 24*(a^2 + 2*I*a)*d^4)*(b*x + a)^2 + (16*b^3*
c^3*d - (48*a + 48*I)*b^2*c^2*d^2 + 48*(a^2 + 2*I*a)*b*c*d^3 - 16*(a^3 + 3*
I*a^2)*d^4)*(b*x + a))*sin(3*b*x + 3*a) + (4*(b*x + a)^4*d^4 + 16*I*b^3*c^3
*d - 48*I*a*b^2*c^2*d^2 + 48*I*a^2*b*c*d^3 - 16*I*a^3*d^4 + (16*b*c*d^3 - (
16*a - 16*I)*d^4)*(b*x + a)^3 + (24*b^2*c^2*d^2 - (48*a - 48*I)*b*c*d^3 + 2
4*(a^2 - 2*I*a)*d^4)*(b*x + a)^2 + (16*b^3*c^3*d - (48*a - 48*I)*b^2*c^2*d^
2 + 48*(a^2 - 2*I*a)*b*c*d^3 - 16*(a^3 - 3*I*a^2)*d^4)*(b*x + a))*sin(b*x +
 a))/(-4*I*b^4*cos(4*b*x + 4*a) + 8*I*b^4*cos(2*b*x + 2*a) + 4*b^4*sin(4*b*
x + 4*a) - 8*b^4*sin(2*b*x + 2*a) - 4*I*b^4))/b
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x)^4)/sin(a + b*x),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*cot(a + b*x)**2*csc(a + b*x), x)
```

# 3.113 $\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx$

**Optimal**. Leaf size=308

$$\frac{3id^3\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_2\left(e^{i(a+bx)}\right)}{b^4}+\frac{3id^3\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}-\frac{3d^2(c}{}$$

```
[Out] -6*d^2*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^3+(d*x+c)^3*arctanh(exp(I*(b*x+a))
)/b-3/2*d*(d*x+c)^2*csc(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)*csc(b*x+a)/b+3*
I*d^3*polylog(2,-exp(I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*
x+a)))/b^2-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(
2,exp(I*(b*x+a)))/b^2+3*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^3-3*d^2*(d
*x+c)*polylog(3,exp(I*(b*x+a)))/b^3+3*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4-
3*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4
```

**Rubi [A]** time = 0.34, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {4415, 4183, 2531, 6609, 2282, 6589, 4186, 2279, 2391}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}-\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}+\frac{3id(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^3 + ((c + d*x)^3*ArcTanh[E^(I
*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot
[a + b*x]*Csc[a + b*x])/(2*b) + ((3*I)*d^3*PolyLog[2, -E^(I*(a + b*x))])/b^
4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d^3
*PolyLog[2, E^(I*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^(
I*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 - (
3*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[4, -E
^(I*(a + b*x))])/b^4 - ((3*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4415

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \cot^2(a+bx) \csc(a+bx)\,dx = -\int (c+dx)^3 \csc(a+bx)\,dx + \int (c+dx)^3 \csc^3(a+bx)\,dx$$

$$= \frac{2(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2 \csc(a+bx)}{2b^2} - \frac{(c+dx)^3 \text{co}}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+}{}$$

**Mathematica** [A] time = 4.74, size = 528, normalized size = 1.71

$$-\frac{b^3c^3\log\left(1-e^{i(a+bx)}\right) - b^3c^3\log\left(1+e^{i(a+bx)}\right) + 3b^3c^2dx\log\left(1-e^{i(a+bx)}\right) - 3b^3c^2dx\log\left(1+e^{i(a+bx)}\right) + 3b^3cd^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -1/2*(b^2*(c + d*x)^2*(3*d + b*(c + d*x)*Cot[a + b*x])*Csc[a + b*x] + b^3*c
^3*Log[1 - E^(I*(a + b*x))] - 6*b*c*d^2*Log[1 - E^(I*(a + b*x))] + 3*b^3*c^
2*d*x*Log[1 - E^(I*(a + b*x))] - 6*b*d^3*x*Log[1 - E^(I*(a + b*x))] + 3*b^3
*c*d^2*x^2*Log[1 - E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 - E^(I*(a + b*x))]
- b^3*c^3*Log[1 + E^(I*(a + b*x))] + 6*b*c*d^2*Log[1 + E^(I*(a + b*x))] - 3
*b^3*c^2*d*x*Log[1 + E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 + E^(I*(a + b*x))]
- 3*b^3*c*d^2*x^2*Log[1 + E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + E^(I*(a +
b*x))] + (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*x))] -
(3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))] - 6*b*c*d^2*
PolyLog[3, -E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, -E^(I*(a + b*x))] + 6*b
*c*d^2*PolyLog[3, E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, E^(I*(a + b*x))]
- (6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, E^(I*(a + b
*x))])/b^4
```

**fricas** [C] time = 1.19, size = 1734, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x +
a) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3 + (3*I*b
^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2)*dil
og(cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*
I*b^2*c^2*d - 6*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d
 + 6*I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^
2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3 + (3*I*b^2*d^3*x^2 +
6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x +
```

```
a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d
- 6*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3)*
cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^
3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d -
2*b*d^3)*x)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 -
 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b
*d^3)*x)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2
 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/
2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 -
(a^3 - 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6
*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)
+ (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6
*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
(a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d
- 2*b*d^3)*x)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 - (b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(
b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*log(-co
s(b*x + a) - I*sin(b*x + a) + 1) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*poly
log(4, cos(b*x + a) + I*sin(b*x + a)) + (6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*
polylog(4, cos(b*x + a) - I*sin(b*x + a)) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*
d^3)*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + (6*I*d^3*cos(b*x + a)^2 -
 6*I*d^3)*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2
 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a) + I*sin(b*x
+ a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(
3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d
^2)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x
 + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) -
 I*sin(b*x + a)) + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(b*x + a)
)/(b^4*cos(b*x + a)^2 - b^4)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cot(bx+a)^2 \csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cot(b*x + a)^2*csc(b*x + a), x)
```

**maple [B]** time = 0.14, size = 1056, normalized size = 3.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x)
```

```
[Out] 3*I*d^3*polylog(2,-exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(4,-exp(I*(b*x+a)))/b
^4+3/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))-3/b^3*c*d^2*polylog(3,exp(I*(b*x+
a)))-3/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+3/b^3*d^3*polylog(3,-exp(I*(b*x+
a)))*x-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(4,exp(I*(b*x+a
)))/b^4+1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^3*x^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^
2*b*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))+d^3*x^3*b*exp(I*(b*x+a))+
c^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(3*I*(b*
x+a))+3*c^2*d*x*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(3*I*(b*x+a))+c^3*b*exp(I*(
b*x+a))-3*I*c^2*d*exp(3*I*(b*x+a))+3*I*d^3*x^2*exp(I*(b*x+a))+6*I*c*d^2*x*e
```

```
xp(I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a)))+1/b*c^3*arctanh(exp(I*(b*x+a)))-3/b
^3*d^3*ln(exp(I*(b*x+a))+1)*x+3/b^3*d^3*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln
(1-exp(I*(b*x+a)))*a-1/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))+3/2/b^2*c^2*d*ln
(exp(I*(b*x+a))+1)*a-3/2/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))+1)+1/2/b^4*d^3*ln(
exp(I*(b*x+a))+1)*a^3+3/2*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+3/b^3*c*d
^2*a^2*arctanh(exp(I*(b*x+a)))-3/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))-3/2*I/
b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3/2*I/b^2*c^2*d*polylog(2,-exp(I*(b*
x+a)))+3/2*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+3/2/b*c^2*d*ln(exp(I*(b*x+
a))+1)*x-3/2/b*c^2*d*ln(1-exp(I*(b*x+a)))*x-3/2/b^2*c^2*d*ln(1-exp(I*(b*x+a
)))*a+3/2/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))-3/2/b*c*d^2*ln(1-exp(I*(b*x+a)
))*x^2+3/2/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-1/2/b*d^3*ln(1-exp(I*(b*x+a)))*
x^3-1/2/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+1/2/b*d^3*ln(exp(I*(b*x+a))+1)*x^3
-3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a+6/b^4*d^3*a*arctanh(exp(I*(b*x+a)))-6/b^3
*c*d^2*arctanh(exp(I*(b*x+a)))-3*I/b^2*polylog(2,-exp(I*(b*x+a)))*c*d^2*x+3
*I/b^2*polylog(2,exp(I*(b*x+a)))*c*d^2*x
```

**maxima [B]** time = 1.92, size = 3872, normalized size = 12.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(c^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(
cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*cos(b*x + a)/
(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^2 -
 a^3*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1))/b^3 + 4*((2*(b*x + a)^3*d^3 - 12*b*c*d^2 + 12*a*d^3 + 6
*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3
)*(b*x + a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(
4*b*x + 4*a) - 4*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos
(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (6*I*b*c
*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6*I*a^2
- 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^3*d^3 + 24*I*b*c
*d^2 - 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c
^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a)
)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (12*b*c*d^2 - 12*a*d^3 + 12*(b*
c*d^2 - a*d^3)*cos(4*b*x + 4*a) - 24*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) + (
12*I*b*c*d^2 - 12*I*a*d^3)*sin(4*b*x + 4*a) + (-24*I*b*c*d^2 + 24*I*a*d^3)*
sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*(b*x + a)^3*
d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 -
 2)*d^3)*(b*x + a) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) -
4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b
*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3
+ (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 +
 (6*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^3*d^3
+ (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*
d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x
+ a), -cos(b*x + a) + 1) + (-4*I*(b*x + a)^3*d^3 - 12*b^2*c^2*d + 24*a*b*c*
d^2 - 12*a^2*d^3 - 12*(I*b*c*d^2 + (-I*a + 1)*d^3)*(b*x + a)^2 + (-12*I*b^2
*c^2*d - 24*(-I*a + 1)*b*c*d^2 + (-12*I*a^2 + 24*a)*d^3)*(b*x + a))*cos(3*b
*x + 3*a) + (-4*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^
3 + (-12*I*b*c*d^2 - 12*(-I*a - 1)*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d - 24
*(-I*a - 1)*b*c*d^2 + (-12*I*a^2 - 24*a)*d^3)*(b*x + a))*cos(b*x + a) - (6*
b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^
2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2
```

```
- 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 12*(b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
+ a))*cos(2*b*x + 2*a) - (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2
*d^3 + (-6*I*a^2 + 12*I)*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(
4*b*x + 4*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + (1
2*I*a^2 - 24*I)*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*
a))*dilog(-e^(I*b*x + I*a)) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d
^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*
b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(4*b*x + 4*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2
)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*b^2*c^2*d -
12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + (6*I*a^2 - 12*I)*d^3 + (12*I*b*c*d^2
 - 12*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-12*I*b^2*c^2*d + 24*I*a*b*c*
d^2 - 12*I*(b*x + a)^2*d^3 + (-12*I*a^2 + 24*I)*d^3 + (-24*I*b*c*d^2 + 24*I
*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)
^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 +
 (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 + (-3*I*a^2 + 6*I)*d^3)*(b*x + a) + (-I*(b
*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x +
 a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 + (-3*I*a^2 + 6*I)*d^3)*(b*x + a))*
cos(4*b*x + 4*a) + (2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (6*I*
b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6*I*a
^2 - 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 - 6*b*c*d^2
+ 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 +
(a^2 - 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 - 6*b*c*d^2
 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 +
 (a^2 - 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3
+ (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 +
(3*I*a^2 - 6*I)*d^3)*(b*x + a) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d
^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2
 + (3*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^3*d^3
 + 12*I*b*c*d^2 - 12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6
*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a))*cos(2*b*x
 + 2*a) - ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
 + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(4*b*x
+ 4*a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*
x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(2*b*x
 + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (12*d^
3*cos(4*b*x + 4*a) - 24*d^3*cos(2*b*x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a) -
24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)*polylog(4, -e^(I*b*x + I*a)) - (12*d^3*
cos(4*b*x + 4*a) - 24*d^3*cos(2*b*x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a) - 24
*I*d^3*sin(2*b*x + 2*a) + 12*d^3)*polylog(4, e^(I*b*x + I*a)) + (-12*I*b*c*
d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3
 + 12*I*a*d^3)*cos(4*b*x + 4*a) + (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I
*a*d^3)*cos(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x +
 4*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -
e^(I*b*x + I*a)) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*
b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(4*b*x + 4*a) + (-24*I*b*c*d^
2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x
+ a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*s
in(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) + (4*(b*x + a)^3*d^3 - 12*I*b^
2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (12*b*c*d^2 - (12*a + 12*I)*d^3)*
(b*x + a)^2 + (12*b^2*c^2*d - (24*a + 24*I)*b*c*d^2 + 12*(a^2 + 2*I*a)*d^3)
*(b*x + a))*sin(3*b*x + 3*a) + (4*(b*x + a)^3*d^3 + 12*I*b^2*c^2*d - 24*I*a
*b*c*d^2 + 12*I*a^2*d^3 + (12*b*c*d^2 - (12*a - 12*I)*d^3)*(b*x + a)^2 + (1
2*b^2*c^2*d - (24*a - 24*I)*b*c*d^2 + 12*(a^2 - 2*I*a)*d^3)*(b*x + a))*sin(
b*x + a))/(-4*I*b^3*cos(4*b*x + 4*a) + 8*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin
(4*b*x + 4*a) - 8*b^3*sin(2*b*x + 2*a) - 4*I*b^3))/b
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x)^3)/sin(a + b*x),x)
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cot(b*x+a)**2*csc(b*x+a),x)
[Out] Integral((c + d*x)**3*cot(a + b*x)**2*csc(a + b*x), x)
```

# 3.114 $\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx) \, dx$

**Optimal**. Leaf size=179

$$\frac{d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}-\frac{d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{d(c$$

```
[Out] (d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-d^2*arctanh(cos(b*x+a))/b^3-d*(d*x+c)*c
sc(b*x+a)/b^2-1/2*(d*x+c)^2*cot(b*x+a)*csc(b*x+a)/b-I*d*(d*x+c)*polylog(2,-
exp(I*(b*x+a)))/b^2+I*d*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^2+d^2*polylog(3
,-exp(I*(b*x+a)))/b^3-d^2*polylog(3,exp(I*(b*x+a)))/b^3
```

**Rubi [A]** time = 0.22, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4415, 4183, 2531, 2282, 6589, 4186, 3770}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}-\frac{d^2\text{PolyLog}\left(3,e^{i(a-}\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] ((c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]])/b^3
- (d*(c + d*x)*Csc[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]*Csc[a + b*x])/
(2*b) - (I*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 + (I*d*(c + d*x)*P
olyLog[2, E^(I*(a + b*x))])/b^2 + (d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 -
(d^2*PolyLog[3, E^(I*(a + b*x))])/b^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4415

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)^2 \csc(a+bx)\,dx + \int (c+dx)^2 \csc^3(a+bx)\,dx$$

$$= \frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d(c+dx)\csc(a+bx)}{b^2} - \frac{(c+dx)^2 \cot(}{}$$

$$= \frac{(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc}{b^2}$$

$$= \frac{(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc}{b^2}$$

$$= \frac{(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc}{b^2}$$

$$= \frac{(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2 \tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc}{b^2}$$

**Mathematica [B]** time = 7.58, size = 471, normalized size = 2.63

$$\frac{\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\left(cd\sin\left(\frac{bx}{2}\right)+d^2x\sin\left(\frac{bx}{2}\right)\right)}{2b^2}+\frac{\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\left(d^2(-x)\sin\left(\frac{bx}{2}\right)-cd\sin\left(\frac{bx}{2}\right)\right)}{2b^2}-\frac{d\csc(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -((d*(c + d*x)*Csc[a])/b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/2
]^2)/(8*b) + (-(b^2*c^2*Log[1 - E^(I*(a + b*x))]) + 2*d^2*Log[1 - E^(I*(a +
 b*x))] - 2*b^2*c*d*x*Log[1 - E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 - E^(I*(
a + b*x))] + b^2*c^2*Log[1 + E^(I*(a + b*x))] - 2*d^2*Log[1 + E^(I*(a + b*x
))] + 2*b^2*c*d*x*Log[1 + E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 + E^(I*(a +
b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + (2*I)*b*d*(c +
d*x)*PolyLog[2, E^(I*(a + b*x))] + 2*d^2*PolyLog[3, -E^(I*(a + b*x))] - 2*d
^2*PolyLog[3, E^(I*(a + b*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a/
2 + (b*x)/2]^2)/(8*b) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*Sin[(b*x)/2]) -
```

```
d^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d*Sin[(b*x)
/2] + d^2*x*Sin[(b*x)/2]))/(2*b^2)
```

**fricas [C]** time = 0.71, size = 966, normalized size = 5.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) + (-2*I*b*d^2*x -
 2*I*b*c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) +
 I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*co
s(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*
c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin
(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x
+ a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 -
2*d^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 -
2*d^2)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + (a^2
 - 2)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2)*log(-1/2*
cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)
*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b
*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*
d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a
)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x +
 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*co
s(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 2*(d^2*cos(b*x + a)
^2 - d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 2*(d^2*cos(b*x + a)^2
 - d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^2 -
 d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^2 -
d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 4*(b*d^2*x + b*c*d)*sin(b
*x + a))/(b^3*cos(b*x + a)^2 - b^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \cot (bx+a)^2 \csc (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cot(b*x + a)^2*csc(b*x + a), x)
```

**maple [B]** time = 0.12, size = 546, normalized size = 3.05

$$\frac{d^2x^2b\,e^{3i(bx+a)}+2cdxb\,e^{3i(bx+a)}+c^2b\,e^{3i(bx+a)}+d^2x^2b\,e^{i(bx+a)}+2cdxb\,e^{i(bx+a)}-2id^2x\,e^{3i(bx+a)}+c^2b\,e^{i(bx+a)}-2idc}{b^2\left(e^{2i(bx+a)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*
(b*x+a))+c^2*b*exp(3*I*(b*x+a))+d^2*x^2*b*exp(I*(b*x+a))+2*c*d*x*b*exp(I*(b
*x+a))-2*I*d^2*x*exp(3*I*(b*x+a))+c^2*b*exp(I*(b*x+a))-2*I*d*c*exp(3*I*(b*x
+a))+2*I*d^2*x*exp(I*(b*x+a))+2*I*d*c*exp(I*(b*x+a)))+1/b*c*d*ln(exp(I*(b*x
+a))+1)*x+1/b^2*c*d*ln(exp(I*(b*x+a))+1)*a-1/b*c*d*ln(1-exp(I*(b*x+a)))*x-1
/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+d^2*polylog(3,-exp(I*(b*x+a)))/b^3-d^2*poly
```

```
log(3,exp(I*(b*x+a)))/b^3-2/b^3*d^2*arctanh(exp(I*(b*x+a)))+1/b*c^2*arctanh
(exp(I*(b*x+a)))+1/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))+1/2/b*d^2*ln(exp(I*(
b*x+a))+1)*x^2-I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-1/2/b*d^2*ln(1-exp(I*(b
*x+a)))*x^2+I/b^2*polylog(2,exp(I*(b*x+a)))*d^2*x-2/b^2*c*d*a*arctanh(exp(I
*(b*x+a)))-I/b^2*polylog(2,-exp(I*(b*x+a)))*d^2*x+I/b^2*c*d*polylog(2,exp(I
*(b*x+a)))-1/2/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2+1/2/b^3*d^2*ln(1-exp(I*(b*x
+a)))*a^2
```

**maxima [B]** time = 0.81, size = 1932, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(c^2*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1)) - 2*a*c*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(co
s(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + a^2*d^2*(2*cos(b*x + a)/(cos(b
*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^2 + 4*((2
*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - 4*d^2 + 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) - 4*((b*x + a)^2*d
^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) + (2*I*(b*x + a)
^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(4*b*x + 4*a) + (-
4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) + 8*I*d^2)*sin(2*b
*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (4*d^2*cos(4*b*x + 4*a
) - 8*d^2*cos(2*b*x + 2*a) + 4*I*d^2*sin(4*b*x + 4*a) - 8*I*d^2*sin(2*b*x +
 2*a) + 4*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*(b*x + a)^2*d^2
 + 4*(b*c*d - a*d^2)*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*
x + a))*cos(4*b*x + 4*a) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)
)*cos(2*b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x +
a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*
x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(I*(
b*x + a)^2*d^2 + 2*b*c*d - 2*a*d^2 + 2*(I*b*c*d + (-I*a + 1)*d^2)*(b*x + a)
)*cos(3*b*x + 3*a) + (-4*I*(b*x + a)^2*d^2 + 8*b*c*d - 8*a*d^2 + (-8*I*b*c*
d - 8*(-I*a - 1)*d^2)*(b*x + a))*cos(b*x + a) - (4*b*c*d + 4*(b*x + a)*d^2
- 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 8*(b*c*d +
 (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2
+ 4*I*a*d^2)*sin(4*b*x + 4*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)
*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (4*b*c*d + 4*(b*x + a)*d^2 - 4
*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 8*(b*c*d + (b
*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*
I*a*d^2)*sin(4*b*x + 4*a) + (-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*si
n(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d
+ 2*I*a*d^2)*(b*x + a) + 2*I*d^2 + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*
a*d^2)*(b*x + a) + 2*I*d^2)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (4*I*
b*c*d - 4*I*a*d^2)*(b*x + a) - 4*I*d^2)*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d
^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a
)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b*c*
d - 2*I*a*d^2)*(b*x + a) - 2*I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*
a*d^2)*(b*x + a) - 2*I*d^2)*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*
I*b*c*d + 4*I*a*d^2)*(b*x + a) + 4*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d
^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2
*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (-4*I*d^2*cos(4*b*x + 4*a) +
 8*I*d^2*cos(2*b*x + 2*a) + 4*d^2*sin(4*b*x + 4*a) - 8*d^2*sin(2*b*x + 2*a)
 - 4*I*d^2)*polylog(3, -e^(I*b*x + I*a)) + (4*I*d^2*cos(4*b*x + 4*a) - 8*I*
d^2*cos(2*b*x + 2*a) - 4*d^2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a) + 4*
I*d^2)*polylog(3, e^(I*b*x + I*a)) + (4*(b*x + a)^2*d^2 - 8*I*b*c*d + 8*I*a
*d^2 + (8*b*c*d - (8*a + 8*I)*d^2)*(b*x + a))*sin(3*b*x + 3*a) + (4*(b*x +
```

```
a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (8*b*c*d - (8*a - 8*I)*d^2)*(b*x + a))*s
in(b*x + a))/(-4*I*b^2*cos(4*b*x + 4*a) + 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*
sin(4*b*x + 4*a) - 8*b^2*sin(2*b*x + 2*a) - 4*I*b^2))/b
```

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x)^2)/sin(a + b*x),x)
```

```
[Out] \text{Hanged}
```

**sympy** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \cot^2 (a + bx) \csc (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cot(a + b*x)**2*csc(a + b*x), x)
```

# 3.115 $\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$

**Optimal**. Leaf size=108

$$-\frac{id\text{Li}_2\left(-e^{i(a+bx)}\right)}{2b^2}+\frac{id\text{Li}_2\left(e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}+\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

```
[Out] (d*x+c)*arctanh(exp(I*(b*x+a)))/b-1/2*d*csc(b*x+a)/b^2-1/2*(d*x+c)*cot(b*x+
a)*csc(b*x+a)/b-1/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2+1/2*I*d*polylog(2,ex
p(I*(b*x+a)))/b^2
```

**Rubi** [A] time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4415, 4183, 2279, 2391, 4185}

$$-\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}+\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}+\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)\cot(a+\ldots}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] ((c + d*x)*ArcTanh[E^(I*(a + b*x))])/b - (d*Csc[a + b*x])/(2*b^2) - ((c + d
*x)*Cot[a + b*x]*Csc[a + b*x])/(2*b) - ((I/2)*d*PolyLog[2, -E^(I*(a + b*x))
])/b^2 + ((I/2)*d*PolyLog[2, E^(I*(a + b*x))])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4415

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rubi steps

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)\csc(a+bx)\,dx + \int (c+dx)\csc^3(a+bx)\,dx$$
$$= \frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+$$
$$= \frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+$$
$$= \frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+$$
$$= \frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+$$

**Mathematica [B]** time = 1.59, size = 260, normalized size = 2.41

$$-\frac{d\left(i\left(\text{Li}_2\left(-e^{i(a+bx)}\right)-\text{Li}_2\left(e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)\right)}{2b^2}-\frac{d\tan\left(\frac{1}{2}(a+bx)\right)}{4b^2}-\frac{d\cot\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -1/4*(d*Cot[(a + b*x)/2])/b^2 - (c*Csc[(a + b*x)/2]^2)/(8*b) - (d*x*Csc[(a
+ b*x)/2]^2)/(8*b) + (c*Log[Cos[(a + b*x)/2]])/(2*b) - (c*Log[Sin[(a + b*x)
/2]])/(2*b) + (a*d*Log[Tan[(a + b*x)/2]])/(2*b^2) - (d*((a + b*x)*(Log[1 -
E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x
))] - PolyLog[2, E^(I*(a + b*x))])))/(2*b^2) + (c*Sec[(a + b*x)/2]^2)/(8*b)
 + (d*x*Sec[(a + b*x)/2]^2)/(8*b) - (d*Tan[(a + b*x)/2])/(4*b^2)
```

**fricas [B]** time = 0.57, size = 454, normalized size = 4.20

$$\frac{2\,(bdx+bc)\cos(bx+a)+\left(i\,d\cos(bx+a)^2-i\,d\right)\text{Li}_2\left(\cos(bx+a)+i\,\sin(bx+a)\right)+\left(-i\,d\cos(bx+a)^2+i\,d\right)\text{Li}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*d*x + b*c)*cos(b*x + a) + (I*d*cos(b*x + a)^2 - I*d)*dilog(cos(b*
x + a) + I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(cos(b*x + a) -
 I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) + I*sin(b
*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)
) - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*sin(b
*x + a) + 1) - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a
) - I*sin(b*x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2
*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - ((b*c - a*d)*cos(b*x + a)^2 - b
*c + a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x - (b*d
*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (
b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) - I*sin(b*x +
 a) + 1) + 2*d*sin(b*x + a))/(b^2*cos(b*x + a)^2 - b^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\cot(bx+a)^2\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cot(b*x + a)^2*csc(b*x + a), x)
```

**maple [B]** time = 0.10, size = 246, normalized size = 2.28

$$\frac{bdx\,e^{3i(bx+a)}+cb\,e^{3i(bx+a)}+bdx\,e^{i(bx+a)}+cb\,e^{i(bx+a)}-id\,e^{3i(bx+a)}+id\,e^{i(bx+a)}}{b^2\left(e^{2i(bx+a)}-1\right)^2}+\frac{c\operatorname{arctanh}\left(e^{i(bx+a)}\right)}{b}+\frac{d\ln\left(e^{i(bx+}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2*(b*d*x*exp(3*I*(b*x+a))+c*b*exp(3*I*(b*x+a))+b
*d*x*exp(I*(b*x+a))+c*b*exp(I*(b*x+a))-I*d*exp(3*I*(b*x+a))+I*d*exp(I*(b*x+
a)))+1/b*c*arctanh(exp(I*(b*x+a)))+1/2/b*d*ln(exp(I*(b*x+a))+1)*x+1/2/b^2*d
*ln(exp(I*(b*x+a))+1)*a-1/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/2/b*d*ln(1
-exp(I*(b*x+a)))*x-1/2/b^2*d*ln(1-exp(I*(b*x+a)))*a+1/2*I*d*polylog(2,exp(I
*(b*x+a)))/b^2-1/b^2*d*a*arctanh(exp(I*(b*x+a)))
```

**maxima [B]** time = 0.54, size = 770, normalized size = 7.13

$$\frac{(2\,bdx+2\,bc+2\,(bdx+bc)\cos\left(4\,bx+4\,a\right)-4\,(bdx+bc)\cos\left(2\,bx+2\,a\right)+(2i\,bdx+2i\,bc)\sin\left(4\,bx+4\,a\right)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] ((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*cos(
2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*a) + (-4*I*b*d*x - 4*I*b
*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos(
4*b*x + 4*a) - 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin(4*b*x + 4*a) - 4*I*b*c*
sin(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*
x*cos(4*b*x + 4*a) - 4*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*sin(4*b*x + 4*a)
- 4*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a)
 + 1) + (-4*I*b*d*x - 4*I*b*c - 4*d)*cos(3*b*x + 3*a) + (-4*I*b*d*x - 4*I*b
*c + 4*d)*cos(b*x + a) - (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I
*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*a))
 + (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a) -
4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c +
(-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*cos(2*b*x + 2*a
) + (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(
cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b*d*x + I*b*c +
(I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*cos(2*b*x + 2*a
) - (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(
cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (4*b*d*x + 4*b*c -
4*I*d)*sin(3*b*x + 3*a) + (4*b*d*x + 4*b*c + 4*I*d)*sin(b*x + a))/(-4*I*b^2
*cos(4*b*x + 4*a) + 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) - 8*b
^2*sin(2*b*x + 2*a) - 4*I*b^2)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x))/sin(a + b*x),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \cot^2(a + bx) \csc(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)*cot(a + b*x)**2*csc(a + b*x), x)
```

# 3.116 $\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=39

$$\mathrm{Int}\left(\frac{\csc^3(a+bx)}{c+dx},x\right)-\mathrm{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right)$$

```
[Out] -Unintegrable(csc(b*x+a)/(d*x+c),x)+Unintegrable(csc(b*x+a)^3/(d*x+c),x)
```

**Rubi** [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x),x]

[Out] -Defer[Int][Csc[a + b*x]/(c + d*x), x] + Defer[Int][Csc[a + b*x]^3/(c + d*x
), x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx=-\int \frac{\csc(a+bx)}{c+dx}\,dx+\int \frac{\csc^3(a+bx)}{c+dx}\,dx$$

**Mathematica** [A] time = 35.89, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x),x]

[Out] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x), x]
```

**fricas** [A] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cot(bx+a)^2\csc(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] integral(cot(b*x + a)^2*csc(b*x + a)/(d*x + c), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)^2\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] integrate(cot(b*x + a)^2*csc(b*x + a)/(d*x + c), x)
```

**maple** [A] time = 2.76, size = 0, normalized size = 0.00

$$\int \frac{\left(\cot^2(bx+a)\right)\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x)
```

```
[Out] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] (((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - d*sin(3*b*x
 + 3*a) + d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x + b*c)
*cos(2*b*x + 2*a) - 2*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x + b*
c)*cos(b*x + a) + d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*cos(b*x
+ a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 -
4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) +
 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b
*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*si
n(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) -
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x +
2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*
a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x +
2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x +
 a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 - 2*(b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) + (d*cos(
3*b*x + 3*a) - d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) + (b*d*x + b
*c)*sin(b*x + a))*sin(4*b*x + 4*a) + (2*d*cos(2*b*x + 2*a) - 2*(b*d*x + b*c
)*sin(2*b*x + 2*a) - d)*sin(3*b*x + 3*a) + 2*(d*cos(b*x + a) - (b*d*x + b*c
)*sin(b*x + a))*sin(2*b*x + 2*a) + d*sin(b*x + a))/(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(a+b\,x)^2}{\sin(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)),x)
```

```
[Out] int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2*csc(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cot(a + b*x)**2*csc(a + b*x)/(c + d*x), x)
```

# 3.117 $\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2} dx$

**Optimal**. Leaf size=39

$$\text{Int}\left(\frac{\csc^3(a+bx)}{(c+dx)^2},x\right)-\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] -Unintegrable(csc(b*x+a)/(d*x+c)^2,x)+Unintegrable(csc(b*x+a)^3/(d*x+c)^2,x
)
```

**Rubi** [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -Defer[Int][Csc[a + b*x]/(c + d*x)^2, x] + Defer[Int][Csc[a + b*x]^3/(c + d
*x)^2, x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2} dx = -\int \frac{\csc(a+bx)}{(c+dx)^2} dx + \int \frac{\csc^3(a+bx)}{(c+dx)^2} dx$$

**Mathematica** [A] time = 41.72, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2, x]
```

**fricas** [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)^2\csc(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^2*csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)^2\csc(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^2*csc(b*x + a)/(d*x + c)^2, x)
```

**maple** [A] time = 4.19, size = 0, normalized size = 0.00

$$\int \frac{\left(\cot^{2}\left(bx+a\right)\right)\csc\left(bx+a\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - 2*d*sin(3*b
*x + 3*a) + 2*d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x +
b*c)*cos(2*b*x + 2*a) - 4*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x
+ b*c)*cos(b*x + a) + 2*d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*co
s(b*x + a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 +
 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*
a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x
+ 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)*sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x
^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d
^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*s
in(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^
2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^
3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4
*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)
)*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)*sin(b*x + a)/
(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^
4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^
2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2
+ 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 - 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^
3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) + (2*d*c
os(3*b*x + 3*a) - 2*d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) + (b*d*
x + b*c)*sin(b*x + a))*sin(4*b*x + 4*a) + 2*(2*d*cos(2*b*x + 2*a) - (b*d*x
+ b*c)*sin(2*b*x + 2*a) - d)*sin(3*b*x + 3*a) + 2*(2*d*cos(b*x + a) - (b*d*
x + b*c)*sin(b*x + a))*sin(2*b*x + 2*a) + 2*d*sin(b*x + a))/(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
```

```
+ 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x +
 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*c
os(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^
3)*cos(2*b*x + 2*a))
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(a+bx)^2}{\sin(a+bx)\,(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)^2), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2*csc(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cot(a + b*x)**2*csc(a + b*x)/(c + d*x)**2, x)
```

# 3.118 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=406

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)}{144b^{7/2}}$$

```
[Out] -1/4*(d*x+c)^(5/2)*cos(b*x+a)/b-1/12*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+5/8*d*(
d*x+c)^(3/2)*sin(b*x+a)/b^2+5/72*d*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-5/864*d
^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/864*d^(5/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)-15/32*
d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2
))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/16*d^2*cos
(b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

**Rubi [A]** time = 0.67, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}\,d^{5}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(4*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(12*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*Fres
nelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)
*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x]
)/Sqrt[d]])/(144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/
2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a -
(b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(8*b^2) + (5*d*
(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(72*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{5/2}\sin(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{5/2}\sin(a+bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(3a+3bx)\,dx$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{(5d)\int(c+dx)}{}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{5d(c+dx)^{3/2}\,\mathrm{s}}{8b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,\mathrm{co}}{144b}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,\mathrm{co}}{144b}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,\mathrm{co}}{144b}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,\mathrm{co}}{144b}$$

**Mathematica [C]** time = 15.95, size = 1168, normalized size = 2.88

$$\frac{e^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)c^2}{8b}-\frac{\left(2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\cos(3(a+bx))-\sqrt{2\pi}\,\cos\left(3a-\frac{3bc}{d}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c^2*(2*Sqrt[3]*Sqrt
[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*Fres
nelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sq
rt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (
c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d
*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]
*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d])
+ 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(8*b^
3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d]) - S
qrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (
b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) - 2*Sqrt[b/d]*d*Sqrt[c + d
*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*d*x)*Sin[a + b*x])))/(32
*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d
*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*S
in[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a
+ b*x)] - Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*P
i]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2 - 5*d^2)*Cos[3
*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c)/d] + (12*b^2*c
^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*
(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)*Sin[3*(a + b*x)])))/(28
8*Sqrt[3]*b^5)
```

**fricas [A]** time = 0.54, size = 341, normalized size = 0.84

$$-\frac{5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+405\,\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(30*b*d^2*cos(b
*x + a) - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x +
a)^3 + 10*(2*b^2*d^2*x + 2*b^2*c*d + (b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*
sin(b*x + a))*sqrt(d*x + c))/b^4
```

**giac [C]** time = 3.08, size = 2465, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/1728*(72*(I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) + 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
```

```
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*((I*sqrt(6)*sqrt(p
i)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*
b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2
)/d^2 + 9*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*
b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I
*b*c - I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d -
 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2
*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2
)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (-I*sqrt(6)*sqrt(pi)*(12
*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c
*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^
2) + d^3*((-I*sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 -
5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) -
 6*I*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt
(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 -
 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d
^3 + 27*(-I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*
I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(
4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x +
c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*s
qrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 27*(I*s
qrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(
-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b
*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x +
 c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2
*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x +
c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + (I*sqrt(6)*sqrt(pi
)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)
*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(
d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^
((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^3) + 36*(-I*sqrt(6)*sqrt(p
i)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)*b) - 9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b
*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b)
 + 6*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqr
t(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e
^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x +
 c)*b + 3*I*b*c - 3*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.04, size = 476, normalized size = 1.17

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\cdots\right)}{4b\sqrt{\frac{b}{d}}}\cdots\right)}{2b}\cdots\right)}{4b}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+5/8/b*d*(1/2/b*d
*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/
2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1/
24/b*d*(d*x+c)^(5/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/24/b*d*(1/6/b*d*(d*
x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/2)
*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.53, size = 543, normalized size = 1.34

$$\frac{\left(240\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)+2160\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)-24\left(\frac{12\,(dx+c)^{\frac{5}{2}}b^{4}}{d}-5\,\sqrt{dx+c}\,b^{2}d\right)\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3456*(240*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d) + 2160*(
d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d) - 24*(12*(d*x + c)^(5/2
)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*(
4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(((d*x + c)*b - b*c +
a*d)/d) + ((5*I - 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*
(b*c - a*d)/d) + (5*I + 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*s
in(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + ((405*I - 405)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (405*I + 405)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(I*b/d)) + (-(405*I + 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-
(b*c - a*d)/d) - (405*I - 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(5*I + 5)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (5*I - 5)*9^(1/4)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(-3*I*b/d)))*d/b^5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2), x)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

## 3.119 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=353

$$-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(-\right.}{8b^{5/2}}$$

```
[Out] -1/4*(d*x+c)^(3/2)*cos(b*x+a)/b-1/12*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b-1/144*d
^(3/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d
^(3/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*sin(b*x+a
)*(d*x+c)^(1/2)/b^2+1/24*d*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.53, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b
*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) -
 (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (3*d*Sq
rt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(24*
b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{3/2}\sin(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{3/2}\sin(a+bx)\,dx + \frac{1}{4}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{d\int\sqrt{c+dx}\,\mathrm{c}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{3d\sqrt{c+dx}\,\mathrm{si}}{8b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{3d\sqrt{c+dx}\,\mathrm{si}}{8b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{3d\sqrt{c+dx}\,\mathrm{si}}{8b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos(}{}$$

**Mathematica [C]** time = 9.04, size = 676, normalized size = 1.92

$$-\frac{d\sqrt{\frac{b}{d}}\left(\sqrt{2\pi}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(3d\sin\left(a-\frac{bc}{d}\right)+2bc\cos\left(a-\frac{bc}{d}\right)\right)+\sqrt{2\pi}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(3d\cos\left(a-\right.\right.}{16b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c*(2*Sqrt[3]*Sqrt[b/d
]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC
[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6
/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (Sqrt
[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d*Cos[a
 - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[
2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*Sq
rt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(16*b^3) -
(Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Co
s[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (
3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] -
Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)
```

**fricas [A]** time = 0.70, size = 280, normalized size = 0.79

$$-\frac{\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)S\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-
(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(
2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b^2*c)*
cos(b*x + a)^3 - (b*d*cos(b*x + a)^2 + 2*b*d)*sin(b*x + a))*sqrt(d*x + c))/
b^3
```

**giac [C]** time = 4.78, size = 1538, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/288*(12*(I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) + 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((I*sqrt(6)*sqrt(pi)*(12
*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d
+ sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2
+ 9*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/
(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d -
4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c -
 I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2
)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
```

```
e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*
I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((
I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (-I*sqrt(6)*sqrt(pi)*(12*b^2*c
^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*
d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - s
qrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2) + 4
*(-I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqr
t(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(2*b*c
- I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sq
rt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I
*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 1
8*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c
)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b)*c)/d
```

**maple [A]** time = 0.04, size = 384, normalized size = 1.09

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)S\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{4b}-\frac{d(dx+c)^{\frac{3}{2}}\cdots}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/8/b*d*(1/2/b*d
*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))-1/24/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(1
/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1
/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

**maxima [C]** time = 0.51, size = 499, normalized size = 1.41

$$-\frac{\left(\frac{48\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{144\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}-24\sqrt{dx+c}\,b^2\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)-216\sqrt{dx+c}\,b^2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/576*(48*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 144*(
d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d - 24*sqrt(d*x + c)*b^
2*sin(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*sin(((d*x + c)
*b - b*c + a*d)/d) - (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)
*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/
```

```
4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - (-(27*I + 27)*
sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (27*I - 27)*sqrt
(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqr
t(I*b/d)) - ((27*I - 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) - (27*I + 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/
d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b
*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d))
)*d/b^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}} \sin(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**2, x)
```

# 3.120 $\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}}{\sqrt{}}\right.}{12b^{3/2}}$$

```
[Out] 1/72*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/72*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1
/8*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^
(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/4*cos(b*x+
a)*(d*x+c)^(1/2)/b-1/12*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

**Rubi [A]** time = 0.42, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(4*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12*
b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt
[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
 - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\,\sin(a+bx)+\frac{1}{4}\sqrt{c+dx}\,\sin(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int \sqrt{c+dx}\,\sin(a+bx)\,dx+\frac{1}{4}\int \sqrt{c+dx}\,\sin(3a+3bx)\,dx$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}+\frac{d\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}\,dx}{24b}$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}+\frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right.}{2}$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}+\frac{\cos\left(3a-\frac{3bc}{d}\right)S}{}$$

$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\right.}{}$$

**Mathematica [C]** time = 6.62, size = 278, normalized size = 0.91

$$-\frac{-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+\sqrt{2\pi}\sin\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\,c}{24\sqrt{3}\,b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[
(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (2*Sqrt[3]*Sqrt[b/d]*Sqr
t[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*
Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(24*Sqrt[3]*b*Sqrt[b/d])
```

**fricas [A]** time = 0.73, size = 235, normalized size = 0.77

$$\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+9\,\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{2}\,\pi d\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*sqrt(d*x + c)*b*cos(b*x + a)^3)/b
^2
```

**giac [C]** time = 2.78, size = 842, normalized size = 2.77

$$-\frac{-\frac{i\sqrt{6}\,\sqrt{\pi}\,(6\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{6}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{3i\,bc-3i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{9i\,\sqrt{2}\,\sqrt{\pi}\,(2\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{9i\,\sqrt{2}\,\sqrt{\pi}\,(2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/144*(-I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)
*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1
/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(
2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b)
+ I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) + 6*(I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(s
qrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*
I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c + 6*sqrt(d*x
+ c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e
^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c
)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*
c - 3*I*a*d)/d)/b)/d
```

**maple [A]** time = 0.03, size = 296, normalized size = 0.97

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\cos\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{12b}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/16/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/
d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)
/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima** [C] time = 0.50, size = 422, normalized size = 1.39

$$-\frac{\left(\frac{24\sqrt{dx+c}\,b^2\cos\left(\frac{3\left((dx+c)b-bc+ad\right)}{d}\right)}{d}+\frac{72\sqrt{dx+c}\,b^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left((i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)+(i+1)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/288*(24*sqrt(d*x + c)*b^2*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 72*sqrt
(d*x + c)*b^2*cos(((d*x + c)*b - b*c + a*d)/d)/d + ((I - 1)*9^(1/4)*sqrt(2)
*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)
*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3
*I*b/d)) + ((9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d
) + (9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(I*b/d)) + (-(9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*
cos(-(b*c - a*d)/d) - (9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*
c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I
*b/d)))*d/b^3
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\,\sin(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**2, x)
```

# 3.121 $\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}$$

```
[Out] 1/72*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/72*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1
/8*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^
(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/4*cos(b*x+
a)*(d*x+c)^(1/2)/b-1/12*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

**Rubi [A]** time = 0.42, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(4*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12*
b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt
[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
 - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\sin(a+bx)+\frac{1}{4}\sqrt{c+dx}\sin(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int \sqrt{c+dx}\sin(a+bx)\,dx+\frac{1}{4}\int \sqrt{c+dx}\sin(3a+3bx)\,dx$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{d\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}\,dx}{24b} \cdots$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\cos\left(3a-\frac{3bc}{d}\right)S\cdots}{\cdots}$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\cdots\right.}{\cdots}$$

**Mathematica [C]** time = 6.39, size = 264, normalized size = 0.87

$$\frac{\frac{\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-\sqrt{6\pi}\sin\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-6\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(3(a+bx))}{\sqrt{\frac{b}{d}}}+9\sqrt{c+dx}\,e^{-\frac{i(ad+bc)}{d}}\left(-\frac{e^{2ia}\Gamma\left(\frac{3}{2},\cdots\right)}{\sqrt{-\cdots}}\cdots\right.}{72b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] ((9*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((
-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sq
rt[(I*b*(c + d*x))/d]))/E^((I*(b*c + a*d))/d) + (-6*Sqrt[b/d]*Sqrt[c + d*x]
*Cos[3*(a + b*x)] + Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt
[6/Pi]*Sqrt[c + d*x]] - Sqrt[6*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d
*x]]*Sin[3*a - (3*b*c)/d])/Sqrt[b/d])/(72*b)
```

**fricas [A]** time = 0.72, size = 235, normalized size = 0.77

$$\frac{\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+9\,\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{2}\,\pi d\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*sqrt(d*x + c)*b*cos(b*x + a)^3)/b
^2
```

**giac [C]** time = 4.92, size = 842, normalized size = 2.77

$$-\frac{-\frac{i\sqrt{6}\sqrt{\pi}\,(6\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{3i\,bc-3i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{9i\sqrt{2}\sqrt{\pi}\,(2\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{9i\sqrt{2}\sqrt{\pi}\,(2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/144*(-I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)
*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1
/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(
2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b)
+ I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) + 6*(I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(s
qrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*
I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c + 6*sqrt(d*x
+ c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e
^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c
)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*
c - 3*I*a*d)/d)/b)/d
```

**maple [A]** time = 0.00, size = 296, normalized size = 0.97

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\cos\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{12b}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/16/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/
d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)
/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima [C]** time = 0.50, size = 422, normalized size = 1.39

$$-\frac{\left(\frac{24\sqrt{dx+c}\,b^2\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{72\sqrt{dx+c}\,b^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left((i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i+1)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/288*(24*sqrt(d*x + c)*b^2*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 72*sqrt
(d*x + c)*b^2*cos(((d*x + c)*b - b*c + a*d)/d)/d + ((I - 1)*9^(1/4)*sqrt(2)
*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)
*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3
*I*b/d)) + ((9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d
) + (9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(I*b/d)) + (-(9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*
cos(-(b*c - a*d)/d) - (9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*
c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I
*b/d)))*d/b^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\sin\left(a+bx\right)\cos^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**2, x)
```

# 3.122 $\int (c+dx)^{3/2} \cos^2(a+bx) \sin(a+bx) \, dx$

**Optimal**. Leaf size=353

$$-\frac{\sqrt{\frac{\pi}{6}} d^{3/2} \sin\left(3a-\frac{3bc}{d}\right) C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}} d^{3/2} \sin\left(a-\frac{bc}{d}\right) C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}} d^{3/2} \cos\left(a-\frac{bc}{d}\right) S\left(\frac{\sqrt{b}\sqrt{\cdots}}{\cdots}\right)}{8b^{5/2}}$$

```
[Out] -1/4*(d*x+c)^(3/2)*cos(b*x+a)/b-1/12*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b-1/144*d
^(3/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d
^(3/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*sin(b*x+a
)*(d*x+c)^(1/2)/b^2+1/24*d*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.53, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{6}} d^{3/2} \sin\left(3a-\frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}} d^{3/2} \sin\left(a-\frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}} d^{3/2} \cos \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b
*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) -
 (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (3*d*Sq
rt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(24*
b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{3/2}\sin(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{3/2}\sin(a+bx)\,dx+\frac{1}{4}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{d\int \sqrt{c+d\ldots}}{\ldots}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{3d\sqrt{c+dx}\ldots}{8\ldots}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{3d\sqrt{c+dx}\ldots}{8\ldots}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{3d\sqrt{c+dx}\ldots}{8\ldots}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\mathrm{c}\ldots}{\ldots}$$

**Mathematica [C]** time = 8.93, size = 676, normalized size = 1.92

$$-\frac{d\sqrt{\frac{b}{d}}\left(\sqrt{2\pi}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(3d\sin\left(a-\frac{bc}{d}\right)+2bc\cos\left(a-\frac{bc}{d}\right)\right)+\sqrt{2\pi}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(3d\cos\left(a\ldots\right.\right.\right.}{16b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c*(2*Sqrt[3]*Sqrt[b/d
]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC
[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6
/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (Sqrt
[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d*Cos[a
 - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[
2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*Sq
rt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(16*b^3) -
(Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Co
s[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (
3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] -
Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)
```

**fricas [A]** time = 0.72, size = 280, normalized size = 0.79

$$-\frac{\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)S\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-
(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(
2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b^2*c)*
cos(b*x + a)^3 - (b*d*cos(b*x + a)^2 + 2*b*d)*sin(b*x + a))*sqrt(d*x + c))/
b^3
```

**giac [C]** time = 1.93, size = 1538, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/288*(12*(I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) + 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((I*sqrt(6)*sqrt(pi)*(12
*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d
+ sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2
+ 9*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/
(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d -
4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c -
 I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2
)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
```

```
e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*
I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((
I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (-I*sqrt(6)*sqrt(pi)*(12*b^2*c
^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*
d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - s
qrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2) + 4
*(-I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqr
t(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(2*b*c
- I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sq
rt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I
*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 1
8*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c
)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b)*c)/d
```

**maple** [A] time = 0.00, size = 384, normalized size = 1.09

$$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)S\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{4b}-\frac{d(dx\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/8/b*d*(1/2/b*d
*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))-1/24/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(1
/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1
/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima** [C] time = 0.53, size = 499, normalized size = 1.41

$$-\frac{\left(\frac{48\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{144\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}-24\sqrt{dx+c}\,b^2\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)-216\sqrt{dx+c}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/576*(48*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 144*(
d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d - 24*sqrt(d*x + c)*b^
2*sin(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*sin(((d*x + c)
*b - b*c + a*d)/d) - (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)
*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/
```

```
4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - (-(27*I + 27)*
sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (27*I - 27)*sqrt
(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqr
t(I*b/d)) - ((27*I - 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) - (27*I + 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/
d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b
*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d))
)*d/b^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)^{\frac{3}{2}}\sin\left(a+bx\right)\cos^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**2, x)
```

# 3.123 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=406

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)}{144b^{7/2}}$$

```
[Out] -1/4*(d*x+c)^(5/2)*cos(b*x+a)/b-1/12*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+5/8*d*(
d*x+c)^(3/2)*sin(b*x+a)/b^2+5/72*d*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-5/864*d
^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/864*d^(5/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)-15/32*
d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2
))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/16*d^2*cos
(b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

**Rubi** [A] time = 0.63, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}\,d^{5}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(4*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(12*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*Fres
nelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)
*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x]
)/Sqrt[d]])/(144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/
2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a -
(b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(8*b^2) + (5*d*
(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(72*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{5/2}\sin(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{5/2}\sin(a+bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(3a+3bx)\,dx$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{(5d)\int (c+dx)}{}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{5d(c+dx)^{3/2}}{8b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,\mathrm{co}}{144b}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,\mathrm{co}}{144b}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,\mathrm{co}}{144b}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,\mathrm{co}}{144b}$$

**Mathematica [C]** time = 15.31, size = 1168, normalized size = 2.88

$$\frac{e^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)c^2}{8b}-\frac{\left(2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\cos(3(a+bx))-\sqrt{2\pi}\,\cos\left(3a-\frac{3bc}{d}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c^2*(2*Sqrt[3]*Sqrt
[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*Fres
nelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sq
rt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (
c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d
*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]
*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d])
+ 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(8*b^
3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d]) - S
qrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (
b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) - 2*Sqrt[b/d]*d*Sqrt[c + d
*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*d*x)*Sin[a + b*x])))/(32
*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d
*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*S
in[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a
+ b*x)] - Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*P
i]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2 - 5*d^2)*Cos[3
*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c)/d] + (12*b^2*c
^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*
(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)*Sin[3*(a + b*x)])))/(28
8*Sqrt[3]*b^5)
```

**fricas [A]** time = 0.52, size = 341, normalized size = 0.84

$$-\frac{5\,\sqrt{6}\,\pi d^{3}\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+405\,\sqrt{2}\,\pi d^{3}\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(30*b*d^2*cos(b
*x + a) - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x +
a)^3 + 10*(2*b^2*d^2*x + 2*b^2*c*d + (b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*
sin(b*x + a))*sqrt(d*x + c))/b^4
```

**giac [C]** time = 4.90, size = 2465, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/1728*(72*(I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) + 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
```

```
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*((I*sqrt(6)*sqrt(p
i)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*
b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2
)/d^2 + 9*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*
b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I
*b*c - I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d -
 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2
*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2
)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (-I*sqrt(6)*sqrt(pi)*(12
*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c
*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^
2) + d^3*((-I*sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 -
5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) -
 6*I*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt
(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 -
 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d
^3 + 27*(-I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*
I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(
4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x +
c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*s
qrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 27*(I*s
qrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(
-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b
*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x +
 c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2
*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x +
c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + (I*sqrt(6)*sqrt(pi
)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)
*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(
d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^
((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^3) + 36*(-I*sqrt(6)*sqrt(p
i)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)*b) - 9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b
*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b)
 + 6*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqr
t(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e
^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x +
 c)*b + 3*I*b*c - 3*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.00, size = 476, normalized size = 1.17

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\cdots\right.}{4b\sqrt{\frac{b}{d}}}\right.}{2b}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+5/8/b*d*(1/2/b*d
*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/
2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1/
24/b*d*(d*x+c)^(5/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/24/b*d*(1/6/b*d*(d*
x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/2)
*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.52, size = 543, normalized size = 1.34

$$\frac{\left(240\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)+2160\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)-24\left(\frac{12\,(dx+c)^{\frac{5}{2}}b^4}{d}-5\sqrt{dx+c}\,b^2d\right)\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3456*(240*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d) + 2160*(
d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d) - 24*(12*(d*x + c)^(5/2
)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*(
4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(((d*x + c)*b - b*c +
a*d)/d) + ((5*I - 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*
(b*c - a*d)/d) + (5*I + 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*s
in(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + ((405*I - 405)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (405*I + 405)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(I*b/d)) + (-(405*I + 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-
(b*c - a*d)/d) - (405*I - 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(5*I + 5)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (5*I - 5)*9^(1/4)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(-3*I*b/d)))*d/b^5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2), x)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

# 3.124 $\int (c+dx)^{5/2} \cos^2(a+bx) \sin^2(a+bx) \, dx$

**Optimal**. Leaf size=228

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\sin(4a}{2048b^3}$$

```
[Out] 1/28*(d*x+c)^(7/2)/d-5/256*d*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)^
(5/2)*sin(4*b*x+4*a)/b-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*
2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192*d^
(5/2)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*
b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/2048*d^2*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^
3
```

**Rubi [A]** time = 0.40, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+d}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(7/2)/(28*d) - (5*d*(c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(256*b^2) -
 (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4096*b
^(7/2)) + (15*d^2*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^(
5/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}-\frac{1}{8}(c+dx)^{5/2}\cos(4a+4bx)\right)dx$$

$$=\frac{(c+dx)^{7/2}}{28d}-\frac{1}{8}\int (c+dx)^{5/2}\cos(4a+4bx)\,dx$$

$$=\frac{(c+dx)^{7/2}}{28d}-\frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b}+\frac{(5d)\int (c+dx)^{3/2}\sin(4a+\cdots}{64b}$$

$$=\frac{(c+dx)^{7/2}}{28d}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\sin(4a+4bx\cdots}{32b}$$

$$=\frac{(c+dx)^{7/2}}{28d}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}+\frac{15d^2\sqrt{c+dx}\,\sin(4a+4\cdots}{2048b^3}$$

$$=\frac{(c+dx)^{7/2}}{28d}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}+\frac{15d^2\sqrt{c+dx}\,\sin(4a+4\cdots}{2048b^3}$$

$$=\frac{(c+dx)^{7/2}}{28d}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}+\frac{15d^2\sqrt{c+dx}\,\sin(4a+4\cdots}{2048b^3}$$

$$=\frac{(c+dx)^{7/2}}{28d}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}-\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\cdots\right.}{4096b^{7/\cdots}}$$

**Mathematica [A]** time = 3.42, size = 206, normalized size = 0.90

$$\frac{\sqrt{\frac{b}{d}}\left(4\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(-7d\sin(4(a+bx))\left(64b^2(c+dx)^2-15d^2\right)-280bd^2(c+dx)\cos(4(a+bx))+512b^3(c+dx)^3\cdots\right.\right.}{57344b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(-105*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*S
qrt[2/Pi]*Sqrt[c + d*x]] - 105*d^3*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(512*b^3
*(c + d*x)^3 - 280*b*d^2*(c + d*x)*Cos[4*(a + b*x)] - 7*d*(-15*d^2 + 64*b^2
*(c + d*x)^2)*Sin[4*(a + b*x)])))/(57344*b^4)
```

**fricas [A]** time = 0.51, size = 347, normalized size = 1.52

$$-\frac{105\,\sqrt{2}\,\pi d^4\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+105\,\sqrt{2}\,\pi d^4\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(b}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/57344*(105*sqrt(2)*pi*d^4*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_s
in(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 105*sqrt(2)*pi*d^4*sqrt(b/(pi*
d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/
d) - 16*(128*b^4*d^3*x^3 + 384*b^4*c*d^2*x^2 + 128*b^4*c^3 - 70*b^2*c*d^2 -
 560*(b^2*d^3*x + b^2*c*d^2)*cos(b*x + a)^4 + 560*(b^2*d^3*x + b^2*c*d^2)*c
os(b*x + a)^2 + 2*(192*b^4*c^2*d - 35*b^2*d^3)*x - 7*(2*(64*b^3*d^3*x^2 + 1
28*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a)^3 - (64*b^3*d^3*x^2
+ 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a))*sin(b*x + a))*sq
rt(d*x + c))/(b^4*d)
```

**giac [C]** time = 3.30, size = 1358, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/573440*(17920*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*c^3 + 56*c*d^2*(512*(3*(d*x + c)^(5/
2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 + 15*(sqrt(2)*sqrt(pi
)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b^2) - 4*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)
/d^2 + 15*(sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*(-8*I*(d*x + c)^(3/2
)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b
 - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2) + d^3*(4096*(5*(d*x + c)^(7/2) - 21*(d*x
 + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)/d^3 - 35*(sq
rt(2)*sqrt(pi)*(512*b^3*c^3 + 192*I*b^2*c^2*d - 72*b*c*d^2 - 15*I*d^3)*d*er
f(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c
 - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 4*(64*I*(d*x + c
)^(5/2)*b^2*d - 192*I*(d*x + c)^(3/2)*b^2*c*d + 192*I*sqrt(d*x + c)*b^2*c^2
*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x +
c)*d^3)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^3)/d^3 - 35*(sqrt(2)
*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*b*c*d^2 + 15*I*d^3)*d*erf(-sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c +
4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 4*(-64*I*(d*x + c)
^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d - 192*I*sqrt(d*x + c)*b^2*c^2*
d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c
)*d^3)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^3)/d^3) - 2240*(3*sqrt
(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c
```

```
- 12*I*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*
I*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.05, size = 251, normalized size = 1.10

$$\frac{\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4da-4cb}{d}\right)S\left(\frac{2\sqrt{2}}{\sqrt{\pi}}\right.\right.}{}\right.}{8b}\right.}{32b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/56*(d*x+c)^(7/2)-1/64/b*d*(d*x+c)^(5/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+5/64/b*d*(-1/8/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/8/
b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d
)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(
b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.47, size = 285, normalized size = 1.25

$$\sqrt{2}\left(\frac{4096\sqrt{2}\,(dx+c)^{\frac{7}{2}}b^4}{d}-2240\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-\left((105i+105)\,\sqrt{\pi}\,d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/229376*sqrt(2)*(4096*sqrt(2)*(d*x + c)^(7/2)*b^4/d - 2240*sqrt(2)*(d*x +
c)^(3/2)*b^2*d*cos(4*((d*x + c)*b - b*c + a*d)/d) - ((105*I + 105)*sqrt(pi)
*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (105*I - 105)*sqrt(pi)*d^3*(b^
2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (-(1
05*I - 105)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (105*I + 1
05)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)
*sqrt(-I*b/d)) - 56*(64*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x +
 c)*b*d^2)*sin(4*((d*x + c)*b - b*c + a*d)/d))/b^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.125 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=200

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2}$$

```
[Out] 1/20*(d*x+c)^(5/2)/d-1/32*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b+3/1024*d^(3/2)*cos
(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^
(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)-3/256*d*cos
(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.33, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\cos(4a}{256b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(5/2)/(20*d) - (3*d*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(256*b^2) + (
3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(2*Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)
) - ((c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^{3/2}-\frac{1}{8}(c+dx)^{3/2}\cos(4a+4bx)\right)dx \\
&= \frac{(c+dx)^{5/2}}{20d}-\frac{1}{8}\int (c+dx)^{3/2}\cos(4a+4bx)\,dx \\
&= \frac{(c+dx)^{5/2}}{20d}-\frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}+\frac{(3d)\int \sqrt{c+dx}\,\sin(4a+}{64b} \\
&= \frac{(c+dx)^{5/2}}{20d}-\frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\sin(4a+4b}{32b} \\
&= \frac{(c+dx)^{5/2}}{20d}-\frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\sin(4a+4b}{32b} \\
&= \frac{(c+dx)^{5/2}}{20d}-\frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\sin(4a+4b}{32b} \\
&= \frac{(c+dx)^{5/2}}{20d}-\frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2}+\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right.}{512b^5}
\end{aligned}$$

**Mathematica [A]** time = 2.80, size = 187, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{d}}\left(15\sqrt{2\pi}\,d^2\cos\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)-15\sqrt{2\pi}\,d^2\sin\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+4\sqrt{\frac{b}{d}}\right.}{5120b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(15*d^2*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqr
t[2/Pi]*Sqrt[c + d*x]] - 15*d^2*Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*
Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(-15*d^2*Co
s[4*(a + b*x)] + 8*b*(c + d*x)*(8*b*(c + d*x) - 5*d*Sin[4*(a + b*x)]))))/(5
120*b^3)
```

**fricas [A]** time = 0.74, size = 249, normalized size = 1.24

$$\frac{15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5120*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
 4*(64*b^3*d^2*x^2 - 120*b*d^2*cos(b*x + a)^4 + 128*b^3*c*d*x + 64*b^3*c^2
+ 120*b*d^2*cos(b*x + a)^2 - 15*b*d^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*
x + a)^3 - (b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))
/(b^3*d)
```

**giac [C]** time = 1.99, size = 842, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/30720*(960*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*c^2 + d^2*(512*(3*(d*x + c)^(5/2) - 10*
(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 + 15*(sqrt(2)*sqrt(pi)*(64*b^
2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)*b^2) - 4*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*s
qrt(d*x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + 1
5*(sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*(-8*I*(d*x + c)^(3/2)*b*d +
16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b
*c + 4*I*a*d)/d)/b^2)/d^2) - 80*(3*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b
*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6
4*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c - 12*I*sqrt(d*x + c)*d*e^((4*I*(d*x
 + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)
*b + 4*I*b*c - 4*I*a*d)/d)/b)*c)/d
```

**maple [A]** time = 0.05, size = 206, normalized size = 1.03

$$\frac{\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4da-4cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{4da-4cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{2}}{\sqrt{\pi}}\right.\right.}{32b\sqrt{\frac{b}{d}}}\right.}{32b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/40*(d*x+c)^(5/2)-1/64/b*d*(d*x+c)^(3/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+3/64/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/32
/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)
/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima [C]** time = 0.48, size = 264, normalized size = 1.32

$$\sqrt{2}\left(\frac{512\sqrt{2}(dx+c)^{\frac{5}{2}}b^3}{d} - 320\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right) - 120\sqrt{2}\sqrt{dx+c}\,bd\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right) - \left((15\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/20480*sqrt(2)*(512*sqrt(2)*(d*x + c)^(5/2)*b^3/d - 320*sqrt(2)*(d*x + c)^
(3/2)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 120*sqrt(2)*sqrt(d*x + c)*b*
d*cos(4*((d*x + c)*b - b*c + a*d)/d) - ((15*I - 15)*sqrt(pi)*d^2*(b^2/d^2)^
(1/4)*cos(-4*(b*c - a*d)/d) + (15*I + 15)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(
-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (-(15*I + 15)*sqrt(pi
)*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (15*I - 15)*sqrt(pi)*d^2*(b^2
/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

# 3.126 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=174

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{(c+$$

```
[Out] 1/12*(d*x+c)^(3/2)/d+1/128*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*FresnelC
(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)
*2^(1/2)*Pi^(1/2)/b^(3/2)-1/32*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

**Rubi [A]** time = 0.27, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4b}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt
[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*
b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}-\frac{1}{8}\sqrt{c+dx}\cos(4a+4bx)\right)dx$$

$$=\frac{(c+dx)^{3/2}}{12d}-\frac{1}{8}\int \sqrt{c+dx}\cos(4a+4bx)\,dx$$

$$=\frac{(c+dx)^{3/2}}{12d}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{d\int \frac{\sin(4a+4bx)}{\sqrt{c+dx}}\,dx}{64b}$$

$$=\frac{(c+dx)^{3/2}}{12d}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\sin\left(\frac{4bc}{d}\right.}{\sqrt{c+d}}}{64b}$$

$$=\frac{(c+dx)^{3/2}}{12d}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Subst}\left(\int \sin\right.}{32b}$$

$$=\frac{(c+dx)^{3/2}}{12d}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}C}{}$$

**Mathematica [A]** time = 0.82, size = 161, normalized size = 0.93

$$\frac{3\sqrt{2\pi}\,d\sin\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+3\sqrt{2\pi}\,d\cos\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+4\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(}{384d^2\left(\frac{b}{d}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c
 + d*x]] + 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Si
n[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(8*b*(c + d*x) - 3*d*Sin[4*(
a + b*x)]))/(384*(b/d)^(3/2)*d^2)
```

**fricas [A]** time = 0.52, size = 175, normalized size = 1.01

$$\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)}{384\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2*
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fre
snel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 16
*(2*b^2*d*x + 2*b^2*c - 3*(2*b*d*cos(b*x + a)^3 - b*d*cos(b*x + a))*sin(b*x
 + a))*sqrt(d*x + c))/(b^2*d)
```

**giac [C]** time = 0.99, size = 452, normalized size = 2.60

$$-\frac{\frac{3\sqrt{2}\sqrt{\pi}(8bc+id)d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4ibc-4iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{3\sqrt{2}\sqrt{\pi}(8bc-id)d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4ibc+4iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-24\left(\frac{\sqrt{2}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/768*(3*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 24*(sqrt(2)*sqrt(pi)*d*erf
(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c
- 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*er
f(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b
*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*
c - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c - 12*I*sqrt(d*x + c)*d*e^((4*I
*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*e^((-4*I*(d*x
 + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)/d
```

**maple [A]** time = 0.05, size = 159, normalized size = 0.91

$$\frac{\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4da-4cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4da-4cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/24*(d*x+c)^(3/2)-1/64/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+1/256/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*
2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC
(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima [C]** time = 0.46, size = 219, normalized size = 1.26

$$\frac{\sqrt{2}\left(\frac{64\sqrt{2}(dx+c)^{\frac{3}{2}}b^2}{d}-24\sqrt{2}\sqrt{dx+c}\,b\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right)-\left(-(3i+3)\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)+(3i-3)\sqrt{\pi}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*sqrt(2)*(64*sqrt(2)*(d*x + c)^(3/2)*b^2/d - 24*sqrt(2)*sqrt(d*x + c)
*b*sin(4*((d*x + c)*b - b*c + a*d)/d) - (-(3*I + 3)*sqrt(pi)*d*(b^2/d^2)^(1
```

```
/4)*cos(-4*(b*c - a*d)/d) + (3*I - 3)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-4*(b*
c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((3*I - 3)*sqrt(pi)*d*(b^2/
d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (3*I + 3)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin
(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^2
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^2(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

# 3.127 $\int \sqrt{c+dx} \cos^2(a+bx) \sin^2(a+bx) \, dx$

**Optimal**. Leaf size=174

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{(c+}{}$$

```
[Out] 1/12*(d*x+c)^(3/2)/d+1/128*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*FresnelC
(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)
*2^(1/2)*Pi^(1/2)/b^(3/2)-1/32*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

**Rubi [A]** time = 0.25, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4b}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt
[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*
b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}-\frac{1}{8}\sqrt{c+dx}\cos(4a+4bx)\right)dx$$
$$=\frac{(c+dx)^{3/2}}{12d}-\frac{1}{8}\int \sqrt{c+dx}\cos(4a+4bx)\,dx$$
$$=\frac{(c+dx)^{3/2}}{12d}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{d\int \frac{\sin(4a+4bx)}{\sqrt{c+dx}}\,dx}{64b}$$
$$=\frac{(c+dx)^{3/2}}{12d}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\sin\left(\frac{4bc}{d}\right.}{\sqrt{c+d}}}{64b}$$
$$=\frac{(c+dx)^{3/2}}{12d}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Subst}\left(\int \sin\right.}{32b}$$
$$=\frac{(c+dx)^{3/2}}{12d}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}C}{}$$

**Mathematica [A]** time = 0.11, size = 161, normalized size = 0.93

$$\frac{3\sqrt{2\pi}\,d\sin\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+3\sqrt{2\pi}\,d\cos\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+4\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(}{384d^2\left(\frac{b}{d}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c
 + d*x]] + 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Si
n[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(8*b*(c + d*x) - 3*d*Sin[4*(
a + b*x)]))/(384*(b/d)^(3/2)*d^2)
```

**fricas [A]** time = 0.75, size = 175, normalized size = 1.01

$$\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)}{384\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2*
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fre
snel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 16
*(2*b^2*d*x + 2*b^2*c - 3*(2*b*d*cos(b*x + a)^3 - b*d*cos(b*x + a))*sin(b*x
 + a))*sqrt(d*x + c))/(b^2*d)
```

**giac [C]** time = 1.01, size = 452, normalized size = 2.60

$$-\frac{\frac{3\sqrt{2}\sqrt{\pi}(8bc+id)d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4ibc-4iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{3\sqrt{2}\sqrt{\pi}(8bc-id)d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4ibc+4iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-24\left(\frac{\sqrt{2}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/768*(3*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 24*(sqrt(2)*sqrt(pi)*d*erf
(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c
- 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*er
f(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b
*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*
c - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c - 12*I*sqrt(d*x + c)*d*e^((4*I
*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*e^((-4*I*(d*x
 + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)/d
```

**maple [A]** time = 0.00, size = 159, normalized size = 0.91

$$\frac{\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4da-4cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4da-4cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/24*(d*x+c)^(3/2)-1/64/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+1/256/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*
2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC
(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima [C]** time = 0.46, size = 219, normalized size = 1.26

$$\frac{\sqrt{2}\left(\frac{64\sqrt{2}(dx+c)^{\frac{3}{2}}b^2}{d}-24\sqrt{2}\sqrt{dx+c}\,b\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right)-\left(-(3i+3)\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)+(3i-3)\sqrt{\pi}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*sqrt(2)*(64*sqrt(2)*(d*x + c)^(3/2)*b^2/d - 24*sqrt(2)*sqrt(d*x + c)
*b*sin(4*((d*x + c)*b - b*c + a*d)/d) - (-(3*I + 3)*sqrt(pi)*d*(b^2/d^2)^(1
```

```
/4)*cos(-4*(b*c - a*d)/d) + (3*I - 3)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-4*(b*
c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((3*I - 3)*sqrt(pi)*d*(b^2/
d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (3*I + 3)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin
(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

# 3.128 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=200

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2}$$

```
[Out] 1/20*(d*x+c)^(5/2)/d-1/32*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b+3/1024*d^(3/2)*cos
(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^
(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)-3/256*d*cos
(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.32, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\cos(4a}{256b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(5/2)/(20*d) - (3*d*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(256*b^2) + (
3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(2*Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)
) - ((c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^{3/2} - \frac{1}{8}(c+dx)^{3/2}\cos(4a+4bx)\right)dx \\
&= \frac{(c+dx)^{5/2}}{20d} - \frac{1}{8}\int (c+dx)^{3/2}\cos(4a+4bx)\,dx \\
&= \frac{(c+dx)^{5/2}}{20d} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(3d)\int \sqrt{c+dx}\,\sin(4a+}{64b} \\
&= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4b}{32b} \\
&= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4b}{32b} \\
&= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4b}{32b} \\
&= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right.}{512b^5}
\end{aligned}$$

**Mathematica [A]** time = 1.27, size = 187, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{d}}\left(15\sqrt{2\pi}\,d^2\cos\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) - 15\sqrt{2\pi}\,d^2\sin\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) + 4\sqrt{\frac{b}{d}}\right.}{5120b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(15*d^2*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqr
t[2/Pi]*Sqrt[c + d*x]] - 15*d^2*Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*
Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(-15*d^2*Co
s[4*(a + b*x)] + 8*b*(c + d*x)*(8*b*(c + d*x) - 5*d*Sin[4*(a + b*x)]))))/(5
120*b^3)
```

**fricas [A]** time = 0.52, size = 249, normalized size = 1.24

$$\frac{15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5120*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
 4*(64*b^3*d^2*x^2 - 120*b*d^2*cos(b*x + a)^4 + 128*b^3*c*d*x + 64*b^3*c^2
+ 120*b*d^2*cos(b*x + a)^2 - 15*b*d^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*
x + a)^3 - (b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))
/(b^3*d)
```

**giac [C]** time = 4.12, size = 842, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/30720*(960*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*c^2 + d^2*(512*(3*(d*x + c)^(5/2) - 10*
(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 + 15*(sqrt(2)*sqrt(pi)*(64*b^
2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)*b^2) - 4*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*s
qrt(d*x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + 1
5*(sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*(-8*I*(d*x + c)^(3/2)*b*d +
16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b
*c + 4*I*a*d)/d)/b^2)/d^2) - 80*(3*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b
*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6
4*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c - 12*I*sqrt(d*x + c)*d*e^((4*I*(d*x
 + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)
*b + 4*I*b*c - 4*I*a*d)/d)/b)*c)/d
```

**maple [A]** time = 0.00, size = 206, normalized size = 1.03

$$\frac{\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4da-4cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{4da-4cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{2}}{\sqrt{\pi}}\right.\right.}{32b\sqrt{\frac{b}{d}}}\right.}{32b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/40*(d*x+c)^(5/2)-1/64/b*d*(d*x+c)^(3/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+3/64/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/32
/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)
/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima [C]** time = 0.78, size = 264, normalized size = 1.32

$$\sqrt{2}\left(\frac{512\,\sqrt{2}\,(dx+c)^{\frac{5}{2}}b^3}{d}-320\,\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-120\,\sqrt{2}\,\sqrt{dx+c}\,bd\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-\left((15\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/20480*sqrt(2)*(512*sqrt(2)*(d*x + c)^(5/2)*b^3/d - 320*sqrt(2)*(d*x + c)^
(3/2)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 120*sqrt(2)*sqrt(d*x + c)*b*
d*cos(4*((d*x + c)*b - b*c + a*d)/d) - ((15*I - 15)*sqrt(pi)*d^2*(b^2/d^2)^
(1/4)*cos(-4*(b*c - a*d)/d) + (15*I + 15)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(
-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (-(15*I + 15)*sqrt(pi
)*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (15*I - 15)*sqrt(pi)*d^2*(b^2
/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

# 3.129 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=228

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\sin(4a+}{2048b^3}$$

```
[Out] 1/28*(d*x+c)^(7/2)/d-5/256*d*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)^
(5/2)*sin(4*b*x+4*a)/b-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*
2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192*d^
(5/2)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*
b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/2048*d^2*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^
3
```

**Rubi [A]** time = 0.38, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,s}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(7/2)/(28*d) - (5*d*(c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(256*b^2) -
 (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4096*b
^(7/2)) + (15*d^2*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^(
5/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2} - \frac{1}{8}(c+dx)^{5/2}\cos(4a+4bx)\right)dx$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{1}{8}\int (c+dx)^{5/2}\cos(4a+4bx)\,dx$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b} + \frac{(5d)\int (c+dx)^{3/2}\sin(4a}{64b}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{5/2}\sin(4a+4}{32b}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\,\sin(4a}{2048b^3}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\,\sin(4a}{2048b^3}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\,\sin(4a}{2048b^3}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\right.}{409}$$

**Mathematica [A]** time = 2.32, size = 206, normalized size = 0.90

$$\frac{\sqrt{\frac{b}{d}}\left(4\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(-7d\sin(4(a+bx))\left(64b^2(c+dx)^2-15d^2\right)-280bd^2(c+dx)\cos(4(a+bx))+512b^3(c+d\right.\right.}{5734}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(-105*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*S
qrt[2/Pi]*Sqrt[c + d*x]] - 105*d^3*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(512*b^3
*(c + d*x)^3 - 280*b*d^2*(c + d*x)*Cos[4*(a + b*x)] - 7*d*(-15*d^2 + 64*b^2
*(c + d*x)^2)*Sin[4*(a + b*x)])))/(57344*b^4)
```

**fricas [A]** time = 0.61, size = 347, normalized size = 1.52

$$-\frac{105\,\sqrt{2}\,\pi d^4\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+105\,\sqrt{2}\,\pi d^4\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-a}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/57344*(105*sqrt(2)*pi*d^4*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_s
in(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 105*sqrt(2)*pi*d^4*sqrt(b/(pi*
d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/
d) - 16*(128*b^4*d^3*x^3 + 384*b^4*c*d^2*x^2 + 128*b^4*c^3 - 70*b^2*c*d^2 -
 560*(b^2*d^3*x + b^2*c*d^2)*cos(b*x + a)^4 + 560*(b^2*d^3*x + b^2*c*d^2)*c
os(b*x + a)^2 + 2*(192*b^4*c^2*d - 35*b^2*d^3)*x - 7*(2*(64*b^3*d^3*x^2 + 1
28*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a)^3 - (64*b^3*d^3*x^2
+ 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a))*sin(b*x + a))*sq
rt(d*x + c))/(b^4*d)
```

**giac [C]** time = 4.93, size = 1358, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/573440*(17920*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*c^3 + 56*c*d^2*(512*(3*(d*x + c)^(5/
2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 + 15*(sqrt(2)*sqrt(pi
)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b^2) - 4*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)
/d^2 + 15*(sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*(-8*I*(d*x + c)^(3/2
)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b
 - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2) + d^3*(4096*(5*(d*x + c)^(7/2) - 21*(d*x
 + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)/d^3 - 35*(sq
rt(2)*sqrt(pi)*(512*b^3*c^3 + 192*I*b^2*c^2*d - 72*b*c*d^2 - 15*I*d^3)*d*er
f(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c
 - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 4*(64*I*(d*x + c
)^(5/2)*b^2*d - 192*I*(d*x + c)^(3/2)*b^2*c*d + 192*I*sqrt(d*x + c)*b^2*c^2
*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x +
c)*d^3)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^3)/d^3 - 35*(sqrt(2)
*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*b*c*d^2 + 15*I*d^3)*d*erf(-sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c +
4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 4*(-64*I*(d*x + c)
^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d - 192*I*sqrt(d*x + c)*b^2*c^2*
d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c
)*d^3)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^3)/d^3) - 2240*(3*sqrt
(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c
```

```
- 12*I*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*
I*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.00, size = 251, normalized size = 1.10

$$\frac{\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4da-4cb}{d}\right)\mathrm{S}\left(2\right.\right.}{}\right)}{8b}\right)}{32b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/56*(d*x+c)^(7/2)-1/64/b*d*(d*x+c)^(5/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+5/64/b*d*(-1/8/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/8/
b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d
)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(
b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 2.02, size = 285, normalized size = 1.25

$$\sqrt{2}\left(\frac{4096\sqrt{2}\,(dx+c)^{\frac{7}{2}}b^4}{d}-2240\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)-\left((105i+105)\,\sqrt{\pi}\,d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/229376*sqrt(2)*(4096*sqrt(2)*(d*x + c)^(7/2)*b^4/d - 2240*sqrt(2)*(d*x +
c)^(3/2)*b^2*d*cos(4*((d*x + c)*b - b*c + a*d)/d) - ((105*I + 105)*sqrt(pi)
*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (105*I - 105)*sqrt(pi)*d^3*(b^
2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (-(1
05*I - 105)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (105*I + 1
05)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)
*sqrt(-I*b/d)) - 56*(64*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x +
 c)*b*d^2)*sin(4*((d*x + c)*b - b*c + a*d)/d))/b^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\sin(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.130 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=615

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)}{1600b^{7/2}}$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(b*x+a)/b-1/48*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+1/80*(d
*x+c)^(5/2)*cos(5*b*x+5*a)/b+5/16*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2+5/288*d*(d
*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-1/160*d*(d*x+c)^(3/2)*sin(5*b*x+5*a)/b^2+3/1
6000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelS(b^(1/2)*10^
(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^
(7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/3456*d^(5/2)*FresnelS(b^(1/2
)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)
/b^(7/2)-15/64*d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*FresnelS(b^(1/2)*2
^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2
)+15/32*d^2*cos(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*cos(3*b*x+3*a)*(d*x+c)^(
1/2)/b^3-3/1600*d^2*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b^3
```

**Rubi** [A] time = 1.15, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+3\sqrt{\frac{\pi}{10}}\,d^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1600
*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(32*
b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqr
t[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/10]*Cos
[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1
600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/
6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c + d*x)^(3/2)*S
in[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(288*b^2) -
(d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{5/2}\sin(3a+3bx)-\frac{1}{16}(c+$$

$$= \frac{1}{16}\int (c+dx)^{5/2}\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^{5/2}\sin(5a+5bx)\,d$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}\text{co}}{80}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}\text{co}}{80}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}\,\text{co}}{576}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}\,\text{co}}{576}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}\,\text{co}}{576}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}\,\text{co}}{576}$$

**Mathematica [C]** time = 24.10, size = 3348, normalized size = 5.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c^2*(2*Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fre
snelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*
Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d])
 - (c^2*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Co
s[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi
]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*S
qrt[3]*b*Sqrt[b/d]) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/
Pi]*Sqrt[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] +
 3*d*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] -
3*Sin[a + b*x])))/(16*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d
]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c
*d*Sin[a - (b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) -
2*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*
d*x)*Sin[a + b*x])))/(64*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/
d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b
*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Co
s[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c
 + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) + ((
b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((1
2*b^2*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) -
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a
- (3*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[
b/d]*d*Sqrt[c + d*x]*(d*(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)
*Sin[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) + (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*Fresne
lS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*b*c)/d] - 10*b*c*
Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c +
d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*
Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin[5*(a + b*x)])))/
(800*Sqrt[5]*b^3) - (d^2*(Sin[5*a]*((c^2*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]
*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c
 + d*x]])*Sin[(5*b*c)/d])/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (c^2*Cos[(5*b*c)/d]
*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) -
 (2*c*Cos[(5*b*c)/d]*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c +
d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 +
5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*
(b/d)^(5/2)*d^3) - (2*c*Sin[(5*b*c)/d]*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3
/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi
]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d])
)/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (Sin[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/
2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10
/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c
 + d*x))/d]))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3) + (Cos[(5*b*c)/d]*(25*Sqrt[
5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Sin[(5*b*(c + d*x))/d] - (5*(-5*Sqrt[5]*(b/d
)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[
Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin
```

```
[(5*b*(c + d*x))/d]))/2))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3)) + Cos[5*a]*((c
^2*Cos[(5*b*c)/d]*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]
) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/(5*Sqrt[5]*(
b/d)^(3/2)*d^3) - (c^2*Sin[(5*b*c)/d]*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x
))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (2*c*Sin[(5*b*c)/d]*((-3*(-(Sqrt[5]*S
qrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b
/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*
Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) - (2*c*Cos[(5*b*c)/d]
*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqr
t[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*
Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (
Cos[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))
/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) +
Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b
/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c + d*x))/d]))/2))/(125*Sqrt[5]*(b/d)^(
7/2)*d^3) - (Sin[(5*b*c)/d]*(25*Sqrt[5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Sin[(5*
b*(c + d*x))/d] - (5*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c +
d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]])
+ Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/2))/2))/(125*Sqr
t[5]*(b/d)^(7/2)*d^3))))/16
```

**fricas [A]** time = 1.01, size = 521, normalized size = 0.85

$$\frac{81\,\sqrt{10}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{5\,(bc-ad)}{d}\right)C\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-625\,\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_c
os(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d
))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
- 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fr
esnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*s
qrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-3*(b*c - a*d)/d) - 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(9*(20*b^
3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^5 + 390*b*d^2
*cos(b*x + a) - 5*(60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 - 13*b*d^2)*
cos(b*x + a)^3 + 10*(26*b^2*d^2*x - 9*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^4
+ 26*b^2*c*d + 13*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(
d*x + c))/b^4
```

**giac [C]** time = 15.99, size = 3689, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/864000*(1800*(-3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(s
qrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt
(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/
```

```
2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c
+ I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*sqrt(pi)*d
*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(1
0)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^3 + 18*c*d^2*(9*(-I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^
2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I
*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^
2)*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(I*sqrt(6)*s
qrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x
+ c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d
)/b^2)/d^2 + 2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf
(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c
)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x +
c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 2250*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 -
4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d
*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(-I*sqrt(
6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sq
rt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*
a*d)/d)/b^2)/d^2 + 9*(I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2
)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10
*I*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*
d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2)/d^2) + d^3*(27*(I*sqr
t(10)*sqrt(pi)*(200*b^3*c^3 + 60*I*b^2*c^2*d - 18*b*c*d^2 - 3*I*d^3)*d*erf(
-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*
b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(
d*x + c)^(5/2)*b^2*d + 60*I*(d*x + c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^
2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*
x + c)*d^3)*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^3)/d^3 + 125*(-I
*sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*d^3)*d*er
f(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I
*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d
*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2
*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x
 + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^3 + 6750*(-I
*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b
*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x +
c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*
d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c
)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 6750*(I*sqrt(2)*sq
rt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt
(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)
*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(
d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e
^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + 125*(I*sqrt(6)*sqrt(pi)*(72
*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/
```

```
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2*
d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x +
 c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((3*I
*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^3 + 27*(-I*sqrt(10)*sqrt(pi)*(2
00*b^3*c^3 - 60*I*b^2*c^2*d - 18*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(d*x + c)^(5/2)
*b^2*d + 60*I*(d*x + c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^2*c^2*d + 10*(
d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^
((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^3)/d^3) + 180*(9*I*sqrt(10)*sqr
t(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*
d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*
sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*(10*b*c - I*
d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*
sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b + 150*sqrt(d*
x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*sqrt(d*x + c)*d*e^((-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b +
3*I*b*c - 3*I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c
 - 5*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.05, size = 719, normalized size = 1.17

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-}{}\right.\right.}{4b\sqrt{\frac{b}{d}}}\right)}{2b}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+5/16/b*d*(1/2/b
*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(
1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(c
os((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-si
n((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-
1/96/b*d*(d*x+c)^(5/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/96/b*d*(1/6/b*d*(
d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/
2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d
*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)
```

```
^(1/2)*(d*x+c)^(1/2)*b/d))))+1/160/b*d*(d*x+c)^(5/2)*cos(5/d*(d*x+c)*b+5*(a
*d-b*c)/d)-1/32/b*d*(1/10/b*d*(d*x+c)^(3/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d
)-3/10/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/100/
b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(5*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.90, size = 820, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3456000*sqrt(2)*(10800*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b -
b*c + a*d)/d)/d - 30000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b*
c + a*d)/d)/d - 540000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c +
 a*d)/d)/d - 1080*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sqrt(d*x
+ c)*b^3)*cos(5*((d*x + c)*b - b*c + a*d)/d) + 3000*(12*sqrt(2)*(d*x + c)^(
5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*cos(3*((d*x + c)*b - b*c + a*d)
/d) + 54000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x + c)*b
^3)*cos(((d*x + c)*b - b*c + a*d)/d) + ((162*I - 162)*25^(1/4)*sqrt(pi)*b^2
*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (162*I + 162)*25^(1/4)*sqrt(pi)*
b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d
)) + (-(1250*I - 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c -
 a*d)/d) - (1250*I + 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (-(202500*I - 202500)*sqrt
(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (202500*I + 202500)*sqrt(p
i)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)
) + ((202500*I + 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)
 + (202500*I - 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((1250*I + 1250)*9^(1/4)*sqrt(pi)*b^2*d*(
b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (1250*I - 1250)*9^(1/4)*sqrt(pi)*b^2
*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d))
 + (-(162*I + 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*
d)/d) - (162*I - 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^6
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.131 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=534

$$\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\cdots}{\cdots}\right)}{16b^{5/2}}\cdots$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(b*x+a)/b-1/48*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b+1/80*(d
*x+c)^(3/2)*cos(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/
2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)+3/800
0*d^(3/2)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a
-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)-1/576*d^(3/2)*cos(3*a-3*b*c/d)*FresnelS
(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1
/576*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3
*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*FresnelS(b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/32*
d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/
d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*sin(b*x+a)*(d*x+c)^(1/2)/b^2+1/96*d*sin(
3*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.88, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*
b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[
6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*Cos[5*
a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(800*
b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*
x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Fres
nelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(96*
b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x]
)/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d*Sqrt[c + d*
x]*Sin[5*a + 5*b*x])/(800*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{3/2}\sin(3a+3bx)-\frac{1}{16}(c$$

$$= \frac{1}{16}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^{3/2}\sin(5a+5bx$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}}{}$$

**Mathematica [C]** time = 11.68, size = 1041, normalized size = 1.95

$$\frac{ce^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b}+\frac{c\left(2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(5(a+bx))-\sqrt{2\pi}\cos\left(5a-\frac{5b}{d}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c*(2*Sqrt[5]*Sqrt[b/
d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fresnel
C[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (
c*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a
- (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*Fres
nelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*Sqrt[3]
*b*Sqrt[b/d]) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt
[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin
[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a
+ b*x])))/(32*b^3) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]
*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*
c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*
x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) + (Sqrt[b/d]*d*(
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*
b*c)/d] - 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)
/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin
[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

**fricas [A]** time = 0.99, size = 427, normalized size = 0.80

$$\frac{27\,\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{5\,(bc-ad)}{d}\right)S\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right) - 125\,\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)S\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_si
n(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) + 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(30*(b^2*d*x +
 b^2*c)*cos(b*x + a)^5 - 50*(b^2*d*x + b^2*c)*cos(b*x + a)^3 - (9*b*d*cos(b
*x + a)^4 - 13*b*d*cos(b*x + a)^2 - 26*b*d)*sin(b*x + a))*sqrt(d*x + c))/b^
3
```

**giac [C]** time = 4.63, size = 2300, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/144000*(300*(-3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*
```

```
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
 I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*sqrt(pi)*d*
erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10
)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1))
)*c^2 + d^2*(9*(-I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*e
rf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5
*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-10*
I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(
(-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(I*sqrt(6)*sqrt(pi
)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b
*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)
/d^2 + 2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2
)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b +
 I*b*c - I*a*d)/d)/b^2)/d^2 + 2250*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*
c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2
) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c
)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(-I*sqrt(6)*sqr
t(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x
 + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d
)/b^2)/d^2 + 9*(I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*er
f(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-1
0*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e
^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2)/d^2) + 20*(9*I*sqrt(10)*sqr
t(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*
d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*
sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*(10*b*c - I*
d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*
sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b + 150*sqrt(d*
x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*sqrt(d*x + c)*d*e^((-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b +
3*I*b*c - 3*I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c
 - 5*I*a*d)/d)/b)*c)/d
```

**maple [A]** time = 0.04, size = 580, normalized size = 1.09

$$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)S\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{8b}-d(dx+c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/16/b*d*(1/2/b
*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1
/2)*b/d)))-1/96/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d
*(1/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3
^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))+1/160/b*d*(d*x+c)^(3/2)*co
s(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-3/160/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*(d
*x+c)*b+5*(a*d-b*c)/d)-1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(
5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*
b/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+
c)^(1/2)*b/d))))
```

**maxima [C]** time = 0.92, size = 760, normalized size = 1.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/576000*sqrt(2)*(3600*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 6000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*c
+ a*d)/d)/d^2 - 36000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c +
a*d)/d)/d^2 - 1080*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c + a*d
)/d)/d + 3000*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/
d + 54000*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d - (-
(54*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (
54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(5*I*b/d)) - ((250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*cos(-3*(b*c - a*d)/d) - (250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - ((13500*
I + 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (13500*I - 13
500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(I*b/d)) - (-(13500*I - 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) + (13500*I + 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)
)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - (-(250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - ((
54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (5
4*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\,\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.132 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{1}\cdots}{\cdots}\right.}{80b^{3/2}}$$

```
[Out] -1/800*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelS(b^(1/2)*10^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)/b^
(3/2)+1/288*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2
)/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/288*FresnelS(b^(1/2)*6^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b
^(3/2)+1/16*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/
8*cos(b*x+a)*(d*x+c)^(1/2)/b-1/48*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b+1/80*cos(5
*b*x+5*a)*(d*x+c)^(1/2)/b
```

**Rubi [A]** time = 0.67, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(8*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48*
b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2))
+ (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqr
t[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\,\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\,\sin(a+bx)+\frac{1}{16}\sqrt{c+dx}\,\sin(3a+3bx)-\frac{1}{16}\sqrt{c+d$$

$$=\frac{1}{16}\int \sqrt{c+dx}\,\sin(3a+3bx)\,dx-\frac{1}{16}\int \sqrt{c+dx}\,\sin(5a+5bx)\,dx$$

$$=-\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\,\cos(5\ldots}{80b}$$

$$=-\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\,\cos(5\ldots}{80b}$$

$$=-\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\,\cos(5\ldots}{80b}$$

$$=-\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\,\cos(5\ldots}{80b}$$

**Mathematica [C]** time = 7.34, size = 432, normalized size = 0.94

$$\frac{-\sqrt{2\pi}\cos\left(5a-\frac{5bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)+\sqrt{2\pi}\sin\left(5a-\frac{5bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)+2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\,c\ldots}{160\sqrt{5}\,b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[
(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (2*Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelC[Sqr
t[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/P
i]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d])/(160*Sqrt[5]*b*Sqrt[b/d]) - (2*Sqrt
[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)
/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(96*Sqrt[3]*b*Sqrt[b/
d])
```

**fricas** [A] time = 1.01, size = 356, normalized size = 0.78

$$-\frac{9\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-25\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(
-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*s
qrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b
/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*
d)/d) - 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^5 - 5*b*cos(b*
x + a)^3)*sqrt(d*x + c))/b^2
```

**giac** [C] time = 2.74, size = 1258, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/14400*(9*I*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d
*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(
2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d
*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt
(10)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)*b) + 30*(-3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt
(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*I*sqrt(2)*sqrt(pi)*d*erf
(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)
*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
```

```
^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*
sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) +
3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)))*c - 90*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d
)/d)/b + 150*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b
+ 900*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*sqrt(d*
x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^(
(-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^((-5*I*(
d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)/d
```

**maple [A]** time = 0.04, size = 447, normalized size = 0.97

$$-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\cos\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)
/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fres
nelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c
)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/16
0/b*d*(d*x+c)^(1/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/1600/b*d*2^(1/2)*Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima [C]** time = 0.84, size = 674, normalized size = 1.47

$$\sqrt{2}\left(\frac{360\,\sqrt{2}\,\sqrt{dx+c}\,b^3\cos\left(\frac{5\,((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{600\,\sqrt{2}\,\sqrt{dx+c}\,b^3\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{3600\,\sqrt{2}\,\sqrt{dx+c}\,b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2}+\left(\frac{(18i-18)\cdots}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/57600*sqrt(2)*(360*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c + a
*d)/d)/d^2 - 600*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/
d)/d^2 - 3600*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d^
2 + ((18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d
)/d + (18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (-(50*I - 50)*9^(1/4)*sqrt(pi)*b^2
*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d - (50*I + 50)*9^(1/4)*sqrt(pi)*b^2
*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d))
+ (-(900*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (900
*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x
+ c)*sqrt(I*b/d)) + ((900*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c -
 a*d)/d)/d + (900*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)
/d)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((50*I + 50)*9^(1/4)*sqrt(pi)*b^2*(b^
```

```
2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d + (50*I - 50)*9^(1/4)*sqrt(pi)*b^2*(b^
2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + (
-(18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d
- (18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d
)*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\sin^3\left(a+bx\right)\cos^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

# 3.133 $\int \sqrt{c+dx}\,\cos^2(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}}{}\right.}{80b^{3/2}}$$

```
[Out] -1/800*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelS(b^(1/2)*10^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)/b^
(3/2)+1/288*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2
)/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/288*FresnelS(b^(1/2)*6^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b
^(3/2)+1/16*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/
8*cos(b*x+a)*(d*x+c)^(1/2)/b-1/48*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b+1/80*cos(5
*b*x+5*a)*(d*x+c)^(1/2)/b
```

**Rubi** [A] time = 0.66, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\sqrt{\frac{\pi}{10}}\sqrt{d}\cos$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(8*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48*
b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2))
+ (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqr
t[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\sin(a+bx)+\frac{1}{16}\sqrt{c+dx}\sin(3a+3bx)-\frac{1}{16}\sqrt{c+dx}\,\mathrm{s}$$

$$= \frac{1}{16}\int \sqrt{c+dx}\sin(3a+3bx)\,dx-\frac{1}{16}\int \sqrt{c+dx}\sin(5a+5bx)\,dx+$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a+}{80b}$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a+}{80b}$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a+}{80b}$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a+}{80b}$$

**Mathematica [C]** time = 7.26, size = 432, normalized size = 0.94

$$\frac{-\sqrt{2\pi}\cos\left(5a-\frac{5bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)+\sqrt{2\pi}\sin\left(5a-\frac{5bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)+2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(}{160\sqrt{5}\,b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (2*Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d])/(160*Sqrt[5]*b*Sqrt[b/d]) - (2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(96*Sqrt[3]*b*Sqrt[b/d])

**fricas [A]** time = 0.67, size = 356, normalized size = 0.78

$$-\frac{9\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-25\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")

[Out] -1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^5 - 5*b*cos(b*x + a)^3)*sqrt(d*x + c))/b^2

**giac [C]** time = 2.85, size = 1258, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")

[Out] -1/14400*(9*I*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 30*(-3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e

```
^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*
sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) +
3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)))*c - 90*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d
)/d)/b + 150*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b
+ 900*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*sqrt(d*
x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^(
(-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^((-5*I*(
d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)/d
```

**maple** [A] time = 0.00, size = 447, normalized size = 0.97

$$-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\operatorname{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\cos\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{48b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)
/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fres
nelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c
)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/16
0/b*d*(d*x+c)^(1/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/1600/b*d*2^(1/2)*Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima** [C] time = 0.57, size = 674, normalized size = 1.47

$$\sqrt{2}\left(\frac{360\sqrt{2}\,\sqrt{dx+c}\,b^3\cos\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{600\sqrt{2}\,\sqrt{dx+c}\,b^3\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{3600\sqrt{2}\,\sqrt{dx+c}\,b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2}+\left(\frac{(18i-18)\cdot 25^{\frac{1}{4}}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/57600*sqrt(2)*(360*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c + a
*d)/d)/d^2 - 600*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/
d)/d^2 - 3600*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d^
2 + ((18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d
)/d + (18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (-(50*I - 50)*9^(1/4)*sqrt(pi)*b^2
*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d - (50*I + 50)*9^(1/4)*sqrt(pi)*b^2
*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d))
+ (-(900*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (900
*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x
+ c)*sqrt(I*b/d)) + ((900*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c -
 a*d)/d)/d + (900*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)
/d)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((50*I + 50)*9^(1/4)*sqrt(pi)*b^2*(b^
```

```
2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d + (50*I - 50)*9^(1/4)*sqrt(pi)*b^2*(b^
2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + (
-(18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d
- (18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d
)*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^3\left(a+bx\right)\cos^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

# 3.134 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=534

$$\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\ldots}{\ldots}\right)}{16b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(b*x+a)/b-1/48*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b+1/80*(d
*x+c)^(3/2)*cos(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/
2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)+3/800
0*d^(3/2)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a
-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)-1/576*d^(3/2)*cos(3*a-3*b*c/d)*FresnelS
(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1
/576*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3
*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*FresnelS(b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/32*
d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/
d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*sin(b*x+a)*(d*x+c)^(1/2)/b^2+1/96*d*sin(
3*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.80, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*
b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[
6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*Cos[5*
a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(800*
b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*
x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Fres
nelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(96*
b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x]
)/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d*Sqrt[c + d*
x]*Sin[5*a + 5*b*x])/(800*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{3/2}\sin(3a+3bx)-\frac{1}{16}(c$$

$$=\frac{1}{16}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^{3/2}\sin(5a+5bx$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}}{}$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}}{}$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}}{}$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}}{}$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}}{}$$

**Mathematica [C]** time = 11.45, size = 1041, normalized size = 1.95

$$\frac{ce^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b}+\frac{c\left(2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(5(a+bx))-\sqrt{2\pi}\cos\left(5a-\frac{5b}{d}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c*(2*Sqrt[5]*Sqrt[b/
d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fresnel
C[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (
c*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a
- (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*Fres
nelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*Sqrt[3]
*b*Sqrt[b/d]) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt
[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin
[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a
+ b*x])))/(32*b^3) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]
*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*
c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*
x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) + (Sqrt[b/d]*d*(
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*
b*c)/d] - 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)
/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin
[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

**fricas [A]** time = 0.63, size = 427, normalized size = 0.80

$$27\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{5\,(bc-ad)}{d}\right)S\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-125\,\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)S\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_si
n(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) + 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(30*(b^2*d*x +
 b^2*c)*cos(b*x + a)^5 - 50*(b^2*d*x + b^2*c)*cos(b*x + a)^3 - (9*b*d*cos(b
*x + a)^4 - 13*b*d*cos(b*x + a)^2 - 26*b*d)*sin(b*x + a))*sqrt(d*x + c))/b^
3
```

**giac [C]** time = 4.21, size = 2300, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/144000*(300*(-3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*
```

```
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
 I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*sqrt(pi)*d*
erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10
)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1))
)*c^2 + d^2*(9*(-I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*e
rf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5
*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-10*
I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(
(-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(I*sqrt(6)*sqrt(pi
)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b
*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)
/d^2 + 2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2
)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b +
 I*b*c - I*a*d)/d)/b^2)/d^2 + 2250*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*
c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2
) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c
)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(-I*sqrt(6)*sqr
t(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x
 + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d
)/b^2)/d^2 + 9*(I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*er
f(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-1
0*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e
^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2)/d^2) + 20*(9*I*sqrt(10)*sqr
t(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*
d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*
sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*(10*b*c - I*
d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*
sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b + 150*sqrt(d*
x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*sqrt(d*x + c)*d*e^((-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b +
3*I*b*c - 3*I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c
 - 5*I*a*d)/d)/b)*c)/d
```

**maple [A]** time = 0.00, size = 580, normalized size = 1.09

$$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{8b}-\frac{d(dx+c)^{\frac{3}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/16/b*d*(1/2/b
*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1
/2)*b/d)))-1/96/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d
*(1/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3
^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))+1/160/b*d*(d*x+c)^(3/2)*co
s(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-3/160/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*(d
*x+c)*b+5*(a*d-b*c)/d)-1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(
5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*
b/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+
c)^(1/2)*b/d))))
```

**maxima [C]** time = 0.80, size = 760, normalized size = 1.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/576000*sqrt(2)*(3600*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 6000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*c
+ a*d)/d)/d^2 - 36000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c +
a*d)/d)/d^2 - 1080*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c + a*d
)/d)/d + 3000*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/
d + 54000*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d - (-
(54*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (
54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(5*I*b/d)) - ((250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*cos(-3*(b*c - a*d)/d) - (250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - ((13500*
I + 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (13500*I - 13
500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(I*b/d)) - (-(13500*I - 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) + (13500*I + 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)
)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - (-(250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - ((
54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (5
4*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.135 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=615

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)C}{1600b^{7/2}}$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(b*x+a)/b-1/48*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+1/80*(d
*x+c)^(5/2)*cos(5*b*x+5*a)/b+5/16*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2+5/288*d*(d
*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-1/160*d*(d*x+c)^(3/2)*sin(5*b*x+5*a)/b^2+3/1
6000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelS(b^(1/2)*10^
(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^
(7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/3456*d^(5/2)*FresnelS(b^(1/2
)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)
/b^(7/2)-15/64*d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*FresnelS(b^(1/2)*2
^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2
)+15/32*d^2*cos(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*cos(3*b*x+3*a)*(d*x+c)^(
1/2)/b^3-3/1600*d^2*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b^3
```

**Rubi [A]** time = 0.95, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+3\sqrt{\frac{\pi}{10}}\,d^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1600
*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(32*
b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqr
t[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/10]*Cos
[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1
600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/
6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c + d*x)^(3/2)*S
in[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(288*b^2) -
(d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{5/2}\sin(3a+3bx)-\frac{1}{16}(c\right.$$

$$= \frac{1}{16}\int (c+dx)^{5/2}\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^{5/2}\sin(5a+5bx$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}}{}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}}{5}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}}{5}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}}{5}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}}{5}$$

**Mathematica [C]** time = 22.97, size = 3348, normalized size = 5.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c^2*(2*Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fre
snelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*
Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d])
 - (c^2*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Co
s[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi
]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*S
qrt[3]*b*Sqrt[b/d]) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/
Pi]*Sqrt[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] +
 3*d*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] -
3*Sin[a + b*x])))/(16*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d
]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c
*d*Sin[a - (b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) -
2*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*
d*x)*Sin[a + b*x])))/(64*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/
d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b
*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Co
s[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c
 + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) + ((
b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((1
2*b^2*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) -
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a
- (3*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[
b/d]*d*Sqrt[c + d*x]*(d*(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)
*Sin[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) + (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*Fresne
lS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*b*c)/d] - 10*b*c*
Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c +
d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*
Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin[5*(a + b*x)])))/
(800*Sqrt[5]*b^3) - (d^2*(Sin[5*a]*((c^2*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]
*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c
 + d*x]])*Sin[(5*b*c)/d])/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (c^2*Cos[(5*b*c)/d]
*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) -
 (2*c*Cos[(5*b*c)/d]*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c +
d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 +
5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*
(b/d)^(5/2)*d^3) - (2*c*Sin[(5*b*c)/d]*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3
/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi
]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d])
)/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (Sin[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/
2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10
/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c
 + d*x))/d]))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3) + (Cos[(5*b*c)/d]*(25*Sqrt[
5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Sin[(5*b*(c + d*x))/d] - (5*(-5*Sqrt[5]*(b/d
)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[
Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin
```

```
[(5*b*(c + d*x))/d]))/2))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3)) + Cos[5*a]*((c
^2*Cos[(5*b*c)/d]*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]
) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/(5*Sqrt[5]*(
b/d)^(3/2)*d^3) - (c^2*Sin[(5*b*c)/d]*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x
))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (2*c*Sin[(5*b*c)/d]*((-3*(-(Sqrt[5]*S
qrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b
/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*
Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) - (2*c*Cos[(5*b*c)/d]
*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqr
t[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*
Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (
Cos[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))
/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) +
Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b
/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c + d*x))/d]))/2))/(125*Sqrt[5]*(b/d)^(
7/2)*d^3) - (Sin[(5*b*c)/d]*(25*Sqrt[5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Sin[(5*
b*(c + d*x))/d] - (5*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c +
d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]])
+ Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/2))/2))/(125*Sqr
t[5]*(b/d)^(7/2)*d^3))))/16
```

**fricas [A]** time = 0.82, size = 521, normalized size = 0.85

$$\frac{81\sqrt{10}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-625\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_c
os(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d
))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
- 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fr
esnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*s
qrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-3*(b*c - a*d)/d) - 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(9*(20*b^
3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^5 + 390*b*d^2
*cos(b*x + a) - 5*(60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 - 13*b*d^2)*
cos(b*x + a)^3 + 10*(26*b^2*d^2*x - 9*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^4
+ 26*b^2*c*d + 13*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(
d*x + c))/b^4
```

**giac [C]** time = 7.58, size = 3689, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/864000*(1800*(-3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(s
qrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt
(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/
```

```
2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c
+ I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*sqrt(pi)*d
*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(1
0)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^3 + 18*c*d^2*(9*(-I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^
2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I
*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^
2)*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(I*sqrt(6)*s
qrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x
+ c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d
)/b^2)/d^2 + 2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf
(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c
)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x +
c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 2250*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 -
4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d
*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(-I*sqrt(
6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sq
rt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*
a*d)/d)/b^2)/d^2 + 9*(I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2
)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10
*I*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*
d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2)/d^2) + d^3*(27*(I*sqr
t(10)*sqrt(pi)*(200*b^3*c^3 + 60*I*b^2*c^2*d - 18*b*c*d^2 - 3*I*d^3)*d*erf(
-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*
b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(
d*x + c)^(5/2)*b^2*d + 60*I*(d*x + c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^
2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*
x + c)*d^3)*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^3)/d^3 + 125*(-I
*sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*d^3)*d*er
f(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I
*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d
*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2
*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x
 + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^3 + 6750*(-I
*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b
*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x +
c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*
d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c
)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 6750*(I*sqrt(2)*sq
rt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt
(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)
*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(
d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e
^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + 125*(I*sqrt(6)*sqrt(pi)*(72
*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/
```

```
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2*
d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x +
 c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((3*I
*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^3 + 27*(-I*sqrt(10)*sqrt(pi)*(2
00*b^3*c^3 - 60*I*b^2*c^2*d - 18*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(d*x + c)^(5/2)
*b^2*d + 60*I*(d*x + c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^2*c^2*d + 10*(
d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^
((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^3)/d^3) + 180*(9*I*sqrt(10)*sqr
t(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*
d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*
sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*(10*b*c - I*
d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*
sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b + 150*sqrt(d*
x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*sqrt(d*x + c)*d*e^((-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b +
3*I*b*c - 3*I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c
 - 5*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.00, size = 719, normalized size = 1.17

$$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\cdots\right.}{4b\sqrt{\frac{b}{d}}}\right.}{2b}\right.}{8b}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+5/16/b*d*(1/2/b
*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(
1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(c
os((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-si
n((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-
1/96/b*d*(d*x+c)^(5/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/96/b*d*(1/6/b*d*(
d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/
2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d
*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)
```

```
^(1/2)*(d*x+c)^(1/2)*b/d))))+1/160/b*d*(d*x+c)^(5/2)*cos(5/d*(d*x+c)*b+5*(a
*d-b*c)/d)-1/32/b*d*(1/10/b*d*(d*x+c)^(3/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d
)-3/10/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/100/
b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(5*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 1.79, size = 820, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3456000*sqrt(2)*(10800*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b -
b*c + a*d)/d)/d - 30000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b*
c + a*d)/d)/d - 540000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c +
 a*d)/d)/d - 1080*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sqrt(d*x
+ c)*b^3)*cos(5*((d*x + c)*b - b*c + a*d)/d) + 3000*(12*sqrt(2)*(d*x + c)^(
5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*cos(3*((d*x + c)*b - b*c + a*d)
/d) + 54000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x + c)*b
^3)*cos(((d*x + c)*b - b*c + a*d)/d) + ((162*I - 162)*25^(1/4)*sqrt(pi)*b^2
*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (162*I + 162)*25^(1/4)*sqrt(pi)*
b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d
)) + (-(1250*I - 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c -
 a*d)/d) - (1250*I + 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (-(202500*I - 202500)*sqrt
(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (202500*I + 202500)*sqrt(p
i)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)
) + ((202500*I + 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)
 + (202500*I - 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((1250*I + 1250)*9^(1/4)*sqrt(pi)*b^2*d*(
b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (1250*I - 1250)*9^(1/4)*sqrt(pi)*b^2
*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d))
 + (-(162*I + 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*
d)/d) - (162*I - 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^6
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.136 $\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=273

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\right.}{b}$$

```
[Out] -2^(-4-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b
*(d*x+c)/d)^m)-2^(-4-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b
*c/d))/((I*b*(d*x+c)/d)^m)-exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b*(d
*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)-(d*x+c)^m*GAMMA(1+m,4*I*b*(d*x+
c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

**Rubi [A]** time = 0.29, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3308, 2181}

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Ga}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-4 - m)*(c + d*x)^m*Gamma[
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m) - (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c + d*
x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 +
 m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*((I*b*
(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^m \sin(2a+2bx) + \frac{1}{8}(c+dx)^m \sin(4a+4bx)\right) dx$$
$$= \frac{1}{8}\int (c+dx)^m \sin(4a+4bx)\,dx + \frac{1}{4}\int (c+dx)^m \sin(2a+2bx)\,dx$$
$$= \frac{1}{16}i\int e^{-i(4a+4bx)}(c+dx)^m\,dx - \frac{1}{16}i\int e^{i(4a+4bx)}(c+dx)^m\,dx + \frac{1}{8}i\int e^{-i}$$
$$= -\frac{2^{-4-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-4-m}e^{-}}{}$$

**Mathematica [A]** time = 0.24, size = 245, normalized size = 0.90

$$-\frac{4^{-m-3}e^{-\frac{4i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(2^{m+2}e^{2i\left(a+\frac{3bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right)+2^{m+2}e^{2i\left(3a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((4^(-3 - m)*(c + d*x)^m*(2^(2 + m)*E^((2*I)*(3*a + (b*c)/d))*((I*b*(c + d
*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d] + 2^(2 + m)*E^((2*I)*(a + (3
*b*c)/d))*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d] + E^
((8*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d] + E^((
(8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])
)/(b*E^(((4*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m))
```

**fricas [A]** time = 0.60, size = 184, normalized size = 0.67

$$-\frac{e^{\left(-\frac{dm\log\left(\frac{4ib}{d}\right)-4ibc+4iad}{d}\right)}\Gamma\left(m+1,\frac{4ibdx+4ibc}{d}\right)+4e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2ibc+2iad}{d}\right)}\Gamma\left(m+1,\frac{2ibdx+2ibc}{d}\right)+4e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}\Gamma}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/64*(e^(-(d*m*log(4*I*b/d) - 4*I*b*c + 4*I*a*d)/d)*gamma(m + 1, (4*I*b*d*
x + 4*I*b*c)/d) + 4*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m +
 1, (2*I*b*d*x + 2*I*b*c)/d) + 4*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d
)/d)*gamma(m + 1, (-2*I*b*d*x - 2*I*b*c)/d) + e^(-(d*m*log(-4*I*b/d) + 4*I*
b*c - 4*I*a*d)/d)*gamma(m + 1, (-4*I*b*d*x - 4*I*b*c)/d))/b
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a), x)
```

**maple [F]** time = 0.18, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^3(bx+a)\right)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3 \sin(a+b\,x)\,(c+d\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^m, x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

## 3.137 $\int (c+dx)^4 \cos^3(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=260

$$-\frac{3d^4\cos^4(a+bx)}{128b^5}-\frac{45d^4\cos^2(a+bx)}{128b^5}-\frac{3d^3(c+dx)\sin(a+bx)\cos^3(a+bx)}{32b^4}-\frac{45d^3(c+dx)\sin(a+bx)\cos(a+b}{64b^4}$$

```
[Out] -45/64*c*d^3*x/b^3-45/128*d^4*x^2/b^3+3/32*(d*x+c)^4/b-45/128*d^4*cos(b*x+a
)^2/b^5+9/16*d^2*(d*x+c)^2*cos(b*x+a)^2/b^3-3/128*d^4*cos(b*x+a)^4/b^5+3/16
*d^2*(d*x+c)^2*cos(b*x+a)^4/b^3-1/4*(d*x+c)^4*cos(b*x+a)^4/b-45/64*d^3*(d*x
+c)*cos(b*x+a)*sin(b*x+a)/b^4+3/8*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2-3/3
2*d^3*(d*x+c)*cos(b*x+a)^3*sin(b*x+a)/b^4+1/4*d*(d*x+c)^3*cos(b*x+a)^3*sin(
b*x+a)/b^2
```

**Rubi** [A] time = 0.23, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4405, 3311, 32, 3310}

$$\frac{3d^2(c+dx)^2\cos^4(a+bx)}{16b^3}+\frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3}-\frac{3d^3(c+dx)\sin(a+bx)\cos^3(a+bx)}{32b^4}-\frac{45d^3(c+dx)\sin(a}{64b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-45*c*d^3*x)/(64*b^3) - (45*d^4*x^2)/(128*b^3) + (3*(c + d*x)^4)/(32*b) -
(45*d^4*Cos[a + b*x]^2)/(128*b^5) + (9*d^2*(c + d*x)^2*Cos[a + b*x]^2)/(16*
b^3) - (3*d^4*Cos[a + b*x]^4)/(128*b^5) + (3*d^2*(c + d*x)^2*Cos[a + b*x]^4
)/(16*b^3) - ((c + d*x)^4*Cos[a + b*x]^4)/(4*b) - (45*d^3*(c + d*x)*Cos[a +
 b*x]*Sin[a + b*x])/(64*b^4) + (3*d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/
(8*b^2) - (3*d^3*(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x])/(32*b^4) + (d*(c +
d*x)^3*Cos[a + b*x]^3*Sin[a + b*x])/(4*b^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
```

```
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^4\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^4\cos^4(a+bx)}{4b}+\frac{d\int (c+dx)^3\cos^4(a+bx)\,dx}{b}$$
$$=\frac{3d^2(c+dx)^2\cos^4(a+bx)}{16b^3}-\frac{(c+dx)^4\cos^4(a+bx)}{4b}+\frac{d(c+dx)^3\cos$$
$$=\frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3}-\frac{3d^4\cos^4(a+bx)}{128b^5}+\frac{3d^2(c+dx)^2\cos^4(a}{16b^3}$$
$$=\frac{3(c+dx)^4}{32b}-\frac{45d^4\cos^2(a+bx)}{128b^5}+\frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3}-\frac{3d^4 c}{}$$
$$=-\frac{45cd^3x}{64b^3}-\frac{45d^4x^2}{128b^3}+\frac{3(c+dx)^4}{32b}-\frac{45d^4\cos^2(a+bx)}{128b^5}+\frac{9d^2(c+dx)}{1}$$

**Mathematica [A]** time = 1.85, size = 158, normalized size = 0.61

$$-\frac{-8bd(c+dx)\sin(2(a+bx))\left(\cos(2(a+bx))\left(8b^2(c+dx)^2-3d^2\right)+16\left(2b^2(c+dx)^2-3d^2\right)\right)+64\cos(2(a+b}{1024b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -1/1024*(64*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a +
b*x)] + (3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Cos[4*(a + b*
x)] - 8*b*d*(c + d*x)*(16*(-3*d^2 + 2*b^2*(c + d*x)^2) + (-3*d^2 + 8*b^2*(c
 + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)])/b^5
```

**fricas [A]** time = 0.70, size = 378, normalized size = 1.45

$$\frac{12\,b^4d^4x^4+48\,b^4cd^3x^3-\left(32\,b^4d^4x^4+128\,b^4cd^3x^3+32\,b^4c^4-24\,b^2c^2d^2+3\,d^4+24\left(8\,b^4c^2d^2-b^2d^4\right)x^2+16}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/128*(12*b^4*d^4*x^4 + 48*b^4*c*d^3*x^3 - (32*b^4*d^4*x^4 + 128*b^4*c*d^3*
x^3 + 32*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^
2 + 16*(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^4 + 9*(8*b^4*c^2*d^2 - 5
*b^2*d^4)*x^2 + 9*(8*b^2*d^4*x^2 + 16*b^2*c*d^3*x + 8*b^2*c^2*d^2 - 5*d^4)*
cos(b*x + a)^2 + 6*(8*b^4*c^3*d - 15*b^2*c*d^3)*x + 2*(2*(8*b^3*d^4*x^3 + 2
4*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c^2*d^2 - b*d^4)*x)*co
s(b*x + a)^3 + 3*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 15*b*c*d
^3 + 3*(8*b^3*c^2*d^2 - 5*b*d^4)*x)*cos(b*x + a))*sin(b*x + a))/b^5
```

**giac [A]** time = 0.26, size = 361, normalized size = 1.39

$$-\frac{\left(32\,b^4d^4x^4+128\,b^4cd^3x^3+192\,b^4c^2d^2x^2+128\,b^4c^3dx+32\,b^4c^4-24\,b^2d^4x^2-48\,b^2cd^3x-24\,b^2c^2d^2+3\,d^4\right)}{1024\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/1024*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4
*c^3*d*x + 32*b^4*c^4 - 24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 +
3*d^4)*cos(4*b*x + 4*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^
4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x
- 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 + 1/256*(8*b^3*d^4*x^3 + 24*b
^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(
4*b*x + 4*a)/b^5 + 1/8*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x +
 2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

**maple** [B] time = 0.09, size = 1150, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^4*d^4*(-1/4*(b*x+a)^4*cos(b*x+a)^4+(b*x+a)^3*(1/4*(cos(b*x+a)^3+3/
2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/16*(b*x+a)^2*cos(b*x+a)^4-3/8*(b*
x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+45/128*(b
*x+a)^2-3/128*cos(b*x+a)^4-9/128*cos(b*x+a)^2+9/16*(b*x+a)^2*cos(b*x+a)^2-9
/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+9/32*sin(b*x+a)^2-9/32
*(b*x+a)^4)-4/b^4*a*d^4*(-1/4*(b*x+a)^3*cos(b*x+a)^4+3/4*(b*x+a)^2*(1/4*(co
s(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*cos(b*x+a
)^4-3/128*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)-45/256*b*x-45/256*a+9/32
*(b*x+a)*cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x+a)-3/16*(b*x+a)^3)+4/b^3*c*d^
3*(-1/4*(b*x+a)^3*cos(b*x+a)^4+3/4*(b*x+a)^2*(1/4*(cos(b*x+a)^3+3/2*cos(b*x
+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*cos(b*x+a)^4-3/128*(cos(b*x+a)^
3+3/2*cos(b*x+a))*sin(b*x+a)-45/256*b*x-45/256*a+9/32*(b*x+a)*cos(b*x+a)^2-
9/64*cos(b*x+a)*sin(b*x+a)-3/16*(b*x+a)^3)+6/b^4*a^2*d^4*(-1/4*(b*x+a)^2*co
s(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*
x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*cos(b*x+a)^2)-12/b^3*a*c*d^3
*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a)
)*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*cos(b*x+a
)^2)+6/b^2*c^2*d^2*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a
)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a
)^4+3/32*cos(b*x+a)^2)-4/b^4*a^3*d^4*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b
*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+12/b^3*a^2*c*d^3*(-1/4*
(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x
+3/32*a)-12/b^2*a*c^2*d^2*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2
*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+4/b*c^3*d*(-1/4*(b*x+a)*cos(b*x+a)
^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)-1/4/b^4*a
^4*d^4*cos(b*x+a)^4+1/b^3*a^3*c*d^3*cos(b*x+a)^4-3/2/b^2*a^2*c^2*d^2*cos(b*
x+a)^4+1/b*a*c^3*d*cos(b*x+a)^4-1/4*c^4*cos(b*x+a)^4)
```

**maxima** [B] time = 0.39, size = 967, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*(256*c^4*cos(b*x + a)^4 - 1024*a*c^3*d*cos(b*x + a)^4/b + 1536*a^2*
c^2*d^2*cos(b*x + a)^4/b^2 - 1024*a^3*c*d^3*cos(b*x + a)^4/b^3 + 256*a^4*d^
4*cos(b*x + a)^4/b^4 + 32*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(
2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*c^3*d/b - 96*(4*(b*x
+ a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) -
8*sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 96*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(
b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a^2*c*d^
3/b^3 - 32*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) -
sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a^3*d^4/b^4 + 24*((8*(b*x + a)^2 - 1
```

```
)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*
sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 48*((8*(b*x
 + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4
*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 +
24*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x
 + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a^2
*d^4/b^4 + 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x
 + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x +
 4*a) - 96*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 4*(4*(8*(b*x +
 a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*co
s(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) - 96*(2*(b*x + a)^2
 - 1)*sin(2*b*x + 2*a))*a*d^4/b^4 + ((32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*
cos(4*b*x + 4*a) + 64*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a)
- 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a) - 128*(2*(b*x + a)^3 - 3
*b*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b
```

**mupad [B]** time = 1.34, size = 576, normalized size = 2.22

$$-\frac{192\,d^4\,\cos\left(2\,a+2\,b\,x\right)+3\,d^4\,\cos\left(4\,a+4\,b\,x\right)+128\,b^4\,c^4\,\cos\left(2\,a+2\,b\,x\right)+32\,b^4\,c^4\,\cos\left(4\,a+4\,b\,x\right)-256\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^4,x)
```

```
[Out] -(192*d^4*cos(2*a + 2*b*x) + 3*d^4*cos(4*a + 4*b*x) + 128*b^4*c^4*cos(2*a +
 2*b*x) + 32*b^4*c^4*cos(4*a + 4*b*x) - 256*b^3*c^3*d*sin(2*a + 2*b*x) - 32
*b^3*c^3*d*sin(4*a + 4*b*x) - 384*b^2*c^2*d^2*cos(2*a + 2*b*x) - 24*b^2*c^2
*d^2*cos(4*a + 4*b*x) - 384*b^2*d^4*x^2*cos(2*a + 2*b*x) - 24*b^2*d^4*x^2*c
os(4*a + 4*b*x) + 128*b^4*d^4*x^4*cos(2*a + 2*b*x) + 32*b^4*d^4*x^4*cos(4*a
 + 4*b*x) - 256*b^3*d^4*x^3*sin(2*a + 2*b*x) - 32*b^3*d^4*x^3*sin(4*a + 4*b
*x) + 384*b*c*d^3*sin(2*a + 2*b*x) + 12*b*c*d^3*sin(4*a + 4*b*x) + 384*b*d^
4*x*sin(2*a + 2*b*x) + 12*b*d^4*x*sin(4*a + 4*b*x) + 768*b^4*c^2*d^2*x^2*co
s(2*a + 2*b*x) + 192*b^4*c^2*d^2*x^2*cos(4*a + 4*b*x) - 768*b^2*c*d^3*x*cos
(2*a + 2*b*x) + 512*b^4*c^3*d*x*cos(2*a + 2*b*x) - 48*b^2*c*d^3*x*cos(4*a +
 4*b*x) + 128*b^4*c^3*d*x*cos(4*a + 4*b*x) + 512*b^4*c*d^3*x^3*cos(2*a + 2*
b*x) + 128*b^4*c*d^3*x^3*cos(4*a + 4*b*x) - 768*b^3*c^2*d^2*x*sin(2*a + 2*b
*x) - 768*b^3*c*d^3*x^2*sin(2*a + 2*b*x) - 96*b^3*c^2*d^2*x*sin(4*a + 4*b*x
) - 96*b^3*c*d^3*x^2*sin(4*a + 4*b*x))/(1024*b^5)
```

**sympy [A]** time = 13.23, size = 935, normalized size = 3.60

$$\begin{cases} -\frac{c^4\cos^4(a+bx)}{4b} + \frac{3c^3dx\sin^4(a+bx)}{8b} + \frac{3c^3dx\sin^2(a+bx)\cos^2(a+bx)}{4b} - \frac{5c^3dx\cos^4(a+bx)}{8b} + \frac{9c^2d^2x^2\sin^4(a+bx)}{16b} + \frac{9c^2d^2x^2\sin^2(a+bx)\cos}{8b} \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sin(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**4*cos(a + b*x)**4/(4*b) + 3*c**3*d*x*sin(a + b*x)**4/(8*b) +
 3*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 5*c**3*d*x*cos(a + b*x)
**4/(8*b) + 9*c**2*d**2*x**2*sin(a + b*x)**4/(16*b) + 9*c**2*d**2*x**2*sin(
a + b*x)**2*cos(a + b*x)**2/(8*b) - 15*c**2*d**2*x**2*cos(a + b*x)**4/(16*b
) + 3*c*d**3*x**3*sin(a + b*x)**4/(8*b) + 3*c*d**3*x**3*sin(a + b*x)**2*cos
(a + b*x)**2/(4*b) - 5*c*d**3*x**3*cos(a + b*x)**4/(8*b) + 3*d**4*x**4*sin(
a + b*x)**4/(32*b) + 3*d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5
*d**4*x**4*cos(a + b*x)**4/(32*b) + 3*c**3*d*sin(a + b*x)**3*cos(a + b*x)/(
8*b**2) + 5*c**3*d*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 9*c**2*d**2*x*si
n(a + b*x)**3*cos(a + b*x)/(8*b**2) + 15*c**2*d**2*x*sin(a + b*x)*cos(a + b
```

```
*x)**3/(8*b**2) + 9*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 15*
c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 3*d**4*x**3*sin(a + b*x
)**3*cos(a + b*x)/(8*b**2) + 5*d**4*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b*
*2) - 9*c**2*d**2*sin(a + b*x)**4/(32*b**3) + 15*c**2*d**2*cos(a + b*x)**4/
(32*b**3) - 45*c*d**3*x*sin(a + b*x)**4/(64*b**3) - 9*c*d**3*x*sin(a + b*x)
**2*cos(a + b*x)**2/(32*b**3) + 51*c*d**3*x*cos(a + b*x)**4/(64*b**3) - 45*
d**4*x**2*sin(a + b*x)**4/(128*b**3) - 9*d**4*x**2*sin(a + b*x)**2*cos(a +
b*x)**2/(64*b**3) + 51*d**4*x**2*cos(a + b*x)**4/(128*b**3) - 45*c*d**3*sin
(a + b*x)**3*cos(a + b*x)/(64*b**4) - 51*c*d**3*sin(a + b*x)*cos(a + b*x)**
3/(64*b**4) - 45*d**4*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 51*d**4*x*
sin(a + b*x)*cos(a + b*x)**3/(64*b**4) + 45*d**4*sin(a + b*x)**4/(256*b**5)
 - 51*d**4*cos(a + b*x)**4/(256*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2
+ 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*cos(a)**3, True))
```

# 3.138 $\int (c+dx)^3 \cos^3(a+bx) \sin(a+bx)\,dx$

**Optimal**. Leaf size=196

$$-\frac{3d^3 \sin(a+bx)\cos^3(a+bx)}{128b^4}-\frac{45d^3 \sin(a+bx)\cos(a+bx)}{256b^4}+\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}+\frac{9d^2(c+dx)\cos^2(a+$$

```
[Out] -45/256*d^3*x/b^3+3/32*(d*x+c)^3/b+9/32*d^2*(d*x+c)*cos(b*x+a)^2/b^3+3/32*d
^2*(d*x+c)*cos(b*x+a)^4/b^3-1/4*(d*x+c)^3*cos(b*x+a)^4/b-45/256*d^3*cos(b*x
+a)*sin(b*x+a)/b^4+9/32*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-3/128*d^3*cos
(b*x+a)^3*sin(b*x+a)/b^4+3/16*d*(d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)/b^2
```

**Rubi [A]** time = 0.16, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4405, 3311, 32, 2635, 8}

$$\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}+\frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d(c+dx)^2\sin(a+bx)\cos^3(a+bx)}{16b^2}+\frac{9d(c+dx)^2\sin(a}{32}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-45*d^3*x)/(256*b^3) + (3*(c + d*x)^3)/(32*b) + (9*d^2*(c + d*x)*Cos[a + b
*x]^2)/(32*b^3) + (3*d^2*(c + d*x)*Cos[a + b*x]^4)/(32*b^3) - ((c + d*x)^3*
Cos[a + b*x]^4)/(4*b) - (45*d^3*Cos[a + b*x]*Sin[a + b*x])/(256*b^4) + (9*d
*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) - (3*d^3*Cos[a + b*x]^3*Si
n[a + b*x])/(128*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x])/(16*b
^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
```

```
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^3 \cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^3\cos^4(a+bx)}{4b} + \frac{(3d)\int (c+dx)^2\cos^4(a+bx)\,dx}{4b}$$
$$= \frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3} - \frac{(c+dx)^3\cos^4(a+bx)}{4b} + \frac{3d(c+dx)^2\cos^3(a}{16}$$
$$= \frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3} + \frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3} - \frac{(c+dx)^3\cos^4(a}{4b}$$
$$= \frac{3(c+dx)^3}{32b} + \frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3} + \frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3} - \frac{(c}{}$$
$$= -\frac{45d^3x}{256b^3} + \frac{3(c+dx)^3}{32b} + \frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3} + \frac{3d^2(c+dx)\cos^4(a}{32b^3}$$

**Mathematica [A]** time = 0.92, size = 135, normalized size = 0.69

$$\frac{-64b(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)-4b(c+dx)\cos(4(a+bx))\left(8b^2(c+dx)^2-3d^2\right)+6d\sin(2(a+}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-64*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 4*b*(c + d
*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + 6*d*(16*(-d^2 + 2*b^2*(
c + d*x)^2) + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)]
)/(1024*b^4)
```

**fricas [A]** time = 0.67, size = 238, normalized size = 1.21

$$\frac{24\,b^3d^3x^3+72\,b^3cd^2x^2-8\left(8\,b^3d^3x^3+24\,b^3cd^2x^2+8\,b^3c^3-3\,bcd^2+3\left(8\,b^3c^2d-bd^3\right)x\right)\cos\left(bx+a\right)^4+72\left(bd^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/256*(24*b^3*d^3*x^3 + 72*b^3*c*d^2*x^2 - 8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*
x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^4 + 7
2*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 9*(8*b^3*c^2*d - 5*b*d^3)*x + 3*(2*(
8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 + 3*(8*b
^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - 5*d^3)*cos(b*x + a))*sin(b*x +
a))/b^4
```

**giac [A]** time = 4.51, size = 241, normalized size = 1.23

$$-\frac{\left(8\,b^3d^3x^3+24\,b^3cd^2x^2+24\,b^3c^2dx+8\,b^3c^3-3\,bd^3x-3\,bcd^2\right)\cos\left(4\,bx+4\,a\right)}{256\,b^4}-\frac{\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b
*d^3*x - 3*b*c*d^2)*cos(4*b*x + 4*a)/b^4 - 1/16*(2*b^3*d^3*x^3 + 6*b^3*c*d^
2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)
```

```
/b^4 + 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*sin(4*b*
x + 4*a)/b^4 + 3/32*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin
(2*b*x + 2*a)/b^4
```

**maple [B]** time = 0.02, size = 594, normalized size = 3.03

$$\frac{d^3\left(-\frac{(bx+a)^3(\cos^4(bx+a))}{4}+\frac{3(bx+a)^2\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)}{4}+\frac{3(bx+a)(\cos^4(bx+a))}{32}-\frac{3\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{128}-\frac{45bx}{256}-\frac{45a}{256}+\frac{9(b}{}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^3*d^3*(-1/4*(b*x+a)^3*cos(b*x+a)^4+3/4*(b*x+a)^2*(1/4*(cos(b*x+a)^
3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*cos(b*x+a)^4-3/128
*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)-45/256*b*x-45/256*a+9/32*(b*x+a)*
cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x+a)-3/16*(b*x+a)^3)-3/b^3*a*d^3*(-1/4*(
b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*
x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*cos(b*x+a)^2)+3/b
^2*c*d^2*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*co
s(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*c
os(b*x+a)^2)+3/b^3*a^2*d^3*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/
2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)-6/b^2*a*c*d^2*(-1/4*(b*x+a)*cos(b
*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+3/b*
c^2*d*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x
+a)+3/32*b*x+3/32*a)+1/4/b^3*a^3*d^3*cos(b*x+a)^4-3/4/b^2*a^2*c*d^2*cos(b*x
+a)^4+3/4/b*a*c^2*d*cos(b*x+a)^4-1/4*c^3*cos(b*x+a)^4)
```

**maxima [B]** time = 0.36, size = 549, normalized size = 2.80

$$-\frac{256\,c^3\cos\left(bx+a\right)^4-\frac{768\,ac^2d\cos(bx+a)^4}{b}+\frac{768\,a^2cd^2\cos(bx+a)^4}{b^2}-\frac{256\,a^3d^3\cos(bx+a)^4}{b^3}+\frac{24\,(4\,(bx+a)\cos(4\,bx+4\,a)+16\,(bx+a)\cos}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*(256*c^3*cos(b*x + a)^4 - 768*a*c^2*d*cos(b*x + a)^4/b + 768*a^2*c*
d^2*cos(b*x + a)^4/b^2 - 256*a^3*d^3*cos(b*x + a)^4/b^3 + 24*(4*(b*x + a)*c
os(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(
2*b*x + 2*a))*c^2*d/b - 48*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos
(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a*c*d^2/b^2 + 24*(4*
(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*
a) - 8*sin(2*b*x + 2*a))*a^2*d^3/b^3 + 12*((8*(b*x + a)^2 - 1)*cos(4*b*x +
4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*
a) - 32*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 12*((8*(b*x + a)^2 - 1)*cos
(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4
*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (4*(8*(b*x + a)^3
- 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*
x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) - 96*(2*(b*x + a)^2 - 1)*
sin(2*b*x + 2*a))*d^3/b^3)/b
```

**mupad [B]** time = 2.06, size = 366, normalized size = 1.87

$$-\frac{24\,d^3\,\sin\left(2\,a+2\,b\,x\right)+\frac{3\,d^3\,\sin(4\,a+4\,b\,x)}{4}+32\,b^3\,c^3\,\cos\left(2\,a+2\,b\,x\right)+8\,b^3\,c^3\,\cos\left(4\,a+4\,b\,x\right)-48\,b^2\,c^2\,d\,\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^3,x)
```

```
[Out] -(24*d^3*sin(2*a + 2*b*x) + (3*d^3*sin(4*a + 4*b*x))/4 + 32*b^3*c^3*cos(2*a
 + 2*b*x) + 8*b^3*c^3*cos(4*a + 4*b*x) - 48*b^2*c^2*d*sin(2*a + 2*b*x) - 6*
b^2*c^2*d*sin(4*a + 4*b*x) + 32*b^3*d^3*x^3*cos(2*a + 2*b*x) + 8*b^3*d^3*x^
3*cos(4*a + 4*b*x) - 48*b^2*d^3*x^2*sin(2*a + 2*b*x) - 6*b^2*d^3*x^2*sin(4*
a + 4*b*x) - 48*b*c*d^2*cos(2*a + 2*b*x) - 3*b*c*d^2*cos(4*a + 4*b*x) - 48*
b*d^3*x*cos(2*a + 2*b*x) - 3*b*d^3*x*cos(4*a + 4*b*x) + 96*b^3*c^2*d*x*cos(
2*a + 2*b*x) + 24*b^3*c^2*d*x*cos(4*a + 4*b*x) - 96*b^2*c*d^2*x*sin(2*a + 2
*b*x) - 12*b^2*c*d^2*x*sin(4*a + 4*b*x) + 96*b^3*c*d^2*x^2*cos(2*a + 2*b*x)
 + 24*b^3*c*d^2*x^2*cos(4*a + 4*b*x))/(256*b^4)
```

**sympy** [A] time = 7.63, size = 602, normalized size = 3.07

$$\begin{cases} -\frac{c^3\cos^4(a+bx)}{4b} + \frac{9c^2dx\sin^4(a+bx)}{32b} + \frac{9c^2dx\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{15c^2dx\cos^4(a+bx)}{32b} + \frac{9cd^2x^2\sin^4(a+bx)}{32b} + \frac{9cd^2x^2\sin^2(a+bx)\cos^2(a \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**3*cos(a + b*x)**4/(4*b) + 9*c**2*d*x*sin(a + b*x)**4/(32*b)
+ 9*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 15*c**2*d*x*cos(a + b
*x)**4/(32*b) + 9*c*d**2*x**2*sin(a + b*x)**4/(32*b) + 9*c*d**2*x**2*sin(a
+ b*x)**2*cos(a + b*x)**2/(16*b) - 15*c*d**2*x**2*cos(a + b*x)**4/(32*b) +
3*d**3*x**3*sin(a + b*x)**4/(32*b) + 3*d**3*x**3*sin(a + b*x)**2*cos(a + b*
x)**2/(16*b) - 5*d**3*x**3*cos(a + b*x)**4/(32*b) + 9*c**2*d*sin(a + b*x)**
3*cos(a + b*x)/(32*b**2) + 15*c**2*d*sin(a + b*x)*cos(a + b*x)**3/(32*b**2)
 + 9*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 15*c*d**2*x*sin(a +
b*x)*cos(a + b*x)**3/(16*b**2) + 9*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(
32*b**2) + 15*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) - 9*c*d**2*s
in(a + b*x)**4/(64*b**3) + 15*c*d**2*cos(a + b*x)**4/(64*b**3) - 45*d**3*x*
sin(a + b*x)**4/(256*b**3) - 9*d**3*x*sin(a + b*x)**2*cos(a + b*x)**2/(128*
b**3) + 51*d**3*x*cos(a + b*x)**4/(256*b**3) - 45*d**3*sin(a + b*x)**3*cos(
a + b*x)/(256*b**4) - 51*d**3*sin(a + b*x)*cos(a + b*x)**3/(256*b**4), Ne(b
, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)*cos(a
)**3, True))
```

# 3.139 $\int (c+dx)^2 \cos^3(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=134

$$\frac{d^2\cos^4(a+bx)}{32b^3}+\frac{3d^2\cos^2(a+bx)}{32b^3}+\frac{d(c+dx)\sin(a+bx)\cos^3(a+bx)}{8b^2}+\frac{3d(c+dx)\sin(a+bx)\cos(a+bx)}{16b^2}-\frac{(c$$

```
[Out] 3/16*c*d*x/b+3/32*d^2*x^2/b+3/32*d^2*cos(b*x+a)^2/b^3+1/32*d^2*cos(b*x+a)^4
/b^3-1/4*(d*x+c)^2*cos(b*x+a)^4/b+3/16*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2+
1/8*d*(d*x+c)*cos(b*x+a)^3*sin(b*x+a)/b^2
```

**Rubi [A]** time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4405, 3310}

$$\frac{d(c+dx)\sin(a+bx)\cos^3(a+bx)}{8b^2}+\frac{3d(c+dx)\sin(a+bx)\cos(a+bx)}{16b^2}+\frac{d^2\cos^4(a+bx)}{32b^3}+\frac{3d^2\cos^2(a+bx)}{32b^3}-\frac{(c$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (3*c*d*x)/(16*b) + (3*d^2*x^2)/(32*b) + (3*d^2*Cos[a + b*x]^2)/(32*b^3) + (
d^2*Cos[a + b*x]^4)/(32*b^3) - ((c + d*x)^2*Cos[a + b*x]^4)/(4*b) + (3*d*(c
 + d*x)*Cos[a + b*x]*Sin[a + b*x])/(16*b^2) + (d*(c + d*x)*Cos[a + b*x]^3*S
in[a + b*x])/(8*b^2)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\cos^3(a+bx)\sin(a+bx)\,dx &= -\frac{(c+dx)^2\cos^4(a+bx)}{4b}+\frac{d\int (c+dx)\cos^4(a+bx)\,dx}{2b}\\ &= \frac{d^2\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}+\frac{d(c+dx)\cos^3(a+bx)\,\mathrm{s}}{8b^2}\\ &= \frac{3d^2\cos^2(a+bx)}{32b^3}+\frac{d^2\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}+\frac{3d(c}{}\\ &= \frac{3cdx}{16b}+\frac{3d^2x^2}{32b}+\frac{3d^2\cos^2(a+bx)}{32b^3}+\frac{d^2\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^2\cos}{4b}\end{aligned}$$

**Mathematica [A]** time = 0.46, size = 89, normalized size = 0.66

$$\frac{-16\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+\cos(4(a+bx))\left(d^2-8b^2(c+dx)^2\right)+4bd(c+dx)(8\sin(2(a+bx))+\mathrm{s}}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-16*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (d^2 - 8*b^2*(c + d*x)^2
)*Cos[4*(a + b*x)] + 4*b*d*(c + d*x)*(8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)]
))/(256*b^3)
```

**fricas [A]** time = 0.64, size = 130, normalized size = 0.97

$$\frac{3\,b^2d^2x^2+6\,b^2cdx-\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\cos\left(bx+a\right)^4+3\,d^2\cos\left(bx+a\right)^2+2\left(2\left(bd^2x+bcd\right)\mathrm{co}\right.}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(3*b^2*d^2*x^2 + 6*b^2*c*d*x - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c
^2 - d^2)*cos(b*x + a)^4 + 3*d^2*cos(b*x + a)^2 + 2*(2*(b*d^2*x + b*c*d)*co
s(b*x + a)^3 + 3*(b*d^2*x + b*c*d)*cos(b*x + a))*sin(b*x + a))/b^3
```

**giac [A]** time = 2.96, size = 145, normalized size = 1.08

$$-\frac{\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\cos\left(4\,bx+4\,a\right)}{256\,b^3}-\frac{\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2\right)\cos\left(2\,bx+2\,a\right)}{16\,b^3}+\frac{\left(bd^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(4*b*x + 4*a)/b^
3 - 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b
^3 + 1/64*(b*d^2*x + b*c*d)*sin(4*b*x + 4*a)/b^3 + 1/8*(b*d^2*x + b*c*d)*si
n(2*b*x + 2*a)/b^3
```

**maple [B]** time = 0.02, size = 260, normalized size = 1.94

$$\frac{\frac{d^2\left(-\frac{(bx+a)^2\left(\cos^4(bx+a)\right)}{4}+\frac{(bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)}{2}-\frac{3(bx+a)^2}{32}+\frac{\left(\cos^4(bx+a)\right)}{32}+\frac{3\left(\cos^2(bx+a)\right)}{32}\right)}{b^2}-\frac{2a\,d^2\left(-\frac{(bx+a)\left(\cos^4(bx+a)\right)}{4}+\frac{\left(\cos^3(bx\right.}{}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^2*d^2*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+
3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+
3/32*cos(b*x+a)^2)-2/b^2*a*d^2*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^
3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+2/b*c*d*(-1/4*(b*x+a)*cos(b*x
+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)-1/4*d^
2/b^2*a^2*cos(b*x+a)^4+1/2*c*d/b*a*cos(b*x+a)^4-1/4*c^2*cos(b*x+a)^4)
```

**maxima [B]** time = 0.34, size = 263, normalized size = 1.96

$$-\frac{64\,c^2\cos\left(bx+a\right)^4-\frac{128\,acd\cos(bx+a)^4}{b}+\frac{64\,a^2d^2\cos(bx+a)^4}{b^2}+\frac{4\left(4\left(bx+a\right)\cos(4\,bx+4\,a)+16\left(bx+a\right)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)-8\,\sin(2\right.}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/256*(64*c^2*cos(b*x + a)^4 - 128*a*c*d*cos(b*x + a)^4/b + 64*a^2*d^2*cos
(b*x + a)^4/b^2 + 4*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x
+ 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*c*d/b - 4*(4*(b*x + a)*cos(
4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b
*x + 2*a))*a*d^2/b^2 + ((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*s
in(2*b*x + 2*a))*d^2/b^2)/b
```

**mupad** [B] time = 1.63, size = 202, normalized size = 1.51

$$\frac{8\,d^2\,\cos\left(2\,a+2\,b\,x\right)+\frac{d^2\,\cos\left(4\,a+4\,b\,x\right)}{2}-16\,b^2\,c^2\,\cos\left(2\,a+2\,b\,x\right)-4\,b^2\,c^2\,\cos\left(4\,a+4\,b\,x\right)+16\,b\,c\,d\,\sin\left(2\,a+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2,x)
```

```
[Out] (8*d^2*cos(2*a + 2*b*x) + (d^2*cos(4*a + 4*b*x))/2 - 16*b^2*c^2*cos(2*a + 2
*b*x) - 4*b^2*c^2*cos(4*a + 4*b*x) + 16*b*c*d*sin(2*a + 2*b*x) + 2*b*c*d*si
n(4*a + 4*b*x) - 16*b^2*d^2*x^2*cos(2*a + 2*b*x) - 4*b^2*d^2*x^2*cos(4*a +
4*b*x) + 16*b*d^2*x*sin(2*a + 2*b*x) + 2*b*d^2*x*sin(4*a + 4*b*x) - 32*b^2*
c*d*x*cos(2*a + 2*b*x) - 8*b^2*c*d*x*cos(4*a + 4*b*x))/(128*b^3)
```

**sympy** [A] time = 3.79, size = 320, normalized size = 2.39

$$\begin{cases} -\frac{c^2\cos^4(a+bx)}{4b}+\frac{3cdx\sin^4(a+bx)}{16b}+\frac{3cdx\sin^2(a+bx)\cos^2(a+bx)}{8b}-\frac{5cdx\cos^4(a+bx)}{16b}+\frac{3d^2x^2\sin^4(a+bx)}{32b}+\frac{3d^2x^2\sin^2(a+bx)\cos^2(a+b}{16b} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**2*cos(a + b*x)**4/(4*b) + 3*c*d*x*sin(a + b*x)**4/(16*b) + 3
*c*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(8*b) - 5*c*d*x*cos(a + b*x)**4/(16*
b) + 3*d**2*x**2*sin(a + b*x)**4/(32*b) + 3*d**2*x**2*sin(a + b*x)**2*cos(a
 + b*x)**2/(16*b) - 5*d**2*x**2*cos(a + b*x)**4/(32*b) + 3*c*d*sin(a + b*x)
**3*cos(a + b*x)/(16*b**2) + 5*c*d*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) +
 3*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 5*d**2*x*sin(a + b*x)*co
s(a + b*x)**3/(16*b**2) - 3*d**2*sin(a + b*x)**4/(64*b**3) + 5*d**2*cos(a +
 b*x)**4/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)*co
s(a)**3, True))
```

# 3.140 $\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=72

$$\frac{d\sin(a+bx)\cos^3(a+bx)}{16b^2}+\frac{3d\sin(a+bx)\cos(a+bx)}{32b^2}-\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{3dx}{32b}$$

```
[Out] 3/32*d*x/b-1/4*(d*x+c)*cos(b*x+a)^4/b+3/32*d*cos(b*x+a)*sin(b*x+a)/b^2+1/16
*d*cos(b*x+a)^3*sin(b*x+a)/b^2
```

**Rubi [A]** time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4405, 2635, 8}

$$\frac{d\sin(a+bx)\cos^3(a+bx)}{16b^2}+\frac{3d\sin(a+bx)\cos(a+bx)}{32b^2}-\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{3dx}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (3*d*x)/(32*b) - ((c + d*x)*Cos[a + b*x]^4)/(4*b) + (3*d*Cos[a + b*x]*Sin[a
 + b*x])/(32*b^2) + (d*Cos[a + b*x]^3*Sin[a + b*x])/(16*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx &= -\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{d\int\cos^4(a+bx)\,dx}{4b}\\ &= -\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{d\cos^3(a+bx)\sin(a+bx)}{16b^2}+\frac{(3d)\int\cos^2(a+b}{16b}\\ &= -\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{3d\cos(a+bx)\sin(a+bx)}{32b^2}+\frac{d\cos^3(a+bx)\,\mathrm{si}}{16b^2}\\ &= \frac{3dx}{32b}-\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{3d\cos(a+bx)\sin(a+bx)}{32b^2}+\frac{d\cos^3(a+}{}\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 75, normalized size = 1.04

$$\frac{d(\sin(2(a+bx))-2bx\cos(2(a+bx)))}{16b^2}+\frac{d(\sin(4(a+bx))-4bx\cos(4(a+bx)))}{128b^2}-\frac{c\cos^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -1/4*(c*Cos[a + b*x]^4)/b + (d*(-2*b*x*Cos[2*(a + b*x)] + Sin[2*(a + b*x)])
)/(16*b^2) + (d*(-4*b*x*Cos[4*(a + b*x)] + Sin[4*(a + b*x)]))/(128*b^2)
```

**fricas [A]** time = 0.62, size = 58, normalized size = 0.81

$$-\frac{8\,(bdx+bc)\cos\left(bx+a\right)^{4}-3\,bdx-\left(2\,d\cos\left(bx+a\right)^{3}+3\,d\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{32\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/32*(8*(b*d*x + b*c)*cos(b*x + a)^4 - 3*b*d*x - (2*d*cos(b*x + a)^3 + 3*d
*cos(b*x + a))*sin(b*x + a))/b^2
```

**giac [A]** time = 0.20, size = 75, normalized size = 1.04

$$-\frac{(bdx+bc)\cos\left(4\,bx+4\,a\right)}{32\,b^{2}}-\frac{(bdx+bc)\cos\left(2\,bx+2\,a\right)}{8\,b^{2}}+\frac{d\sin\left(4\,bx+4\,a\right)}{128\,b^{2}}+\frac{d\sin\left(2\,bx+2\,a\right)}{16\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/32*(b*d*x + b*c)*cos(4*b*x + 4*a)/b^2 - 1/8*(b*d*x + b*c)*cos(2*b*x + 2*
a)/b^2 + 1/128*d*sin(4*b*x + 4*a)/b^2 + 1/16*d*sin(2*b*x + 2*a)/b^2
```

**maple [A]** time = 0.02, size = 85, normalized size = 1.18

$$\frac{\frac{d\left(-\frac{(bx+a)\left(\cos^{4}(bx+a)\right)}{4}+\frac{\left(\cos^{3}(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{16}+\frac{3bx}{32}+\frac{3a}{32}\right)}{b}+\frac{da\left(\cos^{4}(bx+a)\right)}{4b}-\frac{c\left(\cos^{4}(bx+a)\right)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b*d*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*si
n(b*x+a)+3/32*b*x+3/32*a)+1/4/b*d*a*cos(b*x+a)^4-1/4*c*cos(b*x+a)^4)
```

**maxima [A]** time = 0.32, size = 92, normalized size = 1.28

$$-\frac{32\,c\cos\left(bx+a\right)^{4}-\frac{32\,ad\cos(bx+a)^{4}}{b}+\frac{(4\,(bx+a)\cos(4\,bx+4\,a)+16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)-8\,\sin(2\,bx+2\,a))d}{b}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/128*(32*c*cos(b*x + a)^4 - 32*a*d*cos(b*x + a)^4/b + (4*(b*x + a)*cos(4*
b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x
 + 2*a))*d/b)/b
```

**mupad [B]** time = 0.31, size = 94, normalized size = 1.31

$$\frac{4\,d\,\sin\left(2\,a+2\,b\,x\right)+\frac{d\,\sin(4\,a+4\,b\,x)}{2}+4\,b\,c\sin\left(2\,a+2\,b\,x\right)^{2}+16\,b\,c\sin\left(a+b\,x\right)^{2}+8\,b\,d\,x\,\left(2\sin\left(a+b\,x\right)^{2}-1\right)}{64\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x),x)
```

```
[Out] (4*d*sin(2*a + 2*b*x) + (d*sin(4*a + 4*b*x))/2 + 4*b*c*sin(2*a + 2*b*x)^2 +
 16*b*c*sin(a + b*x)^2 + 8*b*d*x*(2*sin(a + b*x)^2 - 1) + 2*b*d*x*(2*sin(2*
a + 2*b*x)^2 - 1))/(64*b^2)
```

**sympy** [A] time = 1.94, size = 138, normalized size = 1.92

$$\begin{cases} -\frac{c\cos^4(a+bx)}{4b} + \frac{3dx\sin^4(a+bx)}{32b} + \frac{3dx\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{5dx\cos^4(a+bx)}{32b} + \frac{3d\sin^3(a+bx)\cos(a+bx)}{32b^2} + \frac{5d\sin(a+bx)\cos^3(a+bx)}{32b^2} \\ \left(cx + \frac{dx^2}{2}\right)\sin(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-c*cos(a + b*x)**4/(4*b) + 3*d*x*sin(a + b*x)**4/(32*b) + 3*d*x*
sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5*d*x*cos(a + b*x)**4/(32*b) + 3*d
*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 5*d*sin(a + b*x)*cos(a + b*x)**3/
(32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)*cos(a)**3, True))
```

## 3.141 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=129

$$\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Si}}{8d}$$

```
[Out] 1/4*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d+1/8*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*
b*x)/d+1/8*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d+1/4*Ci(2*b*c/d+2*b*x)*sin(2
*a-2*b*c/d)/d
```

**Rubi** [A] time = 0.21, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral[
(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*SinI
ntegral[(2*b*c)/d + 2*b*x])/(4*d) + (Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*
c)/d + 4*b*x])/(8*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)} + \frac{\sin(4a+4bx)}{8(c+dx)}\right)dx$$
$$= \frac{1}{8}\int \frac{\sin(4a+4bx)}{c+dx}\,dx + \frac{1}{4}\int \frac{\sin(2a+2bx)}{c+dx}\,dx$$
$$= \frac{1}{8}\cos\left(4a-\frac{4bc}{d}\right)\int \frac{\sin\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx + \frac{1}{4}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,d$$
$$= \frac{\text{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{8d} + \frac{\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{4d} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\text{S}}{4a}$$

**Mathematica [A]** time = 0.34, size = 110, normalized size = 0.85

$$\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4b(c+dx)}{d}\right) + 2\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right) + 2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + \cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c}{}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 2*CosIntegral[(2*b*(
c + d*x))/d]*Sin[2*a - (2*b*c)/d] + 2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b
*(c + d*x))/d] + Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d)
```

**fricas [A]** time = 0.42, size = 155, normalized size = 1.20

$$\frac{2\left(\text{Ci}\left(\frac{2(bdx+bc)}{d}\right) + \text{Ci}\left(-\frac{2(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right) + \left(\text{Ci}\left(\frac{4(bdx+bc)}{d}\right) + \text{Ci}\left(-\frac{4(bdx+bc)}{d}\right)\right)\sin\left(-\frac{4(bc-ad)}{d}\right) + 2\cos\left(-\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/16*(2*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d)
)*sin(-2*(b*c - a*d)/d) + (cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-
4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d) + 2*cos(-4*(b*c - a*d)/d)*sin_int
egral(4*(b*d*x + b*c)/d) + 4*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
b*c)/d))/d
```

**giac [C]** time = 1.92, size = 6046, normalized size = 46.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/16*(imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*
c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2
*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_in
tegral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b
*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d
)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*ta
n(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integra
```

```
l(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real_
part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b
*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(
2*b*c/d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2
*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(4*b*x +
 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^
2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*
c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*ta
n(2*b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^
2*tan(2*b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*ta
n(2*b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)^2 + 8*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan
(2*b*c/d)^2*tan(b*c/d) - 8*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*
a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 16*sin_integral(2*(b*d*x + b*c)/d)*
tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + imag_part(cos_integra
l(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(b
*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x +
 b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*imag_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*imag_part(cos_i
ntegral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 8*
sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2
 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2*tan(b
*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d
)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*t
an(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(
2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(
2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d
))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x +
2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_inte
gral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*sin_integr
al(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*sin_integral
(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*real_part(cos_
integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) + 2*real_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) + 4*real_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 + 4*real
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 - 2*
real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2 -
 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)
^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c
/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b
*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^
2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(2
*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*
tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(2*a)^2*tan(a)*tan(b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(2*a)^2*tan(a)*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*c
/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*
b*c/d))*tan(2*a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_inte
gral(4*b*x + 4*b*c/d))*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos
```

```
_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*
real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2*tan(b*c/d
)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(
b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d)^
2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)
^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2 + 2*ima
g_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2 + imag_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2 - 2*sin_integral(4*(b*d*x +
 b*c)/d)*tan(2*a)^2*tan(a)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2
*tan(a)^2 + 4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*ta
n(2*b*c/d) - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*
tan(2*b*c/d) + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b*
c/d) + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 +
 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - 2*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - imag_p
art(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*sin_integ
ral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 + 4*sin_integral(2*(b*d*x
+ b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/
d))*tan(a)^2*tan(2*b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^
2*tan(2*b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2
*b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2 - 4*s
in_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2 + 8*imag_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(b*c/d) - 8*imag_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(b*c/d) + 16*sin_integral(2*(
b*d*x + b*c)/d)*tan(2*a)^2*tan(a)*tan(b*c/d) + 8*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 8*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 16*sin_integral(2*(b*
d*x + b*c)/d)*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(2*a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(2*a)^2*tan(b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(b*c
/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(b*c/d)^2 + imag_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^
2*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 +
4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)*tan(b*c/d)
^2 - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)*tan(
b*c/d)^2 + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)*tan(b*c/
d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2
 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 +
imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*s
in_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*sin_integral
(2*(b*d*x + b*c)/d)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(
2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a) + 4*real_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(2*a)^2*tan(a) + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*a)*tan(a)^2 + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^
2 + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 2*
real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 2*real_
part(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(2*b*c/d) - 2*real_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d) - 2*real_part(cos_integr
al(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 2*real_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 + 4*real_part(cos_integral(2*b*x +
2*b*c/d))*tan(a)*tan(2*b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d
```

```
))*tan(a)*tan(2*b*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2
*a)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*t
an(b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)
+ 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) - 4*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part
(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(co
s_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(b*c/d)^2 + 2*real_part(cos_integr
al(-4*b*x - 4*b*c/d))*tan(2*a)*tan(b*c/d)^2 - 4*real_part(cos_integral(2*b*
x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(a)*tan(b*c/d)^2 - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*
c/d)*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + 2
*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2 - 2*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(2*a)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/
d))*tan(2*a)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 + 4*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(2*a)^2 + imag_part(cos_integral(4*b*x + 4*b*c/d)
)*tan(a)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + 2*imag_p
art(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - imag_part(cos_integral(-4*b*
x - 4*b*c/d))*tan(a)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^2 - 4*sin
_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + 4*imag_part(cos_integral(4*b*x + 4*
b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))
*tan(2*a)*tan(2*b*c/d) + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b
*c/d) - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 + 2*imag_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2 - 2*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/
d))*tan(2*b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 + 4*s
in_integral(2*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 + 8*imag_part(cos_integral(2*
b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 8*imag_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(a)*tan(b*c/d) + 16*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/
d) + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(b*c/d)^2 - 2*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(b
*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(b*c/d)^2 - 4*sin_integral(2
*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*c/d))
*tan(2*a) + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) + 4*real_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 4*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(a) - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/
d) - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d) - 4*real_part
(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 4*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_integral(4*b*x + 4*b*c/d)) + 2*ima
g_part(cos_integral(2*b*x + 2*b*c/d)) - 2*imag_part(cos_integral(-2*b*x - 2
*b*c/d)) - imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 2*sin_integral(4*(b*
d*x + b*c)/d) + 4*sin_integral(2*(b*d*x + b*c)/d))/(d*tan(2*a)^2*tan(a)^2*t
an(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + d*tan(2
*a)^2*tan(a)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*
tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(a)^2 + d*tan(2*a)^2
*tan(2*b*c/d)^2 + d*tan(a)^2*tan(2*b*c/d)^2 + d*tan(2*a)^2*tan(b*c/d)^2 + d
*tan(a)^2*tan(b*c/d)^2 + d*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2 + d*t
an(a)^2 + d*tan(2*b*c/d)^2 + d*tan(b*c/d)^2 + d)
```

**maple [A]** time = 0.02, size = 178, normalized size = 1.38

$$\frac{\frac{b\left(\frac{4\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}-\frac{4\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}\right)}{32}+\frac{b\left(\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\cdots\right.}{d}\right.}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x)
```

```
[Out] 1/b*(1/32*b*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*
x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)+1/8*b*(2*Si(2*b*x+2*a+2*(-a*d+
b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*
c)/d)/d))
```

**maxima [C]** time = 0.49, size = 274, normalized size = 2.12

$$-\frac{b\left(2i\,E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-2i\,E_1\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(i\,E_1\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_1\left(-\frac{4}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/16*(b*(2*I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) -
2*I*exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*
c - a*d)/d) + b*(I*exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/
d) - I*exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*
(b*c - a*d)/d) + 2*b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*
d)/d) + exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2
*(b*c - a*d)/d) + b*(exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d
)/d) + exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*
(b*c - a*d)/d))/(b*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)\cos^3\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x), x)
```

# 3.142 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=179

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\sin\left(4a-\right.}{}$$

```
[Out] 1/2*b*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^2+1/2*b*Ci(2*b*c/d+2*b*x)*cos(2*
a-2*b*c/d)/d^2-1/2*b*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^2-1/2*b*Si(2*b*c/
d+2*b*x)*sin(2*a-2*b*c/d)/d^2-1/4*sin(2*b*x+2*a)/d/(d*x+c)-1/8*sin(4*b*x+4*
a)/d/(d*x+c)
```

**Rubi [A]** time = 0.27, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Cos[4*
a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]/(
4*d*(c + d*x)) - Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]
*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Sin[4*a - (4*b*c)/d]*SinInteg
ral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^2}+\frac{\sin(4a+4bx)}{8(c+dx)^2}\right)dx$$
$$= \frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^2}\,dx+\frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx$$
$$= -\frac{\sin(2a+2bx)}{4d(c+dx)}-\frac{\sin(4a+4bx)}{8d(c+dx)}+\frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{2d}+\frac{b\int \frac{\cos(4a+4bx)}{c+dx}\,dx}{2d}$$
$$= -\frac{\sin(2a+2bx)}{4d(c+dx)}-\frac{\sin(4a+4bx)}{8d(c+dx)}+\frac{\left(b\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\cos\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx}{2d}+\frac{\left(b\cos\right.}{}$$
$$= \frac{b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{\sin(2a+2b}{4d(c+dx}$$

**Mathematica [A]** time = 1.63, size = 151, normalized size = 0.84

$$-\frac{-4b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2b(c+dx)}{d}\right)-4b\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4b(c+dx)}{d}\right)+4b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right)+4b\sin\left(4a-\right.}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -1/8*(-4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 4*b*Cos[4*
a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] + (2*d*Sin[2*(a + b*x)])/(c +
 d*x) + (d*Sin[4*(a + b*x)])/(c + d*x) + 4*b*Sin[2*a - (2*b*c)/d]*SinIntegr
al[(2*b*(c + d*x))/d] + 4*b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x)
)/d])/d^2
```

**fricas [A]** time = 0.74, size = 235, normalized size = 1.31

$$-\frac{4\,d\cos\left(bx+a\right)^3\sin\left(bx+a\right)+2\left(bdx+bc\right)\sin\left(-\frac{4\left(bc-ad\right)}{d}\right)\mathrm{Si}\left(\frac{4\left(bdx+bc\right)}{d}\right)+2\left(bdx+bc\right)\sin\left(-\frac{2\left(bc-ad\right)}{d}\right)\mathrm{Si}\left(\frac{2\left(bdx+\right.}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*d*cos(b*x + a)^3*sin(b*x + a) + 2*(b*d*x + b*c)*sin(-4*(b*c - a*d)/
d)*sin_integral(4*(b*d*x + b*c)/d) + 2*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*
sin_integral(2*(b*d*x + b*c)/d) - ((b*d*x + b*c)*cos_integral(2*(b*d*x + b*
c)/d) + (b*d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/
d) - ((b*d*x + b*c)*cos_integral(4*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_int
egral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [A] time = 0.02, size = 256, normalized size = 1.43

$$\frac{\frac{b^2\left(-\frac{4\sin(4bx+4a)}{((bx+a)d-da+cb)d}+\frac{\frac{16\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}+\frac{16\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}}{d}\right)}{32}+\frac{b^2\left(-\frac{2\sin(2bx+2a)}{((bx+a)d-da+cb)d}+\frac{\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right.}{d}}{}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/32*b^2*(-4*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)/d+4*(4*Si(4*b*x+4*a+4*
(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-
a*d+b*c)/d)/d)/d)+1/8*b^2*(-2*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si(
2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)
/d)*cos(2*(-a*d+b*c)/d)/d)/d))
```

**maxima** [C] time = 0.56, size = 301, normalized size = 1.68

$$-\frac{b^2\left(2i\,E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-2i\,E_2\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^2\left(i\,E_2\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(b^2*(2*I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
- 2*I*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^2*(I*exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) + 2*b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^2*(exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^3\,\sin(a+b\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**2, x)
```

# 3.143 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=231

$$-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(4a-\right.}{}$$

```
[Out] -1/4*b*cos(2*b*x+2*a)/d^2/(d*x+c)-1/4*b*cos(4*b*x+4*a)/d^2/(d*x+c)-1/2*b^2*
cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3-b^2*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*b*
x)/d^3-b^2*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^3-1/2*b^2*Ci(2*b*c/d+2*b*x)
*sin(2*a-2*b*c/d)/d^3-1/8*sin(2*b*x+2*a)/d/(d*x+c)^2-1/16*sin(4*b*x+4*a)/d/
(d*x+c)^2
```

**Rubi [A]** time = 0.33, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(4*d^2*(c + d*x)) - (b*Cos[4*a + 4*b*x])/(4*d^2*(c +
d*x)) - (b^2*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d^3 - (b^
2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d^3) - Sin[2*a +
2*b*x]/(8*d*(c + d*x)^2) - Sin[4*a + 4*b*x]/(16*d*(c + d*x)^2) - (b^2*Cos[2
*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^3) - (b^2*Cos[4*a - (4
*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/d^3
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^3}+\frac{\sin(4a+4bx)}{8(c+dx)^3}\right)dx$$
$$= \frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^3}\,dx+\frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx$$
$$= -\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}+\frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{4d}+\frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx}{4d}$$
$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}-\frac{b^2 \ldots}{\ldots}$$
$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}-\frac{(b^2 \ldots}{\ldots}$$
$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{b^2\operatorname{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{d^3}-\frac{b^2 \ldots}{\ldots}$$

**Mathematica [A]** time = 3.75, size = 197, normalized size = 0.85

$$-\frac{16b^2\sin\left(4a-\frac{4bc}{d}\right)\operatorname{Ci}\left(\frac{4b(c+dx)}{d}\right)+8b^2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2b(c+dx)}{d}\right)+8b^2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)+16b^2\cos \ldots}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -1/16*(16*b^2*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 8*b^2*C
osIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (2*d*(2*b*(c + d*x)*Co
s[2*(a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + (d*(4*b*(c + d*x)*Cos[4
*(a + b*x)] + d*Sin[4*(a + b*x)]))/(c + d*x)^2 + 8*b^2*Cos[2*a - (2*b*c)/d]
*SinIntegral[(2*b*(c + d*x))/d] + 16*b^2*Cos[4*a - (4*b*c)/d]*SinIntegral[(
4*b*(c + d*x))/d])/d^3
```

**fricas [A]** time = 0.75, size = 397, normalized size = 1.72

$$-\frac{2\,d^2\cos\left(bx+a\right)^3\sin\left(bx+a\right)+8\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4-6\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2+4\left(b^2d^2x^2+2\,b \ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*cos(b*x + a)^3*sin(b*x + a) + 8*(b*d^2*x + b*c*d)*cos(b*x + a)^
4 - 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*
c)/d) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)
/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d
```

```
))*sin(-2*(b*c - a*d)/d) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_int
egral(4*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integr
al(-4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d
^3)
```

**giac** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [**A**] time = 0.02, size = 329, normalized size = 1.42

$$\frac{\frac{b^3\left(-\frac{2\sin(4bx+4a)}{((bx+a)d-da+cb)^2d}+\frac{-\frac{8\cos(4bx+4a)}{((bx+a)d-da+cb)d}-\frac{8\left(\frac{4\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}-\frac{4\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}\right)}{d}}{d}\right)}{32}+\frac{b^3\left(-\frac{\sin(2bx+2a)}{((bx+a)d-da+cb)^2d}+\cdots\right)}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/32*b^3*(-2*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)^2/d+2*(-4*cos(4*b*x+4*
a)/((b*x+a)*d-d*a+c*b)/d-4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)
/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d)+1/8*b^3*(
-sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a)*d-d*a+c
*b)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*
a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d))
```

**maxima** [**C**] time = 0.69, size = 336, normalized size = 1.45

$$-\frac{b^3\left(2i\,E_3\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-2i\,E_3\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^3\left(i\,E_3\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_3\left(\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(b^3*(2*I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
 - 2*I*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^3*(I*exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) + 2*b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^3*(exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^3, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos^3(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**3, x)
```

# 3.144 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

**Optimal**. Leaf size=287

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\sin\left(4\right.}{}$$

```
[Out] -4/3*b^3*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^4-1/3*b^3*Ci(2*b*c/d+2*b*x)*c
os(2*a-2*b*c/d)/d^4-1/12*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2-1/12*b*cos(4*b*x+4*
a)/d^2/(d*x+c)^2+4/3*b^3*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^4+1/3*b^3*Si(
2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/12*sin(2*b*x+2*a)/d/(d*x+c)^3+1/6*b^2
*sin(2*b*x+2*a)/d^3/(d*x+c)-1/24*sin(4*b*x+4*a)/d/(d*x+c)^3+1/3*b^2*sin(4*b
*x+4*a)/d^3/(d*x+c)
```

**Rubi [A]** time = 0.45, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\right.}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(12*d^2*(c + d*x)^2) - (b*Cos[4*a + 4*b*x])/(12*d^2*(
c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) - (4*b^3*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
- Sin[2*a + 2*b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(6*d^3*(c +
d*x)) - Sin[4*a + 4*b*x]/(24*d*(c + d*x)^3) + (b^2*Sin[4*a + 4*b*x])/(3*d^3
*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) + (4*b^3*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^4}+\frac{\sin(4a+4bx)}{8(c+dx)^4}\right)dx$$

$$= \frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^4}\,dx+\frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\sin(2a+2bx)}{12d(c+dx)^3}-\frac{\sin(4a+4bx)}{24d(c+dx)^3}+\frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{6d}+\frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^3}\,dx}{6d}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}-\frac{\sin(4a+4bx)}{24d(c+dx)^3}-\frac{b^2}{\quad}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}-$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}-$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b}{\quad}$$

**Mathematica [A]** time = 2.49, size = 316, normalized size = 1.10

$$-\frac{8b^3(c+dx)^3\left(\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2b(c+dx)}{d}\right)-\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)\right)+32b^3(c+dx)^3\left(\cos\left(4a-\frac{4bc}{d}\right)\operatorname{Ci}\left(\right.\right.}{\quad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -1/24*(2*d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*S
in[2*a]) + d*Cos[4*b*x]*(2*b*d*(c + d*x)*Cos[4*a] + (d^2 - 8*b^2*(c + d*x)^
2)*Sin[4*a]) - 2*d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin
[2*a])*Sin[2*b*x] - d*((-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*a] + 2*b*d*(c + d*x
)*Sin[4*a])*Sin[4*b*x] + 8*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegra
l[(2*b*(c + d*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])
 + 32*b^3*(c + d*x)^3*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d]
- Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d]))/(d^4*(c + d*x)^3)
```

**fricas [B]** time = 0.70, size = 568, normalized size = 1.98

$$-\frac{4\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^4-3\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^2-8\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\sin\left(\right.}{\quad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(4*(b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - 3*(b*d^3*x + b*c*d^2)*cos(b*x
+ a)^2 - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-4
*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*
d*x + b*c)/d) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*
cos_integral(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^
2*d*x + b^3*c^3)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) +
4*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(4
*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^
3)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/d) - 2*((8*b^2*d^3*
x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 - 3*(b^2*d^3*x^2 +
 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a))*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*
x^2 + 3*c^2*d^5*x + c^3*d^4)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.02, size = 404, normalized size = 1.41

$$\frac{\frac{b^4\left(-\frac{4\sin(4bx+4a)}{3((bx+a)d-da+cb)^3 d}+\frac{-\frac{8\cos(4bx+4a)}{3((bx+a)d-da+cb)^2 d}-\frac{8\left(-\frac{4\sin(4bx+4a)}{((bx+a)d-da+cb)d}+\frac{\frac{16\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}+\frac{16\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}}{d}\right)}{3d}}{d}\right)}{32}+\frac{b^4 \cdots}{\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x)
```

```
[Out] 1/b*(1/32*b^4*(-4/3*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)^3/d+4/3*(-2*cos(4*b*
x+4*a)/((b*x+a)*d-d*a+c*b)^2/d-2*(-4*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)/d+4
*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a
*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)/d)/d)+1/8*b^4*(-2/3*sin(2*b*x+2*a)/((b
*x+a)*d-d*a+c*b)^3/d+2/3*(-cos(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d-(-2*sin(2
*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a
*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d)
)
```

**maxima [C]** time = 1.90, size = 386, normalized size = 1.34

$$-\frac{b^4\left(2i\,E_4\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-2i\,E_4\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^4\left(i\,E_4\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_4\left(\cdots\right.\right.}{16\left(b^3c^3d-3\,ab^2c^2d^2+3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/16*(b^4*(2*I*exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
- 2*I*exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
```

```
b*c - a*d)/d) + b^4*(I*exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) + 2*b^4*(exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^4*(exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)^3\,\sin(a+b\,x)}{(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos^3(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**4, x)
```

# 3.145 $\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=419

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{32b}-$$

```
[Out] -1/16*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+
c)/d)^m)+1/16*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b
*(d*x+c)/d)^m)+1/32*I*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*
b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/32*I*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I
*b*(d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)+1/32*I*5^(-1-m)*exp(
5*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-5*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)
-1/32*I*5^(-1-m)*(d*x+c)^m*GAMMA(1+m,5*I*b*(d*x+c)/d)/b/exp(5*I*(a-b*c/d))/
((I*b*(d*x+c)/d)^m)
```

**Rubi [A]** time = 0.43, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3307, 2181}

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/16)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d]
)/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/16)*(c + d*x)^m*Gamma[1 + m, (I*b*(c +
 d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/32)*3^(-1 -
m)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d]
)/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m
, ((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
 + ((I/32)*5^(-1 - m)*E^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-5
*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*5^(-1 - m)*(c +
 d*x)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d])/(b*E^((5*I)*(a - (b*c)/d))*((I
*b*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^m\cos(a+bx) - \frac{1}{16}(c+dx)^m\cos(3a+3bx) - \frac{1}{16}(c+$$

$$= -\left(\frac{1}{16}\int (c+dx)^m\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^m\cos(5a+5b$$

$$= -\left(\frac{1}{32}\int e^{-i(3a+3bx)}(c+dx)^m\,dx\right) - \frac{1}{32}\int e^{i(3a+3bx)}(c+dx)^m\,dx - \frac{1}{3}$$

$$= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{16b} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c}{}$$

**Mathematica [A]** time = 0.57, size = 409, normalized size = 0.98

$$-\frac{i3^{-m-1}e^{-\frac{3i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{\frac{6ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right) - e^{6ia}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,-\frac{3ib(c+}{d}\right.\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*(c + d*x)^m*((E^((2*I)*a)*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/((
(-I)*b*(c + d*x))/d)^m - (E^(((2*I)*b*c)/d)*Gamma[1 + m, (I*b*(c + d*x))/d]
)/((I*b*(c + d*x))/d)^m))/(b*E^((I*(b*c + a*d))/d)) - ((I/32)*3^(-1 - m)*(c
 + d*x)^m*(-(E^((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c +
d*x))/d]) + E^(((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*
b*(c + d*x))/d]))/(b*E^(((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m) -
 ((I/32)*5^(-1 - m)*(c + d*x)^m*(-(E^((10*I)*a)*((I*b*(c + d*x))/d)^m*Gamma
[1 + m, ((-5*I)*b*(c + d*x))/d]) + E^(((10*I)*b*c)/d)*(((-I)*b*(c + d*x))/d
)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d]))/(b*E^(((5*I)*(b*c + a*d))/d)*((b^
2*(c + d*x)^2)/d^2)^m)
```

**fricas [A]** time = 0.87, size = 276, normalized size = 0.66

$$\frac{-3ie^{\left(-\frac{dm\log\left(\frac{5ib}{d}\right)-5ibc+5iad}{d}\right)}\Gamma\left(m+1,\frac{5ibdx+5ibc}{d}\right) - 5ie^{\left(-\frac{dm\log\left(\frac{3ib}{d}\right)-3ibc+3iad}{d}\right)}\Gamma\left(m+1,\frac{3ibdx+3ibc}{d}\right) + 30ie^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+}{d}\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/480*(-3*I*e^(-(d*m*log(5*I*b/d) - 5*I*b*c + 5*I*a*d)/d)*gamma(m + 1, (5*I
*b*d*x + 5*I*b*c)/d) - 5*I*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*ga
mma(m + 1, (3*I*b*d*x + 3*I*b*c)/d) + 30*I*e^(-(d*m*log(I*b/d) - I*b*c + I*
a*d)/d)*gamma(m + 1, (I*b*d*x + I*b*c)/d) - 30*I*e^(-(d*m*log(-I*b/d) + I*b
*c - I*a*d)/d)*gamma(m + 1, (-I*b*d*x - I*b*c)/d) + 5*I*e^(-(d*m*log(-3*I*b
/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d) + 3*I*e^
(-(d*m*log(-5*I*b/d) + 5*I*b*c - 5*I*a*d)/d)*gamma(m + 1, (-5*I*b*d*x - 5*I
*b*c)/d))/b
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^2, x)
```

**maple** **[F]** time = 0.32, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^3(bx+a)\right)\left(\sin^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)
[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

**maxima** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^2, x)
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3 \sin(a+b\,x)^2 (c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^m,x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^m, x)
```

**sympy** **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a)**2,x)
[Out] Exception raised: HeuristicGCDFailed
```

# 3.146 $\int (c+dx)^4 \cos^3(a+bx) \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=330

$$\frac{3d^4\sin(a+bx)}{b^5}-\frac{d^4\sin(3a+3bx)}{162b^5}-\frac{3d^4\sin(5a+5bx)}{6250b^5}-\frac{3d^3(c+dx)\cos(a+bx)}{b^4}+\frac{d^3(c+dx)\cos(3a+3bx)}{54b^4}+\frac{3c}{}$$

```
[Out] -3*d^3*(d*x+c)*cos(b*x+a)/b^4+1/2*d*(d*x+c)^3*cos(b*x+a)/b^2+1/54*d^3*(d*x+
c)*cos(3*b*x+3*a)/b^4-1/36*d*(d*x+c)^3*cos(3*b*x+3*a)/b^2+3/1250*d^3*(d*x+c
)*cos(5*b*x+5*a)/b^4-1/100*d*(d*x+c)^3*cos(5*b*x+5*a)/b^2+3*d^4*sin(b*x+a)/
b^5-3/2*d^2*(d*x+c)^2*sin(b*x+a)/b^3+1/8*(d*x+c)^4*sin(b*x+a)/b-1/162*d^4*s
in(3*b*x+3*a)/b^5+1/36*d^2*(d*x+c)^2*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^4*sin(
3*b*x+3*a)/b-3/6250*d^4*sin(5*b*x+5*a)/b^5+3/500*d^2*(d*x+c)^2*sin(5*b*x+5*
a)/b^3-1/80*(d*x+c)^4*sin(5*b*x+5*a)/b
```

**Rubi [A]** time = 0.37, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2637}

$$-\frac{3d^2(c+dx)^2\sin(a+bx)}{2b^3}+\frac{d^2(c+dx)^2\sin(3a+3bx)}{36b^3}+\frac{3d^2(c+dx)^2\sin(5a+5bx)}{500b^3}-\frac{3d^3(c+dx)\cos(a+bx)}{b^4}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (-3*d^3*(c + d*x)*Cos[a + b*x])/b^4 + (d*(c + d*x)^3*Cos[a + b*x])/(2*b^2)
+ (d^3*(c + d*x)*Cos[3*a + 3*b*x])/(54*b^4) - (d*(c + d*x)^3*Cos[3*a + 3*b*
x])/(36*b^2) + (3*d^3*(c + d*x)*Cos[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)
^3*Cos[5*a + 5*b*x])/(100*b^2) + (3*d^4*Sin[a + b*x])/b^5 - (3*d^2*(c + d*x
)^2*Sin[a + b*x])/(2*b^3) + ((c + d*x)^4*Sin[a + b*x])/(8*b) - (d^4*Sin[3*a
 + 3*b*x])/(162*b^5) + (d^2*(c + d*x)^2*Sin[3*a + 3*b*x])/(36*b^3) - ((c +
d*x)^4*Sin[3*a + 3*b*x])/(48*b) - (3*d^4*Sin[5*a + 5*b*x])/(6250*b^5) + (3*
d^2*(c + d*x)^2*Sin[5*a + 5*b*x])/(500*b^3) - ((c + d*x)^4*Sin[5*a + 5*b*x]
)/(80*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^4 \cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^4\cos(a+bx) - \frac{1}{16}(c+dx)^4\cos(3a+3bx) - \frac{1}{16}(c+dx)^4\right.$$

$$= -\left(\frac{1}{16}\int (c+dx)^4\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^4\cos(5a+5bx)\,d$$

$$= \frac{(c+dx)^4\sin(a+bx)}{8b} - \frac{(c+dx)^4\sin(3a+3bx)}{48b} - \frac{(c+dx)^4\sin(5a+5}{80b}$$

$$= \frac{d(c+dx)^3\cos(a+bx)}{2b^2} - \frac{d(c+dx)^3\cos(3a+3bx)}{36b^2} - \frac{d(c+dx)^3\cos(5}{100b^2}$$

$$= \frac{d(c+dx)^3\cos(a+bx)}{2b^2} - \frac{d(c+dx)^3\cos(3a+3bx)}{36b^2} - \frac{d(c+dx)^3\cos(5}{100b^2}$$

$$= -\frac{3d^3(c+dx)\cos(a+bx)}{b^4} + \frac{d(c+dx)^3\cos(a+bx)}{2b^2} + \frac{d^3(c+dx)\cos(3}{54b^4}$$

$$= -\frac{3d^3(c+dx)\cos(a+bx)}{b^4} + \frac{d(c+dx)^3\cos(a+bx)}{2b^2} + \frac{d^3(c+dx)\cos(3}{54b^4}$$

**Mathematica** [A] time = 3.48, size = 563, normalized size = 1.71

$$-\frac{-506250b^4c^4\sin(a+bx) + 84375b^4c^4\sin(3(a+bx)) + 50625b^4c^4\sin(5(a+bx)) - 2025000b^3c^3d(bx\sin(a+bx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/4050000*(-506250*b^4*c^4*Sin[a + b*x] - 2025000*b^3*c^3*d*(Cos[a + b*x]
+ b*x*Sin[a + b*x]) - 2025000*b*c*d^3*(3*(-2 + b^2*x^2)*Cos[a + b*x] + b*x*
(-6 + b^2*x^2)*Sin[a + b*x]) - 3037500*b^2*c^2*d^2*(2*b*x*Cos[a + b*x] + (-
2 + b^2*x^2)*Sin[a + b*x]) - 506250*d^4*(4*b*x*(-6 + b^2*x^2)*Cos[a + b*x]
+ (24 - 12*b^2*x^2 + b^4*x^4)*Sin[a + b*x]) + 84375*b^4*c^4*Sin[3*(a + b*x)
] + 112500*b^3*c^3*d*(Cos[3*(a + b*x)] + 3*b*x*Sin[3*(a + b*x)]) + 37500*b*
c*d^3*((-2 + 9*b^2*x^2)*Cos[3*(a + b*x)] + 3*b*x*(-2 + 3*b^2*x^2)*Sin[3*(a
+ b*x)]) + 56250*b^2*c^2*d^2*(6*b*x*Cos[3*(a + b*x)] + (-2 + 9*b^2*x^2)*Sin
[3*(a + b*x)]) + 3125*d^4*(12*b*x*(-2 + 3*b^2*x^2)*Cos[3*(a + b*x)] + (8 -
36*b^2*x^2 + 27*b^4*x^4)*Sin[3*(a + b*x)]) + 50625*b^4*c^4*Sin[5*(a + b*x)]
 + 40500*b^3*c^3*d*(Cos[5*(a + b*x)] + 5*b*x*Sin[5*(a + b*x)]) + 1620*b*c*d
^3*((-6 + 75*b^2*x^2)*Cos[5*(a + b*x)] + 5*b*x*(-6 + 25*b^2*x^2)*Sin[5*(a +
 b*x)]) + 12150*b^2*c^2*d^2*(10*b*x*Cos[5*(a + b*x)] + (-2 + 25*b^2*x^2)*Si
n[5*(a + b*x)]) + 81*d^4*(20*b*x*(-6 + 25*b^2*x^2)*Cos[5*(a + b*x)] + (24 -
 300*b^2*x^2 + 625*b^4*x^4)*Sin[5*(a + b*x)]))/b^5
```

**fricas** [A] time = 1.20, size = 527, normalized size = 1.60

$$-\frac{1620\left(25\,b^3d^4x^3 + 75\,b^3cd^3x^2 + 25\,b^3c^3d - 6\,bcd^3 + 3\left(25\,b^3c^2d^2 - 2\,bd^4\right)x\right)\cos(bx+a)^5 - 300\left(75\,b^3d^4x^3 + 22\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/253125*(1620*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 25*b^3*c^3*d - 6*b*c*d
^3 + 3*(25*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^5 - 300*(75*b^3*d^4*x^3 +
 225*b^3*c*d^3*x^2 + 75*b^3*c^3*d + 22*b*c*d^3 + (225*b^3*c^2*d^2 + 22*b*d^
4)*x)*cos(b*x + a)^3 - 1800*(75*b^3*d^4*x^3 + 225*b^3*c*d^3*x^2 + 75*b^3*c^
3*d - 428*b*c*d^3 + (225*b^3*c^2*d^2 - 428*b*d^4)*x)*cos(b*x + a) - (33750*
b^4*d^4*x^4 + 135000*b^4*c*d^3*x^3 + 33750*b^4*c^4 - 385200*b^2*c^2*d^2 - 8
```

```
1*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 625*b^4*c^4 - 300*b^2*c^2*d^2 + 2
4*d^4 + 150*(25*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 100*(25*b^4*c^3*d - 6*b^2*c*
d^3)*x)*cos(b*x + a)^4 + 760816*d^4 + 900*(225*b^4*c^2*d^2 - 428*b^2*d^4)*x
^2 + (16875*b^4*d^4*x^4 + 67500*b^4*c*d^3*x^3 + 16875*b^4*c^4 + 9900*b^2*c^
2*d^2 - 4792*d^4 + 450*(225*b^4*c^2*d^2 + 22*b^2*d^4)*x^2 + 900*(75*b^4*c^3
*d + 22*b^2*c*d^3)*x)*cos(b*x + a)^2 + 1800*(75*b^4*c^3*d - 428*b^2*c*d^3)*
x)*sin(b*x + a))/b^5
```

**giac [A]** time = 0.29, size = 531, normalized size = 1.61

$$-\frac{\left(25\,b^3d^4x^3+75\,b^3cd^3x^2+75\,b^3c^2d^2x+25\,b^3c^3d-6\,bd^4x-6\,bcd^3\right)\cos\left(5\,bx+5\,a\right)}{2500\,b^5}-\frac{\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2500*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 75*b^3*c^2*d^2*x + 25*b^3*c^3*
d - 6*b*d^4*x - 6*b*c*d^3)*cos(5*b*x + 5*a)/b^5 - 1/108*(3*b^3*d^4*x^3 + 9*
b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*cos(
3*b*x + 3*a)/b^5 + 1/2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b
^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5 - 1/50000*(625*b^4*d^4*x
^4 + 2500*b^4*c*d^3*x^3 + 3750*b^4*c^2*d^2*x^2 + 2500*b^4*c^3*d*x + 625*b^4
*c^4 - 300*b^2*d^4*x^2 - 600*b^2*c*d^3*x - 300*b^2*c^2*d^2 + 24*d^4)*sin(5*
b*x + 5*a)/b^5 - 1/1296*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d
^2*x^2 + 108*b^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 3
6*b^2*c^2*d^2 + 8*d^4)*sin(3*b*x + 3*a)/b^5 + 1/8*(b^4*d^4*x^4 + 4*b^4*c*d^
3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b
^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

**maple [B]** time = 0.07, size = 1842, normalized size = 5.58

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/3*(b*x+a)^4*(2+cos(b*x+a)^2)*sin(b*x+a)+8/15*(b*x+a)^3*co
s(b*x+a)-8/5*(b*x+a)^2*sin(b*x+a)+3424/1125*sin(b*x+a)-3424/1125*(b*x+a)*co
s(b*x+a)+4/45*(b*x+a)^3*cos(b*x+a)^3-4/45*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*
x+a)+88/3375*(b*x+a)*cos(b*x+a)^3-88/10125*(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*
(b*x+a)^4*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-4/25*(b*x+a)^3*cos
(b*x+a)^5+12/125*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)+2
4/625*(b*x+a)*cos(b*x+a)^5-24/3125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(
b*x+a))-4/b^4*a*d^4*(1/3*(b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2/5*(b*x+a)^
2*cos(b*x+a)-856/1125*cos(b*x+a)-4/5*(b*x+a)*sin(b*x+a)+1/15*(b*x+a)^2*cos(
b*x+a)^3-2/45*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+22/3375*cos(b*x+a)^3-1/5*
(b*x+a)^3*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-3/25*(b*x+a)^2*cos
(b*x+a)^5+6/125*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)+6/62
5*cos(b*x+a)^5)+4/b^3*c*d^3*(1/3*(b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2/5*
(b*x+a)^2*cos(b*x+a)-856/1125*cos(b*x+a)-4/5*(b*x+a)*sin(b*x+a)+1/15*(b*x+a
)^2*cos(b*x+a)^3-2/45*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+22/3375*cos(b*x+a
)^3-1/5*(b*x+a)^3*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-3/25*(b*x+
a)^2*cos(b*x+a)^5+6/125*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x
+a)+6/625*cos(b*x+a)^5)+6/b^4*a^2*d^4*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b
*x+a)-4/15*sin(b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/1
35*(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+
a)^2)*sin(b*x+a)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(
b*x+a)^2)*sin(b*x+a))-12/b^3*a*c*d^3*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*
x+a)-4/15*sin(b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/13
```

```
5*(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a
)^2)*sin(b*x+a)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b
*x+a)^2)*sin(b*x+a))+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+
a)-4/15*sin(b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*
(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^
2)*sin(b*x+a)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x
+a)^2)*sin(b*x+a))-4/b^4*a^3*d^4*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1
/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+
a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)+12/b^3*a^2*c*d^3*(1/3*(b*x+a)*(2+cos(b*
x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b
*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)-12/b^2*a*c^2*d^2*(1
/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/
5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)
+4/b*c^3*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*
cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*
cos(b*x+a)^5)+1/b^4*a^4*d^4*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+a
)^2)*sin(b*x+a))-4/b^3*a^3*c*d^3*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos(
b*x+a)^2)*sin(b*x+a))+6/b^2*a^2*c^2*d^2*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*
(2+cos(b*x+a)^2)*sin(b*x+a))-4/b*a*c^3*d*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15
*(2+cos(b*x+a)^2)*sin(b*x+a))+c^4*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos
(b*x+a)^2)*sin(b*x+a)))
```

**maxima [B]** time = 0.43, size = 1339, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4050000*(270000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^4 - 1080000*(3*s
in(b*x + a)^5 - 5*sin(b*x + a)^3)*a*c^3*d/b + 1620000*(3*sin(b*x + a)^5 - 5
*sin(b*x + a)^3)*a^2*c^2*d^2/b^2 - 1080000*(3*sin(b*x + a)^5 - 5*sin(b*x +
a)^3)*a^3*c*d^3/b^3 + 270000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a^4*d^4/
b^4 + 4500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) -
 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 45
0*cos(b*x + a))*c^3*d/b - 13500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x +
a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*
cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*c^2*d^2/b^2 + 13500*(45*(b*x + a)*si
n(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a)
 + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a^2*c*d^3/b
^3 - 4500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) -
450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450
*cos(b*x + a))*a^3*d^4/b^4 + 450*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x
 + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2
- 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b
*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 - 900*(270*(b*x + a)*cos(5*b*x + 5
*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(2
5*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3
*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b^3 + 450*(270*(b*x + a)
*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*
x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)
*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*a^2*d^4/b^4 + 60*(
81*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*cos(3*b*
x + 3*a) - 101250*((b*x + a)^2 - 2)*cos(b*x + a) + 135*(25*(b*x + a)^3 - 6*
b*x - 6*a)*sin(5*b*x + 5*a) + 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x
+ 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*c*d^3/b^3 - 60*(81
*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*cos(3*b*x
+ 3*a) - 101250*((b*x + a)^2 - 2)*cos(b*x + a) + 135*(25*(b*x + a)^3 - 6*b*
x - 6*a)*sin(5*b*x + 5*a) + 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x +
3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*a*d^4/b^4 + (1620*(2
```

```
5*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) + 37500*(3*(b*x + a)^3 - 2*b*
x - 2*a)*cos(3*b*x + 3*a) - 2025000*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a
) + 81*(625*(b*x + a)^4 - 300*(b*x + a)^2 + 24)*sin(5*b*x + 5*a) + 3125*(27
*(b*x + a)^4 - 36*(b*x + a)^2 + 8)*sin(3*b*x + 3*a) - 506250*((b*x + a)^4 -
 12*(b*x + a)^2 + 24)*sin(b*x + a))*d^4/b^4)/b
```

**mupad** [B] time = 4.38, size = 816, normalized size = 2.47

$$-\frac{\frac{d^4\,\sin(3a+3bx)}{162} - 3\,d^4\,\sin(a+b\,x) + \frac{3\,d^4\,\sin(5a+5bx)}{6250} - \frac{b^4\,c^4\,\sin(a+bx)}{8} + \frac{b^4\,c^4\,\sin(3a+3bx)}{48} + \frac{b^4\,c^4\,\sin(5a+5bx)}{80} + b^3\,c^3\,d \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] -((d^4*sin(3*a + 3*b*x))/162 - 3*d^4*sin(a + b*x) + (3*d^4*sin(5*a + 5*b*x)
)/6250 - (b^4*c^4*sin(a + b*x))/8 + (b^4*c^4*sin(3*a + 3*b*x))/48 + (b^4*c^
4*sin(5*a + 5*b*x))/80 + (b^3*c^3*d*cos(3*a + 3*b*x))/36 + (b^3*c^3*d*cos(5
*a + 5*b*x))/100 + (3*b^2*c^2*d^2*sin(a + b*x))/2 - (b^3*d^4*x^3*cos(a + b*
x))/2 + (3*b^2*d^4*x^2*sin(a + b*x))/2 - (b^4*d^4*x^4*sin(a + b*x))/8 + 3*b
*c*d^3*cos(a + b*x) + 3*b*d^4*x*cos(a + b*x) - (b^2*c^2*d^2*sin(3*a + 3*b*x
))/36 - (3*b^2*c^2*d^2*sin(5*a + 5*b*x))/500 + (b^3*d^4*x^3*cos(3*a + 3*b*x
))/36 + (b^3*d^4*x^3*cos(5*a + 5*b*x))/100 - (b^2*d^4*x^2*sin(3*a + 3*b*x))
/36 - (3*b^2*d^4*x^2*sin(5*a + 5*b*x))/500 + (b^4*d^4*x^4*sin(3*a + 3*b*x))
/48 + (b^4*d^4*x^4*sin(5*a + 5*b*x))/80 - (b*c*d^3*cos(3*a + 3*b*x))/54 - (
3*b*c*d^3*cos(5*a + 5*b*x))/1250 - (b^3*c^3*d*cos(a + b*x))/2 - (b*d^4*x*co
s(3*a + 3*b*x))/54 - (3*b*d^4*x*cos(5*a + 5*b*x))/1250 + 3*b^2*c*d^3*x*sin(
a + b*x) - (b^4*c^3*d*x*sin(a + b*x))/2 + (b^4*c^2*d^2*x^2*sin(3*a + 3*b*x)
)/8 + (3*b^4*c^2*d^2*x^2*sin(5*a + 5*b*x))/40 - (3*b^3*c^2*d^2*x*cos(a + b*
x))/2 - (3*b^3*c*d^3*x^2*cos(a + b*x))/2 - (b^2*c*d^3*x*sin(3*a + 3*b*x))/1
8 + (b^4*c^3*d*x*sin(3*a + 3*b*x))/12 - (3*b^2*c*d^3*x*sin(5*a + 5*b*x))/25
0 + (b^4*c^3*d*x*sin(5*a + 5*b*x))/20 - (b^4*c*d^3*x^3*sin(a + b*x))/2 + (b
^3*c^2*d^2*x*cos(3*a + 3*b*x))/12 + (b^3*c*d^3*x^2*cos(3*a + 3*b*x))/12 + (
3*b^3*c^2*d^2*x*cos(5*a + 5*b*x))/100 + (3*b^3*c*d^3*x^2*cos(5*a + 5*b*x))/
100 + (b^4*c*d^3*x^3*sin(3*a + 3*b*x))/12 + (b^4*c*d^3*x^3*sin(5*a + 5*b*x)
)/20 - (3*b^4*c^2*d^2*x^2*sin(a + b*x))/4)/b^5
```

**sympy** [A] time = 20.95, size = 1098, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c**4*sin(a + b*x)**5/(15*b) + c**4*sin(a + b*x)**3*cos(a + b*x
)**2/(3*b) + 8*c**3*d*x*sin(a + b*x)**5/(15*b) + 4*c**3*d*x*sin(a + b*x)**3
*cos(a + b*x)**2/(3*b) + 4*c**2*d**2*x**2*sin(a + b*x)**5/(5*b) + 2*c**2*d*
*2*x**2*sin(a + b*x)**3*cos(a + b*x)**2/b + 8*c*d**3*x**3*sin(a + b*x)**5/(
15*b) + 4*c*d**3*x**3*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*d**4*x**4*s
in(a + b*x)**5/(15*b) + d**4*x**4*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 8
*c**3*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 52*c**3*d*sin(a + b*x)**2*
cos(a + b*x)**3/(45*b**2) + 104*c**3*d*cos(a + b*x)**5/(225*b**2) + 8*c**2*
d**2*x*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 52*c**2*d**2*x*sin(a + b*x)*
*2*cos(a + b*x)**3/(15*b**2) + 104*c**2*d**2*x*cos(a + b*x)**5/(75*b**2) +
8*c*d**3*x**2*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 52*c*d**3*x**2*sin(a
+ b*x)**2*cos(a + b*x)**3/(15*b**2) + 104*c*d**3*x**2*cos(a + b*x)**5/(75*b
**2) + 8*d**4*x**3*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 52*d**4*x**3*si
n(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 104*d**4*x**3*cos(a + b*x)**5/(22
5*b**2) - 1712*c**2*d**2*sin(a + b*x)**5/(1125*b**3) - 676*c**2*d**2*sin(a
+ b*x)**3*cos(a + b*x)**2/(225*b**3) - 104*c**2*d**2*sin(a + b*x)*cos(a + b
```

```
*x)**4/(75*b**3) - 3424*c*d**3*x*sin(a + b*x)**5/(1125*b**3) - 1352*c*d**3*
x*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 208*c*d**3*x*sin(a + b*x)*co
s(a + b*x)**4/(75*b**3) - 1712*d**4*x**2*sin(a + b*x)**5/(1125*b**3) - 676*
d**4*x**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 104*d**4*x**2*sin(a
+ b*x)*cos(a + b*x)**4/(75*b**3) - 3424*c*d**3*sin(a + b*x)**4*cos(a + b*x)
/(1125*b**4) - 20456*c*d**3*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b**4) - 5
0272*c*d**3*cos(a + b*x)**5/(16875*b**4) - 3424*d**4*x*sin(a + b*x)**4*cos(
a + b*x)/(1125*b**4) - 20456*d**4*x*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b
**4) - 50272*d**4*x*cos(a + b*x)**5/(16875*b**4) + 760816*d**4*sin(a + b*x)
**5/(253125*b**5) + 303368*d**4*sin(a + b*x)**3*cos(a + b*x)**2/(50625*b**5
) + 50272*d**4*sin(a + b*x)*cos(a + b*x)**4/(16875*b**5), Ne(b, 0)), ((c**4
*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**
2*cos(a)**3, True))
```

# 3.147 $\int (c+dx)^3 \cos^3(a+bx) \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=259

$$-\frac{3d^3\cos(a+bx)}{4b^4}+\frac{d^3\cos(3a+3bx)}{216b^4}+\frac{3d^3\cos(5a+5bx)}{5000b^4}-\frac{3d^2(c+dx)\sin(a+bx)}{4b^3}+\frac{d^2(c+dx)\sin(3a+3bx)}{72b^3}+$$

```
[Out] -3/4*d^3*cos(b*x+a)/b^4+3/8*d*(d*x+c)^2*cos(b*x+a)/b^2+1/216*d^3*cos(3*b*x+
3*a)/b^4-1/48*d*(d*x+c)^2*cos(3*b*x+3*a)/b^2+3/5000*d^3*cos(5*b*x+5*a)/b^4-
3/400*d*(d*x+c)^2*cos(5*b*x+5*a)/b^2-3/4*d^2*(d*x+c)*sin(b*x+a)/b^3+1/8*(d*
x+c)^3*sin(b*x+a)/b+1/72*d^2*(d*x+c)*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^3*sin(
3*b*x+3*a)/b+3/1000*d^2*(d*x+c)*sin(5*b*x+5*a)/b^3-1/80*(d*x+c)^3*sin(5*b*x
+5*a)/b
```

**Rubi** [A] time = 0.27, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$-\frac{3d^2(c+dx)\sin(a+bx)}{4b^3}+\frac{d^2(c+dx)\sin(3a+3bx)}{72b^3}+\frac{3d^2(c+dx)\sin(5a+5bx)}{1000b^3}+\frac{3d(c+dx)^2\cos(a+bx)}{8b^2}-\frac{d(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (-3*d^3*Cos[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x])/(8*b^2) + (d
^3*Cos[3*a + 3*b*x])/(216*b^4) - (d*(c + d*x)^2*Cos[3*a + 3*b*x])/(48*b^2)
+ (3*d^3*Cos[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x)^2*Cos[5*a + 5*b*x])/
(400*b^2) - (3*d^2*(c + d*x)*Sin[a + b*x])/(4*b^3) + ((c + d*x)^3*Sin[a + b
*x])/(8*b) + (d^2*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*Sin[3
*a + 3*b*x])/(48*b) + (3*d^2*(c + d*x)*Sin[5*a + 5*b*x])/(1000*b^3) - ((c +
 d*x)^3*Sin[5*a + 5*b*x])/(80*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^3\cos^3(a+bx)\sin^2(a+bx)\,dx = \int\left(\frac{1}{8}(c+dx)^3\cos(a+bx) - \frac{1}{16}(c+dx)^3\cos(3a+3bx) - \frac{1}{16}(c+dx)^3\right.$$

$$= -\left(\frac{1}{16}\int(c+dx)^3\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int(c+dx)^3\cos(5a+5bx)\,d$$

$$= \frac{(c+dx)^3\sin(a+bx)}{8b} - \frac{(c+dx)^3\sin(3a+3bx)}{48b} - \frac{(c+dx)^3\sin(5a+5}{80b}$$

$$= \frac{3d(c+dx)^2\cos(a+bx)}{8b^2} - \frac{d(c+dx)^2\cos(3a+3bx)}{48b^2} - \frac{3d(c+dx)^2\cos}{400b^2}$$

$$= \frac{3d(c+dx)^2\cos(a+bx)}{8b^2} - \frac{d(c+dx)^2\cos(3a+3bx)}{48b^2} - \frac{3d(c+dx)^2\cos}{400b^2}$$

$$= -\frac{3d^3\cos(a+bx)}{4b^4} + \frac{3d(c+dx)^2\cos(a+bx)}{8b^2} + \frac{d^3\cos(3a+3bx)}{216b^4} - \frac{d(c}{}$$

**Mathematica [A]** time = 2.18, size = 195, normalized size = 0.75

$$-\frac{30b(c+dx)\sin(a+bx)\left(8\cos(2(a+bx))\left(75b^2(c+dx)^2-38d^2\right)+9\cos(4(a+bx))\left(25b^2(c+dx)^2-6d^2\right)-825}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/270000*(-101250*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + 625*d*(-2*d^
2 + 9*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 81*d*(-2*d^2 + 25*b^2*(c + d*x)^2
)*Cos[5*(a + b*x)] + 30*b*(c + d*x)*(-825*b^2*c^2 + 6598*d^2 - 1650*b^2*c*d
*x - 825*b^2*d^2*x^2 + 8*(-38*d^2 + 75*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] +
9*(-6*d^2 + 25*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[a + b*x])/b^4
```

**fricas [A]** time = 0.77, size = 342, normalized size = 1.32

$$-\frac{81\left(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^5-5\left(225\,b^2d^3x^2+450\,b^2cd^2x+225\,b^2c^2d+22\,d^3\right)\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16875*(81*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(b
*x + a)^5 - 5*(225*b^2*d^3*x^2 + 450*b^2*c*d^2*x + 225*b^2*c^2*d + 22*d^3)*
cos(b*x + a)^3 - 30*(225*b^2*d^3*x^2 + 450*b^2*c*d^2*x + 225*b^2*c^2*d - 42
8*d^3)*cos(b*x + a) - 15*(150*b^3*d^3*x^3 + 450*b^3*c*d^2*x^2 + 150*b^3*c^3
 - 9*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 25*b^3*c^3 - 6*b*c*d^2 + 3*(25*b^
3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^4 - 856*b*c*d^2 + (75*b^3*d^3*x^3 + 225*
b^3*c*d^2*x^2 + 75*b^3*c^3 + 22*b*c*d^2 + (225*b^3*c^2*d + 22*b*d^3)*x)*cos
(b*x + a)^2 + 2*(225*b^3*c^2*d - 428*b*d^3)*x)*sin(b*x + a))/b^4
```

**giac [A]** time = 0.36, size = 351, normalized size = 1.36

$$-\frac{3\left(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3\right)\cos\left(5\,bx+5\,a\right)}{10000\,b^4}-\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(3\,bx+\right.}{432\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -3/10000*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(5*b*x
 + 5*a)/b^4 - 1/432*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*
```

```
cos(3*b*x + 3*a)/b^4 + 3/8*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3
)*cos(b*x + a)/b^4 - 1/2000*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 75*b^3*c^2
*d*x + 25*b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(5*b*x + 5*a)/b^4 - 1/144*(3*
b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x - 2*b
*c*d^2)*sin(3*b*x + 3*a)/b^4 + 1/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c
^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4
```

**maple [B]** time = 0.02, size = 1016, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/3*(b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2/5*(b*x+a)^2*cos
(b*x+a)-856/1125*cos(b*x+a)-4/5*(b*x+a)*sin(b*x+a)+1/15*(b*x+a)^2*cos(b*x+a
)^3-2/45*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+22/3375*cos(b*x+a)^3-1/5*(b*x+
a)^3*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-3/25*(b*x+a)^2*cos(b*x+
a)^5+6/125*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)+6/625*cos
(b*x+a)^5)-3/b^3*a*d^3*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/15*sin(
b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*(2+cos(b*x+a
)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a
)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b
*x+a))+3/b^2*c*d^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/15*sin(b*x+
a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*(2+cos(b*x+a)^2)
*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-2/
25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a
))+3/b^3*a^2*d^3*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3
+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)
-1/25*cos(b*x+a)^5)-6/b^2*a*c*d^2*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+
1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x
+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)+3/b*c^2*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2
)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^
4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)-1/b^3*a^3*d^3*(-1/5*cos(b
*x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))+3/b^2*a^2*c*d^2*(-1/5*
cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))-3/b*a*c^2*d*(-1/5
*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))+c^3*(-1/5*cos(b*
x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+a)^2)*sin(b*x+a)))
```

**maxima [B]** time = 0.85, size = 766, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/270000*(18000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^3 - 54000*(3*sin(b
*x + a)^5 - 5*sin(b*x + a)^3)*a*c^2*d/b + 54000*(3*sin(b*x + a)^5 - 5*sin(b
*x + a)^3)*a^2*c*d^2/b^2 - 18000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a^3*
d^3/b^3 + 225*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a
) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) -
 450*cos(b*x + a))*c^2*d/b - 450*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x +
 a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25
*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*c*d^2/b^2 + 225*(45*(b*x + a)*sin(5
*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) +
9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a^2*d^3/b^3 +
15*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500
*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9
*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*c
*d^2/b^2 - 15*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3
```

```
*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*
a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(
b*x + a))*a*d^3/b^3 + (81*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) + 625*(9*(b
*x + a)^2 - 2)*cos(3*b*x + 3*a) - 101250*((b*x + a)^2 - 2)*cos(b*x + a) + 1
35*(25*(b*x + a)^3 - 6*b*x - 6*a)*sin(5*b*x + 5*a) + 1875*(3*(b*x + a)^3 -
2*b*x - 2*a)*sin(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x +
 a))*d^3/b^3)/b
```

**mupad** [**B**] time = 2.43, size = 516, normalized size = 1.99

$$-\frac{\frac{3\,d^3\,\cos(a+b\,x)}{4}-\frac{d^3\,\cos(3\,a+3\,b\,x)}{216}-\frac{3\,d^3\,\cos(5\,a+5\,b\,x)}{5000}-\frac{b^3\,c^3\,\sin(a+b\,x)}{8}+\frac{b^3\,c^3\,\sin(3\,a+3\,b\,x)}{48}+\frac{b^3\,c^3\,\sin(5\,a+5\,b\,x)}{80}+\frac{b^2\,c^2\,d\,\cos(3\,a+3}{48}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] -((3*d^3*cos(a + b*x))/4 - (d^3*cos(3*a + 3*b*x))/216 - (3*d^3*cos(5*a + 5*
b*x))/5000 - (b^3*c^3*sin(a + b*x))/8 + (b^3*c^3*sin(3*a + 3*b*x))/48 + (b^
3*c^3*sin(5*a + 5*b*x))/80 + (b^2*c^2*d*cos(3*a + 3*b*x))/48 + (3*b^2*c^2*d
*cos(5*a + 5*b*x))/400 - (3*b^2*d^3*x^2*cos(a + b*x))/8 - (b^3*d^3*x^3*sin(
a + b*x))/8 + (3*b*c*d^2*sin(a + b*x))/4 + (3*b*d^3*x*sin(a + b*x))/4 + (b^
2*d^3*x^2*cos(3*a + 3*b*x))/48 + (3*b^2*d^3*x^2*cos(5*a + 5*b*x))/400 + (b^
3*d^3*x^3*sin(3*a + 3*b*x))/48 + (b^3*d^3*x^3*sin(5*a + 5*b*x))/80 - (3*b^2
*c^2*d*cos(a + b*x))/8 - (b*c*d^2*sin(3*a + 3*b*x))/72 - (3*b*c*d^2*sin(5*a
 + 5*b*x))/1000 - (b*d^3*x*sin(3*a + 3*b*x))/72 - (3*b*d^3*x*sin(5*a + 5*b*
x))/1000 - (3*b^2*c*d^2*x*cos(a + b*x))/4 - (3*b^3*c^2*d*x*sin(a + b*x))/8
+ (b^2*c*d^2*x*cos(3*a + 3*b*x))/24 + (3*b^2*c*d^2*x*cos(5*a + 5*b*x))/200
+ (b^3*c^2*d*x*sin(3*a + 3*b*x))/16 + (3*b^3*c^2*d*x*sin(5*a + 5*b*x))/80 -
 (3*b^3*c*d^2*x^2*sin(a + b*x))/8 + (b^3*c*d^2*x^2*sin(3*a + 3*b*x))/16 + (
3*b^3*c*d^2*x^2*sin(5*a + 5*b*x))/80)/b^4
```

**sympy** [**A**] time = 11.35, size = 690, normalized size = 2.66

$$\begin{cases}\frac{2c^3\sin^5(a+bx)}{15b}+\frac{c^3\sin^3(a+bx)\cos^2(a+bx)}{3b}+\frac{2c^2dx\sin^5(a+bx)}{5b}+\frac{c^2dx\sin^3(a+bx)\cos^2(a+bx)}{b}+\frac{2cd^2x^2\sin^5(a+bx)}{5b}+\frac{cd^2x^2\sin^3(a+bx)\cos}{b} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^2(a)\cos^3(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c**3*sin(a + b*x)**5/(15*b) + c**3*sin(a + b*x)**3*cos(a + b*x
)**2/(3*b) + 2*c**2*d*x*sin(a + b*x)**5/(5*b) + c**2*d*x*sin(a + b*x)**3*co
s(a + b*x)**2/b + 2*c*d**2*x**2*sin(a + b*x)**5/(5*b) + c*d**2*x**2*sin(a +
 b*x)**3*cos(a + b*x)**2/b + 2*d**3*x**3*sin(a + b*x)**5/(15*b) + d**3*x**3
*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*c**2*d*sin(a + b*x)**4*cos(a + b
*x)/(5*b**2) + 13*c**2*d*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 26*c**
2*d*cos(a + b*x)**5/(75*b**2) + 4*c*d**2*x*sin(a + b*x)**4*cos(a + b*x)/(5*
b**2) + 26*c*d**2*x*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 52*c*d**2*x
*cos(a + b*x)**5/(75*b**2) + 2*d**3*x**2*sin(a + b*x)**4*cos(a + b*x)/(5*b*
*2) + 13*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 26*d**3*x**2
*cos(a + b*x)**5/(75*b**2) - 856*c*d**2*sin(a + b*x)**5/(1125*b**3) - 338*c
*d**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 52*c*d**2*sin(a + b*x)*c
os(a + b*x)**4/(75*b**3) - 856*d**3*x*sin(a + b*x)**5/(1125*b**3) - 338*d**
3*x*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 52*d**3*x*sin(a + b*x)*cos
(a + b*x)**4/(75*b**3) - 856*d**3*sin(a + b*x)**4*cos(a + b*x)/(1125*b**4)
- 5114*d**3*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b**4) - 12568*d**3*cos(a
+ b*x)**5/(16875*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3
 + d**3*x**4/4)*sin(a)**2*cos(a)**3, True))
```

# 3.148 $\int (c+dx)^2 \cos^3(a+bx) \sin^2(a+bx)\,dx$

**Optimal**. Leaf size=184

$$-\frac{d^2\sin(a+bx)}{4b^3}+\frac{d^2\sin(3a+3bx)}{216b^3}+\frac{d^2\sin(5a+5bx)}{1000b^3}+\frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+$$

```
[Out] 1/4*d*(d*x+c)*cos(b*x+a)/b^2-1/72*d*(d*x+c)*cos(3*b*x+3*a)/b^2-1/200*d*(d*x
+c)*cos(5*b*x+5*a)/b^2-1/4*d^2*sin(b*x+a)/b^3+1/8*(d*x+c)^2*sin(b*x+a)/b+1/
216*d^2*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^2*sin(3*b*x+3*a)/b+1/1000*d^2*sin(5
*b*x+5*a)/b^3-1/80*(d*x+c)^2*sin(5*b*x+5*a)/b
```

**Rubi [A]** time = 0.19, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2637}

$$\frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+dx)\cos(5a+5bx)}{200b^2}-\frac{d^2\sin(a+bx)}{4b^3}+\frac{d^2\sin(3a+3bx)}{216b^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (d*(c + d*x)*Cos[a + b*x])/(4*b^2) - (d*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^2
) - (d*(c + d*x)*Cos[5*a + 5*b*x])/(200*b^2) - (d^2*Sin[a + b*x])/(4*b^3) +
 ((c + d*x)^2*Sin[a + b*x])/(8*b) + (d^2*Sin[3*a + 3*b*x])/(216*b^3) - ((c
+ d*x)^2*Sin[3*a + 3*b*x])/(48*b) + (d^2*Sin[5*a + 5*b*x])/(1000*b^3) - ((c
 + d*x)^2*Sin[5*a + 5*b*x])/(80*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2\cos^3(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^2\cos(a+bx)-\frac{1}{16}(c+dx)^2\cos(3a+3bx)-\frac{1}{16}(c+d\right.\\
&= -\left(\frac{1}{16}\int (c+dx)^2\cos(3a+3bx)\,dx\right)-\frac{1}{16}\int (c+dx)^2\cos(5a+5bx\\
&= \frac{(c+dx)^2\sin(a+bx)}{8b}-\frac{(c+dx)^2\sin(3a+3bx)}{48b}-\frac{(c+dx)^2\sin(5a\\
&= \frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+dx)\cos(5\\
&= \frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+dx)\cos(5
\end{aligned}$$

**Mathematica [A]** time = 0.96, size = 252, normalized size = 1.37

$$-\frac{-6750b^2c^2\sin(a+bx)+1125b^2c^2\sin(3(a+bx))+675b^2c^2\sin(5(a+bx))-13500b^2cdx\sin(a+bx)+2250b^2c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/54000*(-13500*b*d*(c + d*x)*Cos[a + b*x] + 750*b*d*(c + d*x)*Cos[3*(a +
b*x)] + 270*b*c*d*Cos[5*(a + b*x)] + 270*b*d^2*x*Cos[5*(a + b*x)] - 6750*b^
2*c^2*Sin[a + b*x] + 13500*d^2*Sin[a + b*x] - 13500*b^2*c*d*x*Sin[a + b*x]
- 6750*b^2*d^2*x^2*Sin[a + b*x] + 1125*b^2*c^2*Sin[3*(a + b*x)] - 250*d^2*S
in[3*(a + b*x)] + 2250*b^2*c*d*x*Sin[3*(a + b*x)] + 1125*b^2*d^2*x^2*Sin[3*
(a + b*x)] + 675*b^2*c^2*Sin[5*(a + b*x)] - 54*d^2*Sin[5*(a + b*x)] + 1350*
b^2*c*d*x*Sin[5*(a + b*x)] + 675*b^2*d^2*x^2*Sin[5*(a + b*x)])/b^3
```

**fricas [A]** time = 0.51, size = 193, normalized size = 1.05

$$-\frac{270\left(bd^2x+bcd\right)\cos\left(bx+a\right)^5-150\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3-900\left(bd^2x+bcd\right)\cos\left(bx+a\right)-\left(450\,b^2d^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3375*(270*(b*d^2*x + b*c*d)*cos(b*x + a)^5 - 150*(b*d^2*x + b*c*d)*cos(b
*x + a)^3 - 900*(b*d^2*x + b*c*d)*cos(b*x + a) - (450*b^2*d^2*x^2 + 900*b^2
*c*d*x - 27*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(b*x +
a)^4 + 450*b^2*c^2 + (225*b^2*d^2*x^2 + 450*b^2*c*d*x + 225*b^2*c^2 + 22*d^
2)*cos(b*x + a)^2 - 856*d^2)*sin(b*x + a))/b^3
```

**giac [A]** time = 2.11, size = 209, normalized size = 1.14

$$-\frac{\left(bd^2x+bcd\right)\cos\left(5\,bx+5\,a\right)}{200\,b^3}-\frac{\left(bd^2x+bcd\right)\cos\left(3\,bx+3\,a\right)}{72\,b^3}+\frac{\left(bd^2x+bcd\right)\cos\left(bx+a\right)}{4\,b^3}-\frac{\left(25\,b^2d^2x^2+50\,b^2cdx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/200*(b*d^2*x + b*c*d)*cos(5*b*x + 5*a)/b^3 - 1/72*(b*d^2*x + b*c*d)*cos(
3*b*x + 3*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*cos(b*x + a)/b^3 - 1/2000*(25*b^2*
d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*sin(5*b*x + 5*a)/b^3 - 1/432*(
9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3*b*x + 3*a)/b^3 + 1/
8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x + a)/b^3
```

**maple [B]** time = 0.02, size = 484, normalized size = 2.63

$$\frac{\frac{d^2\left(\frac{(bx+a)^2\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}-\frac{4\sin(bx+a)}{15}+\frac{4(bx+a)\cos(bx+a)}{15}+\frac{2(bx+a)\left(\cos^3(bx+a)\right)}{45}-\frac{2\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{135}-\frac{(bx+a)^2\left(\frac{8}{3}+\cos^4(bx+a)+\frac{4\left(\cos^2(bx+a)\right)}{3}\right)\sin(bx+}{5}\right.}{b^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/15*sin(b*x+a)+4
/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*(2+cos(b*x+a)^2)*sin
(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-2/25*(
```

```
b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a))-2
/b^2*a*d^2*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*
cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*
cos(b*x+a)^5)+2/b*c*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x
+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b
*x+a)-1/25*cos(b*x+a)^5)+d^2/b^2*a^2*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+
cos(b*x+a)^2)*sin(b*x+a))-2*c*d/b*a*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+c
os(b*x+a)^2)*sin(b*x+a))+c^2*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+
a)^2)*sin(b*x+a)))
```

**maxima [B]** time = 0.60, size = 375, normalized size = 2.04

$$-\frac{3600\left(3\,\sin\left(bx+a\right)^5-5\,\sin\left(bx+a\right)^3\right)c^2-\frac{7200\left(3\,\sin(bx+a)^5-5\,\sin(bx+a)^3\right)acd}{b}+\frac{3600\left(3\,\sin(bx+a)^5-5\,\sin(bx+a)^3\right)a^2d^2}{b^2}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/54000*(3600*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^2 - 7200*(3*sin(b*x
+ a)^5 - 5*sin(b*x + a)^3)*a*c*d/b + 3600*(3*sin(b*x + a)^5 - 5*sin(b*x + a
)^3)*a^2*d^2/b^2 + 30*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b
*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x
+ 3*a) - 450*cos(b*x + a))*c*d/b - 30*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(
b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a)
 + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*d^2/b^2 + (270*(b*x + a)*cos(5
*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a)
 + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3
*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2)/b
```

**mupad [B]** time = 0.85, size = 295, normalized size = 1.60

$$\frac{52\,d^2\,x\cos\left(a+b\,x\right)^5}{225\,b^2}-\frac{52\,d^2\cos\left(a+b\,x\right)^4\,\sin\left(a+b\,x\right)}{225\,b^3}-\frac{\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3\,\left(338\,d^2-225\,b^2\,c^2\right)}{675\,b^3}-\frac{2\,\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] (52*d^2*x*cos(a + b*x)^5)/(225*b^2) - (52*d^2*cos(a + b*x)^4*sin(a + b*x))/
(225*b^3) - (cos(a + b*x)^2*sin(a + b*x)^3*(338*d^2 - 225*b^2*c^2))/(675*b^
3) - (2*sin(a + b*x)^5*(428*d^2 - 225*b^2*c^2))/(3375*b^3) + (2*d^2*x^2*sin
(a + b*x)^5)/(15*b) + (52*c*d*cos(a + b*x)^5)/(225*b^2) + (4*c*d*cos(a + b*
x)*sin(a + b*x)^4)/(15*b^2) + (4*c*d*x*sin(a + b*x)^5)/(15*b) + (d^2*x^2*co
s(a + b*x)^2*sin(a + b*x)^3)/(3*b) + (26*c*d*cos(a + b*x)^3*sin(a + b*x)^2)
/(45*b^2) + (4*d^2*x*cos(a + b*x)*sin(a + b*x)^4)/(15*b^2) + (26*d^2*x*cos(
a + b*x)^3*sin(a + b*x)^2)/(45*b^2) + (2*c*d*x*cos(a + b*x)^2*sin(a + b*x)^
3)/(3*b)
```

**sympy [A]** time = 6.23, size = 382, normalized size = 2.08

$$\begin{cases}\frac{2c^2\sin^5(a+bx)}{15b}+\frac{c^2\sin^3(a+bx)\cos^2(a+bx)}{3b}+\frac{4cdx\sin^5(a+bx)}{15b}+\frac{2cdx\sin^3(a+bx)\cos^2(a+bx)}{3b}+\frac{2d^2x^2\sin^5(a+bx)}{15b}+\frac{d^2x^2\sin^3(a+bx)\mathrm{cc}}{3b}\\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin^2(a)\cos^3(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c**2*sin(a + b*x)**5/(15*b) + c**2*sin(a + b*x)**3*cos(a + b*x
)**2/(3*b) + 4*c*d*x*sin(a + b*x)**5/(15*b) + 2*c*d*x*sin(a + b*x)**3*cos(a
 + b*x)**2/(3*b) + 2*d**2*x**2*sin(a + b*x)**5/(15*b) + d**2*x**2*sin(a + b
*x)**3*cos(a + b*x)**2/(3*b) + 4*c*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2)
 + 26*c*d*sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 52*c*d*cos(a + b*x)**
5/(225*b**2) + 4*d**2*x*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 26*d**2*x*
sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 52*d**2*x*cos(a + b*x)**5/(225*
b**2) - 856*d**2*sin(a + b*x)**5/(3375*b**3) - 338*d**2*sin(a + b*x)**3*cos
(a + b*x)**2/(675*b**3) - 52*d**2*sin(a + b*x)*cos(a + b*x)**4/(225*b**3),
Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**2*cos(a)**3, True))
```

# 3.149 $\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=109

$$\frac{d\cos(a+bx)}{8b^2}-\frac{d\cos(3a+3bx)}{144b^2}-\frac{d\cos(5a+5bx)}{400b^2}+\frac{(c+dx)\sin(a+bx)}{8b}-\frac{(c+dx)\sin(3a+3bx)}{48b}-\frac{(c+dx)\sin(5}{80b}$$

```
[Out] 1/8*d*cos(b*x+a)/b^2-1/144*d*cos(3*b*x+3*a)/b^2-1/400*d*cos(5*b*x+5*a)/b^2+
1/8*(d*x+c)*sin(b*x+a)/b-1/48*(d*x+c)*sin(3*b*x+3*a)/b-1/80*(d*x+c)*sin(5*b
*x+5*a)/b
```

**Rubi [A]** time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3296, 2638}

$$\frac{d\cos(a+bx)}{8b^2}-\frac{d\cos(3a+3bx)}{144b^2}-\frac{d\cos(5a+5bx)}{400b^2}+\frac{(c+dx)\sin(a+bx)}{8b}-\frac{(c+dx)\sin(3a+3bx)}{48b}-\frac{(c+dx)\sin(5}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (d*Cos[a + b*x])/(8*b^2) - (d*Cos[3*a + 3*b*x])/(144*b^2) - (d*Cos[5*a + 5*
b*x])/(400*b^2) + ((c + d*x)*Sin[a + b*x])/(8*b) - ((c + d*x)*Sin[3*a + 3*b
*x])/(48*b) - ((c + d*x)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)\cos(a+bx)-\frac{1}{16}(c+dx)\cos(3a+3bx)-\frac{1}{16}(c+dx)\right.\\ &= -\left(\frac{1}{16}\int (c+dx)\cos(3a+3bx)\,dx\right)-\frac{1}{16}\int (c+dx)\cos(5a+5bx)\,d\\ &= \frac{(c+dx)\sin(a+bx)}{8b}-\frac{(c+dx)\sin(3a+3bx)}{48b}-\frac{(c+dx)\sin(5a+5b}{80b}\\ &= \frac{d\cos(a+bx)}{8b^2}-\frac{d\cos(3a+3bx)}{144b^2}-\frac{d\cos(5a+5bx)}{400b^2}+\frac{(c+dx)\sin(a}{8b}\end{aligned}$$

**Mathematica [A]** time = 0.36, size = 110, normalized size = 1.01

$$-\frac{-450bc\sin(a+bx)+75bc\sin(3(a+bx))+45bc\sin(5(a+bx))-450bdx\sin(a+bx)+75bdx\sin(3(a+bx))}{3600b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/3600*(-450*d*Cos[a + b*x] + 25*d*Cos[3*(a + b*x)] + 9*d*Cos[5*(a + b*x)]
 - 450*b*c*Sin[a + b*x] - 450*b*d*x*Sin[a + b*x] + 75*b*c*Sin[3*(a + b*x)]
+ 75*b*d*x*Sin[3*(a + b*x)] + 45*b*c*Sin[5*(a + b*x)] + 45*b*d*x*Sin[5*(a +
 b*x)])/b^2
```

**fricas [A]** time = 0.76, size = 91, normalized size = 0.83

$$-\frac{9\,d\cos\left(bx+a\right)^{5}-5\,d\cos\left(bx+a\right)^{3}-30\,d\cos\left(bx+a\right)+15\,\left(3\,\left(bdx+bc\right)\cos\left(bx+a\right)^{4}-2\,bdx-\left(bdx+bc\right)\cos\right.}{225\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/225*(9*d*cos(b*x + a)^5 - 5*d*cos(b*x + a)^3 - 30*d*cos(b*x + a) + 15*(3
*(b*d*x + b*c)*cos(b*x + a)^4 - 2*b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 - 2*
b*c)*sin(b*x + a))/b^2
```

**giac [A]** time = 2.97, size = 106, normalized size = 0.97

$$-\frac{d\cos\left(5\,bx+5\,a\right)}{400\,b^{2}}-\frac{d\cos\left(3\,bx+3\,a\right)}{144\,b^{2}}+\frac{d\cos\left(bx+a\right)}{8\,b^{2}}-\frac{\left(bdx+bc\right)\sin\left(5\,bx+5\,a\right)}{80\,b^{2}}-\frac{\left(bdx+bc\right)\sin\left(3\,bx+3\,a\right)}{48\,b^{2}}+\frac{(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/400*d*cos(5*b*x + 5*a)/b^2 - 1/144*d*cos(3*b*x + 3*a)/b^2 + 1/8*d*cos(b*
x + a)/b^2 - 1/80*(b*d*x + b*c)*sin(5*b*x + 5*a)/b^2 - 1/48*(b*d*x + b*c)*s
in(3*b*x + 3*a)/b^2 + 1/8*(b*d*x + b*c)*sin(b*x + a)/b^2
```

**maple [A]** time = 0.02, size = 175, normalized size = 1.61

$$\frac{\frac{d\left(\frac{(bx+a)\left(2+\cos^{2}(bx+a)\right)\sin(bx+a)}{3}+\frac{\left(\cos^{3}(bx+a)\right)}{45}+\frac{2\cos(bx+a)}{15}-\frac{(bx+a)\left(\frac{8}{3}+\cos^{4}(bx+a)+\frac{4\left(\cos^{2}(bx+a)\right)}{3}\right)\sin(bx+a)}{5}-\frac{\left(\cos^{5}(bx+a)\right)}{25}\right)}{b}-\frac{da\left(-\frac{\left(\cos^{4}(bx+a)\right)\sin(bx+a)}{5}+\frac{(2+\cos}{}\right.}{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*
cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*
cos(b*x+a)^5)-1/b*d*a*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+a)^2)*s
in(b*x+a))+c*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+a)^2)*sin(b*x+a)
))
```

**maxima [A]** time = 0.95, size = 139, normalized size = 1.28

$$-\frac{240\,\left(3\,\sin\left(bx+a\right)^{5}-5\,\sin\left(bx+a\right)^{3}\right)c-\frac{240\,\left(3\,\sin(bx+a)^{5}-5\,\sin(bx+a)^{3}\right)ad}{b}+\frac{(45\,(bx+a)\sin(5\,bx+5\,a)+75\,(bx+a)\sin(3\,bx+3\,a)-45}{}}{3600\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3600*(240*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c - 240*(3*sin(b*x + a)^
5 - 5*sin(b*x + a)^3)*a*d/b + (45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)
*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*co
s(3*b*x + 3*a) - 450*cos(b*x + a))*d/b)/b
```

**mupad [B]** time = 0.47, size = 119, normalized size = 1.09

$$\frac{26\,d\cos\left(a+b\,x\right)^5+65\,d\cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^2+30\,d\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^4+30\,b\,c\sin\left(a+b\,x\right)^5+}{225\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x),x)
```

```
[Out] (26*d*cos(a + b*x)^5 + 65*d*cos(a + b*x)^3*sin(a + b*x)^2 + 30*d*cos(a + b*
x)*sin(a + b*x)^4 + 30*b*c*sin(a + b*x)^5 + 30*b*d*x*sin(a + b*x)^5 + 75*b*
c*cos(a + b*x)^2*sin(a + b*x)^3 + 75*b*d*x*cos(a + b*x)^2*sin(a + b*x)^3)/(
225*b^2)
```

**sympy [A]** time = 3.10, size = 163, normalized size = 1.50

$$\begin{cases} \frac{2c\sin^5(a+bx)}{15b} + \frac{c\sin^3(a+bx)\cos^2(a+bx)}{3b} + \frac{2dx\sin^5(a+bx)}{15b} + \frac{dx\sin^3(a+bx)\cos^2(a+bx)}{3b} + \frac{2d\sin^4(a+bx)\cos(a+bx)}{15b^2} + \frac{13d\sin^2(a+bx)\cos}{45b^2} \\ \left(cx + \frac{dx^2}{2}\right)\sin^2(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c*sin(a + b*x)**5/(15*b) + c*sin(a + b*x)**3*cos(a + b*x)**2/(
3*b) + 2*d*x*sin(a + b*x)**5/(15*b) + d*x*sin(a + b*x)**3*cos(a + b*x)**2/(
3*b) + 2*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 13*d*sin(a + b*x)**2*co
s(a + b*x)**3/(45*b**2) + 26*d*cos(a + b*x)**5/(225*b**2), Ne(b, 0)), ((c*x
 + d*x**2/2)*sin(a)**2*cos(a)**3, True))
```

# 3.150 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=185

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{8d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d}-\frac{\cos\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5bc}{d}+5bx\right)}{16d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d}$$

```
[Out] -1/16*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d-1/16*Ci(3*b*c/d+3*b*x)*cos(3*a-3
*b*c/d)/d+1/8*Ci(b*c/d+b*x)*cos(a-b*c/d)/d+1/16*Si(5*b*c/d+5*b*x)*sin(5*a-5
*b*c/d)/d+1/16*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d-1/8*Si(b*c/d+b*x)*sin(a
-b*c/d)/d
```

**Rubi [A]** time = 0.28, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}-\frac{\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d) - (Cos[3*a - (3*b*c)/d]
*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (Cos[5*a - (5*b*c)/d]*CosIntegral
[(5*b*c)/d + 5*b*x])/(16*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])
/(8*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) + (Si
n[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \left(\frac{\cos(a+bx)}{8(c+dx)} - \frac{\cos(3a+3bx)}{16(c+dx)} - \frac{\cos(5a+5bx)}{16(c+dx)}\right)dx$$
$$= -\left(\frac{1}{16}\int \frac{\cos(3a+3bx)}{c+dx}\,dx\right) - \frac{1}{16}\int \frac{\cos(5a+5bx)}{c+dx}\,dx + \frac{1}{8}\int \frac{\cos(a+bx)}{c+dx}$$
$$= -\left(\frac{1}{16}\cos\left(5a-\frac{5bc}{d}\right)\int \frac{\cos\left(\frac{5bc}{d}+5bx\right)}{c+dx}\,dx\right) - \frac{1}{16}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\cos\left(\right.}{c}$$
$$= \frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{8d} - \frac{\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d} - \frac{\cos\left(5a-\frac{5bc}{d}\right)}{16}$$

**Mathematica** [A] time = 0.51, size = 154, normalized size = 0.83

$$\frac{2\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(b\left(\frac{c}{d}+x\right)\right) - \cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3b(c+dx)}{d}\right) - \cos\left(5a-\frac{5bc}{d}\right)\mathrm{Ci}\left(\frac{5b(c+dx)}{d}\right) - 2\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\right.\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Cos[3*a - (3*b*c)/d]*CosInte
gral[(3*b*(c + d*x))/d] - Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*(c + d*x))/
d] - 2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Sin[3*a - (3*b*c)/d]*Sin
Integral[(3*b*(c + d*x))/d] + Sin[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*
x))/d])/(16*d)
```

**fricas** [A] time = 0.61, size = 229, normalized size = 1.24

$$2\left(\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right) - \left(\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right) - \left(\mathrm{Ci}\left(\frac{5\,(bdx+bc}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/32*(2*(cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*co
s(-(b*c - a*d)/d) - (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d
*x + b*c)/d))*cos(-3*(b*c - a*d)/d) - (cos_integral(5*(b*d*x + b*c)/d) + co
s_integral(-5*(b*d*x + b*c)/d))*cos(-5*(b*c - a*d)/d) + 2*sin(-5*(b*c - a*d
)/d)*sin_integral(5*(b*d*x + b*c)/d) + 2*sin(-3*(b*c - a*d)/d)*sin_integral
(3*(b*d*x + b*c)/d) - 4*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/
d
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [A] time = 0.02, size = 252, normalized size = 1.36

$$\frac{\frac{b\left(\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}\right)}{8} - \frac{b\left(\frac{5\,\mathrm{Si}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\sin\left(\frac{-5da+5cb}{d}\right)}{d}+\frac{5\,\mathrm{Ci}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\cos\left(\frac{-5da+5cb}{d}\right)}{d}\right)}{80} - }{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] 1/b*(1/8*b*(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/
d)*cos((-a*d+b*c)/d)/d)-1/80*b*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+
b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)-1/48*b*(3*S
i(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*
c)/d)*cos(3*(-a*d+b*c)/d)/d))
```

**maxima [C]** time = 0.51, size = 408, normalized size = 2.21

$$-\frac{2\,b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(E_1\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_1\left(-\frac{3i\,bc+3i\,(bx+a)d-3}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/32*(2*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(exp
_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1,
 -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b*(exp_
integral_e(1, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_integral_e(1,
-(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c - a*d)/d) + b*(-2*I*
exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*exp_integral_e(1
, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(I*exp_integ
ral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_integral_e(1, -(3
*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) + b*(I*exp_in
tegral_e(1, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) - I*exp_integral_e(1,
-(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d))/(b*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}{\left(a+bx\right)}\cos^{3}{\left(a+bx\right)}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x), x)
```

## 3.151 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=257

$$\frac{5b\sin\left(5a-\frac{5bc}{d}\right)\mathrm{Ci}\left(\frac{5bc}{d}+5bx\right)}{16d^2}+\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{8d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\mathrm{S}}{8d^2}$$

```
[Out] -1/8*cos(b*x+a)/d/(d*x+c)+1/16*cos(3*b*x+3*a)/d/(d*x+c)+1/16*cos(5*b*x+5*a)
/d/(d*x+c)-1/8*b*cos(a-b*c/d)*Si(b*c/d+b*x)/d^2+3/16*b*cos(3*a-3*b*c/d)*Si(
3*b*c/d+3*b*x)/d^2+5/16*b*cos(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d^2+5/16*b*Ci(
5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^2+3/16*b*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/
d)/d^2-1/8*b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

**Rubi [A]** time = 0.34, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{5b\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d^2}+\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosInte}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -Cos[a + b*x]/(8*d*(c + d*x)) + Cos[3*a + 3*b*x]/(16*d*(c + d*x)) + Cos[5*a
 + 5*b*x]/(16*d*(c + d*x)) + (5*b*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a -
(5*b*c)/d])/(16*d^2) + (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c
)/d])/(16*d^2) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^2) -
(b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Cos[5*a - (5*b*c)/
d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \left(\frac{\cos(a+bx)}{8(c+dx)^2} - \frac{\cos(3a+3bx)}{16(c+dx)^2} - \frac{\cos(5a+5bx)}{16(c+dx)^2}\right) dx$$

$$= -\left(\frac{1}{16}\int \frac{\cos(3a+3bx)}{(c+dx)^2}\,dx\right) - \frac{1}{16}\int \frac{\cos(5a+5bx)}{(c+dx)^2}\,dx + \frac{1}{8}\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} - \frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{8d} + \frac{(3b)\int \frac{\sin}{}}{1}$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} + \frac{\left(5b\cos\left(5a-\frac{5bc}{d}\right)\right)\int \frac{\sin\left(\frac{5b}{d}\right)}{c+}}{16d}$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} + \frac{5b\,\mathrm{Ci}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\right.}{16d^2}$$

**Mathematica [A]** time = 2.11, size = 212, normalized size = 0.82

$$\frac{-2\left(b\sin\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+b\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\frac{d\cos(a+bx)}{c+dx}\right)+5b\sin\left(5a-\frac{5bc}{d}\right)\mathrm{Ci}\left(\frac{5b(c+dx)}{d}\right)+3b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] ((d*Cos[3*(a + b*x)])/(c + d*x) + (d*Cos[5*(a + b*x)])/(c + d*x) + 5*b*CosI
ntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] + 3*b*CosIntegral[(3*b*(c +
 d*x))/d]*Sin[3*a - (3*b*c)/d] - 2*((d*Cos[a + b*x])/(c + d*x) + b*CosInteg
ral[b*(c/d + x)]*Sin[a - (b*c)/d] + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d +
 x)]) + 3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 5*b*Cos[5
*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d^2)
```

**fricas [A]** time = 0.51, size = 339, normalized size = 1.32

$$\frac{32\,d\cos(bx+a)^5 - 32\,d\cos(bx+a)^3 + 10\,(bdx+bc)\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{5\,(bdx+bc)}{d}\right) + 6\,(bdx+bc)\cos\left(-\frac{3\,(bc-ad)}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(32*d*cos(b*x + a)^5 - 32*d*cos(b*x + a)^3 + 10*(b*d*x + b*c)*cos(-5*(
b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 6*(b*d*x + b*c)*cos(-3*(b*c
 - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 4*(b*d*x + b*c)*cos(-(b*c - a*
d)/d)*sin_integral((b*d*x + b*c)/d) - 2*((b*d*x + b*c)*cos_integral((b*d*x
+ b*c)/d) + (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/
d) + 3*((b*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_i
ntegral(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d) + 5*((b*d*x + b*c)*cos_i
ntegral(5*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-5*(b*d*x + b*c)/d)
)*sin(-5*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.02, size = 367, normalized size = 1.43

$$\frac{\frac{b^2\left(-\frac{\cos(bx+a)}{((bx+a)d-da+cb)d}-\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}}{d}\right)}{8}-\frac{b^2\left(-\frac{5\cos(5bx+5a)}{((bx+a)d-da+cb)d}-\frac{5\left(\frac{5\,\mathrm{Si}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\cos\left(\frac{-5da+5cb}{d}\right)}{d}-\right.}{d}\right.}{80}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/8*b^2*(-cos(b*x+a)/((b*x+a)*d-d*a+c*b)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos
((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)-1/80*b^2*(-
5*cos(5*b*x+5*a)/((b*x+a)*d-d*a+c*b)/d-5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*co
s(5*(-a*d+b*c)/d)/d-5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d
)-1/48*b^2*(-3*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d-3*(3*Si(3*b*x+3*a+3*(-a
*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d
+b*c)/d)/d)/d))
```

**maxima [C]** time = 0.57, size = 439, normalized size = 1.71

$$-\frac{1073741824\,b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-536870912\,b^2\left(E_2\left(\frac{3i\,bc+3i\,(bx+a)d-}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/17179869184*(1073741824*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d -
I*a*d)/d) + exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*
c - a*d)/d) - 536870912*b^2*(exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d -
 3*I*a*d)/d) + exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*cos(-3*(b*c - a*d)/d) - 536870912*b^2*(exp_integral_e(2, (5*I*b*c + 5*I*(b
*x + a)*d - 5*I*a*d)/d) + exp_integral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5
*I*a*d)/d))*cos(-5*(b*c - a*d)/d) + b^2*(-1073741824*I*exp_integral_e(2, (I
*b*c + I*(b*x + a)*d - I*a*d)/d) + 1073741824*I*exp_integral_e(2, -(I*b*c +
 I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(536870912*I*exp_inte
gral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 536870912*I*exp_integr
al_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) +
b^2*(536870912*I*exp_integral_e(2, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d)
 - 536870912*I*exp_integral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))
*sin(-5*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2}{{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^2, x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**2, x)
```

# 3.152 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=338

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{16d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{32d^3}+\frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5bc}{d}+5bx\right)}{32d^3}+\frac{b^2\sin\left(\right.}{}$$

```
[Out] 25/32*b^2*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d^3+9/32*b^2*Ci(3*b*c/d+3*b*x)
*cos(3*a-3*b*c/d)/d^3-1/16*b^2*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^3-1/16*cos(b*x+
a)/d/(d*x+c)^2+1/32*cos(3*b*x+3*a)/d/(d*x+c)^2+1/32*cos(5*b*x+5*a)/d/(d*x+c
)^2-25/32*b^2*Si(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^3-9/32*b^2*Si(3*b*c/d+3*
b*x)*sin(3*a-3*b*c/d)/d^3+1/16*b^2*Si(b*c/d+b*x)*sin(a-b*c/d)/d^3+1/16*b*si
n(b*x+a)/d^2/(d*x+c)-3/32*b*sin(3*b*x+3*a)/d^2/(d*x+c)-5/32*b*sin(5*b*x+5*a
)/d^2/(d*x+c)
```

**Rubi [A]** time = 0.44, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{16d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}+\frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{Co}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] -Cos[a + b*x]/(16*d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(32*d*(c + d*x)^2) + Co
s[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/d]*CosIntegral[(b*c)
/d + b*x])/(16*d^3) + (9*b^2*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3
*b*x])/(32*d^3) + (25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*c)/d + 5*b*
x])/(32*d^3) + (b*Sin[a + b*x])/(16*d^2*(c + d*x)) - (3*b*Sin[3*a + 3*b*x])
/(32*d^2*(c + d*x)) - (5*b*Sin[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (b^2*Sin[
a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Sin[3*a - (3*b*c
)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) - (25*b^2*Sin[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\cos(a+bx)}{8(c+dx)^3} - \frac{\cos(3a+3bx)}{16(c+dx)^3} - \frac{\cos(5a+5bx)}{16(c+dx)^3}\right) dx$$

$$= -\left(\frac{1}{16}\int \frac{\cos(3a+3bx)}{(c+dx)^3}\,dx\right) - \frac{1}{16}\int \frac{\cos(5a+5bx)}{(c+dx)^3}\,dx + \frac{1}{8}\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx$$

$$= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx}{16d} + \frac{(3b)\int \text{si}}{3}$$

$$= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} + \frac{b\sin(a+bx)}{16d^2(c+dx)} - \frac{3b\sin(3a}{32d^2(c+}$$

$$= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} + \frac{b\sin(a+bx)}{16d^2(c+dx)} - \frac{3b\sin(3a}{32d^2(c+}$$

$$= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right.}{16d^3}$$

**Mathematica [A]** time = 3.37, size = 283, normalized size = 0.84

$$-2b^2\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right) + 9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right) + 25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5b(c+dx)}{d}\right) + 2b^2\sin\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] ((d^2*Cos[3*(a + b*x)])/(c + d*x)^2 + (d^2*Cos[5*(a + b*x)])/(c + d*x)^2 -
2*b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 9*b^2*Cos[3*a - (3*b*c)/d
]*CosIntegral[(3*b*(c + d*x))/d] + 25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegral[
(5*b*(c + d*x))/d] + (2*d*(-(d*Cos[a + b*x]) + b*(c + d*x)*Sin[a + b*x]))/(
c + d*x)^2 - (3*b*d*Sin[3*(a + b*x)])/(c + d*x) - (5*b*d*Sin[5*(a + b*x)])/
(c + d*x) + 2*b^2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 9*b^2*Sin[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] - 25*b^2*Sin[5*a - (5*b*c)/d]*
SinIntegral[(5*b*(c + d*x))/d])/(32*d^3)
```

**fricas [A]** time = 0.54, size = 567, normalized size = 1.68

$$32\,d^2\cos(bx+a)^5 - 32\,d^2\cos(bx+a)^3 - 50\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)\text{Si}\left(\frac{5\,(bdx+bc)}{d}\right) - 18\left(b^2d^2x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(32*d^2*cos(b*x + a)^5 - 32*d^2*cos(b*x + a)^3 - 50*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) -
 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_integra
l(3*(b*d*x + b*c)/d) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*c -
a*d)/d)*sin_integral((b*d*x + b*c)/d) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos_integral((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*cos_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) +
 25*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(5*(b*d*x + b*c)/d)
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-5*(b*d*x + b*c)/d))*c
os(-5*(b*c - a*d)/d) - 32*(5*(b*d^2*x + b*c*d)*cos(b*x + a)^4 - 3*(b*d^2*x
+ b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.02, size = 473, normalized size = 1.40

$$\frac{b^3\left(-\frac{\cos(bx+a)}{2((bx+a)d-da+cb)^2 d}-\frac{-\frac{\sin(bx+a)}{((bx+a)d-da+cb)d}+\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}}{d}}{2d}\right)}{8}-\frac{b^3\left(-\frac{5\cos(5bx+5a)}{2((bx+a)d-da+cb)^2 d}-\frac{5\left(-\frac{5\sin(5bx+5\cdots}{((bx+a)d-da+\cdots}\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/8*b^3*(-1/2*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d-1/2*(-sin(b*x+a)/((b*
x+a)*d-d*a+c*b)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*
d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)-1/80*b^3*(-5/2*cos(5*b*x+5*a)/((b*x+a)
*d-d*a+c*b)^2/d-5/2*(-5*sin(5*b*x+5*a)/((b*x+a)*d-d*a+c*b)/d+5*(5*Si(5*b*x+
5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*co
s(5*(-a*d+b*c)/d)/d)/d)/d)-1/48*b^3*(-3/2*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b
)^2/d-3/2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d+3*(3*Si(3*b*x+3*a+3*(-a*
d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+
b*c)/d)/d)/d)/d))
```

**maxima [C]** time = 1.37, size = 474, normalized size = 1.40

$$-\frac{1073741824\, b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-536870912\, b^3\left(E_3\left(\frac{3i\,bc+3i\,(bx+a)d-\cdots}{d}\right.\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/17179869184*(1073741824*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d -
I*a*d)/d) + exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*
c - a*d)/d) - 536870912*b^3*(exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d -
```

```
 3*I*a*d)/d) + exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*cos(-3*(b*c - a*d)/d) - 536870912*b^3*(exp_integral_e(3, (5*I*b*c + 5*I*(b
*x + a)*d - 5*I*a*d)/d) + exp_integral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5
*I*a*d)/d))*cos(-5*(b*c - a*d)/d) + b^3*(-1073741824*I*exp_integral_e(3, (I
*b*c + I*(b*x + a)*d - I*a*d)/d) + 1073741824*I*exp_integral_e(3, -(I*b*c +
 I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(536870912*I*exp_inte
gral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 536870912*I*exp_integr
al_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) +
b^3*(536870912*I*exp_integral_e(3, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d)
 - 536870912*I*exp_integral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))
*sin(-5*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos \left(a+b\,x\right)^3\,\sin \left(a+b\,x\right)^2}{{\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}{\left (a + b x \right )} \cos^{3}{\left (a + b x \right )}}{\left (c + d x\right )^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**3, x)
```

# 3.153 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

**Optimal**. Leaf size=413

$$-\frac{125b^3\sin\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5bc}{d}+5bx\right)}{96d^4}-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{48d^4}+\frac{b^3\cos\left(\right.}{}$$

```
[Out] -1/24*cos(b*x+a)/d/(d*x+c)^3+1/48*b^2*cos(b*x+a)/d^3/(d*x+c)+1/48*cos(3*b*x
+3*a)/d/(d*x+c)^3-3/32*b^2*cos(3*b*x+3*a)/d^3/(d*x+c)+1/48*cos(5*b*x+5*a)/d
/(d*x+c)^3-25/96*b^2*cos(5*b*x+5*a)/d^3/(d*x+c)+1/48*b^3*cos(a-b*c/d)*Si(b*
c/d+b*x)/d^4-9/32*b^3*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^4-125/96*b^3*cos
(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d^4-125/96*b^3*Ci(5*b*c/d+5*b*x)*sin(5*a-5*
b*c/d)/d^4-9/32*b^3*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/48*b^3*Ci(b*c/
d+b*x)*sin(a-b*c/d)/d^4+1/48*b*sin(b*x+a)/d^2/(d*x+c)^2-1/32*b*sin(3*b*x+3*
a)/d^2/(d*x+c)^2-5/96*b*sin(5*b*x+5*a)/d^2/(d*x+c)^2
```

**Rubi [A]** time = 0.54, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{125b^3\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{96d^4}-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{C}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -Cos[a + b*x]/(24*d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(48*d^3*(c + d*x)) +
Cos[3*a + 3*b*x]/(48*d*(c + d*x)^3) - (3*b^2*Cos[3*a + 3*b*x])/(32*d^3*(c +
 d*x)) + Cos[5*a + 5*b*x]/(48*d*(c + d*x)^3) - (25*b^2*Cos[5*a + 5*b*x])/(9
6*d^3*(c + d*x)) - (125*b^3*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c
)/d])/(96*d^4) - (9*b^3*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d]
)/(32*d^4) + (b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(48*d^4) + (
b*Sin[a + b*x])/(48*d^2*(c + d*x)^2) - (b*Sin[3*a + 3*b*x])/(32*d^2*(c + d*
x)^2) - (5*b*Sin[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) + (b^3*Cos[a - (b*c)/d]
*SinIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*Cos[3*a - (3*b*c)/d]*SinInte
gral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*Cos[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*c)/d + 5*b*x])/(96*d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\cos(a+bx)}{8(c+dx)^4} - \frac{\cos(3a+3bx)}{16(c+dx)^4} - \frac{\cos(5a+5bx)}{16(c+dx)^4}\right) dx$$

$$= -\left(\frac{1}{16}\int \frac{\cos(3a+3bx)}{(c+dx)^4}\,dx\right) - \frac{1}{16}\int \frac{\cos(5a+5bx)}{(c+dx)^4}\,dx + \frac{1}{8}\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} + \frac{\cos(5a+5bx)}{48d(c+dx)^3} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx}{24d} + \frac{b\int \frac{\sin(3a}{(c+}}{16}$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} + \frac{\cos(5a+5bx)}{48d(c+dx)^3} + \frac{b\sin(a+bx)}{48d^2(c+dx)^2} - \frac{b\sin(3a}{32d^2(c+}$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2\cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)} + \frac{\cos(}{48d}$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2\cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)} + \frac{\cos(}{48d}$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2\cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)} + \frac{\cos(}{48d}$$

**Mathematica [A]** time = 3.40, size = 451, normalized size = 1.09

$$-\frac{27b^3(c+dx)^3\left(\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right)+\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)\right)+125b^3(c+dx)^3\left(\sin\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5b}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -1/96*(d*Cos[3*b*x]*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x
)*Sin[3*a]) + d*Cos[5*b*x]*((-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*a] + 5*b*d*
(c + d*x)*Sin[5*a]) + d*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*
x)^2)*Sin[3*a])*Sin[3*b*x] + d*(5*b*d*(c + d*x)*Cos[5*a] - (-2*d^2 + 25*b^2
*(c + d*x)^2)*Sin[5*a])*Sin[5*b*x] - 2*(d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)
^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) + d*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^
2*(c + d*x)^2)*Sin[a])*Sin[b*x] + b^3*(c + d*x)^3*(CosIntegral[b*(c/d + x)]
*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)])) + 27*b^3*(c
 + d*x)^3*(CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + Cos[3*a -
(3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]) + 125*b^3*(c + d*x)^3*(CosIntegr
al[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] + Cos[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*(c + d*x))/d]))/(d^4*(c + d*x)^3)
```

**fricas** [B] time = 0.63, size = 811, normalized size = 1.96

$$-\frac{32\left(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^5-32\left(29\,b^2d^3x^2+58\,b^2cd^2x+29\,b^2c^2d-2\,d^3\right)\mathrm{co}\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/192*(32*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(b*x
 + a)^5 - 32*(29*b^2*d^3*x^2 + 58*b^2*c*d^2*x + 29*b^2*c^2*d - 2*d^3)*cos(b
*x + a)^3 + 250*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*c
os(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 54*(b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-3*(b*c - a*d)/d)*sin_integral
(3*(b*d*x + b*c)/d) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^
3*c^3)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 192*(b^2*d^3*x^2
 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a) + 32*(5*(b*d^3*x + b*c*d^2)*cos(
b*x + a)^4 - 3*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*sin(b*x + a) - 2*((b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral((b*d*x + b
*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_inte
gral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) + 27*((b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x + b*c)/d) + (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-3*(b*d*x +
b*c)/d))*sin(-3*(b*c - a*d)/d) + 125*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + b^3*c^3)*cos_integral(5*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-5*(b*d*x + b*c)/d))*sin
(-5*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [A] time = 0.02, size = 583, normalized size = 1.41

$$\frac{b^4\left(-\frac{\cos(bx+a)}{3((bx+a)d-da+cb)^3d}-\frac{-\frac{\sin(bx+a)}{2((bx+a)d-da+cb)^2d}+\frac{-\frac{\cos(bx+a)}{((bx+a)d-da+cb)d}-\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}}{d}}{2d}}{3d}\right)}{8}-\frac{b^4\left(-\frac{5\cos(5bx+5\ldots}{3((bx+a)d-da+c\ldots}\ldots\right)}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x)
```

```
[Out] 1/b*(1/8*b^4*(-1/3*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^3/d-1/3*(-1/2*sin(b*x+a)/
((b*x+a)*d-d*a+c*b)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-d*a+c*b)/d-(Si(b*x+a+(-
a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)
```

```
/d)/d)/d)-1/80*b^4*(-5/3*cos(5*b*x+5*a)/((b*x+a)*d-d*a+c*b)^3/d-5/3*(-5/2*s
in(5*b*x+5*a)/((b*x+a)*d-d*a+c*b)^2/d+5/2*(-5*cos(5*b*x+5*a)/((b*x+a)*d-d*a
+c*b)/d-5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d-5*Ci(5*b*x+
5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d)/d)/d)-1/48*b^4*(-cos(3*b*x+3*
a)/((b*x+a)*d-d*a+c*b)^3/d-(-3/2*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)^2/d+3/2
*(-3*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d-3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)
*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d
)/d)/d)/d))
```

**maxima [C]** time = 1.27, size = 524, normalized size = 1.27

$$-\frac{1073741824\,b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-536870912\,b^4\left(E_4\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,a}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/17179869184*(1073741824*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d -
I*a*d)/d) + exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*
c - a*d)/d) - 536870912*b^4*(exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d -
 3*I*a*d)/d) + exp_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*cos(-3*(b*c - a*d)/d) - 536870912*b^4*(exp_integral_e(4, (5*I*b*c + 5*I*(b
*x + a)*d - 5*I*a*d)/d) + exp_integral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5
*I*a*d)/d))*cos(-5*(b*c - a*d)/d) + b^4*(-1073741824*I*exp_integral_e(4, (I
*b*c + I*(b*x + a)*d - I*a*d)/d) + 1073741824*I*exp_integral_e(4, -(I*b*c +
 I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^4*(536870912*I*exp_inte
gral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 536870912*I*exp_integr
al_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) +
b^4*(536870912*I*exp_integral_e(4, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d)
 - 536870912*I*exp_integral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))
*sin(-5*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+bx\right)\cos^3\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**4, x)
```

# 3.154 $\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=285

$$-\frac{3\,2^{-m-7}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-7}3^{-m-1}e^{6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(n\right.}{b}$$

```
[Out] -3*2^(-7-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I
*b*(d*x+c)/d)^m)-3*2^(-7-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*
(a-b*c/d))/((I*b*(d*x+c)/d)^m)+2^(-7-m)*3^(-1-m)*exp(6*I*(a-b*c/d))*(d*x+c)
^m*GAMMA(1+m,-6*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+2^(-7-m)*3^(-1-m)*(d*
x+c)^m*GAMMA(1+m,6*I*b*(d*x+c)/d)/b/exp(6*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

**Rubi** [A] time = 0.32, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3308, 2181}

$$-\frac{3\,2^{-m-7}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-7}3^{-m-1}e^{6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*2^(-7 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(
c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - (3*2^(-7 - m)*(c + d*x)^m*Gamm
a[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x)
)/d)^m) + (2^(-7 - m)*3^(-1 - m)*E^((6*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[
1 + m, ((-6*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-7 - m)*
3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((6*I)*b*(c + d*x))/d])/(b*E^((6*I)*(a
- (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^m \sin(2a+2bx) - \frac{1}{32}(c+dx)^m \sin(6a+6bx)\right) dx$$
$$= -\left(\frac{1}{32}\int (c+dx)^m \sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^m \sin(2a+2bx)$$
$$= -\left(\frac{1}{64} i \int e^{-i(6a+6bx)}(c+dx)^m\,dx\right) + \frac{1}{64} i \int e^{i(6a+6bx)}(c+dx)^m\,dx + \frac{3}{64}$$
$$= -\frac{3\, 2^{-7-m} e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{2ib(c+dx)}{d}\right)}{b} - \frac{3\, 2^{-7}}{}$$

**Mathematica [A]** time = 3.34, size = 255, normalized size = 0.89

$$\frac{2^{-m-7} 3^{-m-1} e^{-\frac{6i(ad+bc)}{d}} (c+dx)^m \left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m} \left(-3^{m+2} e^{4ia+\frac{8ibc}{d}} \left(-\frac{ib(c+dx)}{d}\right)^m \Gamma\left(m+1, \frac{2ib(c+dx)}{d}\right) - 3^{m+2} e^{4i\left(2a+\frac{bc}{d}\right)} \left(\frac{ib(c+d}{d}\right.\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (2^(-7 - m)*3^(-1 - m)*(c + d*x)^m*(-(3^(2 + m)*E^((4*I)*(2*a + (b*c)/d))*(
(I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]) - 3^(2 + m)*E^((
4*I)*a + ((8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c +
 d*x))/d] + E^((12*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-6*I)*b*(c +
d*x))/d] + E^(((12*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((6*I)*
b*(c + d*x))/d]))/(b*E^(((6*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

**fricas [A]** time = 0.50, size = 184, normalized size = 0.65

$$\frac{e^{\left(-\frac{dm\log\left(\frac{6ib}{d}\right)-6ibc+6iad}{d}\right)}\Gamma\left(m+1, \frac{6ibdx+6ibc}{d}\right) - 9e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2ibc+2iad}{d}\right)}\Gamma\left(m+1, \frac{2ibdx+2ibc}{d}\right) - 9e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}\Gamma\left(\right.}{384b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(e^(-(d*m*log(6*I*b/d) - 6*I*b*c + 6*I*a*d)/d)*gamma(m + 1, (6*I*b*d*
x + 6*I*b*c)/d) - 9*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m +
 1, (2*I*b*d*x + 2*I*b*c)/d) - 9*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d
)/d)*gamma(m + 1, (-2*I*b*d*x - 2*I*b*c)/d) + e^(-(d*m*log(-6*I*b/d) + 6*I*
b*c - 6*I*a*d)/d)*gamma(m + 1, (-6*I*b*d*x - 6*I*b*c)/d))/b
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^3, x)
```

**maple [F]** time = 0.25, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^3(bx+a)\right)\left(\sin^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cos(bx + a)^3 \sin(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^3, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a + b\,x)^3 \sin(a + b\,x)^3 \,(c + d\,x)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^m, x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.155 $\int (c+dx)^4 \cos^3(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=233

$$-\frac{9d^4\cos(2a+2bx)}{128b^5}+\frac{d^4\cos(6a+6bx)}{10368b^5}-\frac{9d^3(c+dx)\sin(2a+2bx)}{64b^4}+\frac{d^3(c+dx)\sin(6a+6bx)}{1728b^4}+\frac{9d^2(c+dx)^2\cos(2}{64b^3}$$

```
[Out] -9/128*d^4*cos(2*b*x+2*a)/b^5+9/64*d^2*(d*x+c)^2*cos(2*b*x+2*a)/b^3-3/64*(d
*x+c)^4*cos(2*b*x+2*a)/b+1/10368*d^4*cos(6*b*x+6*a)/b^5-1/576*d^2*(d*x+c)^2
*cos(6*b*x+6*a)/b^3+1/192*(d*x+c)^4*cos(6*b*x+6*a)/b-9/64*d^3*(d*x+c)*sin(2
*b*x+2*a)/b^4+3/32*d*(d*x+c)^3*sin(2*b*x+2*a)/b^2+1/1728*d^3*(d*x+c)*sin(6*
b*x+6*a)/b^4-1/288*d*(d*x+c)^3*sin(6*b*x+6*a)/b^2
```

**Rubi [A]** time = 0.27, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$-\frac{9d^3(c+dx)\sin(2a+2bx)}{64b^4}+\frac{d^3(c+dx)\sin(6a+6bx)}{1728b^4}+\frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3}-\frac{d^2(c+dx)^2\cos(6a+6bx)}{576b^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-9*d^4*Cos[2*a + 2*b*x])/(128*b^5) + (9*d^2*(c + d*x)^2*Cos[2*a + 2*b*x])/
(64*b^3) - (3*(c + d*x)^4*Cos[2*a + 2*b*x])/(64*b) + (d^4*Cos[6*a + 6*b*x])
/(10368*b^5) - (d^2*(c + d*x)^2*Cos[6*a + 6*b*x])/(576*b^3) + ((c + d*x)^4*
Cos[6*a + 6*b*x])/(192*b) - (9*d^3*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^4) + (
3*d*(c + d*x)^3*Sin[2*a + 2*b*x])/(32*b^2) + (d^3*(c + d*x)*Sin[6*a + 6*b*x
])/(1728*b^4) - (d*(c + d*x)^3*Sin[6*a + 6*b*x])/(288*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^4 \cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^4 \sin(2a+2bx) - \frac{1}{32}(c+dx)^4 \sin(6a+6bx)\right) dx$$
$$= -\left(\frac{1}{32}\int (c+dx)^4 \sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^4 \sin(2a+2bx$$
$$= -\frac{3(c+dx)^4\cos(2a+2bx)}{64b} + \frac{(c+dx)^4\cos(6a+6bx)}{192b} - \frac{d\int (c+dx}{}$$
$$= -\frac{3(c+dx)^4\cos(2a+2bx)}{64b} + \frac{(c+dx)^4\cos(6a+6bx)}{192b} + \frac{3d(c+dx)}{}$$
$$= \frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3} - \frac{3(c+dx)^4\cos(2a+2bx)}{64b} - \frac{d^2(c+dx}{}$$
$$= \frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3} - \frac{3(c+dx)^4\cos(2a+2bx)}{64b} - \frac{d^2(c+dx}{}$$
$$= -\frac{9d^4\cos(2a+2bx)}{128b^5} + \frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3} - \frac{3(c+dx)^4\cos}{64b}$$

**Mathematica [A]** time = 1.54, size = 153, normalized size = 0.66

$$\frac{-12bd(c+dx)\sin(2(a+bx))\left(\cos(4(a+bx))\left(6b^2(c+dx)^2-d^2\right)-78b^2(c+dx)^2+121d^2\right)-243\cos(2(a+bx}{10368b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-243*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)]
+ (d^4 - 18*b^2*d^2*(c + d*x)^2 + 54*b^4*(c + d*x)^4)*Cos[6*(a + b*x)] - 12
*b*d*(c + d*x)*(121*d^2 - 78*b^2*(c + d*x)^2 + (-d^2 + 6*b^2*(c + d*x)^2)*C
os[4*(a + b*x)])*Sin[2*(a + b*x)])/(10368*b^5)
```

**fricas [B]** time = 0.50, size = 546, normalized size = 2.34

$$\frac{27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 2\left(54\,b^4d^4x^4 + 216\,b^4cd^3x^3 + 54\,b^4c^4 - 18\,b^2c^2d^2 + d^4 + 18\left(18\,b^4c^2d^2 - b^2d^4\right)x^2 + 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 2*(54*b^4*d^4*x^4 + 216*b^4*c*d
^3*x^3 + 54*b^4*c^4 - 18*b^2*c^2*d^2 + d^4 + 18*(18*b^4*c^2*d^2 - b^2*d^4)*
x^2 + 36*(6*b^4*c^3*d - b^2*c*d^3)*x)*cos(b*x + a)^6 - 3*(54*b^4*d^4*x^4 +
216*b^4*c*d^3*x^3 + 54*b^4*c^4 - 18*b^2*c^2*d^2 + d^4 + 18*(18*b^4*c^2*d^2
- b^2*d^4)*x^2 + 36*(6*b^4*c^3*d - b^2*c*d^3)*x)*cos(b*x + a)^4 + 18*(9*b^4
*c^2*d^2 - 5*b^2*d^4)*x^2 + 18*(9*b^2*d^4*x^2 + 18*b^2*c*d^3*x + 9*b^2*c^2*
d^2 - 5*d^4)*cos(b*x + a)^2 + 36*(3*b^4*c^3*d - 5*b^2*c*d^3)*x - 12*((6*b^3
*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d - b*c*d^3 + (18*b^3*c^2*d^2 - b*d
^4)*x)*cos(b*x + a)^5 - (6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d - b
*c*d^3 + (18*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^3 - 3*(3*b^3*d^4*x^3 + 9*
b^3*c*d^3*x^2 + 3*b^3*c^3*d - 5*b*c*d^3 + (9*b^3*c^2*d^2 - 5*b*d^4)*x)*cos(
b*x + a))*sin(b*x + a))/b^5
```

**giac [A]** time = 1.13, size = 359, normalized size = 1.54

$$\frac{\left(54\,b^4d^4x^4 + 216\,b^4cd^3x^3 + 324\,b^4c^2d^2x^2 + 216\,b^4c^3dx + 54\,b^4c^4 - 18\,b^2d^4x^2 - 36\,b^2cd^3x - 18\,b^2c^2d^2 + d^4\right)\cos}{10368\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/10368*(54*b^4*d^4*x^4 + 216*b^4*c*d^3*x^3 + 324*b^4*c^2*d^2*x^2 + 216*b^4
*c^3*d*x + 54*b^4*c^4 - 18*b^2*d^4*x^2 - 36*b^2*c*d^3*x - 18*b^2*c^2*d^2 +
d^4)*cos(6*b*x + 6*a)/b^5 - 3/128*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4
*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x -
 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 - 1/1728*(6*b^3*d^4*x^3 + 18*b
^3*c*d^3*x^2 + 18*b^3*c^2*d^2*x + 6*b^3*c^3*d - b*d^4*x - b*c*d^3)*sin(6*b*
x + 6*a)/b^5 + 3/64*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x + 2*
b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

**maple [B]** time = 0.13, size = 2061, normalized size = 8.85

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/4*(b*x+a)^4*sin(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/
2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/12*(b*x+a)^2*sin(b*x+a)^4+1/6*(b*
x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/9*(b*x
+a)^2+1/216*sin(b*x+a)^4+5/36*sin(b*x+a)^2+1/4*(b*x+a)^2*cos(b*x+a)^2-1/2*(
b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/8*(b*x+a)^4-1/6*(b*x+a)^
4*sin(b*x+a)^6+2/3*(b*x+a)^3*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(
b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/18*(b*x+a)^2*sin(b*x+a)^6-1/9*(b*x+a)
*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+
5/16*a)-1/324*sin(b*x+a)^6)-4/b^4*a*d^4*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*
x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/24*(
b*x+a)*sin(b*x+a)^4-1/96*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/18*b*x-
1/18*a+1/8*(b*x+a)*cos(b*x+a)^2-1/16*cos(b*x+a)*sin(b*x+a)+1/12*(b*x+a)^3-1
/6*(b*x+a)^3*sin(b*x+a)^6+1/2*(b*x+a)^2*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^
3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/36*(b*x+a)*sin(b*x+a)^6+1/
216*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+4/b^3*c*d^3
*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+
a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/24*(b*x+a)*sin(b*x+a)^4-1/96*(sin(b*x+a)^3+
3/2*sin(b*x+a))*cos(b*x+a)-1/18*b*x-1/18*a+1/8*(b*x+a)*cos(b*x+a)^2-1/16*co
s(b*x+a)*sin(b*x+a)+1/12*(b*x+a)^3-1/6*(b*x+a)^3*sin(b*x+a)^6+1/2*(b*x+a)^2
*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+
5/16*a)+1/36*(b*x+a)*sin(b*x+a)^6+1/216*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8
*sin(b*x+a))*cos(b*x+a))+6/b^4*a^2*d^4*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x
+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x
+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b
*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16
*b*x+5/16*a)+1/108*sin(b*x+a)^6)-12/b^3*a*c*d^3*(1/4*(b*x+a)^2*sin(b*x+a)^4
-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+
1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)
^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*
x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)+6/b^2*c^2*d^2*(1/4*(b*x+a)^2*sin(
b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x
+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*si
n(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a)
)*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)-4/b^4*a^3*d^4*(1/4*(b*x+a
)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*
a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x
+a))*cos(b*x+a))+12/b^3*a^2*c*d^3*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a
)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/3
6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))-12/b^2*a*c^2*
d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)
```

```
-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)
^3+15/8*sin(b*x+a))*cos(b*x+a))+4/b*c^3*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(s
in(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+
a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+1/b^4
*a^4*d^4*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-4/b^3*a^3*c*d^3
*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+6/b^2*a^2*c^2*d^2*(-1/6
*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-4/b*a*c^3*d*(-1/6*sin(b*x+a)^
2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+c^4*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*
cos(b*x+a)^4))
```

**maxima [B]** time = 0.39, size = 1033, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/10368*(864*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^4 - 3456*(2*sin(b*x +
 a)^6 - 3*sin(b*x + a)^4)*a*c^3*d/b + 5184*(2*sin(b*x + a)^6 - 3*sin(b*x +
a)^4)*a^2*c^2*d^2/b^2 - 3456*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^3*c*d^
3/b^3 + 864*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^4*d^4/b^4 - 36*(6*(b*x
+ a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) +
27*sin(2*b*x + 2*a))*c^3*d/b + 108*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x
+ a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a*c^2*d^2/b
^2 - 108*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - si
n(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a^2*c*d^3/b^3 + 36*(6*(b*x + a)*cos(6
*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b
*x + 2*a))*a^3*d^4/b^4 - 18*((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*
(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x
 + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 + 36*((18*(b*x + a)^2 - 1)*cos(6*b*x +
6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*
a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 - 18*((18*(b*x + a)^2 - 1)
*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*s
in(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^4/b^4 - 6*(6*(6*(b*
x + a)^3 - b*x - a)*cos(6*b*x + 6*a) - 162*(2*(b*x + a)^3 - 3*b*x - 3*a)*co
s(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*sin(6*b*x + 6*a) + 243*(2*(b*x + a)^2
 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 + 6*(6*(6*(b*x + a)^3 - b*x - a)*cos(6*b*
x + 6*a) - 162*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - (18*(b*x +
a)^2 - 1)*sin(6*b*x + 6*a) + 243*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^
4/b^4 - ((54*(b*x + a)^4 - 18*(b*x + a)^2 + 1)*cos(6*b*x + 6*a) - 243*(2*(b
*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) - 6*(6*(b*x + a)^3 - b*x -
a)*sin(6*b*x + 6*a) + 486*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))*d
^4/b^4)/b
```

**mupad [B]** time = 2.58, size = 576, normalized size = 2.47

$$-\frac{729\,d^4\,\cos\left(2\,a+2\,b\,x\right)-d^4\,\cos\left(6\,a+6\,b\,x\right)+486\,b^4\,c^4\,\cos\left(2\,a+2\,b\,x\right)-54\,b^4\,c^4\,\cos\left(6\,a+6\,b\,x\right)-972\,b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] -(729*d^4*cos(2*a + 2*b*x) - d^4*cos(6*a + 6*b*x) + 486*b^4*c^4*cos(2*a + 2
*b*x) - 54*b^4*c^4*cos(6*a + 6*b*x) - 972*b^3*c^3*d*sin(2*a + 2*b*x) + 36*b
^3*c^3*d*sin(6*a + 6*b*x) - 1458*b^2*c^2*d^2*cos(2*a + 2*b*x) + 18*b^2*c^2*
d^2*cos(6*a + 6*b*x) - 1458*b^2*d^4*x^2*cos(2*a + 2*b*x) + 486*b^4*d^4*x^4*
cos(2*a + 2*b*x) + 18*b^2*d^4*x^2*cos(6*a + 6*b*x) - 54*b^4*d^4*x^4*cos(6*a
 + 6*b*x) - 972*b^3*d^4*x^3*sin(2*a + 2*b*x) + 36*b^3*d^4*x^3*sin(6*a + 6*b
*x) + 1458*b*c*d^3*sin(2*a + 2*b*x) - 6*b*c*d^3*sin(6*a + 6*b*x) + 1458*b*d
^4*x*sin(2*a + 2*b*x) - 6*b*d^4*x*sin(6*a + 6*b*x) + 2916*b^4*c^2*d^2*x^2*c
```

```
os(2*a + 2*b*x) - 324*b^4*c^2*d^2*x^2*cos(6*a + 6*b*x) - 2916*b^2*c*d^3*x*c
os(2*a + 2*b*x) + 1944*b^4*c^3*d*x*cos(2*a + 2*b*x) + 36*b^2*c*d^3*x*cos(6*
a + 6*b*x) - 216*b^4*c^3*d*x*cos(6*a + 6*b*x) + 1944*b^4*c*d^3*x^3*cos(2*a
+ 2*b*x) - 216*b^4*c*d^3*x^3*cos(6*a + 6*b*x) - 2916*b^3*c^2*d^2*x*sin(2*a
+ 2*b*x) - 2916*b^3*c*d^3*x^2*sin(2*a + 2*b*x) + 108*b^3*c^2*d^2*x*sin(6*a
+ 6*b*x) + 108*b^3*c*d^3*x^2*sin(6*a + 6*b*x))/(10368*b^5)
```

**sympy [A]** time = 31.30, size = 1334, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)**6/(12*b) + c**4*sin(a + b*x)**4*cos(a + b*x)*
*2/(4*b) + c**3*d*x*sin(a + b*x)**6/(6*b) + c**3*d*x*sin(a + b*x)**4*cos(a
+ b*x)**2/(2*b) - c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(2*b) - c**3*d*x
*cos(a + b*x)**6/(6*b) + c**2*d**2*x**2*sin(a + b*x)**6/(4*b) + 3*c**2*d**2
*x**2*sin(a + b*x)**4*cos(a + b*x)**2/(4*b) - 3*c**2*d**2*x**2*sin(a + b*x)
**2*cos(a + b*x)**4/(4*b) - c**2*d**2*x**2*cos(a + b*x)**6/(4*b) + c*d**3*x
**3*sin(a + b*x)**6/(6*b) + c*d**3*x**3*sin(a + b*x)**4*cos(a + b*x)**2/(2*
b) - c*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**4/(2*b) - c*d**3*x**3*cos(a
+ b*x)**6/(6*b) + d**4*x**4*sin(a + b*x)**6/(24*b) + d**4*x**4*sin(a + b*x)
**4*cos(a + b*x)**2/(8*b) - d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**4/(8*b)
 - d**4*x**4*cos(a + b*x)**6/(24*b) + c**3*d*sin(a + b*x)**5*cos(a + b*x)/(
6*b**2) + 4*c**3*d*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + c**3*d*sin(a
+ b*x)*cos(a + b*x)**5/(6*b**2) + c**2*d**2*x*sin(a + b*x)**5*cos(a + b*x)/
(2*b**2) + 4*c**2*d**2*x*sin(a + b*x)**3*cos(a + b*x)**3/(3*b**2) + c**2*d*
*2*x*sin(a + b*x)*cos(a + b*x)**5/(2*b**2) + c*d**3*x**2*sin(a + b*x)**5*co
s(a + b*x)/(2*b**2) + 4*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)**3/(3*b**2
) + c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**5/(2*b**2) + d**4*x**3*sin(a + b
*x)**5*cos(a + b*x)/(6*b**2) + 4*d**4*x**3*sin(a + b*x)**3*cos(a + b*x)**3/
(9*b**2) + d**4*x**3*sin(a + b*x)*cos(a + b*x)**5/(6*b**2) - 7*c**2*d**2*si
n(a + b*x)**6/(36*b**3) - c**2*d**2*sin(a + b*x)**4*cos(a + b*x)**2/(3*b**3
) + c**2*d**2*cos(a + b*x)**6/(12*b**3) - 5*c*d**3*x*sin(a + b*x)**6/(18*b*
*3) - c*d**3*x*sin(a + b*x)**4*cos(a + b*x)**2/(3*b**3) + c*d**3*x*sin(a +
b*x)**2*cos(a + b*x)**4/(3*b**3) + 5*c*d**3*x*cos(a + b*x)**6/(18*b**3) - 5
*d**4*x**2*sin(a + b*x)**6/(36*b**3) - d**4*x**2*sin(a + b*x)**4*cos(a + b*
x)**2/(6*b**3) + d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**4/(6*b**3) + 5*d**
4*x**2*cos(a + b*x)**6/(36*b**3) - 5*c*d**3*sin(a + b*x)**5*cos(a + b*x)/(1
8*b**4) - 31*c*d**3*sin(a + b*x)**3*cos(a + b*x)**3/(54*b**4) - 5*c*d**3*si
n(a + b*x)*cos(a + b*x)**5/(18*b**4) - 5*d**4*x*sin(a + b*x)**5*cos(a + b*x
)/(18*b**4) - 31*d**4*x*sin(a + b*x)**3*cos(a + b*x)**3/(54*b**4) - 5*d**4*
x*sin(a + b*x)*cos(a + b*x)**5/(18*b**4) + 61*d**4*sin(a + b*x)**6/(648*b**
5) + 31*d**4*sin(a + b*x)**4*cos(a + b*x)**2/(216*b**5) - 5*d**4*cos(a + b*
x)**6/(108*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 +
c*d**3*x**4 + d**4*x**5/5)*sin(a)**3*cos(a)**3, True))
```

# 3.156 $\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx)\,dx$

**Optimal**. Leaf size=181

$$-\frac{9d^3\sin(2a+2bx)}{256b^4}+\frac{d^3\sin(6a+6bx)}{6912b^4}+\frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3}-\frac{d^2(c+dx)\cos(6a+6bx)}{1152b^3}+\frac{9d(c+dx)^2\sin}{128b}$$

```
[Out] 9/128*d^2*(d*x+c)*cos(2*b*x+2*a)/b^3-3/64*(d*x+c)^3*cos(2*b*x+2*a)/b-1/1152
*d^2*(d*x+c)*cos(6*b*x+6*a)/b^3+1/192*(d*x+c)^3*cos(6*b*x+6*a)/b-9/256*d^3*
sin(2*b*x+2*a)/b^4+9/128*d*(d*x+c)^2*sin(2*b*x+2*a)/b^2+1/6912*d^3*sin(6*b*
x+6*a)/b^4-1/384*d*(d*x+c)^2*sin(6*b*x+6*a)/b^2
```

**Rubi [A]** time = 0.22, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2637}

$$\frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3}-\frac{d^2(c+dx)\cos(6a+6bx)}{1152b^3}+\frac{9d(c+dx)^2\sin(2a+2bx)}{128b^2}-\frac{d(c+dx)^2\sin(6a+6bx)}{384b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (9*d^2*(c + d*x)*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^3*Cos[2*a + 2*b
*x])/(64*b) - (d^2*(c + d*x)*Cos[6*a + 6*b*x])/(1152*b^3) + ((c + d*x)^3*Co
s[6*a + 6*b*x])/(192*b) - (9*d^3*Sin[2*a + 2*b*x])/(256*b^4) + (9*d*(c + d*
x)^2*Sin[2*a + 2*b*x])/(128*b^2) + (d^3*Sin[6*a + 6*b*x])/(6912*b^4) - (d*(
c + d*x)^2*Sin[6*a + 6*b*x])/(384*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^3\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^3\sin(2a+2bx) - \frac{1}{32}(c+dx)^3\sin(6a+6bx)\right)dx$$
$$= -\left(\frac{1}{32}\int (c+dx)^3\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^3\sin(2a+2bx)\,d$$
$$= -\frac{3(c+dx)^3\cos(2a+2bx)}{64b} + \frac{(c+dx)^3\cos(6a+6bx)}{192b} - \frac{d\int (c+dx)^2 c}{6}$$
$$= -\frac{3(c+dx)^3\cos(2a+2bx)}{64b} + \frac{(c+dx)^3\cos(6a+6bx)}{192b} + \frac{9d(c+dx)^2 \mathrm{si}}{128}$$
$$= \frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3} - \frac{3(c+dx)^3\cos(2a+2bx)}{64b} - \frac{d^2(c+dx)\mathrm{cc}}{115}$$
$$= \frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3} - \frac{3(c+dx)^3\cos(2a+2bx)}{64b} - \frac{d^2(c+dx)\mathrm{cc}}{115}$$

**Mathematica** [A] time = 2.31, size = 132, normalized size = 0.73

$$\frac{-324b(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)+12b(c+dx)\cos(6(a+bx))\left(6b^2(c+dx)^2-d^2\right)-4d\sin(2(a}{13824b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-324*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 12*b*(c +
 d*x)*(-d^2 + 6*b^2*(c + d*x)^2)*Cos[6*(a + b*x)] - 4*d*(121*d^2 - 234*b^2*
(c + d*x)^2 + (-d^2 + 18*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[2*(a + b*x)
])/(13824*b^4)
```

**fricas** [B] time = 0.46, size = 349, normalized size = 1.93

$$9\,b^3d^3x^3 + 27\,b^3cd^2x^2 + 6\left(6\,b^3d^3x^3 + 18\,b^3cd^2x^2 + 6\,b^3c^3 - bcd^2 + \left(18\,b^3c^2d - bd^3\right)x\right)\cos\left(bx+a\right)^6 - 9\left(6\,b^3d^3x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 6*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x
^2 + 6*b^3*c^3 - b*c*d^2 + (18*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^6 - 9*(6*
b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 6*b^3*c^3 - b*c*d^2 + (18*b^3*c^2*d - b*d^
3)*x)*cos(b*x + a)^4 + 27*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 3*(9*b^3*c^2
*d - 5*b*d^3)*x - ((18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*c
os(b*x + a)^5 - (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*cos(
b*x + a)^3 - 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 5*d^3)*cos(b
*x + a))*sin(b*x + a))/b^4
```

**giac** [A] time = 0.38, size = 241, normalized size = 1.33

$$\frac{\left(6\,b^3d^3x^3 + 18\,b^3cd^2x^2 + 18\,b^3c^2dx + 6\,b^3c^3 - bd^3x - bcd^2\right)\cos\left(6\,bx + 6\,a\right)}{1152\,b^4} - \frac{3\left(2\,b^3d^3x^3 + 6\,b^3cd^2x^2 + 6\,b^3c^2dx +\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/1152*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 18*b^3*c^2*d*x + 6*b^3*c^3 - b*d
^3*x - b*c*d^2)*cos(6*b*x + 6*a)/b^4 - 3/128*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x
```

```
^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/b^
4 - 1/6912*(18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*sin(6*b*x
 + 6*a)/b^4 + 9/256*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin
(2*b*x + 2*a)/b^4
```

**maple [B]** time = 0.02, size = 1100, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^
3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/24*(b*x+a)*sin(b*x+a)^4-1/96*
(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/18*b*x-1/18*a+1/8*(b*x+a)*cos(b*
x+a)^2-1/16*cos(b*x+a)*sin(b*x+a)+1/12*(b*x+a)^3-1/6*(b*x+a)^3*sin(b*x+a)^6
+1/2*(b*x+a)^2*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*
x+a)+5/16*b*x+5/16*a)+1/36*(b*x+a)*sin(b*x+a)^6+1/216*(sin(b*x+a)^5+5/4*sin
(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))-3/b^3*a*d^3*(1/4*(b*x+a)^2*sin(b*x+a
)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*
a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x
+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos
(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)+3/b^2*c*d^2*(1/4*(b*x+a)^2*sin
(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*
x+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*s
in(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a
))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)+3/b^3*a^2*d^3*(1/4*(b*x+
a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24
*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*
x+a))*cos(b*x+a))-6/b^2*a*c*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^
3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*
(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+3/b*c^2*d*(1/4*
(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x
-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*s
in(b*x+a))*cos(b*x+a))-1/b^3*a^3*d^3*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*c
os(b*x+a)^4)+3/b^2*a^2*c*d^2*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a
)^4)-3/b*a*c^2*d*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+c^3*(-1
/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4))
```

**maxima [B]** time = 0.35, size = 602, normalized size = 3.33

$$-\frac{576\left(2\sin\left(bx+a\right)^{6}-3\sin\left(bx+a\right)^{4}\right)c^{3}-\frac{1728\left(2\sin(bx+a)^{6}-3\sin(bx+a)^{4}\right)ac^{2}d}{b}+\frac{1728\left(2\sin(bx+a)^{6}-3\sin(bx+a)^{4}\right)a^{2}cd^{2}}{b^{2}}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6912*(576*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^3 - 1728*(2*sin(b*x +
a)^6 - 3*sin(b*x + a)^4)*a*c^2*d/b + 1728*(2*sin(b*x + a)^6 - 3*sin(b*x + a
)^4)*a^2*c*d^2/b^2 - 576*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^3*d^3/b^3
- 18*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*
b*x + 6*a) + 27*sin(2*b*x + 2*a))*c^2*d/b + 36*(6*(b*x + a)*cos(6*b*x + 6*a
) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))
*a*c*d^2/b^2 - 18*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x +
2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a^2*d^3/b^3 - 6*((18*(b*x +
a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b
*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 + 6*((
18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2
*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/
```

```
b^3 - (6*(6*(b*x + a)^3 - b*x - a)*cos(6*b*x + 6*a) - 162*(2*(b*x + a)^3 -
3*b*x - 3*a)*cos(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*sin(6*b*x + 6*a) + 243
*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

**mupad [B]** time = 1.23, size = 366, normalized size = 2.02

$$-\frac{243\,d^3\,\sin\left(2\,a+2\,b\,x\right)-d^3\,\sin\left(6\,a+6\,b\,x\right)+324\,b^3\,c^3\,\cos\left(2\,a+2\,b\,x\right)-36\,b^3\,c^3\,\cos\left(6\,a+6\,b\,x\right)-486\,b^2\,c^2\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] -(243*d^3*sin(2*a + 2*b*x) - d^3*sin(6*a + 6*b*x) + 324*b^3*c^3*cos(2*a + 2
*b*x) - 36*b^3*c^3*cos(6*a + 6*b*x) - 486*b^2*c^2*d*sin(2*a + 2*b*x) + 18*b
^2*c^2*d*sin(6*a + 6*b*x) + 324*b^3*d^3*x^3*cos(2*a + 2*b*x) - 36*b^3*d^3*x
^3*cos(6*a + 6*b*x) - 486*b^2*d^3*x^2*sin(2*a + 2*b*x) + 18*b^2*d^3*x^2*sin
(6*a + 6*b*x) - 486*b*c*d^2*cos(2*a + 2*b*x) + 6*b*c*d^2*cos(6*a + 6*b*x) -
 486*b*d^3*x*cos(2*a + 2*b*x) + 6*b*d^3*x*cos(6*a + 6*b*x) + 972*b^3*c^2*d*
x*cos(2*a + 2*b*x) - 108*b^3*c^2*d*x*cos(6*a + 6*b*x) - 972*b^2*c*d^2*x*sin
(2*a + 2*b*x) + 36*b^2*c*d^2*x*sin(6*a + 6*b*x) + 972*b^3*c*d^2*x^2*cos(2*a
 + 2*b*x) - 108*b^3*c*d^2*x^2*cos(6*a + 6*b*x))/(6912*b^4)
```

**sympy [A]** time = 18.63, size = 857, normalized size = 4.73

$$\begin{cases} \frac{c^3\sin^6\left(a+bx\right)}{12b}+\frac{c^3\sin^4\left(a+bx\right)\cos^2\left(a+bx\right)}{4b}+\frac{c^2dx\sin^6\left(a+bx\right)}{8b}+\frac{3c^2dx\sin^4\left(a+bx\right)\cos^2\left(a+bx\right)}{8b}-\frac{3c^2dx\sin^2\left(a+bx\right)\cos^4\left(a+bx\right)}{8b}-\frac{c^2dx\cos^6\left(a\right.}{8b} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^3\left(a\right)\cos^3\left(a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)**6/(12*b) + c**3*sin(a + b*x)**4*cos(a + b*x)*
*2/(4*b) + c**2*d*x*sin(a + b*x)**6/(8*b) + 3*c**2*d*x*sin(a + b*x)**4*cos(
a + b*x)**2/(8*b) - 3*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - c**2
*d*x*cos(a + b*x)**6/(8*b) + c*d**2*x**2*sin(a + b*x)**6/(8*b) + 3*c*d**2*x
**2*sin(a + b*x)**4*cos(a + b*x)**2/(8*b) - 3*c*d**2*x**2*sin(a + b*x)**2*c
os(a + b*x)**4/(8*b) - c*d**2*x**2*cos(a + b*x)**6/(8*b) + d**3*x**3*sin(a
+ b*x)**6/(24*b) + d**3*x**3*sin(a + b*x)**4*cos(a + b*x)**2/(8*b) - d**3*x
**3*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - d**3*x**3*cos(a + b*x)**6/(24*b
) + c**2*d*sin(a + b*x)**5*cos(a + b*x)/(8*b**2) + c**2*d*sin(a + b*x)**3*c
os(a + b*x)**3/(3*b**2) + c**2*d*sin(a + b*x)*cos(a + b*x)**5/(8*b**2) + c*
d**2*x*sin(a + b*x)**5*cos(a + b*x)/(4*b**2) + 2*c*d**2*x*sin(a + b*x)**3*c
os(a + b*x)**3/(3*b**2) + c*d**2*x*sin(a + b*x)*cos(a + b*x)**5/(4*b**2) +
d**3*x**2*sin(a + b*x)**5*cos(a + b*x)/(8*b**2) + d**3*x**2*sin(a + b*x)**3
*cos(a + b*x)**3/(3*b**2) + d**3*x**2*sin(a + b*x)*cos(a + b*x)**5/(8*b**2)
 - 7*c*d**2*sin(a + b*x)**6/(72*b**3) - c*d**2*sin(a + b*x)**4*cos(a + b*x)
**2/(6*b**3) + c*d**2*cos(a + b*x)**6/(24*b**3) - 5*d**3*x*sin(a + b*x)**6/
(72*b**3) - d**3*x*sin(a + b*x)**4*cos(a + b*x)**2/(12*b**3) + d**3*x*sin(a
 + b*x)**2*cos(a + b*x)**4/(12*b**3) + 5*d**3*x*cos(a + b*x)**6/(72*b**3) -
 5*d**3*sin(a + b*x)**5*cos(a + b*x)/(72*b**4) - 31*d**3*sin(a + b*x)**3*co
s(a + b*x)**3/(216*b**4) - 5*d**3*sin(a + b*x)*cos(a + b*x)**5/(72*b**4), N
e(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**3
*cos(a)**3, True))
```

# 3.157 $\int (c+dx)^2 \cos^3(a+bx) \sin^3(a+bx) \, dx$

**Optimal**. Leaf size=129

$$\frac{3d^2\cos(2a+2bx)}{128b^3}-\frac{d^2\cos(6a+6bx)}{3456b^3}+\frac{3d(c+dx)\sin(2a+2bx)}{64b^2}-\frac{d(c+dx)\sin(6a+6bx)}{576b^2}-\frac{3(c+dx)^2\cos(2a+}{64b}$$

```
[Out] 3/128*d^2*cos(2*b*x+2*a)/b^3-3/64*(d*x+c)^2*cos(2*b*x+2*a)/b-1/3456*d^2*cos
(6*b*x+6*a)/b^3+1/192*(d*x+c)^2*cos(6*b*x+6*a)/b+3/64*d*(d*x+c)*sin(2*b*x+2
*a)/b^2-1/576*d*(d*x+c)*sin(6*b*x+6*a)/b^2
```

**Rubi [A]** time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$\frac{3d(c+dx)\sin(2a+2bx)}{64b^2}-\frac{d(c+dx)\sin(6a+6bx)}{576b^2}+\frac{3d^2\cos(2a+2bx)}{128b^3}-\frac{d^2\cos(6a+6bx)}{3456b^3}-\frac{3(c+dx)^2\cos(2a+}{64b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (3*d^2*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^2*Cos[2*a + 2*b*x])/(64*b
) - (d^2*Cos[6*a + 6*b*x])/(3456*b^3) + ((c + d*x)^2*Cos[6*a + 6*b*x])/(192
*b) + (3*d*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^2) - (d*(c + d*x)*Sin[6*a + 6*
b*x])/(576*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2\cos^3(a+bx)\sin^3(a+bx)\,dx &= \int \left(\frac{3}{32}(c+dx)^2\sin(2a+2bx)-\frac{1}{32}(c+dx)^2\sin(6a+6bx)\right)dx\\
&= -\left(\frac{1}{32}\int (c+dx)^2\sin(6a+6bx)\,dx\right)+\frac{3}{32}\int (c+dx)^2\sin(2a+2bx\\
&= -\frac{3(c+dx)^2\cos(2a+2bx)}{64b}+\frac{(c+dx)^2\cos(6a+6bx)}{192b}-\frac{d\int (c+dx\\
&= -\frac{3(c+dx)^2\cos(2a+2bx)}{64b}+\frac{(c+dx)^2\cos(6a+6bx)}{192b}+\frac{3d(c+dx)\\
&= \frac{3d^2\cos(2a+2bx)}{128b^3}-\frac{3(c+dx)^2\cos(2a+2bx)}{64b}-\frac{d^2\cos(6a+6bx)}{3456b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.56, size = 91, normalized size = 0.71

$$\frac{-81\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+\cos(6(a+bx))\left(18b^2(c+dx)^2-d^2\right)-6bd(c+dx)(\sin(6(a+bx))-27\,\mathrm{si}}{3456b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-81*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (-d^2 + 18*b^2*(c + d*x)
^2)*Cos[6*(a + b*x)] - 6*b*d*(c + d*x)*(-27*Sin[2*(a + b*x)] + Sin[6*(a + b
*x)]))/(3456*b^3)
```

**fricas [A]** time = 0.47, size = 194, normalized size = 1.50

$$2\left(18\,b^2d^2x^2+36\,b^2cdx+18\,b^2c^2-d^2\right)\cos\left(bx+a\right)^6+9\,b^2d^2x^2+18\,b^2cdx-3\left(18\,b^2d^2x^2+36\,b^2cdx+18\,b^2c^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*(2*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*cos(b*x + a)^6
+ 9*b^2*d^2*x^2 + 18*b^2*c*d*x - 3*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*
c^2 - d^2)*cos(b*x + a)^4 + 9*d^2*cos(b*x + a)^2 - 6*(2*(b*d^2*x + b*c*d)*c
os(b*x + a)^5 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 3*(b*d^2*x + b*c*d)*co
s(b*x + a))*sin(b*x + a))/b^3
```

**giac [A]** time = 0.41, size = 145, normalized size = 1.12

$$\frac{\left(18\,b^2d^2x^2+36\,b^2cdx+18\,b^2c^2-d^2\right)\cos\left(6\,bx+6\,a\right)}{3456\,b^3}-\frac{3\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2\right)\cos\left(2\,bx+2\,a\right)}{128\,b^3}-\frac{\left(bd^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/3456*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*cos(6*b*x + 6*a)/
b^3 - 3/128*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a
)/b^3 - 1/576*(b*d^2*x + b*c*d)*sin(6*b*x + 6*a)/b^3 + 3/64*(b*d^2*x + b*c*
d)*sin(2*b*x + 2*a)/b^3
```

**maple [B]** time = 0.02, size = 498, normalized size = 3.86

$$\frac{d^2\left(\frac{(bx+a)^2\left(\sin^4(bx+a)\right)}{4}-\frac{(bx+a)\left(-\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)}{2}+\frac{(bx+a)^2}{24}-\frac{\left(\sin^4(bx+a)\right)}{72}-\frac{\left(\sin^2(bx+a)\right)}{24}-\frac{(bx+a)^2\left(\sin^6(bx+a)\right)}{6}+\frac{(bx+a)\left(-\frac{\left(\sin^5(bx+a)+\ldots\right.}{}\right.}{}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+
3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-
```

```
1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^
5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b
*x+a)^6)-2/b^2*a*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b
*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)
^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+2/b*c*d*(1/4*(b*x+a)*sin(b
*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(
b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*co
s(b*x+a))+d^2/b^2*a^2*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-2*
c*d/b*a*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+c^2*(-1/6*sin(b*
x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4))
```

**maxima [B]** time = 0.41, size = 303, normalized size = 2.35

$$-\frac{288\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)c^2-\frac{576\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)acd}{b}+\frac{288\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)a^2d^2}{b^2}-\frac{6(6}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3456*(288*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^2 - 576*(2*sin(b*x + a
)^6 - 3*sin(b*x + a)^4)*a*c*d/b + 288*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)
*a^2*d^2/b^2 - 6*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2
*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*c*d/b + 6*(6*(b*x + a)*cos(6*
b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*
x + 2*a))*a*d^2/b^2 - ((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*
sin(2*b*x + 2*a))*d^2/b^2)/b
```

**mupad [B]** time = 0.81, size = 202, normalized size = 1.57

$$\frac{81\,d^2\,\cos(2\,a+2\,b\,x)-d^2\,\cos(6\,a+6\,b\,x)-162\,b^2\,c^2\,\cos(2\,a+2\,b\,x)+18\,b^2\,c^2\,\cos(6\,a+6\,b\,x)+162\,b\,c\,d\,}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] (81*d^2*cos(2*a + 2*b*x) - d^2*cos(6*a + 6*b*x) - 162*b^2*c^2*cos(2*a + 2*b
*x) + 18*b^2*c^2*cos(6*a + 6*b*x) + 162*b*c*d*sin(2*a + 2*b*x) - 6*b*c*d*si
n(6*a + 6*b*x) - 162*b^2*d^2*x^2*cos(2*a + 2*b*x) + 18*b^2*d^2*x^2*cos(6*a
+ 6*b*x) + 162*b*d^2*x*sin(2*a + 2*b*x) - 6*b*d^2*x*sin(6*a + 6*b*x) - 324*
b^2*c*d*x*cos(2*a + 2*b*x) + 36*b^2*c*d*x*cos(6*a + 6*b*x))/(3456*b^3)
```

**sympy [A]** time = 9.99, size = 461, normalized size = 3.57

$$\begin{cases}\frac{c^2\sin^6(a+bx)}{12b}+\frac{c^2\sin^4(a+bx)\cos^2(a+bx)}{4b}+\frac{cdx\sin^6(a+bx)}{12b}+\frac{cdx\sin^4(a+bx)\cos^2(a+bx)}{4b}-\frac{cdx\sin^2(a+bx)\cos^4(a+bx)}{4b}-\frac{cdx\cos^6(a+b}{12b} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin^3(a)\cos^3(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)**6/(12*b) + c**2*sin(a + b*x)**4*cos(a + b*x)*
*2/(4*b) + c*d*x*sin(a + b*x)**6/(12*b) + c*d*x*sin(a + b*x)**4*cos(a + b*x
)**2/(4*b) - c*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c*d*x*cos(a + b*
x)**6/(12*b) + d**2*x**2*sin(a + b*x)**6/(24*b) + d**2*x**2*sin(a + b*x)**4
*cos(a + b*x)**2/(8*b) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) -
d**2*x**2*cos(a + b*x)**6/(24*b) + c*d*sin(a + b*x)**5*cos(a + b*x)/(12*b**
```

```
2) + 2*c*d*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + c*d*sin(a + b*x)*cos(
a + b*x)**5/(12*b**2) + d**2*x*sin(a + b*x)**5*cos(a + b*x)/(12*b**2) + 2*d
**2*x*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + d**2*x*sin(a + b*x)*cos(a
+ b*x)**5/(12*b**2) - 7*d**2*sin(a + b*x)**6/(216*b**3) - d**2*sin(a + b*x)
**4*cos(a + b*x)**2/(18*b**3) + d**2*cos(a + b*x)**6/(72*b**3), Ne(b, 0)),
((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a)**3, True))
```

## 3.158 $\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=77

$$\frac{3d\sin(2a+2bx)}{128b^2}-\frac{d\sin(6a+6bx)}{1152b^2}-\frac{3(c+dx)\cos(2a+2bx)}{64b}+\frac{(c+dx)\cos(6a+6bx)}{192b}$$

```
[Out] -3/64*(d*x+c)*cos(2*b*x+2*a)/b+1/192*(d*x+c)*cos(6*b*x+6*a)/b+3/128*d*sin(2
*b*x+2*a)/b^2-1/1152*d*sin(6*b*x+6*a)/b^2
```

**Rubi [A]** time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4406, 3296, 2637}

$$\frac{3d\sin(2a+2bx)}{128b^2}-\frac{d\sin(6a+6bx)}{1152b^2}-\frac{3(c+dx)\cos(2a+2bx)}{64b}+\frac{(c+dx)\cos(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*(c + d*x)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)*Cos[6*a + 6*b*x])/(192*
b) + (3*d*Sin[2*a + 2*b*x])/(128*b^2) - (d*Sin[6*a + 6*b*x])/(1152*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx &= \int \left(\frac{3}{32}(c+dx)\sin(2a+2bx)-\frac{1}{32}(c+dx)\sin(6a+6bx)\right)dx\\ &= -\left(\frac{1}{32}\int (c+dx)\sin(6a+6bx)\,dx\right)+\frac{3}{32}\int (c+dx)\sin(2a+2bx)\,d\\ &= -\frac{3(c+dx)\cos(2a+2bx)}{64b}+\frac{(c+dx)\cos(6a+6bx)}{192b}-\frac{d\int \cos(6a+}{192b}\\ &= -\frac{3(c+dx)\cos(2a+2bx)}{64b}+\frac{(c+dx)\cos(6a+6bx)}{192b}+\frac{3d\sin(2a+2}{128b^2}\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 63, normalized size = 0.82

$$\frac{-54b(c+dx)\cos(2(a+bx))+6b(c+dx)\cos(6(a+bx))+d(27\sin(2(a+bx))-\sin(6(a+bx)))}{1152b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-54*b*(c + d*x)*Cos[2*(a + b*x)] + 6*b*(c + d*x)*Cos[6*(a + b*x)] + d*(27*
Sin[2*(a + b*x)] - Sin[6*(a + b*x)]))/(1152*b^2)
```

**fricas [A]** time = 0.44, size = 87, normalized size = 1.13

$$\frac{12\,(bdx+bc)\cos{(bx+a)}^6-18\,(bdx+bc)\cos{(bx+a)}^4+3\,bdx-\left(2\,d\cos{(bx+a)}^5-2\,d\cos{(bx+a)}^3-3\,d\cos{(}\right.}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72*(12*(b*d*x + b*c)*cos(b*x + a)^6 - 18*(b*d*x + b*c)*cos(b*x + a)^4 + 3
*b*d*x - (2*d*cos(b*x + a)^5 - 2*d*cos(b*x + a)^3 - 3*d*cos(b*x + a))*sin(b
*x + a))/b^2
```

**giac [A]** time = 0.24, size = 75, normalized size = 0.97

$$\frac{(bdx+bc)\cos{(6\,bx+6\,a)}}{192\,b^2}-\frac{3\,(bdx+bc)\cos{(2\,bx+2\,a)}}{64\,b^2}-\frac{d\sin{(6\,bx+6\,a)}}{1152\,b^2}+\frac{3\,d\sin{(2\,bx+2\,a)}}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/192*(b*d*x + b*c)*cos(6*b*x + 6*a)/b^2 - 3/64*(b*d*x + b*c)*cos(2*b*x + 2
*a)/b^2 - 1/1152*d*sin(6*b*x + 6*a)/b^2 + 3/128*d*sin(2*b*x + 2*a)/b^2
```

**maple [B]** time = 0.02, size = 176, normalized size = 2.29

$$\frac{\frac{d\left(\frac{(bx+a)\left(\sin^4(bx+a)\right)}{4}+\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{16}-\frac{bx}{24}-\frac{a}{24}-\frac{(bx+a)\left(\sin^6(bx+a)\right)}{6}-\frac{\left(\sin^5(bx+a)+\frac{5\left(\sin^3(bx+a)\right)}{4}+\frac{15\sin(bx+a)}{8}\right)\cos(bx+a)}{36}\right)}{b}-\frac{da\left(-\frac{\left(\sin^2(bx+a)\right)(}{6}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos
(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin
(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))-1/b*d*a*(-1/6*sin(b*x+a)^2*cos(b*x+a
)^4-1/12*cos(b*x+a)^4)+c*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)
)
```

**maxima [A]** time = 0.80, size = 119, normalized size = 1.55

$$-\frac{96\left(2\sin{(bx+a)}^6-3\sin{(bx+a)}^4\right)c-\frac{96\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)ad}{b}-\frac{(6\,(bx+a)\cos(6\,bx+6\,a)-54\,(bx+a)\cos(2\,bx+2\,a)-\sin(6}{b}}{1152\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/1152*(96*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c - 96*(2*sin(b*x + a)^6
- 3*sin(b*x + a)^4)*a*d/b - (6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*co
s(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*d/b)/b
```

**mupad [B]** time = 0.71, size = 84, normalized size = 1.09

$$\frac{\frac{27\,d\,\sin(2\,a+2\,b\,x)}{4}-\frac{d\,\sin(6\,a+6\,b\,x)}{4}-\frac{27\,b\,c\,\cos(2\,a+2\,b\,x)}{2}+\frac{3\,b\,c\,\cos(6\,a+6\,b\,x)}{2}-\frac{27\,b\,d\,x\,\cos(2\,a+2\,b\,x)}{2}+\frac{3\,b\,d\,x\,\cos(6\,a+6\,b\,x)}{2}}{288\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x),x)
```

```
[Out] ((27*d*sin(2*a + 2*b*x))/4 - (d*sin(6*a + 6*b*x))/4 - (27*b*c*cos(2*a + 2*b
*x))/2 + (3*b*c*cos(6*a + 6*b*x))/2 - (27*b*d*x*cos(2*a + 2*b*x))/2 + (3*b*
d*x*cos(6*a + 6*b*x))/2)/(288*b^2)
```

**sympy [A]** time = 5.33, size = 201, normalized size = 2.61

$$\begin{cases} \frac{c\sin^6(a+bx)}{12b}+\frac{c\sin^4(a+bx)\cos^2(a+bx)}{4b}+\frac{dx\sin^6(a+bx)}{24b}+\frac{dx\sin^4(a+bx)\cos^2(a+bx)}{8b}-\frac{dx\sin^2(a+bx)\cos^4(a+bx)}{8b}-\frac{dx\cos^6(a+bx)}{24b}+ \\ \left(cx+\frac{dx^2}{2}\right)\sin^3(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c*sin(a + b*x)**6/(12*b) + c*sin(a + b*x)**4*cos(a + b*x)**2/(4*
b) + d*x*sin(a + b*x)**6/(24*b) + d*x*sin(a + b*x)**4*cos(a + b*x)**2/(8*b)
 - d*x*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - d*x*cos(a + b*x)**6/(24*b) +
 d*sin(a + b*x)**5*cos(a + b*x)/(24*b**2) + d*sin(a + b*x)**3*cos(a + b*x)*
*3/(9*b**2) + d*sin(a + b*x)*cos(a + b*x)**5/(24*b**2), Ne(b, 0)), ((c*x +
d*x**2/2)*sin(a)**3*cos(a)**3, True))
```

## 3.159 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=129

$$-\frac{\sin\left(6a-\frac{6bc}{d}\right)\text{Ci}\left(\frac{6bc}{d}+6bx\right)}{32d}+\frac{3\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{32d}+\frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{32d}-\frac{\cos\left(6a-\frac{6bc}{d}\right)\text{S}}{32d}$$

```
[Out] 3/32*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/32*cos(6*a-6*b*c/d)*Si(6*b*c/d+
6*b*x)/d-1/32*Ci(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d+3/32*Ci(2*b*c/d+2*b*x)*s
in(2*a-2*b*c/d)/d
```

**Rubi** [A] time = 0.25, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4406, 3303, 3299, 3302}

$$-\frac{\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{32d}+\frac{3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{32d}+\frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -(CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])/(32*d) + (3*CosInteg
ral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(32*d) + (3*Cos[2*a - (2*b*c)/
d]*SinIntegral[(2*b*c)/d + 2*b*x])/(32*d) - (Cos[6*a - (6*b*c)/d]*SinIntegr
al[(6*b*c)/d + 6*b*x])/(32*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)} - \frac{\sin(6a+6bx)}{32(c+dx)}\right)dx$$
$$= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{c+dx}\,dx\right) + \frac{3}{32}\int \frac{\sin(2a+2bx)}{c+dx}\,dx$$
$$= -\left(\frac{1}{32}\cos\left(6a-\frac{6bc}{d}\right)\int \frac{\sin\left(\frac{6bc}{d}+6bx\right)}{c+dx}\,dx\right) + \frac{1}{32}\left(3\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\mathrm{si}}{}$$
$$= -\frac{\mathrm{Ci}\left(\frac{6bc}{d}+6bx\right)\sin\left(6a-\frac{6bc}{d}\right)}{32d} + \frac{3\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{32d} + \frac{3\cos\left(2a\right.}{}$$

**Mathematica [A]** time = 0.31, size = 110, normalized size = 0.85

$$-\frac{\sin\left(6a-\frac{6bc}{d}\right)\mathrm{Ci}\left(\frac{6b(c+dx)}{d}\right) - 3\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2b(c+dx)}{d}\right) - 3\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right) + \cos\left(6a-\frac{6bc}{d}\right)\mathrm{Si}\left(\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -1/32*(CosIntegral[(6*b*(c + d*x))/d]*Sin[6*a - (6*b*c)/d] - 3*CosIntegral[
(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 3*Cos[2*a - (2*b*c)/d]*SinIntegra
l[(2*b*(c + d*x))/d] + Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d])
/d
```

**fricas [A]** time = 0.44, size = 156, normalized size = 1.21

$$\frac{3\left(\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right) + \mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right) - \left(\mathrm{Ci}\left(\frac{6\,(bdx+bc)}{d}\right) + \mathrm{Ci}\left(-\frac{6\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right) - 2\cos}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/64*(3*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d)
)*sin(-2*(b*c - a*d)/d) - (cos_integral(6*(b*d*x + b*c)/d) + cos_integral(-
6*(b*d*x + b*c)/d))*sin(-6*(b*c - a*d)/d) - 2*cos(-6*(b*c - a*d)/d)*sin_int
egral(6*(b*d*x + b*c)/d) + 6*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
b*c)/d))/d
```

**giac [C]** time = 0.57, size = 6046, normalized size = 46.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/64*(imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b
*c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^
2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_i
ntegral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2
+ 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(
b*c/d)^2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3*b*c/
d)^2*tan(b*c/d)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*t
```

an(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integr
al(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 2*real
_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(
b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*tan
(3*b*c/d)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(6*b*x
+ 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos
_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2
+ imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)
^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b
*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*t
an(3*b*c/d)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)
^2*tan(3*b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*t
an(3*b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3
*b*c/d)^2 - 12*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*t
an(3*b*c/d)^2*tan(b*c/d) + 12*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - 24*sin_integral(2*(b*d*x + b*c)/
d)*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(6*b
*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(
2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + 3*imag_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + imag_part(cos_inte
gral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*sin_integral(6
*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 6*sin_integral(2*(b*d*
x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*imag_part(cos_integral(6*b
*x + 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 4*imag_part(co
s_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 +
 8*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d
)^2 + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2*ta
n(b*c/d)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(3*b*
c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^
2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/d))*t
an(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*t
an(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*t
an(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b*
c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*imag_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_i
ntegral(-6*b*x - 6*b*c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*sin_int
egral(6*(b*d*x + b*c)/d)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 2*real_part(c
os_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d) + 2*real_par
t(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d) - 6*real
_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2 - 6*r
eal_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2 -
 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)^
2 - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c
/d)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(
b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*ta
n(b*c/d) + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(3*b*c/
d)^2*tan(b*c/d) + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*ta
n(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)
^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 6*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(3*a)^2*tan(a)*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(3*a)^2*tan(a)*tan(b*c/d)^2 + 2*real_part(cos_integral(6*b*x + 6*
b*c/d))*tan(3*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(-6*b*x -
6*b*c/d))*tan(3*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(6*b*x +
 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(

```
-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_i
ntegral(6*b*x + 6*b*c/d))*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 2*real_part(
cos_integral(-6*b*x - 6*b*c/d))*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 2*real
_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(3*b*c/d)^2*tan(b*c/d)^2 -
 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(3*b*c/d)^2*tan(b*
c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(3*b*c/d)^2*t
an(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(3*b*c/
d)^2*tan(b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan
(a)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2 - 3*
imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2 + imag_part(c
os_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2 - 2*sin_integral(6*(b*d*
x + b*c)/d)*tan(3*a)^2*tan(a)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a
)^2*tan(a)^2 + 4*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(a)^2
*tan(3*b*c/d) - 4*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)
^2*tan(3*b*c/d) + 8*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)*tan(a)^2*tan(3
*b*c/d) + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^
2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2 +
3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2 - ima
g_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2 + 2*sin_in
tegral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(3*b*c/d)^2 - 6*sin_integral(2*(b*d
*x + b*c)/d)*tan(3*a)^2*tan(3*b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b
*c/d))*tan(a)^2*tan(3*b*c/d)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))
*tan(a)^2*tan(3*b*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
a)^2*tan(3*b*c/d)^2 + imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(a)^2*ta
n(3*b*c/d)^2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(a)^2*tan(3*b*c/d)^2 +
6*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(3*b*c/d)^2 - 12*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*tan(b*c/d) + 12*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)*tan(b*c/d) - 24*sin_integra
l(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)*tan(b*c/d) - 12*imag_part(cos_integr
al(2*b*x + 2*b*c/d))*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) + 12*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - 24*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - imag_part(cos_integ
ral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(b*c/d)^2 + 3*imag_part(cos_integral(2*
b*x + 2*b*c/d))*tan(3*a)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(3*a)^2*tan(b*c/d)^2 + imag_part(cos_integral(-6*b*x - 6*b*c/
d))*tan(3*a)^2*tan(b*c/d)^2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*
tan(b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(b*c/d)^2 +
imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 3*imag_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 3*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_part(cos_integral(-6
*b*x - 6*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*
tan(a)^2*tan(b*c/d)^2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/
d)^2 + 4*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(3*b*c/d)*tan
(b*c/d)^2 - 4*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(3*b*c/
d)*tan(b*c/d)^2 + 8*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)*tan(3*b*c/d)*t
an(b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*c/d)^2*tan(b
*c/d)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*c/d)^2*tan(b*c
/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*c/d)^2*tan(b*c/
d)^2 + imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*c/d)^2*tan(b*c/d)^
2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*sin_i
ntegral(2*(b*d*x + b*c)/d)*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*real_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a) - 6*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(3*a)^2*tan(a) + 2*real_part(cos_integral(6*b*x + 6*b*c/d)
)*tan(3*a)*tan(a)^2 + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*
tan(a)^2 + 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(3*b*c/
d) + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d) -
2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(a)^2*tan(3*b*c/d) - 2*real_p
art(cos_integral(-6*b*x - 6*b*c/d))*tan(a)^2*tan(3*b*c/d) - 2*real_part(cos
_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(3*b*c/d)^2 - 2*real_part(cos_integ
```

```
ral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(3*b*c/d)^2 - 6*real_part(cos_integral(2
*b*x + 2*b*c/d))*tan(a)*tan(3*b*c/d)^2 - 6*real_part(cos_integral(-2*b*x -
2*b*c/d))*tan(a)*tan(3*b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(3*a)^2*tan(b*c/d) + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3
*a)^2*tan(b*c/d) - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(
b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) +
6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*c/d)^2*tan(b*c/d) + 6*re
al_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*c/d)^2*tan(b*c/d) + 2*real_
part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(b*c/d)^2 + 2*real_part(cos
_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(b*c/d)^2 + 6*real_part(cos_integr
al(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*real_part(cos_integral(6*b*x + 6*b*c/d)
)*tan(3*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*t
an(3*b*c/d)*tan(b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a
)^2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2 + 3*imag_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2 + imag_part(cos_integral(-6*b*x -
 6*b*c/d))*tan(3*a)^2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2 - 6*si
n_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2 + imag_part(cos_integral(6*b*x + 6
*b*c/d))*tan(a)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 3
*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - imag_part(cos_integra
l(-6*b*x - 6*b*c/d))*tan(a)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(a)^2
+ 6*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + 4*imag_part(cos_integral(6*b
*x + 6*b*c/d))*tan(3*a)*tan(3*b*c/d) - 4*imag_part(cos_integral(-6*b*x - 6*
b*c/d))*tan(3*a)*tan(3*b*c/d) + 8*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)*
tan(3*b*c/d) - imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*c/d)^2 - 3*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*c/d)^2 + 3*imag_part(cos_i
ntegral(-2*b*x - 2*b*c/d))*tan(3*b*c/d)^2 + imag_part(cos_integral(-6*b*x -
 6*b*c/d))*tan(3*b*c/d)^2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*b*c/d)^
2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*b*c/d)^2 - 12*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 12*imag_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(a)*tan(b*c/d) - 24*sin_integral(2*(b*d*x + b*c)/d)*tan(a)
*tan(b*c/d) + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(b*c/d)^2 + 3*ima
g_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 3*imag_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/
d))*tan(b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 6*sin_i
ntegral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(6*b*x +
6*b*c/d))*tan(3*a) + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a) -
 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) - 6*real_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(a) - 2*real_part(cos_integral(6*b*x + 6*b*c/d))*t
an(3*b*c/d) - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*c/d) + 6*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 6*real_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_integral(6*b*x + 6*b*c/d)
) - 3*imag_part(cos_integral(2*b*x + 2*b*c/d)) + 3*imag_part(cos_integral(-
2*b*x - 2*b*c/d)) - imag_part(cos_integral(-6*b*x - 6*b*c/d)) + 2*sin_integ
ral(6*(b*d*x + b*c)/d) - 6*sin_integral(2*(b*d*x + b*c)/d))/(d*tan(3*a)^2*t
an(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + d*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2
+ d*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + d*tan(3*a)^2*tan(3*b*c/d)^2*tan(b*c/
d)^2 + d*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + d*tan(3*a)^2*tan(a)^2 + d*t
an(3*a)^2*tan(3*b*c/d)^2 + d*tan(a)^2*tan(3*b*c/d)^2 + d*tan(3*a)^2*tan(b*c
/d)^2 + d*tan(a)^2*tan(b*c/d)^2 + d*tan(3*b*c/d)^2*tan(b*c/d)^2 + d*tan(3*a
)^2 + d*tan(a)^2 + d*tan(3*b*c/d)^2 + d*tan(b*c/d)^2 + d)
```

**maple [A]** time = 0.02, size = 178, normalized size = 1.38

$$\frac{-\frac{b\left(\frac{6\,\mathrm{Si}\left(6bx+6a+\frac{-6da+6cb}{d}\right)\cos\left(\frac{-6da+6cb}{d}\right)}{d}-\frac{6\,\mathrm{Ci}\left(6bx+6a+\frac{-6da+6cb}{d}\right)\sin\left(\frac{-6da+6cb}{d}\right)}{d}\right)}{192}+\frac{3b\left(\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-\cdots}{}\right.}{d}\right.}{64}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/b*(-1/192*b*(6*Si(6*b*x+6*a+6*(-a*d+b*c)/d)*cos(6*(-a*d+b*c)/d)/d-6*Ci(6*
b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d)+3/64*b*(2*Si(2*b*x+2*a+2*(-a
*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d
+b*c)/d)/d))
```

**maxima [C]** time = 0.72, size = 274, normalized size = 2.12

$$\frac{b\left(-3i\,E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+3i\,E_1\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(i\,E_1\left(\frac{6i\,bc+6i\,(bx+a)d-6i\,ad}{d}\right)-i\,E_1\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/64*(b*(-3*I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) +
3*I*exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*
c - a*d)/d) + b*(I*exp_integral_e(1, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/
d) - I*exp_integral_e(1, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*cos(-6*
(b*c - a*d)/d) - 3*b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*
d)/d) + exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2
*(b*c - a*d)/d) + b*(exp_integral_e(1, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d
)/d) + exp_integral_e(1, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*sin(-6*
(b*c - a*d)/d))/(b*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x), x)
```

# 3.160 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=179

$$\frac{3b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{16d^2}-\frac{3b\cos\left(6a-\frac{6bc}{d}\right)\text{Ci}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{16d^2}+\frac{3b\sin\left(6a-\right.}{}$$

```
[Out] -3/16*b*Ci(6*b*c/d+6*b*x)*cos(6*a-6*b*c/d)/d^2+3/16*b*Ci(2*b*c/d+2*b*x)*cos
(2*a-2*b*c/d)/d^2+3/16*b*Si(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d^2-3/16*b*Si(2
*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^2-3/32*sin(2*b*x+2*a)/d/(d*x+c)+1/32*sin(6
*b*x+6*a)/d/(d*x+c)
```

**Rubi [A]** time = 0.30, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{3b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^2}-\frac{3b\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (3*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) - (3*b*C
os[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(16*d^2) - (3*Sin[2*a +
 2*b*x])/(32*d*(c + d*x)) + Sin[6*a + 6*b*x]/(32*d*(c + d*x)) - (3*b*Sin[2*
a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) + (3*b*Sin[6*a - (6
*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(16*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^2} - \frac{\sin(6a+6bx)}{32(c+dx)^2}\right) dx$$
$$= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{(c+dx)^2}\,dx\right) + \frac{3}{32}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx$$
$$= -\frac{3\sin(2a+2bx)}{32d(c+dx)} + \frac{\sin(6a+6bx)}{32d(c+dx)} + \frac{(3b)\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{16d} - \frac{(3b)\int \frac{\cos(6a+6b}{c+dx}}{16d}$$
$$= -\frac{3\sin(2a+2bx)}{32d(c+dx)} + \frac{\sin(6a+6bx)}{32d(c+dx)} - \frac{\left(3b\cos\left(6a-\frac{6bc}{d}\right)\right)\int \frac{\cos\left(\frac{6bc}{d}+6bx\right)}{c+dx}\,dx}{16d} +$$
$$= \frac{3b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{16d^2} - \frac{3b\cos\left(6a-\frac{6bc}{d}\right)\text{Ci}\left(\frac{6bc}{d}+6bx\right)}{16d^2} - \frac{3\sin}{32d}$$

**Mathematica [A]** time = 0.96, size = 189, normalized size = 1.06

$$6b(c+dx)\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right) - 6b(c+dx)\cos\left(6a-\frac{6bc}{d}\right)\text{Ci}\left(\frac{6b(c+dx)}{d}\right) - 6b(c+dx)\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (6*b*(c + d*x)*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 6*b*(c
 + d*x)*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*(c + d*x))/d] - 3*d*Cos[2*b*x
]*Sin[2*a] + d*Cos[6*b*x]*Sin[6*a] - 3*d*Cos[2*a]*Sin[2*b*x] + d*Cos[6*a]*S
in[6*b*x] - 6*b*(c + d*x)*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/
d] + 6*b*(c + d*x)*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d])/(32
*d^2*(c + d*x))
```

**fricas [A]** time = 0.48, size = 248, normalized size = 1.39

$$6(bdx+bc)\sin\left(-\frac{6(bc-ad)}{d}\right)\text{Si}\left(\frac{6(bdx+bc)}{d}\right) - 6(bdx+bc)\sin\left(-\frac{2(bc-ad)}{d}\right)\text{Si}\left(\frac{2(bdx+bc)}{d}\right) + 3\left((bdx+bc)\text{Ci}\left(\frac{2(bdx+}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(6*(b*d*x + b*c)*sin(-6*(b*c - a*d)/d)*sin_integral(6*(b*d*x + b*c)/d)
 - 6*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) +
3*((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integr
al(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) - 3*((b*d*x + b*c)*cos_integr
al(6*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-6*(b*d*x + b*c)/d))*cos
(-6*(b*c - a*d)/d) + 32*(d*cos(b*x + a)^5 - d*cos(b*x + a)^3)*sin(b*x + a))
/(d^3*x + c*d^2)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.02, size = 256, normalized size = 1.43

$$\frac{-\frac{b^{2}\left(-\frac{6\sin(6bx+6a)}{((bx+a)d-da+cb)d}+\frac{\frac{36\,\mathrm{Si}\left(6bx+6a+\frac{-6da+6cb}{d}\right)\sin\left(\frac{-6da+6cb}{d}\right)}{d}+\frac{36\,\mathrm{Ci}\left(6bx+6a+\frac{-6da+6cb}{d}\right)\cos\left(\frac{-6da+6cb}{d}\right)}{d}}{d}\right)}{192}+\frac{3b^{2}\left(-\frac{2\sin(2bx+2a)}{((bx+a)d-da+cb)d}+\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)}{d}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/b*(-1/192*b^2*(-6*sin(6*b*x+6*a)/((b*x+a)*d-d*a+c*b)/d+6*(6*Si(6*b*x+6*a+
6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d+6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*cos(6*
(-a*d+b*c)/d)/d)/d)+3/64*b^2*(-2*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*
Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b
*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d))
```

**maxima [C]** time = 0.48, size = 301, normalized size = 1.68

$$\frac{b^{2}\left(-3i\,E_{2}\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+3i\,E_{2}\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^{2}\left(i\,E_{2}\left(\frac{6i\,bc+6i\,(bx+a)d-6i\,ad}{d}\right)-i\,E_{2}\left(\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(b^2*(-3*I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 3*I*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^2*(I*exp_integral_e(2, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a
*d)/d) - I*exp_integral_e(2, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*cos
(-6*(b*c - a*d)/d) - 3*b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^2*(exp_integral_e(2, (6*I*b*c + 6*I*(b*x + a)*d -
 6*I*a*d)/d) + exp_integral_e(2, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))
*sin(-6*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3}{{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{3}{\left(a + bx \right)} \cos^{3}{\left(a + bx \right)}}{\left(c + dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x)**2, x)
```

## 3.161 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=235

$$\frac{9b^2\sin\left(6a-\frac{6bc}{d}\right)\text{Ci}\left(\frac{6bc}{d}+6bx\right)}{16d^3}-\frac{3b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{16d^3}-\frac{3b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{16d^3}+\frac{9b^2\text{cc}}{}$$

```
[Out] -3/32*b*cos(2*b*x+2*a)/d^2/(d*x+c)+3/32*b*cos(6*b*x+6*a)/d^2/(d*x+c)-3/16*b
^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3+9/16*b^2*cos(6*a-6*b*c/d)*Si(6*b*
c/d+6*b*x)/d^3+9/16*b^2*Ci(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d^3-3/16*b^2*Ci(
2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3-3/64*sin(2*b*x+2*a)/d/(d*x+c)^2+1/64*si
n(6*b*x+6*a)/d/(d*x+c)^2
```

**Rubi** [A] time = 0.35, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{9b^2\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^3}-\frac{3b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^3}-\frac{3b^2\cos\left(2a-\frac{2bc}{d}\right)\text{S}}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (-3*b*Cos[2*a + 2*b*x])/(32*d^2*(c + d*x)) + (3*b*Cos[6*a + 6*b*x])/(32*d^2
*(c + d*x)) + (9*b^2*CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])/(
16*d^3) - (3*b^2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(16*d
^3) - (3*Sin[2*a + 2*b*x])/(64*d*(c + d*x)^2) + Sin[6*a + 6*b*x]/(64*d*(c +
 d*x)^2) - (3*b^2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(16*
d^3) + (9*b^2*Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(16*d^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^3} - \frac{\sin(6a+6bx)}{32(c+dx)^3}\right)dx$$

$$= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{(c+dx)^3}\,dx\right) + \frac{3}{32}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx$$

$$= -\frac{3\sin(2a+2bx)}{64d(c+dx)^2} + \frac{\sin(6a+6bx)}{64d(c+dx)^2} + \frac{(3b)\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{32d} - \frac{(3b)\int \frac{\cos(6a+6bx)}{(c+dx)^2}\,d}{32d}$$

$$= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)} + \frac{3b\cos(6a+6bx)}{32d^2(c+dx)} - \frac{3\sin(2a+2bx)}{64d(c+dx)^2} + \frac{\sin(6a+6bx)}{64d(c+dx)^2} - $$

$$= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)} + \frac{3b\cos(6a+6bx)}{32d^2(c+dx)} - \frac{3\sin(2a+2bx)}{64d(c+dx)^2} + \frac{\sin(6a+6bx)}{64d(c+dx)^2} + $$

$$= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)} + \frac{3b\cos(6a+6bx)}{32d^2(c+dx)} + \frac{9b^2\operatorname{Ci}\left(\frac{6bc}{d}+6bx\right)\sin\left(6a-\frac{6bc}{d}\right)}{16d^3} - $$

**Mathematica** [A] time = 1.04, size = 239, normalized size = 1.02

$$6b^2(c+dx)^2\left(6\sin\left(6a-\frac{6bc}{d}\right)\operatorname{Ci}\left(\frac{6b(c+dx)}{d}\right) - 2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2b(c+dx)}{d}\right) - 2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right) + 6\cos\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (-3*d*Cos[2*b*x]*(2*b*(c + d*x)*Cos[2*a] + d*Sin[2*a]) + d*Cos[6*b*x]*(6*b*
(c + d*x)*Cos[6*a] + d*Sin[6*a]) + 3*d*(-(d*Cos[2*a]) + 2*b*(c + d*x)*Sin[2
*a])*Sin[2*b*x] + d*(d*Cos[6*a] - 6*b*(c + d*x)*Sin[6*a])*Sin[6*b*x] + 6*b^
2*(c + d*x)^2*(6*CosIntegral[(6*b*(c + d*x))/d]*Sin[6*a - (6*b*c)/d] - 2*Co
sIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 2*Cos[2*a - (2*b*c)/d]*
SinIntegral[(2*b*(c + d*x))/d] + 6*Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c
 + d*x))/d]))/(64*d^3*(c + d*x)^2)
```

**fricas** [A] time = 0.54, size = 434, normalized size = 1.85

$$96\left(bd^2x+bcd\right)\cos\left(bx+a\right)^6 - 144\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4 + 48\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2 + 18\left(b^2d^2x^2+2\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(96*(b*d^2*x + b*c*d)*cos(b*x + a)^6 - 144*(b*d^2*x + b*c*d)*cos(b*x +
 a)^4 + 48*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*cos(-6*(b*c - a*d)/d)*sin_integral(6*(b*d*x + b*c)/d) - 6*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*
x + b*c)/d) + 16*(d^2*cos(b*x + a)^5 - d^2*cos(b*x + a)^3)*sin(b*x + a) - 3
```

```
*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)/d) + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d))*sin(
-2*(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(6
*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-6*(
b*d*x + b*c)/d))*sin(-6*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.02, size = 329, normalized size = 1.40

$$-\frac{b^3\left(-\frac{3\sin(6bx+6a)}{((bx+a)d-da+cb)^2 d}+\frac{-\frac{18\cos(6bx+6a)}{((bx+a)d-da+cb)d}-\frac{18\left(\frac{6\,\mathrm{Si}\left(6bx+6a+\frac{-6da+6cb}{d}\right)\cos\left(\frac{-6da+6cb}{d}\right)}{d}-\frac{6\,\mathrm{Ci}\left(6bx+6a+\frac{-6da+6cb}{d}\right)\sin\left(\frac{-6da+6cb}{d}\right)}{d}\right)}{d}}{d}\right)}{192}+\frac{3b^3\left(-\frac{\sin(2bx+2a)}{((bx+a)d-da+cb}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] 1/b*(-1/192*b^3*(-3*sin(6*b*x+6*a)/((b*x+a)*d-d*a+c*b)^2/d+3*(-6*cos(6*b*x+
6*a)/((b*x+a)*d-d*a+c*b)/d-6*(6*Si(6*b*x+6*a+6*(-a*d+b*c)/d)*cos(6*(-a*d+b*
c)/d)/d-6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d)/d)/d)+3/64*b^
3*(-sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a)*d-d*
a+c*b)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x
+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d))
```

**maxima [C]** time = 0.69, size = 336, normalized size = 1.43

$$\frac{b^3\left(-3i\,E_3\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+3i\,E_3\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^3\left(i\,E_3\left(\frac{6i\,bc+6i\,(bx+a)d-6i\,ad}{d}\right)-i\,E\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(b^3*(-3*I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 3*I*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^3*(I*exp_integral_e(3, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a
*d)/d) - I*exp_integral_e(3, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*cos
(-6*(b*c - a*d)/d) - 3*b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^3*(exp_integral_e(3, (6*I*b*c + 6*I*(b*x + a)*d -
 6*I*a*d)/d) + exp_integral_e(3, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))
*sin(-6*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^3, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Timed out
```

# 3.162 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

**Optimal**. Leaf size=287

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{8d^4}+\frac{9b^3\cos\left(6a-\frac{6bc}{d}\right)\text{Ci}\left(\frac{6bc}{d}+6bx\right)}{8d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{8d^4}-\frac{9b^3\sin}{}$$

```
[Out] 9/8*b^3*Ci(6*b*c/d+6*b*x)*cos(6*a-6*b*c/d)/d^4-1/8*b^3*Ci(2*b*c/d+2*b*x)*co
s(2*a-2*b*c/d)/d^4-1/32*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2+1/32*b*cos(6*b*x+6*a
)/d^2/(d*x+c)^2-9/8*b^3*Si(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d^4+1/8*b^3*Si(2
*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/32*sin(2*b*x+2*a)/d/(d*x+c)^3+1/16*b^2
*sin(2*b*x+2*a)/d^3/(d*x+c)+1/96*sin(6*b*x+6*a)/d/(d*x+c)^3-3/16*b^2*sin(6*
b*x+6*a)/d^3/(d*x+c)
```

**Rubi [A]** time = 0.42, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{8d^4}+\frac{9b^3\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{8d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}}{8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(32*d^2*(c + d*x)^2) + (b*Cos[6*a + 6*b*x])/(32*d^2*(
c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(8*
d^4) + (9*b^3*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(8*d^4)
- Sin[2*a + 2*b*x]/(32*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(16*d^3*(c +
 d*x)) + Sin[6*a + 6*b*x]/(96*d*(c + d*x)^3) - (3*b^2*Sin[6*a + 6*b*x])/(16
*d^3*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])
/(8*d^4) - (9*b^3*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(8*d
^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^4} - \frac{\sin(6a+6bx)}{32(c+dx)^4}\right) dx$$

$$= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{(c+dx)^4}\,dx\right) + \frac{3}{32}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{\sin(6a+6bx)}{96d(c+dx)^3} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{16d} - \frac{b\int \frac{\cos(6a+6bx)}{(c+dx)^3}\,dx}{16d}$$

$$= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{\sin(6a+6bx)}{96d(c+dx)^3} - \frac{b^2\int}{}$$

$$= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{16d^3(c+dx)} + \frac{\sin}{9}$$

$$= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{16d^3(c+dx)} + \frac{\sin}{9}$$

$$= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{8d^4} + \frac{9b^3}{}$$

**Mathematica [A]** time = 4.94, size = 554, normalized size = 1.93

$$\frac{12b^3c^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) - 108b^3c^3\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6b(c+dx)}{d}\right) + 36b^3c^2dx\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) - 3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] (-3*b*c*d^2*Cos[2*(a + b*x)] - 3*b*d^3*x*Cos[2*(a + b*x)] + 3*b*c*d^2*Cos[6
*(a + b*x)] + 3*b*d^3*x*Cos[6*(a + b*x)] - 12*b^3*(c + d*x)^3*Cos[2*a - (2*
b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + 108*b^3*(c + d*x)^3*Cos[6*a - (6*b
*c)/d]*CosIntegral[(6*b*(c + d*x))/d] + 6*b^2*c^2*d*Sin[2*(a + b*x)] - 3*d^
3*Sin[2*(a + b*x)] + 12*b^2*c*d^2*x*Sin[2*(a + b*x)] + 6*b^2*d^3*x^2*Sin[2*
(a + b*x)] - 18*b^2*c^2*d*Sin[6*(a + b*x)] + d^3*Sin[6*(a + b*x)] - 36*b^2*
c*d^2*x*Sin[6*(a + b*x)] - 18*b^2*d^3*x^2*Sin[6*(a + b*x)] + 12*b^3*c^3*Sin
[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 36*b^3*c^2*d*x*Sin[2*a -
 (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 36*b^3*c*d^2*x^2*Sin[2*a - (2*
b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 12*b^3*d^3*x^3*Sin[2*a - (2*b*c)/d
]*SinIntegral[(2*b*(c + d*x))/d] - 108*b^3*c^3*Sin[6*a - (6*b*c)/d]*SinInte
gral[(6*b*(c + d*x))/d] - 324*b^3*c^2*d*x*Sin[6*a - (6*b*c)/d]*SinIntegral[
(6*b*(c + d*x))/d] - 324*b^3*c*d^2*x^2*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*
b*(c + d*x))/d] - 108*b^3*d^3*x^3*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c
+ d*x))/d])/(96*d^4*(c + d*x)^3)
```

**fricas [B]** time = 0.56, size = 638, normalized size = 2.22

$$\frac{48\left(bd^3x+bcd^2\right)\cos(bx+a)^6 - 72\left(bd^3x+bcd^2\right)\cos(bx+a)^4 + 24\left(bd^3x+bcd^2\right)\cos(bx+a)^2 - 54\left(b^3d^3x^3+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(48*(b*d^3*x + b*c*d^2)*cos(b*x + a)^6 - 72*(b*d^3*x + b*c*d^2)*cos(b*
x + a)^4 + 24*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 - 54*(b^3*d^3*x^3 + 3*b^3*
c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-6*(b*c - a*d)/d)*sin_integral(6*(
b*d*x + b*c)/d) + 6*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^
3)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 3*((b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d
) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(
-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) + 27*((b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(6*(b*d*x + b*c)/d) + (b^3*d^3*
x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-6*(b*d*x + b
*c)/d))*cos(-6*(b*c - a*d)/d) - 16*((18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b
^2*c^2*d - d^3)*cos(b*x + a)^5 - (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*
c^2*d - d^3)*cos(b*x + a)^3 + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*c
os(b*x + a))*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.02, size = 404, normalized size = 1.41

$$\frac{\frac{3b^4\left(-\frac{2\sin(2bx+2a)}{3((bx+a)d-da+cb)^3 d}+\frac{-\frac{2\cos(2bx+2a)}{3((bx+a)d-da+cb)^2 d}-\frac{2\left(-\frac{2\sin(2bx+2a)}{((bx+a)d-da+cb)d}+\frac{\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}}{d}\right)}{3d}}{d}\right)}{64}-}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x)
```

```
[Out] 1/b*(3/64*b^4*(-2/3*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^3/d+2/3*(-cos(2*b*x+
2*a)/((b*x+a)*d-d*a+c*b)^2/d-(-2*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*
Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b
*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d)-1/192*b^4*(-2*sin(6*b*x+6*a)/((b*x+a
)*d-d*a+c*b)^3/d+2*(-3*cos(6*b*x+6*a)/((b*x+a)*d-d*a+c*b)^2/d-3*(-6*sin(6*b
*x+6*a)/((b*x+a)*d-d*a+c*b)/d+6*(6*Si(6*b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d
+b*c)/d)/d+6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*cos(6*(-a*d+b*c)/d)/d)/d)/d)/d))
```

**maxima [C]** time = 0.86, size = 386, normalized size = 1.34

$$\frac{b^4\left(-3i\,E_4\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+3i\,E_4\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^4\left(i\,E_4\left(\frac{6i\,bc+6i\,(bx+a)d-6i\,ad}{d}\right)-i\,E\right.}{64\left(b^3c^3d-3\,ab^2c^2d^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*(b^4*(-3*I*exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 3*I*exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^4*(I*exp_integral_e(4, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a
*d)/d) - I*exp_integral_e(4, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*cos
(-6*(b*c - a*d)/d) - 3*b^4*(exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^4*(exp_integral_e(4, (6*I*b*c + 6*I*(b*x + a)*d -
 6*I*a*d)/d) + exp_integral_e(4, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))
*sin(-6*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3}{{\left(c+d\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^4, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**4,x)
```

```
[Out] Timed out
```

# 3.163 $\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$

**Optimal**. Leaf size=152

$$\text{Int}\left(\cot(a+bx)(c+dx)^m,x\right)+\frac{2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+d$$

```
[Out] 2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b*
(d*x+c)/d)^m)+2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b*
c/d))/((I*b*(d*x+c)/d)^m)+Unintegrable((d*x+c)^m*cot(b*x+a),x)
```

**Rubi** [A] time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c +
 d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-3 - m)*(c + d*x)^m*Gamma[1 +
 m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^
m) + Defer[Int][(c + d*x)^m*Cot[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx &= \int (c+dx)^m \cot(a+bx)\,dx-\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx\\ &= \int (c+dx)^m \cot(a+bx)\,dx-\int \frac{1}{2}(c+dx)^m \sin(2a+2bx)\,dx\\ &= -\left(\frac{1}{2}\int (c+dx)^m \sin(2a+2bx)\,dx\right)+\int (c+dx)^m \cot(a+bx)\,dx\\ &= -\left(\frac{1}{4}i\int e^{-i(2a+2bx)}(c+dx)^m\,dx\right)+\frac{1}{4}i\int e^{i(2a+2bx)}(c+dx)^m\,dx+\int (\\ &= \frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-3-m}e}{}\end{aligned}$$

**Mathematica** [A] time = 7.77, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x], x]
```

**fricas** [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m\cos(bx+a)^2\cot(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^2 \cot(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

**maple** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^2(bx+a)\right) \cot(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)
[Out] int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^2 \cot(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)^2 \cot(a+b\,x)\ (c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^m,x)
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^m, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos^2(a+bx) \cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*cot(b*x+a),x)
[Out] Integral((c + d*x)**m*cos(a + b*x)**2*cot(a + b*x), x)
```

# 3.164 $\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx$

**Optimal**. Leaf size=307

$$-\frac{3d^4\text{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}-\frac{3d^4\sin^2(a+bx)}{4b^5}+\frac{3id^3(c+dx)\text{Li}_4\left(e^{2i(a+bx)}\right)}{b^4}+\frac{3d^3(c+dx)\sin(a+bx)\cos(a+bx)}{2b^4}+\frac{3d^2(c\cdots}{\cdots}$$

```
[Out] -3/2*c*d^3*x/b^3-3/4*d^4*x^2/b^3+1/4*(d*x+c)^4/b-1/5*I*(d*x+c)^5/d+(d*x+c)^
4*ln(1-exp(2*I*(b*x+a)))/b-2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2+
3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b^3+3*I*d^3*(d*x+c)*polylog(4,e
xp(2*I*(b*x+a)))/b^4-3/2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5+3/2*d^3*(d*x+c
)*cos(b*x+a)*sin(b*x+a)/b^4-d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2-3/4*d^4*s
in(b*x+a)^2/b^5+3/2*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3-1/2*(d*x+c)^4*sin(b*x+a)
^2/b
```

**Rubi** [A] time = 0.34, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4408, 4404, 3311, 32, 3310, 3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{3d\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (-3*c*d^3*x)/(2*b^3) - (3*d^4*x^2)/(4*b^3) + (c + d*x)^4/(4*b) - ((I/5)*(c
+ d*x)^5)/d + ((c + d*x)^4*Log[1 - E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c +
d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*PolyLog[3,
 E^((2*I)*(a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*(a +
b*x))])/b^4 - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5) + (3*d^3*(c +
 d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b^4) - (d*(c + d*x)^3*Cos[a + b*x]*Sin[
a + b*x])/b^2 - (3*d^4*Sin[a + b*x]^2)/(4*b^5) + (3*d^2*(c + d*x)^2*Sin[a +
 b*x]^2)/(2*b^3) - ((c + d*x)^4*Sin[a + b*x]^2)/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\cos^2(a+bx)\cot(a+bx)\,dx &= \int (c+dx)^4\cot(a+bx)\,dx-\int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx\\ &= -\frac{i(c+dx)^5}{5d}-\frac{(c+dx)^4\sin^2(a+bx)}{2b}-2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1-e^{2i(a+bx)}}\,dx+\cdots\\ &= -\frac{i(c+dx)^5}{5d}+\frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{d(c+dx)^3\cos(a+bx)\,\mathrm{s}\cdots}{b^2}\\ &= \frac{(c+dx)^4}{4b}-\frac{i(c+dx)^5}{5d}+\frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{2id(c+dx)^3\cdots}{b}\\ &= -\frac{3cd^3x}{2b^3}-\frac{3d^4x^2}{4b^3}+\frac{(c+dx)^4}{4b}-\frac{i(c+dx)^5}{5d}+\frac{(c+dx)^4\log\left(1-e^{2i(a+\cdots}\right.}{b}\\ &= -\frac{3cd^3x}{2b^3}-\frac{3d^4x^2}{4b^3}+\frac{(c+dx)^4}{4b}-\frac{i(c+dx)^5}{5d}+\frac{(c+dx)^4\log\left(1-e^{2i(a+\cdots}\right.}{b}\\ &= -\frac{3cd^3x}{2b^3}-\frac{3d^4x^2}{4b^3}+\frac{(c+dx)^4}{4b}-\frac{i(c+dx)^5}{5d}+\frac{(c+dx)^4\log\left(1-e^{2i(a+\cdots}\right.}{b}\\ &= -\frac{3cd^3x}{2b^3}-\frac{3d^4x^2}{4b^3}+\frac{(c+dx)^4}{4b}-\frac{i(c+dx)^5}{5d}+\frac{(c+dx)^4\log\left(1-e^{2i(a+\cdots}\right.}{b}\end{aligned}$$

**Mathematica [B]** time = 6.52, size = 2828, normalized size = 9.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] -((c^2*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*
I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*L
og[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)
*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E
^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a +
 b*x))]))/E^((2*I)*a)))/b^3) - (c*d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a)
 + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3
*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))
*(b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(
a + b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^(
(2*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^
((-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^4 -
(d^4*E^(I*a)*Csc[a]*((2*b^5*x^5)/E^((2*I)*a) + (5*I)*b^4*(1 - E^((-2*I)*a))
*x^4*Log[1 - E^((-I)*(a + b*x))] + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1 +
 E^((-I)*(a + b*x))] - (20*(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, -E^((-I)*
(a + b*x))] - (3*I)*b^2*x^2*PolyLog[3, -E^((-I)*(a + b*x))] - 6*b*x*PolyLog
[4, -E^((-I)*(a + b*x))] + (6*I)*PolyLog[5, -E^((-I)*(a + b*x))]))/E^((2*I)
*a) - (20*(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, E^((-I)*(a + b*x))] - (3*I
)*b^2*x^2*PolyLog[3, E^((-I)*(a + b*x))] - 6*b*x*PolyLog[4, E^((-I)*(a + b*
x))] + (6*I)*PolyLog[5, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(10*b^5) + (c^4
*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b
*(Cos[a]^2 + Sin[a]^2)) + Csc[a]*(Cos[2*a + 2*b*x]/(160*b^5) - ((I/160)*Sin
[2*a + 2*b*x])/b^5)*(80*b^5*c^4*x*Cos[a + 2*b*x] + 160*b^5*c^3*d*x^2*Cos[a
+ 2*b*x] + 160*b^5*c^2*d^2*x^3*Cos[a + 2*b*x] + 80*b^5*c*d^3*x^4*Cos[a + 2*
b*x] + 16*b^5*d^4*x^5*Cos[a + 2*b*x] + 80*b^5*c^4*x*Cos[3*a + 2*b*x] + 160*
b^5*c^3*d*x^2*Cos[3*a + 2*b*x] + 160*b^5*c^2*d^2*x^3*Cos[3*a + 2*b*x] + 80*
b^5*c*d^3*x^4*Cos[3*a + 2*b*x] + 16*b^5*d^4*x^5*Cos[3*a + 2*b*x] + (10*I)*b
^4*c^4*Cos[3*a + 4*b*x] - 20*b^3*c^3*d*Cos[3*a + 4*b*x] - (30*I)*b^2*c^2*d^
2*Cos[3*a + 4*b*x] + 30*b*c*d^3*Cos[3*a + 4*b*x] + (15*I)*d^4*Cos[3*a + 4*b
```

```
*x] + (40*I)*b^4*c^3*d*x*Cos[3*a + 4*b*x] - 60*b^3*c^2*d^2*x*Cos[3*a + 4*b*
x] - (60*I)*b^2*c*d^3*x*Cos[3*a + 4*b*x] + 30*b*d^4*x*Cos[3*a + 4*b*x] + (6
0*I)*b^4*c^2*d^2*x^2*Cos[3*a + 4*b*x] - 60*b^3*c*d^3*x^2*Cos[3*a + 4*b*x] -
 (30*I)*b^2*d^4*x^2*Cos[3*a + 4*b*x] + (40*I)*b^4*c*d^3*x^3*Cos[3*a + 4*b*x
] - 20*b^3*d^4*x^3*Cos[3*a + 4*b*x] + (10*I)*b^4*d^4*x^4*Cos[3*a + 4*b*x] -
 (10*I)*b^4*c^4*Cos[5*a + 4*b*x] + 20*b^3*c^3*d*Cos[5*a + 4*b*x] + (30*I)*b
^2*c^2*d^2*Cos[5*a + 4*b*x] - 30*b*c*d^3*Cos[5*a + 4*b*x] - (15*I)*d^4*Cos[
5*a + 4*b*x] - (40*I)*b^4*c^3*d*x*Cos[5*a + 4*b*x] + 60*b^3*c^2*d^2*x*Cos[5
*a + 4*b*x] + (60*I)*b^2*c*d^3*x*Cos[5*a + 4*b*x] - 30*b*d^4*x*Cos[5*a + 4*
b*x] - (60*I)*b^4*c^2*d^2*x^2*Cos[5*a + 4*b*x] + 60*b^3*c*d^3*x^2*Cos[5*a +
 4*b*x] + (30*I)*b^2*d^4*x^2*Cos[5*a + 4*b*x] - (40*I)*b^4*c*d^3*x^3*Cos[5*
a + 4*b*x] + 20*b^3*d^4*x^3*Cos[5*a + 4*b*x] - (10*I)*b^4*d^4*x^4*Cos[5*a +
 4*b*x] + 20*b^4*c^4*Sin[a] - (40*I)*b^3*c^3*d*Sin[a] - 60*b^2*c^2*d^2*Sin[
a] + (60*I)*b*c*d^3*Sin[a] + 30*d^4*Sin[a] + 80*b^4*c^3*d*x*Sin[a] - (120*I
)*b^3*c^2*d^2*x*Sin[a] - 120*b^2*c*d^3*x*Sin[a] + (60*I)*b*d^4*x*Sin[a] + 1
20*b^4*c^2*d^2*x^2*Sin[a] - (120*I)*b^3*c*d^3*x^2*Sin[a] - 60*b^2*d^4*x^2*S
in[a] + 80*b^4*c*d^3*x^3*Sin[a] - (40*I)*b^3*d^4*x^3*Sin[a] + 20*b^4*d^4*x^
4*Sin[a] + (80*I)*b^5*c^4*x*Sin[a + 2*b*x] + (160*I)*b^5*c^3*d*x^2*Sin[a +
2*b*x] + (160*I)*b^5*c^2*d^2*x^3*Sin[a + 2*b*x] + (80*I)*b^5*c*d^3*x^4*Sin[
a + 2*b*x] + (16*I)*b^5*d^4*x^5*Sin[a + 2*b*x] + (80*I)*b^5*c^4*x*Sin[3*a +
 2*b*x] + (160*I)*b^5*c^3*d*x^2*Sin[3*a + 2*b*x] + (160*I)*b^5*c^2*d^2*x^3*
Sin[3*a + 2*b*x] + (80*I)*b^5*c*d^3*x^4*Sin[3*a + 2*b*x] + (16*I)*b^5*d^4*x
^5*Sin[3*a + 2*b*x] - 10*b^4*c^4*Sin[3*a + 4*b*x] - (20*I)*b^3*c^3*d*Sin[3*
a + 4*b*x] + 30*b^2*c^2*d^2*Sin[3*a + 4*b*x] + (30*I)*b*c*d^3*Sin[3*a + 4*b
*x] - 15*d^4*Sin[3*a + 4*b*x] - 40*b^4*c^3*d*x*Sin[3*a + 4*b*x] - (60*I)*b^
3*c^2*d^2*x*Sin[3*a + 4*b*x] + 60*b^2*c*d^3*x*Sin[3*a + 4*b*x] + (30*I)*b*d
^4*x*Sin[3*a + 4*b*x] - 60*b^4*c^2*d^2*x^2*Sin[3*a + 4*b*x] - (60*I)*b^3*c*
d^3*x^2*Sin[3*a + 4*b*x] + 30*b^2*d^4*x^2*Sin[3*a + 4*b*x] - 40*b^4*c*d^3*x
^3*Sin[3*a + 4*b*x] - (20*I)*b^3*d^4*x^3*Sin[3*a + 4*b*x] - 10*b^4*d^4*x^4*
Sin[3*a + 4*b*x] + 10*b^4*c^4*Sin[5*a + 4*b*x] + (20*I)*b^3*c^3*d*Sin[5*a +
 4*b*x] - 30*b^2*c^2*d^2*Sin[5*a + 4*b*x] - (30*I)*b*c*d^3*Sin[5*a + 4*b*x]
 + 15*d^4*Sin[5*a + 4*b*x] + 40*b^4*c^3*d*x*Sin[5*a + 4*b*x] + (60*I)*b^3*c
^2*d^2*x*Sin[5*a + 4*b*x] - 60*b^2*c*d^3*x*Sin[5*a + 4*b*x] - (30*I)*b*d^4*
x*Sin[5*a + 4*b*x] + 60*b^4*c^2*d^2*x^2*Sin[5*a + 4*b*x] + (60*I)*b^3*c*d^3
*x^2*Sin[5*a + 4*b*x] - 30*b^2*d^4*x^2*Sin[5*a + 4*b*x] + 40*b^4*c*d^3*x^3*
Sin[5*a + 4*b*x] + (20*I)*b^3*d^4*x^3*Sin[5*a + 4*b*x] + 10*b^4*d^4*x^4*Sin
[5*a + 4*b*x]) - (2*c^3*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I
*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTa
n[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2
*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x
 + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^2*Sqrt[Sec[a]^2*(Cos[
a]^2 + Sin[a]^2)])
```

**fricas [C]** time = 0.66, size = 1453, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 48*d^4*polylog(5, cos(b*x + a) + I*si
n(b*x + a)) + 48*d^4*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 48*d^4*pol
ylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 48*d^4*polylog(5, -cos(b*x + a) -
 I*sin(b*x + a)) + 3*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - (2*b^4*d^4*x^4 + 8*b^4
*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*d^4
)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 + 2*(2*b^3*d^4*x^3
+ 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*
cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3*d - 3*b^2*c*d^3)*x - (-8*I*b^3*d^4
*x^3 - 24*I*b^3*c*d^3*x^2 - 24*I*b^3*c^2*d^2*x - 8*I*b^3*c^3*d)*dilog(cos(b
*x + a) + I*sin(b*x + a)) - (8*I*b^3*d^4*x^3 + 24*I*b^3*c*d^3*x^2 + 24*I*b^
```

```
3*c^2*d^2*x + 8*I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (8*I*b^
3*d^4*x^3 + 24*I*b^3*c*d^3*x^2 + 24*I*b^3*c^2*d^2*x + 8*I*b^3*c^3*d)*dilog(
-cos(b*x + a) + I*sin(b*x + a)) - (-8*I*b^3*d^4*x^3 - 24*I*b^3*c*d^3*x^2 -
24*I*b^3*c^2*d^2*x - 8*I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) -
 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4
*c^4)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3
*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*si
n(b*x + a) + 1) - 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b^4
*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/
2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x
^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2
+ 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^4
*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^
3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*si
n(b*x + a) + 1) - (48*I*b*d^4*x + 48*I*b*c*d^3)*polylog(4, cos(b*x + a) + I
*sin(b*x + a)) - (-48*I*b*d^4*x - 48*I*b*c*d^3)*polylog(4, cos(b*x + a) - I
*sin(b*x + a)) - (-48*I*b*d^4*x - 48*I*b*c*d^3)*polylog(4, -cos(b*x + a) +
I*sin(b*x + a)) - (48*I*b*d^4*x + 48*I*b*c*d^3)*polylog(4, -cos(b*x + a) -
I*sin(b*x + a)) - 24*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3,
 cos(b*x + a) + I*sin(b*x + a)) - 24*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2
*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 24*(b^2*d^4*x^2 + 2*b^2*c
*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 24*(b^2*
d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x
 + a)))/b^5
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \cos(bx+a)^2 \cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)^2*cot(b*x + a), x)
```

**maple [B]** time = 0.54, size = 1326, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] 1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))+12/b^3*
c^2*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))
-1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^
2+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+8/5*I/b^5*d^4*a^5-I*c*d^3*x^4-2
*I*c^2*d^2*x^3-2*I*c^3*d*x^2-24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*p
olylog(5,exp(I*(b*x+a)))/b^5+1/8*(2*b^4*d^4*x^4+8*b^4*c*d^3*x^3+12*b^4*c^2*
d^2*x^2+8*b^4*c^3*d*x+2*b^4*c^4-6*b^2*d^4*x^2-12*b^2*c*d^3*x-6*b^2*c^2*d^2+
3*d^4)/b^5*cos(2*b*x+2*a)+I*c^4*x-2/b*c^4*ln(exp(I*(b*x+a)))+1/b*c^4*ln(exp
(I*(b*x+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)-1/5*I*d^4*x^5+4/b*c^3*d*ln(exp(
I*(b*x+a))+1)*x+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*
x+a)))*a+6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+24/b^3*c*d^3*polylog(3,-exp(I
*(b*x+a)))*x-6/b^3*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))+6/b*c^2*d^2*ln(1-exp(I*
(b*x+a)))*x^2+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x+24*I/b^4*c*d^3*polyl
og(4,-exp(I*(b*x+a)))+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+2*I/b^4*d^4*
a^4*x-4*I/b^2*c^3*d*a^2+8*I/b^3*c^2*d^2*a^3-6*I/b^4*c*d^3*a^4-4*I/b^2*d^4*p
olylog(2,exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b
^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)
```

```
))*x-4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,exp(I
*(b*x+a)))+8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a
))-1)+8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)
)-1)-12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))-4/b^2*c^3*d*a*ln(exp(I*(b*x+a))-
1)+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-8*I/b^
3*c*d^3*a^3*x+12*I/b^2*c^2*d^2*a^2*x-8*I/b*c^3*d*a*x-12*I/b^2*c*d^3*polylog
(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-12*I/
b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x
+a)))*x^2+4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+4/b*c*d^3*ln(1-exp(I*(b*x+a)))
*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3-1/4/b^4*d*(2*b^2*d^3*x^3+6*b^2*c*
d^2*x^2+6*b^2*c^2*d*x+2*b^2*c^3-3*d^3*x-3*c*d^2)*sin(2*b*x+2*a)
```

**maxima [B]** time = 1.06, size = 1635, normalized size = 5.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/40*(20*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^4 - 80*(sin(b*x + a)^2 -
 log(sin(b*x + a)^2))*a*c^3*d/b + 120*(sin(b*x + a)^2 - log(sin(b*x + a)^2)
)*a^2*c^2*d^2/b^2 - 80*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^3*c*d^3/b^3
 + 20*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^4*d^4/b^4 - (-8*I*(b*x + a)^
5*d^4 + (-40*I*b*c*d^3 + 40*I*a*d^4)*(b*x + a)^4 - 960*d^4*polylog(5, -e^(I
*b*x + I*a)) - 960*d^4*polylog(5, e^(I*b*x + I*a)) + (-80*I*b^2*c^2*d^2 + 1
60*I*a*b*c*d^3 - 80*I*a^2*d^4)*(b*x + a)^3 + (-80*I*b^3*c^3*d + 240*I*a*b^2
*c^2*d^2 - 240*I*a^2*b*c*d^3 + 80*I*a^3*d^4)*(b*x + a)^2 + (40*I*(b*x + a)^
4*d^4 + (160*I*b*c*d^3 - 160*I*a*d^4)*(b*x + a)^3 + (240*I*b^2*c^2*d^2 - 48
0*I*a*b*c*d^3 + 240*I*a^2*d^4)*(b*x + a)^2 + (160*I*b^3*c^3*d - 480*I*a*b^2
*c^2*d^2 + 480*I*a^2*b*c*d^3 - 160*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x +
a), cos(b*x + a) + 1) + (-40*I*(b*x + a)^4*d^4 + (-160*I*b*c*d^3 + 160*I*a*
d^4)*(b*x + a)^3 + (-240*I*b^2*c^2*d^2 + 480*I*a*b*c*d^3 - 240*I*a^2*d^4)*(
b*x + a)^2 + (-160*I*b^3*c^3*d + 480*I*a*b^2*c^2*d^2 - 480*I*a^2*b*c*d^3 +
160*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 5*(2*(
b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 3*(2*a^2 - 1)*d^4 + 8*(b*c*
d^3 - a*d^4)*(b*x + a)^3 + 6*(2*b^2*c^2*d^2 - 4*a*b*c*d^3 + (2*a^2 - 1)*d^4
)*(b*x + a)^2 + 4*(2*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 3*(2*a^2 - 1)*b*c*d^3 -
(2*a^3 - 3*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (-160*I*b^3*c^3*d + 480*I*
a*b^2*c^2*d^2 - 480*I*a^2*b*c*d^3 - 160*I*(b*x + a)^3*d^4 + 160*I*a^3*d^4 +
 (-480*I*b*c*d^3 + 480*I*a*d^4)*(b*x + a)^2 + (-480*I*b^2*c^2*d^2 + 960*I*a
*b*c*d^3 - 480*I*a^2*d^4)*(b*x + a))*dilog(-e^(I*b*x + I*a)) + (-160*I*b^3*
c^3*d + 480*I*a*b^2*c^2*d^2 - 480*I*a^2*b*c*d^3 - 160*I*(b*x + a)^3*d^4 + 1
60*I*a^3*d^4 + (-480*I*b*c*d^3 + 480*I*a*d^4)*(b*x + a)^2 + (-480*I*b^2*c^2
*d^2 + 960*I*a*b*c*d^3 - 480*I*a^2*d^4)*(b*x + a))*dilog(e^(I*b*x + I*a)) +
 20*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2
*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*
b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b
*x + a) + 1) + 20*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^
2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 - 2*cos(b*x + a) + 1) + (960*I*b*c*d^3 + 960*I*(b*x + a)*d^4 - 960*I*a
*d^4)*polylog(4, -e^(I*b*x + I*a)) + (960*I*b*c*d^3 + 960*I*(b*x + a)*d^4 -
 960*I*a*d^4)*polylog(4, e^(I*b*x + I*a)) + 480*(b^2*c^2*d^2 - 2*a*b*c*d^3
+ (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^
(I*b*x + I*a)) + 480*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4
 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, e^(I*b*x + I*a)) - 10*(2*b^3*c
^3*d - 6*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3*(2*a^2 - 1)*b*c*d^3 - (2*a^3
 - 3*a)*d^4 + 6*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(2*b^2*c^2*d^2 - 4*a*b*c*
d^3 + (2*a^2 - 1)*d^4)*(b*x + a))*sin(2*b*x + 2*a))/b^4)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\cot(a+b\,x)\;(c+d\,x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^4,x)
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4\cos^2(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*cot(b*x+a),x)
[Out] Integral((c + d*x)**4*cos(a + b*x)**2*cot(a + b*x), x)
```

# 3.165 $\int (c+dx)^3 \cos^2(a+bx) \cot(a+bx)\, dx$

**Optimal**. Leaf size=246

$$\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^3\sin(a+bx)\cos(a+bx)}{8b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}-\frac{3id(c+dx)}{}$$

```
[Out] -3/8*d^3*x/b^3+1/4*(d*x+c)^3/b-1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1-exp(2*I*(b*
x+a)))/b-3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c)*
polylog(3,exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4+3
/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4-3/4*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2+
3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3-1/2*(d*x+c)^3*sin(b*x+a)^2/b
```

**Rubi [A]** time = 0.28, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4408, 4404, 3311, 32, 2635, 8, 3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^2(c+dx)\sin}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (-3*d^3*x)/(8*b^3) + (c + d*x)^3/(4*b) - ((I/4)*(c + d*x)^4)/d + ((c + d*x)
^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^
((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])
/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^3*Cos
[a + b*x]*Sin[a + b*x])/(8*b^4) - (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x
])/(4*b^2) + (3*d^2*(c + d*x)*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^3*Sin[a
+ b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^3\cot(a+bx)\,dx - \int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1-e^{2i(a+bx)}}\,dx + \frac{(3d$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cos(a+bx)\sin}{4b^2}$$
$$= \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2}{2b^2}$$
$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c}{}$$
$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c}{}$$
$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c}{}$$

**Mathematica** [**B**] time = 6.41, size = 1918, normalized size = 7.80

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] -1/2*(c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2
*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*
Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I
)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 +
E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a
+ b*x))]))/E^((2*I)*a)))/b^3 - (d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) +
 (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(
1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(
b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a
+ b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2
*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^((
-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(4*b^4)
+ (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a
]))/(b*(Cos[a]^2 + Sin[a]^2)) + Csc[a]*(Cos[2*a + 2*b*x]/(64*b^4) - ((I/64)
*Sin[2*a + 2*b*x])/b^4)*(32*b^4*c^3*x*Cos[a + 2*b*x] + 48*b^4*c^2*d*x^2*Cos
[a + 2*b*x] + 32*b^4*c*d^2*x^3*Cos[a + 2*b*x] + 8*b^4*d^3*x^4*Cos[a + 2*b*x
] + 32*b^4*c^3*x*Cos[3*a + 2*b*x] + 48*b^4*c^2*d*x^2*Cos[3*a + 2*b*x] + 32*
b^4*c*d^2*x^3*Cos[3*a + 2*b*x] + 8*b^4*d^3*x^4*Cos[3*a + 2*b*x] + (4*I)*b^3
*c^3*Cos[3*a + 4*b*x] - 6*b^2*c^2*d*Cos[3*a + 4*b*x] - (6*I)*b*c*d^2*Cos[3*
a + 4*b*x] + 3*d^3*Cos[3*a + 4*b*x] + (12*I)*b^3*c^2*d*x*Cos[3*a + 4*b*x] -
 12*b^2*c*d^2*x*Cos[3*a + 4*b*x] - (6*I)*b*d^3*x*Cos[3*a + 4*b*x] + (12*I)*
b^3*c*d^2*x^2*Cos[3*a + 4*b*x] - 6*b^2*d^3*x^2*Cos[3*a + 4*b*x] + (4*I)*b^3
*d^3*x^3*Cos[3*a + 4*b*x] - (4*I)*b^3*c^3*Cos[5*a + 4*b*x] + 6*b^2*c^2*d*Co
s[5*a + 4*b*x] + (6*I)*b*c*d^2*Cos[5*a + 4*b*x] - 3*d^3*Cos[5*a + 4*b*x] -
(12*I)*b^3*c^2*d*x*Cos[5*a + 4*b*x] + 12*b^2*c*d^2*x*Cos[5*a + 4*b*x] + (6*
I)*b*d^3*x*Cos[5*a + 4*b*x] - (12*I)*b^3*c*d^2*x^2*Cos[5*a + 4*b*x] + 6*b^2
*d^3*x^2*Cos[5*a + 4*b*x] - (4*I)*b^3*d^3*x^3*Cos[5*a + 4*b*x] + 8*b^3*c^3*
Sin[a] - (12*I)*b^2*c^2*d*Sin[a] - 12*b*c*d^2*Sin[a] + (6*I)*d^3*Sin[a] + 2
4*b^3*c^2*d*x*Sin[a] - (24*I)*b^2*c*d^2*x*Sin[a] - 12*b*d^3*x*Sin[a] + 24*b
^3*c*d^2*x^2*Sin[a] - (12*I)*b^2*d^3*x^2*Sin[a] + 8*b^3*d^3*x^3*Sin[a] + (3
2*I)*b^4*c^3*x*Sin[a + 2*b*x] + (48*I)*b^4*c^2*d*x^2*Sin[a + 2*b*x] + (32*I
)*b^4*c*d^2*x^3*Sin[a + 2*b*x] + (8*I)*b^4*d^3*x^4*Sin[a + 2*b*x] + (32*I)*
```

```
b^4*c^3*x*Sin[3*a + 2*b*x] + (48*I)*b^4*c^2*d*x^2*Sin[3*a + 2*b*x] + (32*I)
*b^4*c*d^2*x^3*Sin[3*a + 2*b*x] + (8*I)*b^4*d^3*x^4*Sin[3*a + 2*b*x] - 4*b^
3*c^3*Sin[3*a + 4*b*x] - (6*I)*b^2*c^2*d*Sin[3*a + 4*b*x] + 6*b*c*d^2*Sin[3
*a + 4*b*x] + (3*I)*d^3*Sin[3*a + 4*b*x] - 12*b^3*c^2*d*x*Sin[3*a + 4*b*x]
- (12*I)*b^2*c*d^2*x*Sin[3*a + 4*b*x] + 6*b*d^3*x*Sin[3*a + 4*b*x] - 12*b^3
*c*d^2*x^2*Sin[3*a + 4*b*x] - (6*I)*b^2*d^3*x^2*Sin[3*a + 4*b*x] - 4*b^3*d^
3*x^3*Sin[3*a + 4*b*x] + 4*b^3*c^3*Sin[5*a + 4*b*x] + (6*I)*b^2*c^2*d*Sin[5
*a + 4*b*x] - 6*b*c*d^2*Sin[5*a + 4*b*x] - (3*I)*d^3*Sin[5*a + 4*b*x] + 12*
b^3*c^2*d*x*Sin[5*a + 4*b*x] + (12*I)*b^2*c*d^2*x*Sin[5*a + 4*b*x] - 6*b*d^
3*x*Sin[5*a + 4*b*x] + 12*b^3*c*d^2*x^2*Sin[5*a + 4*b*x] + (6*I)*b^2*d^3*x^
2*Sin[5*a + 4*b*x] + 4*b^3*d^3*x^3*Sin[5*a + 4*b*x]) - (3*c^2*d*Csc[a]*Sec[
a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Lo
g[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + Ar
cTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[
Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1
+ Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

**fricas [C]** time = 0.59, size = 984, normalized size = 4.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 24*I*d^3*polylog(4, cos(b*x + a) +
I*sin(b*x + a)) + 24*I*d^3*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 24*I
*d^3*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) - 24*I*d^3*polylog(4, -cos(
b*x + a) - I*sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3
 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2
 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3*
c^2*d - b*d^3)*x - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*
dilog(cos(b*x + a) + I*sin(b*x + a)) - (12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x
 + 12*I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (12*I*b^2*d^3*x^2
 + 24*I*b^2*c*d^2*x + 12*I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a))
 - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*dilog(-cos(b*x +
 a) - I*sin(b*x + a)) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) -
 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2) - 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 4*(b^3*d^3*x
^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*
d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*
x + a) - I*sin(b*x + a) + 1) - 24*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x +
a) + I*sin(b*x + a)) - 24*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*s
in(b*x + a)) - 24*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x
+ a)) - 24*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/
b^4
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \cos(bx + a)^2 \cot(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)^2*cot(b*x + a), x)
```

**maple [B]** time = 0.46, size = 899, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] 6*I/b^2*c*d^2*a^2*x-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^2
*polylog(2,-exp(I*(b*x+a)))*x-6*I/b*c^2*d*a*x+6*I*d^3*polylog(4,exp(I*(b*x+
a)))/b^4+I*c^3*x+1/8/b^3*(2*b^2*d^3*x^3+6*b^2*c*d^2*x^2+6*b^2*c^2*d*x+2*b^2
*c^3-3*d^3*x-3*c*d^2)*cos(2*b*x+2*a)-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+2/b
^4*d^3*a^3*ln(exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*
c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+6/b^3
*d^3*polylog(3,-exp(I*(b*x+a)))*x-3/2*I/b^4*a^4*d^3+6*I/b^4*d^3*polylog(4,-
exp(I*(b*x+a)))-I*c*d^2*x^3-3/2*I*c^2*d*x^2-1/4*I*d^3*x^4+1/b*c^3*ln(exp(I*
(b*x+a))-1)+1/b*c^3*ln(exp(I*(b*x+a))+1)-2/b*c^3*ln(exp(I*(b*x+a)))+3/b^3*c
*d^2*a^2*ln(exp(I*(b*x+a))-1)-6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-3*I/b^2*c^
2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-3*I/
b^2*c^2*d*a^2-2*I/b^3*a^3*d^3*x+4*I/b^3*c*d^2*a^3-3*I/b^2*d^3*polylog(2,exp
(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+3/b*c^2*d*ln(ex
p(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(
b*x+a)))*a-3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+3/b*c*d^2*ln(1-exp(I*(b*x+a
)))*x^2+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-
1)+6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*
d^3*ln(1-exp(I*(b*x+a)))*a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-3/16*d*(2*b^2
*d^2*x^2+4*b^2*c*d*x+2*b^2*c^2-d^2)/b^4*sin(2*b*x+2*a)
```

**maxima [B]** time = 0.80, size = 967, normalized size = 3.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/16*(8*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^3 - 24*(sin(b*x + a)^2 -
log(sin(b*x + a)^2))*a*c^2*d/b + 24*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*
a^2*c*d^2/b^2 - 8*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^3*d^3/b^3 - (-4*
I*(b*x + a)^4*d^3 + (-16*I*b*c*d^2 + 16*I*a*d^3)*(b*x + a)^3 + 96*I*d^3*pol
ylog(4, -e^(I*b*x + I*a)) + 96*I*d^3*polylog(4, e^(I*b*x + I*a)) + (-24*I*b
^2*c^2*d + 48*I*a*b*c*d^2 - 24*I*a^2*d^3)*(b*x + a)^2 + (16*I*(b*x + a)^3*d
^3 + (48*I*b*c*d^2 - 48*I*a*d^3)*(b*x + a)^2 + (48*I*b^2*c^2*d - 96*I*a*b*c
*d^2 + 48*I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (
-16*I*(b*x + a)^3*d^3 + (-48*I*b*c*d^2 + 48*I*a*d^3)*(b*x + a)^2 + (-48*I*b
^2*c^2*d + 96*I*a*b*c*d^2 - 48*I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a),
-cos(b*x + a) + 1) + 2*(2*(b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 6*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 - 1)*d^3)*(b
*x + a))*cos(2*b*x + 2*a) + (-48*I*b^2*c^2*d + 96*I*a*b*c*d^2 - 48*I*(b*x +
 a)^2*d^3 - 48*I*a^2*d^3 + (-96*I*b*c*d^2 + 96*I*a*d^3)*(b*x + a))*dilog(-e
^(I*b*x + I*a)) + (-48*I*b^2*c^2*d + 96*I*a*b*c*d^2 - 48*I*(b*x + a)^2*d^3
- 48*I*a^2*d^3 + (-96*I*b*c*d^2 + 96*I*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I
*a)) + 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2
*cos(b*x + a) + 1) + 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(
b*x + a)^2 - 2*cos(b*x + a) + 1) + 96*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*pol
ylog(3, -e^(I*b*x + I*a)) + 96*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3,
 e^(I*b*x + I*a)) - 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a
^2 - 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))/b^3)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\mathrm{cot}\left(a+b\,x\right)\,{\left(c+d\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^3,x)
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cos^2(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*cot(b*x+a),x)
[Out] Integral((c + d*x)**3*cos(a + b*x)**2*cot(a + b*x), x)
```

# 3.166 $\int (c+dx)^2\cos^2(a+bx)\cot(a+bx)\,dx$

**Optimal**. Leaf size=181

$$\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\sin^2(a+bx)}{4b^3}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}+\frac{(c+dx)^2\log\left(1-\right.}{b}$$

```
[Out] 1/2*c*d*x/b+1/4*d^2*x^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1-exp(2*I*(b*x+a))
)/b-I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,exp(2*I*(
b*x+a)))/b^3-1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2+1/4*d^2*sin(b*x+a)^2/b
^3-1/2*(d*x+c)^2*sin(b*x+a)^2/b
```

**Rubi [A]** time = 0.23, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4408, 4404, 3310, 3717, 2190, 2531, 2282, 6589}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}+\frac{d^2\sin^2(a+bx)}{4b^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (c*d*x)/(2*b) + (d^2*x^2)/(4*b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[
1 - E^((2*I)*(a + b*x))])/b - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^2 + (d^2*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Cos[a
 + b*x]*Sin[a + b*x])/(2*b^2) + (d^2*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^2
*Sin[a + b*x]^2)/(2*b)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^2\cot(a+bx)\,dx - \int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx$$

$$= -\frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx + \cdots$$

$$= -\frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cos(a+bx)\sin\cdots}{2b^2}$$

$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\cdots}{\cdots}$$

$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\cdots}{\cdots}$$

$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\cdots}{\cdots}$$

**Mathematica [B]** time = 2.90, size = 564, normalized size = 3.12

$$48b^3cdx^2\cot(a) - 48b^3cdx^2e^{i\tan^{-1}(\tan(a))}\cot(a)\sqrt{\sec^2(a)} + 48b^2c^2\log(\sin(a+bx)) - 6b^2c^2\csc(a)\sin(a+2bx)\cdots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] ((48*I)*b^2*c*d*Pi*x + (16*I)*b^3*d^2*x^3 - (96*I)*b^2*c*d*x*ArcTan[Tan[a]]
 + 48*b^3*c*d*x^2*Cot[a] - 6*b*c*d*Cos[a + 2*b*x]*Csc[a] - 6*b*d^2*x*Cos[a
+ 2*b*x]*Csc[a] + 6*b*c*d*Cos[3*a + 2*b*x]*Csc[a] + 6*b*d^2*x*Cos[3*a + 2*b
*x]*Csc[a] + 48*b*c*d*Pi*Log[1 + E^((-2*I)*b*x)] + 48*b^2*d^2*x^2*Log[1 - E
```

```
^((-I)*(a + b*x))] + 48*b^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))] + 96*b^2*c*
d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 96*b*c*d*ArcTan[Tan[a]]*Log
[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - 48*b*c*d*Pi*Log[Cos[b*x]] + 48*b^2
*c^2*Log[Sin[a + b*x]] - 96*b*c*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a
]]]] + (96*I)*b*d^2*x*PolyLog[2, -E^((-I)*(a + b*x))] + (96*I)*b*d^2*x*Poly
Log[2, E^((-I)*(a + b*x))] - (48*I)*b*c*d*PolyLog[2, E^((2*I)*(b*x + ArcTan
[Tan[a]]))] + 96*d^2*PolyLog[3, -E^((-I)*(a + b*x))] + 96*d^2*PolyLog[3, E^
((-I)*(a + b*x))] - 48*b^3*c*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^
2] - 6*b^2*c^2*Csc[a]*Sin[a + 2*b*x] + 3*d^2*Csc[a]*Sin[a + 2*b*x] - 12*b^2
*c*d*x*Csc[a]*Sin[a + 2*b*x] - 6*b^2*d^2*x^2*Csc[a]*Sin[a + 2*b*x] + 6*b^2*
c^2*Csc[a]*Sin[3*a + 2*b*x] - 3*d^2*Csc[a]*Sin[3*a + 2*b*x] + 12*b^2*c*d*x*
Csc[a]*Sin[3*a + 2*b*x] + 6*b^2*d^2*x^2*Csc[a]*Sin[3*a + 2*b*x])/(48*b^3)
```

**fricas [C]** time = 0.56, size = 594, normalized size = 3.28

$$-\frac{b^2d^2x^2 + 2\,b^2cdx - \left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(bx+a\right)^2 - 4\,d^2\text{polylog}\left(3, \cos\left(bx+a\right) + i\,\sin\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2
- d^2)*cos(b*x + a)^2 - 4*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 4
*d^2*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 4*d^2*polylog(3, -cos(b*x
+ a) + I*sin(b*x + a)) - 4*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) +
 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - (-4*I*b*d^2*x - 4*I*b*c*d)
*dilog(cos(b*x + a) + I*sin(b*x + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(cos
(b*x + a) - I*sin(b*x + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(-cos(b*x + a)
 + I*sin(b*x + a)) - (-4*I*b*d^2*x - 4*I*b*c*d)*dilog(-cos(b*x + a) - I*sin
(b*x + a)) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*s
in(b*x + a) + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a)
 - I*sin(b*x + a) + 1) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x
 + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-
1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^2*d^
2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x +
a) + 1))/b^3
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\cos\left(bx+a\right)^2\cot\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)^2*cot(b*x + a), x)
```

**maple [B]** time = 0.49, size = 535, normalized size = 2.96

$$\frac{d^2a^2\ln\left(e^{i(bx+a)}-1\right)}{b^3} - \frac{2d^2a^2\ln\left(e^{i(bx+a)}\right)}{b^3} + \frac{d^2\ln\left(1-e^{i(bx+a)}\right)x^2}{b} - \frac{d^2\ln\left(1-e^{i(bx+a)}\right)a^2}{b^3} + \frac{d^2\ln\left(e^{i(bx+a)}+1\right)x^2}{b} + \frac{4ia}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] 1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+1/b*d^2
*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(exp
(I*(b*x+a))+1)*x^2+4/3*I/b^3*d^2*a^3-I*c*d*x^2-4*I/b*c*d*a*x+2*d^2*polylog(
```

```
3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3+I*c^2*x-2/b*c^2*
ln(exp(I*(b*x+a)))+1/b*c^2*ln(exp(I*(b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1
)-1/3*I*d^2*x^3-1/4*d*(d*x+c)*sin(2*b*x+2*a)/b^2+2/b*c*d*ln(1-exp(I*(b*x+a)
))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+2/b*c*d*ln(exp(I*(b*x+a))+1)*x+4/b^2*
c*d*a*ln(exp(I*(b*x+a)))-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)+2*I/b^2*d^2*a^2*x
-2*I/b^2*c*d*a^2-2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*d^2*polyl
og(2,exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-2*I/b^2*c*d*p
olylog(2,exp(I*(b*x+a)))+1/8*(2*b^2*d^2*x^2+4*b^2*c*d*x+2*b^2*c^2-d^2)/b^3*
cos(2*b*x+2*a)
```

**maxima [B]** time = 0.46, size = 522, normalized size = 2.88

$$-\frac{12\left(\sin\left(bx+a\right)^{2}-\log\left(\sin\left(bx+a\right)^{2}\right)\right)c^{2}-\frac{24\left(\sin(bx+a)^{2}-\log\left(\sin(bx+a)^{2}\right)\right)acd}{b}+\frac{12\left(\sin(bx+a)^{2}-\log\left(\sin(bx+a)^{2}\right)\right)a^{2}d^{2}}{b^{2}}-\frac{-8\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/24*(12*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^2 - 24*(sin(b*x + a)^2 -
 log(sin(b*x + a)^2))*a*c*d/b + 12*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a
^2*d^2/b^2 - (-8*I*(b*x + a)^3*d^2 + (-24*I*b*c*d + 24*I*a*d^2)*(b*x + a)^2
 + 48*d^2*polylog(3, -e^(I*b*x + I*a)) + 48*d^2*polylog(3, e^(I*b*x + I*a))
 + (24*I*(b*x + a)^2*d^2 + (48*I*b*c*d - 48*I*a*d^2)*(b*x + a))*arctan2(sin
(b*x + a), cos(b*x + a) + 1) + (-24*I*(b*x + a)^2*d^2 + (-48*I*b*c*d + 48*I
*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 3*(2*(b*x + a
)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) + (-48*I*b*c*
d - 48*I*(b*x + a)*d^2 + 48*I*a*d^2)*dilog(-e^(I*b*x + I*a)) + (-48*I*b*c*d
 - 48*I*(b*x + a)*d^2 + 48*I*a*d^2)*dilog(e^(I*b*x + I*a)) + 12*((b*x + a)^
2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
2*cos(b*x + a) + 1) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 6*(b*c*d + (b*x +
 a)*d^2 - a*d^2)*sin(2*b*x + 2*a))/b^2)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^2\,\cot\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^2,x)
```

```
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)^{2}\cos^{2}\left(a+bx\right)\cot\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)**2*cot(a + b*x), x)
```

# 3.167 $\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$

**Optimal**. Leaf size=114

$$-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{dx}{4b}-\frac{i(c+dx)^2}{2d}$$

```
[Out] 1/4*d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b-1/2*I*d*polylo
g(2,exp(2*I*(b*x+a)))/b^2-1/4*d*cos(b*x+a)*sin(b*x+a)/b^2-1/2*(d*x+c)*sin(b
*x+a)^2/b
```

**Rubi [A]** time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4408, 4404, 2635, 8, 3717, 2190, 2279, 2391}

$$-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{dx}{4b}-\frac{i(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (d*x)/(4*b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 - E^((2*I)*(a + b*x)
)])/b - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d*Cos[a + b*x]*Sin
[a + b*x])/(4*b^2) - ((c + d*x)*Sin[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
```

```
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)\cot(a+bx)\,dx - \int (c+dx)\cos(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{i(c+dx)^2}{2d} - \frac{(c+dx)\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx + \frac{d\int$$
$$= -\frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)\sin(a+bx)}{4b^2}$$
$$= \frac{dx}{4b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)\sin(a+}{4b^2}$$
$$= \frac{dx}{4b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} - \frac{d\,c$$

**Mathematica [A]** time = 0.35, size = 131, normalized size = 1.15

$$\frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2+\text{Li}_2\left(e^{2i(a+bx)}\right)\right)\right)}{b^2}-\frac{d\sin(2(a+bx))}{8b^2}-\frac{ad\log(\sin(a+bx))}{b^2}-\frac{c\sin^2(}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (d*x*Cos[2*(a + b*x)])/(4*b) + (c*Log[Sin[a + b*x]])/b - (a*d*Log[Sin[a + b
*x]])/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((a + b*x)^2
 + PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 - (c*Sin[a + b*x]^2)/(2*b) - (d*S
in[2*(a + b*x)])/(8*b^2)
```

**fricas [B]** time = 0.59, size = 292, normalized size = 2.56

$$-\frac{bdx - 2\,(bdx+bc)\cos\left(bx+a\right)^2 + d\cos\left(bx+a\right)\sin\left(bx+a\right) + 2i\,d\text{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right) - 2i\,d\text{Li}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a)
+ 2*I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*I*d*dilog(cos(b*x + a) - I
```

```
*sin(b*x + a)) - 2*I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) + 2*I*d*dilog(
-cos(b*x + a) - I*sin(b*x + a)) - 2*(b*d*x + b*c)*log(cos(b*x + a) + I*sin(
b*x + a) + 1) - 2*(b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 2*
(b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b*c - a*
d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b*d*x + a*d)*log(
-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*d*x + a*d)*log(-cos(b*x + a) - I
*sin(b*x + a) + 1))/b^2
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\cos(bx+a)^2\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cos(b*x + a)^2*cot(b*x + a), x)
```

**maple** [B] time = 0.41, size = 249, normalized size = 2.18

$$icx-\frac{id\,x^2}{2}+\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}-\frac{2c\ln\left(e^{i(bx+a)}\right)}{b}-\frac{id\operatorname{polylog}\left(2,-e^{i(bx+a)}\right)}{b^2}-\frac{id\operatorname{polylog}\left(2,e^{i(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] I*c*x-2*I/b*d*a*x+1/b*c*ln(exp(I*(b*x+a))-1)+1/b*c*ln(exp(I*(b*x+a))+1)-2/b
*c*ln(exp(I*(b*x+a)))-1/2*I*d*x^2-I/b^2*d*a^2-I*d*polylog(2,exp(I*(b*x+a)))
/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2+1/b*d*
ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-1/b^2*d*a*ln(exp(I*(b
*x+a))-1)+2/b^2*d*a*ln(exp(I*(b*x+a)))+1/4*(d*x+c)*cos(2*b*x+2*a)/b-1/8*d*s
in(2*b*x+2*a)/b^2
```

**maxima** [B] time = 0.79, size = 222, normalized size = 1.95

$$\frac{-4i\,b^2dx^2-8i\,b^2cx-8i\,bdx\arctan\left(\sin(bx+a),-\cos(bx+a)+1\right)+8i\,bc\arctan\left(\sin(bx+a),\cos(bx+a)-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(-4*I*b^2*d*x^2 - 8*I*b^2*c*x - 8*I*b*d*x*arctan2(sin(b*x + a), -cos(b*
x + a) + 1) + 8*I*b*c*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (8*I*b*d*x
+ 8*I*b*c)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(b*d*x + b*c)*cos(2*
b*x + 2*a) - 8*I*d*dilog(-e^(I*b*x + I*a)) - 8*I*d*dilog(e^(I*b*x + I*a)) +
 4*(b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
+ 4*(b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
 - d*sin(2*b*x + 2*a))/b^2
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)^2\cot(a+b\,x)\,(c+d\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x),x)
```

```
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \cos^2 (a + bx) \cot (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)**2*cot(a + b*x), x)
```

## 3.168 $\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=82

$$\text{Int}\left(\frac{\cot(a+bx)}{c+dx},x\right)-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] -1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b
*c/d)/d+Unintegrable(cot(b*x+a)/(d*x+c),x)
```

**Rubi** [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x),x]
```

```
[Out] -(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) - (Cos[2*a - (
2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Defer[Int][Cot[a + b*x]/(
c + d*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx &= \int \frac{\cot(a+bx)}{c+dx}\,dx-\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx\\
&= \int \frac{\cot(a+bx)}{c+dx}\,dx-\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx\\
&= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\right)+\int \frac{\cot(a+bx)}{c+dx}\,dx\\
&= -\left(\frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx\right)-\frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\\
&= -\frac{\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\int \frac{\cot(a+bx)}{c+dx}
\end{aligned}$$

**Mathematica** [A] time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x), x]
```

**fricas** [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)^2\cot(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^2*cot(b*x + a)/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^2 \cot(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*cot(b*x + a)/(d*x + c), x)
```

**maple [A]** time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2(bx+a)\right)\cot(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(-i\,E_1\left(\frac{2i\,bdx+2i\,bc}{d}\right)+i\,E_1\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+4\,d\int\frac{\sin(bx+a)}{(dx+c)\left(\cos(bx+a)^2+\sin(bx+a)^2+2\,\cos(bx+a)+1\right)}\,dx-4\,d}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*((-I*exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) + I*exp_integral_e(1,
-(2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 4*d*integrate(sin(b*x +
a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)
*cos(b*x + a) + c), x) - 4*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)
^2 + (d*x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) - (
exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(1, -(2*I*b*d*x
+ 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d))/d
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^2\cot(a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*cot(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)**2*cot(a + b*x)/(c + d*x), x)
```

# 3.169 $\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=102

$$\text{Int}\left(\frac{\cot(a+bx)}{(c+dx)^2},x\right)-\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a+2bx)}{2d(c+dx)}$$

```
[Out] -b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2+b*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c
/d)/d^2+1/2*sin(2*b*x+2*a)/d/(d*x+c)+Unintegrable(cot(b*x+a)/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2) + Sin[2*a +
2*b*x]/(2*d*(c + d*x)) + (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*
b*x])/d^2 + Defer[Int][Cot[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx &= \int \frac{\cot(a+bx)}{(c+dx)^2}\,dx-\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx\\
&= \int \frac{\cot(a+bx)}{(c+dx)^2}\,dx-\int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx\\
&= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\right)+\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d}+\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(2\right.}{c}}{d}\\
&= -\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a+2bx)}{2d(c+dx)}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2\right.}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 2.49, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2, x]
```

**fricas [A]** time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)^2\cot(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cos(b*x + a)^2*cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^2 \cot(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(cos(b*x + a)^2*cot(b*x + a)/(d*x + c)^2, x)
```

**maple [A]** time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2(bx+a)\right)\cot(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)
[Out] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(-i\,E_2\left(\frac{2i\,bdx+2i\,bc}{d}\right)+i\,E_2\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+4\left(d^2x+cd\right)\int\frac{\sin(bx+a)}{(dx+c)^2\left(\cos(bx+a)^2+\sin(bx+a)^2+2\,\cos(bx+a)+\right.}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] -1/4*((-I*exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) + I*exp_integral_e(2,
-(2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 4*(d^2*x + c*d)*integrat
e(sin(b*x + a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^
2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x +
 c^2)*cos(b*x + a)), x) - 4*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2 +
 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x +
c^2)*sin(b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) -
 (exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(2, -(2*I*b*d*
x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^2*x + c*d)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^2\cot(a+b\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x)^2,x)
[Out] int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x)^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*cot(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)**2*cot(a + b*x)/(c + d*x)**2, x)
```

# 3.170 $\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$

**Optimal**. Leaf size=154

$$\text{Int}\left(\cot(a+bx)\csc(a+bx)(c+dx)^m,x\right)+\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx}{}$$

```
[Out] CannotIntegrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x)+1/2*I*exp(I*(a-b*c/d))*(
d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/2*I*(d*x+c)^m*G
AMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

**Rubi [A]** time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] ((I/2)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/
(b*(((-I)*b*(c + d*x))/d)^m) - ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*
x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x
)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx &= -\int (c+dx)^m\cos(a+bx)\,dx+\int (c+dx)^m\cot(a+bx)\csc(a+bx)\\ &= -\left(\frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m\,dx\right)-\frac{1}{2}\int e^{i(a+bx)}(c+dx)^m\,dx+\int (c+d\\ &= \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+}{}\end{aligned}$$

**Mathematica [A]** time = 9.89, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2, x]
```

**fricas [A]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m\cos(bx+a)\cot(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cos(bx + a) \cot(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

**maple** [A] time = 0.10, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cos(bx + a) \left(\cot^2(bx + a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)
[Out] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cos(bx + a) \cot(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a + b\,x)\, \cot(a + b\,x)^2\, (c + d\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^m,x)
[Out] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^m, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \cos(a + bx) \cot^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*cot(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*cos(a + b*x)*cot(a + b*x)**2, x)
```

# 3.171 $\int (c+dx)^4 \cos(a+bx)\cot^2(a+bx)\,dx$

**Optimal**. Leaf size=299

$$-\frac{24id^4\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^5}+\frac{24id^4\text{Li}_4\left(e^{i(a+bx)}\right)}{b^5}-\frac{24d^4\sin(a+bx)}{b^5}-\frac{24d^3(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}$$

```
[Out] -8*d*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b^2+24*d^3*(d*x+c)*cos(b*x+a)/b^4-4*
d*(d*x+c)^3*cos(b*x+a)/b^2-(d*x+c)^4*csc(b*x+a)/b+12*I*d^2*(d*x+c)^2*polylo
g(2,-exp(I*(b*x+a)))/b^3-12*I*d^2*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^3-2
4*d^3*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^4+24*d^3*(d*x+c)*polylog(3,exp(I
*(b*x+a)))/b^4-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+24*I*d^4*polylog(4,e
xp(I*(b*x+a)))/b^5-24*d^4*sin(b*x+a)/b^5+12*d^2*(d*x+c)^2*sin(b*x+a)/b^3-(d
*x+c)^4*sin(b*x+a)/b
```

**Rubi [A]** time = 0.29, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4408, 3296, 2637, 4410, 4183, 2531, 6609, 2282, 6589}

$$-\frac{24d^3(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4}+\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (-8*d*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b^2 + (24*d^3*(c + d*x)*Cos[a +
 b*x])/b^4 - (4*d*(c + d*x)^3*Cos[a + b*x])/b^2 - ((c + d*x)^4*Csc[a + b*x]
)/b + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((12*I)*d
^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (24*d^3*(c + d*x)*PolyLog
[3, -E^(I*(a + b*x))])/b^4 + (24*d^3*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])
/b^4 - ((24*I)*d^4*PolyLog[4, -E^(I*(a + b*x))])/b^5 + ((24*I)*d^4*PolyLog[
4, E^(I*(a + b*x))])/b^5 - (24*d^4*Sin[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*
Sin[a + b*x])/b^3 - ((c + d*x)^4*Sin[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4\cos(a+bx)\cot^2(a+bx)\,dx = -\int (c+dx)^4\cos(a+bx)\,dx + \int (c+dx)^4\cot(a+bx)\csc(a+bx)\,dx$$

$$= -\frac{(c+dx)^4\csc(a+bx)}{b} - \frac{(c+dx)^4\sin(a+bx)}{b} + \frac{(4d)\int (c+dx)^3\csc(a$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{4d(c+dx)^3\cos(a+bx)}{b^2} - \frac{(c+dx)^4\text{cs}}{b}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{4d(c+dx)^3\cos(a+bx)}{b^2} - \frac{(c+dx)^4\text{cs}}{b}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^3(c+dx)\cos(a+bx)}{b^4} - \frac{4d(c+dx}{}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^3(c+dx)\cos(a+bx)}{b^4} - \frac{4d(c+dx}{}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^3(c+dx)\cos(a+bx)}{b^4} - \frac{4d(c+dx}{}$$

**Mathematica [B]** time = 1.71, size = 798, normalized size = 2.67

$$\frac{\csc(a+bx)\left(-3c^4b^4-3d^4x^4b^4-12cd^3x^3b^4-18c^2d^2x^2b^4-12c^3dxb^4+c^4\cos(2(a+bx))b^4+d^4x^4\cos(2(a+b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (Csc[a + b*x]*(-3*b^4*c^4 + 12*b^2*c^2*d^2 - 24*d^4 - 12*b^4*c^3*d*x + 24*b
^2*c*d^3*x - 18*b^4*c^2*d^2*x^2 + 12*b^2*d^4*x^2 - 12*b^4*c*d^3*x^3 - 3*b^4
*d^4*x^4 + b^4*c^4*Cos[2*(a + b*x)] - 12*b^2*c^2*d^2*Cos[2*(a + b*x)] + 24*
d^4*Cos[2*(a + b*x)] + 4*b^4*c^3*d*x*Cos[2*(a + b*x)] - 24*b^2*c*d^3*x*Cos[
2*(a + b*x)] + 6*b^4*c^2*d^2*x^2*Cos[2*(a + b*x)] - 12*b^2*d^4*x^2*Cos[2*(a
 + b*x)] + 4*b^4*c*d^3*x^3*Cos[2*(a + b*x)] + b^4*d^4*x^4*Cos[2*(a + b*x)]
- 16*b^3*c^3*d*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 48*b^3
*c^2*d^2*x*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 48*b^3*c*d
^3*x^2*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 16*b^3*d^4*x^3
*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] + (24*I)*b^2*d^2*(c +
d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] - (24*I)*b^2
*d^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 4
8*b*c*d^3*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] - 48*b*d^
4*x*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] + 48*b*c*d^3*Po
lyLog[3, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] + 48*b*d^4*x*PolyLog[3
, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - (48*I)*d^4*PolyLog[4, -Cos[
a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] + (48*I)*d^4*PolyLog[4, Cos[a + b*x
] + I*Sin[a + b*x]]*Sin[a + b*x] - 4*b^3*c^3*d*Sin[2*(a + b*x)] + 24*b*c*d^
3*Sin[2*(a + b*x)] - 12*b^3*c^2*d^2*x*Sin[2*(a + b*x)] + 24*b*d^4*x*Sin[2*(
a + b*x)] - 12*b^3*c*d^3*x^2*Sin[2*(a + b*x)] - 4*b^3*d^4*x^3*Sin[2*(a + b*
x)]))/(2*b^5)
```

**fricas [C]** time = 0.63, size = 1233, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 12*b^2*c^2*d^2 - 12*I*d^4*p
olylog(4, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4,
 cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 12*I*d^4*polylog(4, -cos(b*x
 + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4, -cos(b*x + a) -
I*sin(b*x + a))*sin(b*x + a) + 24*d^4 + 12*(b^4*c^2*d^2 - b^2*d^4)*x^2 - (b
^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c
^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4
*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2 -
2*b*d^4)*x)*cos(b*x + a)*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*
x - 6*I*b^2*c^2*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6
*I*b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(cos(b*x + a) - I
*sin(b*x + a))*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*x - 6*I*b^
2*c^2*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b^2*d^
4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(-cos(b*x + a) - I*sin(b*x
 + a))*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x +
b^3*c^3*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 2*(b^3*d^4
*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x + a) - I*
sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c
*d^3 - a^3*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x +
 a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*co
s(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b^3*d^4*x^3 + 3*b^
3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*
```

```
log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*d^4*x^3 + 3*b
^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)
*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 12*(b*d^4*x + b*c*d
^3)*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 12*(b*d^4*x +
b*c*d^3)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 12*(b*d^4
*x + b*c*d^3)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*
(b*d^4*x + b*c*d^3)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a)
 + 8*(b^4*c^3*d - 3*b^2*c*d^3)*x)/(b^5*sin(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \cos(bx+a)\cot(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*cot(b*x + a)^2, x)
```

**maple [B]** time = 0.16, size = 1056, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x)
```

```
[Out] 8/b^5*d^4*a^3*arctanh(exp(I*(b*x+a)))-24/b^4*d^3*c*polylog(3,-exp(I*(b*x+a)
))+24/b^4*d^3*c*polylog(3,exp(I*(b*x+a)))-8/b^2*d*c^3*arctanh(exp(I*(b*x+a)
))-24/b^4*d^4*polylog(3,-exp(I*(b*x+a)))*x+24/b^4*d^4*polylog(3,exp(I*(b*x+
a)))*x-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+24*I/b^3*d^3*c*polylog(2,-ex
p(I*(b*x+a)))*x-24*I/b^3*d^3*c*polylog(2,exp(I*(b*x+a)))*x+24*I*d^4*polylog
(4,exp(I*(b*x+a)))/b^5+1/2*I*(d^4*x^4*b^4+4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2
+4*b^4*c^3*d*x+4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^4*x^2+12*I*b^3*c*d^3*x^2-24
*b^2*c*d^3*x+12*I*b^3*c^2*d^2*x-12*c^2*d^2*b^2+4*I*b^3*c^3*d-24*I*b*d^4*x+2
4*d^4-24*I*b*c*d^3)/b^5*exp(I*(b*x+a))-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x
^2+4*c^3*d*x+c^4)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-1/2*I*(d^4*x^4*b^4+
4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x-4*I*b^3*d^4*x^3+b^4*c^4-12*
b^2*d^4*x^2-12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x-12*I*b^3*c^2*d^2*x-12*c^2*d^2
*b^2-4*I*b^3*c^3*d+24*I*b*d^4*x+24*d^4+24*I*b*c*d^3)/b^5*exp(-I*(b*x+a))+12
/b^2*d^2*c^2*ln(1-exp(I*(b*x+a)))*x+12/b^3*d^2*c^2*ln(1-exp(I*(b*x+a)))*a-1
2/b^2*d^2*c^2*ln(exp(I*(b*x+a))+1)*x-12/b^3*d^2*c^2*ln(exp(I*(b*x+a))+1)*a+
12/b^2*d^3*c*ln(1-exp(I*(b*x+a)))*x^2-12/b^4*d^3*c*ln(1-exp(I*(b*x+a)))*a^2
-12/b^2*d^3*c*ln(exp(I*(b*x+a))+1)*x^2+12/b^4*d^3*c*ln(exp(I*(b*x+a))+1)*a^
2+4/b^2*d^4*ln(1-exp(I*(b*x+a)))*x^3+4/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^3-4/b
^2*d^4*ln(exp(I*(b*x+a))+1)*x^3-4/b^5*d^4*ln(exp(I*(b*x+a))+1)*a^3-24/b^4*d
^3*c*a^2*arctanh(exp(I*(b*x+a)))+24/b^3*d^2*c^2*a*arctanh(exp(I*(b*x+a)))+1
2*I/b^3*d^4*polylog(2,-exp(I*(b*x+a)))*x^2-12*I/b^3*d^4*polylog(2,exp(I*(b*
x+a)))*x^2+12*I/b^3*d^2*c^2*polylog(2,-exp(I*(b*x+a)))-12*I/b^3*d^2*c^2*pol
ylog(2,exp(I*(b*x+a)))
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a + b\,x)\,\cot(a + b\,x)^2\,(c + d\,x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^4,x)
[Out] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^4 \cos(a + bx)\cot^2(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*cot(b*x+a)**2,x)
[Out] Integral((c + d*x)**4*cos(a + b*x)*cot(a + b*x)**2, x)
```

# 3.172 $\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx)\, dx$

**Optimal**. Leaf size=216

$$-\frac{6d^3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\cos(a+bx)}{b^4}+\frac{6id^2(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}+\frac{6\ldots}{}$$

```
[Out] -6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2+6*d^3*cos(b*x+a)/b^4-3*d*(d*x+c)
^2*cos(b*x+a)/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-exp(I*(
b*x+a)))/b^3-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^3-6*d^3*polylog(3,
-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+6*d^2*(d*x+c)*sin(
b*x+a)/b^3-(d*x+c)^3*sin(b*x+a)/b
```

**Rubi [A]** time = 0.22, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4408, 3296, 2638, 4410, 4183, 2531, 2282, 6589}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{6d^3\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{PolyLog}\left(3,\ldots\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (-6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 + (6*d^3*Cos[a + b*x])/b^4
 - (3*d*(c + d*x)^2*Cos[a + b*x])/b^2 - ((c + d*x)^3*Csc[a + b*x])/b + ((6*I
)*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(c + d*x)*Po
lyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, -E^(I*(a + b*x))])/b^4 +
 (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4 + (6*d^2*(c + d*x)*Sin[a + b*x])/b
^3 - ((c + d*x)^3*Sin[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\cos(a+bx)\cot^2(a+bx)\,dx &= -\int (c+dx)^3\cos(a+bx)\,dx+\int (c+dx)^3\cot(a+bx)\csc(a+bx)\,d\\
&= -\frac{(c+dx)^3\csc(a+bx)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}+\frac{(3d)\int (c+dx)^2\csc}{b}\\
&= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3d(c+dx)^2\cos(a+bx)}{b^2}-\frac{(c+dx)^3}{}\\
&= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3d(c+dx)^2\cos(a+bx)}{b^2}-\frac{(c+dx)^3}{}\\
&= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{6d^3\cos(a+bx)}{b^4}-\frac{3d(c+dx)^2\cos(}{b^2}\\
&= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{6d^3\cos(a+bx)}{b^4}-\frac{3d(c+dx)^2\cos(}{b^2}\end{aligned}$$

**Mathematica [B]** time = 1.42, size = 539, normalized size = 2.50

$$\frac{\csc(a+bx)\left(b^3c^3\cos(2(a+bx))+3b^3c^2dx\cos(2(a+bx))+3b^3cd^2x^2\cos(2(a+bx))+b^3d^3x^3\cos(2(a+bx))-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (Csc[a + b*x]*(-3*b^3*c^3 + 6*b*c*d^2 - 9*b^3*c^2*d*x + 6*b*d^3*x - 9*b^3*c
*d^2*x^2 - 3*b^3*d^3*x^3 + b^3*c^3*Cos[2*(a + b*x)] - 6*b*c*d^2*Cos[2*(a +
b*x)] + 3*b^3*c^2*d*x*Cos[2*(a + b*x)] - 6*b*d^3*x*Cos[2*(a + b*x)] + 3*b^3
*c*d^2*x^2*Cos[2*(a + b*x)] + b^3*d^3*x^3*Cos[2*(a + b*x)] + 6*b^2*c^2*d*Lo
g[1 - E^(I*(a + b*x))]*Sin[a + b*x] + 12*b^2*c*d^2*x*Log[1 - E^(I*(a + b*x)
```

```
)]*Sin[a + b*x] + 6*b^2*d^3*x^2*Log[1 - E^(I*(a + b*x))]*Sin[a + b*x] - 6*b
^2*c^2*d*Log[1 + E^(I*(a + b*x))]*Sin[a + b*x] - 12*b^2*c*d^2*x*Log[1 + E^(
I*(a + b*x))]*Sin[a + b*x] - 6*b^2*d^3*x^2*Log[1 + E^(I*(a + b*x))]*Sin[a +
 b*x] + (12*I)*b*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))]*Sin[a + b*x] -
(12*I)*b*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))]*Sin[a + b*x] - 12*d^3*Po
lyLog[3, -E^(I*(a + b*x))]*Sin[a + b*x] + 12*d^3*PolyLog[3, E^(I*(a + b*x))
]*Sin[a + b*x] - 3*b^2*c^2*d*Sin[2*(a + b*x)] + 6*d^3*Sin[2*(a + b*x)] - 6*
b^2*c*d^2*x*Sin[2*(a + b*x)] - 3*b^2*d^3*x^2*Sin[2*(a + b*x)]))/(2*b^4)
```

**fricas [C]** time = 0.56, size = 797, normalized size = 3.69

$$-\frac{4\,b^3d^3x^3 + 12\,b^3cd^2x^2 + 4\,b^3c^3 - 6\,d^3\mathrm{polylog}\left(3, \cos\left(bx+a\right) + i\,\sin\left(bx+a\right)\right)\sin\left(bx+a\right) - 6\,d^3\mathrm{polylog}\left(3, \cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 4*b^3*c^3 - 6*d^3*polylog(3, cos(b
*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) - I*
sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) + I*sin(b*x + a
))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x
+ a) - 12*b*c*d^2 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2
+ 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*
x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*
d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b*d^3*x + 6*I
*b*c*d^2)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (-6*I*b*d^3*x
 - 6*I*b*c*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b
*d^3*x + 6*I*b*c*d^2)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) +
3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x +
a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(
b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
+ a) + 1/2)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a
^2*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^3*x
^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x +
 a) + 1)*sin(b*x + a) + 12*(b^3*c^2*d - b*d^3)*x)/(b^4*sin(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cos(bx+a)\cot(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*cot(b*x + a)^2, x)
```

**maple [B]** time = 0.13, size = 649, normalized size = 3.00

$$\frac{i\left(d^3x^3b^3 + 3b^3c\,d^2x^2 + 3ib^2d^3x^2 + 3b^3c^2dx + 6ib^2c\,d^2x + b^3c^3 + 3ib^2c^2d - 6b\,d^3x - 6c\,d^2b - 6id^3\right)e^{i(bx+a)}}{2b^4} - \frac{i\left(d^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x)
```

```
[Out] 1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*
b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))
```

```
-1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6
*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a
))+6*I/b^3*d^3*polylog(2,-exp(I*(b*x+a)))*x+6/b^2*d^2*c*ln(1-exp(I*(b*x+a))
)*x+6/b^3*d^2*c*ln(1-exp(I*(b*x+a)))*a-6/b^2*d^2*c*ln(exp(I*(b*x+a))+1)*x-6
/b^3*d^2*c*ln(exp(I*(b*x+a))+1)*a-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^
3*polylog(3,exp(I*(b*x+a)))/b^4-6/b^2*d*c^2*arctanh(exp(I*(b*x+a)))-6/b^4*d
^3*a^2*arctanh(exp(I*(b*x+a)))+12/b^3*d^2*c*a*arctanh(exp(I*(b*x+a)))-6*I/b
^3*d^3*polylog(2,exp(I*(b*x+a)))*x-6*I/b^3*d^2*c*polylog(2,exp(I*(b*x+a)))-
3/b^2*d^3*ln(exp(I*(b*x+a))+1)*x^2+3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^2+6*I/b
^3*d^2*c*polylog(2,-exp(I*(b*x+a)))+3/b^2*d^3*ln(1-exp(I*(b*x+a)))*x^2-3/b^
4*d^3*ln(1-exp(I*(b*x+a)))*a^2-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(
I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)
```

**maxima [B]** time = 1.96, size = 11018, normalized size = 51.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^3*(1/sin(b*x + a) + sin(b*x + a)) - 6*a*c^2*d*(1/sin(b*x + a) + s
in(b*x + a))/b + 6*a^2*c*d^2*(1/sin(b*x + a) + sin(b*x + a))/b^2 - 2*a^3*d^
3*(1/sin(b*x + a) + sin(b*x + a))/b^3 - 3*(((b*x + a)*sin(2*b*x + 2*a) - co
s(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a)
+ a - sin(2*b*x + 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2
*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + c
os(b*x + a))*cos(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*co
s(3*b*x + 3*a)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)
^2 + (8*(b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*
a) - cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) -
 sin(b*x + a))*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a
)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x
 + a)*cos(b*x + a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) +
 cos(b*x + a)^2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2
 + cos(b*x + a)^2 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2
)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3
*a) + 2*(3*(b*x + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a
)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + si
n(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x +
a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 +
sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + c
os(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x
+ a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*
a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)
^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x
 + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x +
 a) + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) +
 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)
^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin
(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*
(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(
2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a
)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*
a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*si
n(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x
+ 2*a) + a - sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2
*a)^2 + (b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x +
 a)*sin(b*x + a)^2 + b*x + 2*(((b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2
```

```
*b*x + 2*a) - (b*x + a)*cos(b*x + a) - ((b*x + a)*sin(b*x + a) - cos(b*x +
a))*sin(2*b*x + 2*a) - sin(b*x + a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x
+ a)^2 + 13*(b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*
(b*x + a)*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(
2*b*x + 2*a) + a)*sin(3*b*x + 3*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a
)*cos(b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a)
^2 + b*x + a)*sin(b*x + a) - cos(b*x + a))*c^2*d/(((cos(2*b*x + 2*a)^2 + si
n(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x +
a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 +
sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + c
os(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x
+ a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*
a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)
^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x
 + 3*a) + sin(b*x + a)^2)*b) + 6*(((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x +
 2*a) + 1)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin
(2*b*x + 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x
+ a)*cos(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x +
a))*cos(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x +
 3*a)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(
b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(
2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x
+ a))*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b
*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos
(b*x + a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x
+ a)^2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(b*
x + a)^2 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(2*b
*x + 2*a) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) + 2*(
3*(b*x + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x
 + a) + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + si
n(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x +
 a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x +
a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos
(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(
b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*
x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) +
 sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
+ ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3
*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos
(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x +
3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*
x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2
*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin
(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(
b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a
) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + s
in(b*x + a)^2 - 2*cos(b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x + 2*a) +
a - sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2*a)^2 + (
b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x + a)*sin(b
*x + a)^2 + b*x + 2*(((b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2*b*x + 2*
a) - (b*x + a)*cos(b*x + a) - ((b*x + a)*sin(b*x + a) - cos(b*x + a))*sin(2
*b*x + 2*a) - sin(b*x + a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x + a)^2 +
13*(b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)
*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(2*b*x + 2
*a) + a)*sin(3*b*x + 3*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a)*cos(b*x
 + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a)^2 + b*x
```

```
+ a)*sin(b*x + a) - cos(b*x + a))*a*c*d^2/(((cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 +
sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)
^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x
 + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x
+ a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*c
os(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + co
s(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(
b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a)
 + sin(b*x + a)^2)*b^2) - 3*(((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
 + 1)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*
x + 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x + a)*
cos(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*c
os(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)
^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x +
 a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x
 + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))
*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b*x +
a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos(b*x
+ a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^
2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(b*x + a
)^2 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(2*b*x +
2*a) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) + 2*(3*(b*
x + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a)
 + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)
^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x
 + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2
)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*si
n(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*
x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x +
 a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a
) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(
b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + ((c
os(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x
+ 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*
x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^
2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2
*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*c
os(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x
+ a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x +
 a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + s
in(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*cos(b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x + 2*a) + a - s
in(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2*a)^2 + (b*x +
 a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x + a)*sin(b*x +
a)^2 + b*x + 2*(((b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2*b*x + 2*a) -
(b*x + a)*cos(b*x + a) - ((b*x + a)*sin(b*x + a) - cos(b*x + a))*sin(2*b*x
+ 2*a) - sin(b*x + a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x + a)^2 + 13*(b
*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(
b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(2*b*x + 2*a) +
 a)*sin(3*b*x + 3*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a)*cos(b*x + a)
*sin(b*x + a)^2)*sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a)^2 + b*x + a)*
sin(b*x + a) - cos(b*x + a))*a^2*d^3/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*
a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b
*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 -
2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)
^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*
sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*
```

```
b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x
 + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x +
 a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + si
n(b*x + a)^2)*b^3) - 2*(-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*(I*a - 1)*d^3
 - 3*(I*b*c*d^2 + (-I*a + 1)*d^3)*(b*x + a)^2 + (I*(b*x + a)^3*d^3 - 6*I*b*c
*d^2 - 6*(-I*a - 1)*d^3 + (3*I*b*c*d^2 - 3*(I*a + 1)*d^3)*(b*x + a)^2 - (6*
b*c*d^2 - (6*a - 6*I)*d^3)*(b*x + a))*cos(3*b*x + 3*a)^2 + (6*I*(b*x + a)^3
*d^3 - 12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (18*I*b*c*d^2 - 18*
I*a*d^3)*(b*x + a)^2)*cos(b*x + a)^2 + (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 -
6*(I*a + 1)*d^3 + (-3*I*b*c*d^2 - 3*(-I*a - 1)*d^3)*(b*x + a)^2 + (6*b*c*d^
2 - (6*a - 6*I)*d^3)*(b*x + a))*sin(3*b*x + 3*a)^2 - 12*((b*x + a)^3*d^3 -
2*b*c*d^2 - 2*(b*x + a)*d^3 + 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*co
s(b*x + a)*sin(b*x + a) + (-6*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 + 12*I*(b*x
+ a)*d^3 - 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2)*sin(b*x +
 a)^2 - (6*b*c*d^2 - (6*a + 6*I)*d^3)*(b*x + a) + ((6*I*(b*x + a)^2*d^3 + (
12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d
^2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*
d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + ((6*I*(b*x + a
)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(b*x + a) - 6*((b*x + a
)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) + (
-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(b*x + a)
 - (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^3 +
 (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a)
- (6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-6*I
*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(b*x + a))*si
n(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*
x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + ((6*I*(b*x + a)^2*d^3 + (
12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d
^2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*
d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + ((6*I*(b*x + a
)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(b*x + a) - 6*((b*x + a
)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) + (
-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(b*x + a)
 - (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^3 +
 (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a)
- (6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-6*I
*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(b*x + a))*si
n(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*
x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + ((-7*I*(b*x + a)^3*d^3 +
 18*I*b*c*d^2 - 6*(3*I*a + 1)*d^3 + (-21*I*b*c*d^2 - 3*(-7*I*a - 1)*d^3)*(b
*x + a)^2 + (6*b*c*d^2 - (6*a - 18*I)*d^3)*(b*x + a))*cos(b*x + a) + (7*(b*
x + a)^3*d^3 - 18*b*c*d^2 + (18*a - 6*I)*d^3 + (21*b*c*d^2 - (21*a - 3*I)*d
^3)*(b*x + a)^2 + (6*I*b*c*d^2 - 6*(I*a + 3)*d^3)*(b*x + a))*sin(b*x + a))*
cos(3*b*x + 3*a) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 - 6*(-I*a + 1)*d^3 - 3*
(-I*b*c*d^2 + (I*a - 1)*d^3)*(b*x + a)^2 + (6*b*c*d^2 - (6*a + 6*I)*d^3)*(b
*x + a))*cos(2*b*x + 2*a) + ((-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d
^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(2*b*x + 2*a) - 12
*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + ((-
12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(b*x + a) + 12*(b*c*d^2
+ (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*cos(2*b*x + 2*a) + (12*I*b*c*d^2 + 1
2*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(b*x + a) + (12*b*c*d^2 + 12*(b*x + a)*d
^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (-1
2*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*sin(3*b*x
+ 3*a) + (12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(b*x + a) + (12*I*b*c*d^2
 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(b*x + a))*sin(2*b*x + 2*a) - 12*(b*
c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*dilog(-e^(I*b*x + I*a)) + ((12
*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x +
```

```
a)*d^3 + 12*I*a*d^3)*cos(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^
3)*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + ((12*I*b*c*d^2 + 12*I*(b*x + a)*d^3
 - 12*I*a*d^3)*cos(b*x + a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x
+ a))*cos(2*b*x + 2*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*
cos(b*x + a) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b
*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3
- 12*I*a*d^3)*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - (12*(b*c*d^2 + (b*x + a)
*d^3 - a*d^3)*cos(b*x + a) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d
^3)*sin(b*x + a))*sin(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*s
in(b*x + a))*dilog(e^(I*b*x + I*a)) + ((3*(b*x + a)^2*d^3 + 6*(b*c*d^2 - a*
d^3)*(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*
b*x + 2*a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*
sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + (3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d
^3)*(b*x + a))*cos(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d
^3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^2*d^3 + 2*(b*c
*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2
 - 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3
)*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b
*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) + ((3*I*(b*x + a)^2*d^3 + (6*I*
b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(b*c*
d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*sin(2*b*x + 2*a) + (-3*I*(b*x + a)^2*
d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*sin(b*x + a))*log(cos(b*x + a)^
2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((3*(b*x + a)^2*d^3 + 6*(b*c*d^2
 - a*d^3)*(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*c
os(2*b*x + 2*a) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a
))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + (3*((b*x + a)^2*d^3 + 2*(b*c*d^2 -
a*d^3)*(b*x + a))*cos(b*x + a) - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*
I*a*d^3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^2*d^3 + 2
*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-3*I*(b*x + a)^2*d^3 + (-6*I*
b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*
a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^
3)*(b*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - ((-3*I*(b*x + a)^2*d^3 +
 (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(b*x + a) + 3*((b*x + a)^2*d^3 +
2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*sin(2*b*x + 2*a) - (3*I*(b*x +
 a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*sin(b*x + a))*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (12*d^3*cos(b*x + a) + 12*I
*d^3*sin(b*x + a) + 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) - d^3
)*cos(3*b*x + 3*a) - 12*(d^3*cos(b*x + a) + I*d^3*sin(b*x + a))*cos(2*b*x +
 2*a) - (-12*I*d^3*cos(2*b*x + 2*a) + 12*d^3*sin(2*b*x + 2*a) + 12*I*d^3)*s
in(3*b*x + 3*a) - (12*I*d^3*cos(b*x + a) - 12*d^3*sin(b*x + a))*sin(2*b*x +
 2*a))*polylog(3, -e^(I*b*x + I*a)) + (12*d^3*cos(b*x + a) + 12*I*d^3*sin(b
*x + a) + 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) - d^3)*cos(3*b*
x + 3*a) - 12*(d^3*cos(b*x + a) + I*d^3*sin(b*x + a))*cos(2*b*x + 2*a) + (1
2*I*d^3*cos(2*b*x + 2*a) - 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*sin(3*b*x +
3*a) + (-12*I*d^3*cos(b*x + a) + 12*d^3*sin(b*x + a))*sin(2*b*x + 2*a))*pol
ylog(3, e^(I*b*x + I*a)) - ((2*(b*x + a)^3*d^3 - 12*b*c*d^2 + (12*a - 12*I)
*d^3 + (6*b*c*d^2 - (6*a - 6*I)*d^3)*(b*x + a)^2 - (-12*I*b*c*d^2 - 12*(-I*
a - 1)*d^3)*(b*x + a))*cos(3*b*x + 3*a) - (7*(b*x + a)^3*d^3 - 18*b*c*d^2 +
 (18*a - 6*I)*d^3 + (21*b*c*d^2 - (21*a - 3*I)*d^3)*(b*x + a)^2 + (6*I*b*c*
d^2 - 6*(I*a + 3)*d^3)*(b*x + a))*cos(b*x + a) - (7*I*(b*x + a)^3*d^3 - 18*
I*b*c*d^2 - 6*(-3*I*a - 1)*d^3 + (21*I*b*c*d^2 - 3*(7*I*a + 1)*d^3)*(b*x +
a)^2 - (6*b*c*d^2 - (6*a - 18*I)*d^3)*(b*x + a))*sin(b*x + a))*sin(3*b*x +
3*a) - ((b*x + a)^3*d^3 - 6*b*c*d^2 + (6*a + 6*I)*d^3 + (3*b*c*d^2 - (3*a +
 3*I)*d^3)*(b*x + a)^2 + 6*(-I*b*c*d^2 + (I*a - 1)*d^3)*(b*x + a))*sin(2*b*
x + 2*a))/(2*b^3*cos(b*x + a) + 2*I*b^3*sin(b*x + a) + (2*b^3*cos(2*b*x + 2
*a) + 2*I*b^3*sin(2*b*x + 2*a) - 2*b^3)*cos(3*b*x + 3*a) - 2*(b^3*cos(b*x +
 a) + I*b^3*sin(b*x + a))*cos(2*b*x + 2*a) - (-2*I*b^3*cos(2*b*x + 2*a) + 2
*b^3*sin(2*b*x + 2*a) + 2*I*b^3)*sin(3*b*x + 3*a) - (2*I*b^3*cos(b*x + a) -
 2*b^3*sin(b*x + a))*sin(2*b*x + 2*a)))/b
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\cot(a+b\,x)^2\,(c+d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^3, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3\cos(a+bx)\cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)*cot(a + b*x)**2, x)
```

# 3.173 $\int (c+dx)^2 \cos(a+bx)\cot^2(a+bx)\,dx$

**Optimal**. Leaf size=139

$$\frac{2id^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\sin(a+bx)}{b^3}-\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-$$

```
[Out] -4*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-2*d*(d*x+c)*cos(b*x+a)/b^2-(d*x+c)
^2*csc(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,ex
p(I*(b*x+a)))/b^3+2*d^2*sin(b*x+a)/b^3-(d*x+c)^2*sin(b*x+a)/b
```

**Rubi [A]** time = 0.15, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4408, 3296, 2637, 4410, 4183, 2279, 2391}

$$\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (-4*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - (2*d*(c + d*x)*Cos[a + b*x]
)/b^2 - ((c + d*x)^2*Csc[a + b*x])/b + ((2*I)*d^2*PolyLog[2, -E^(I*(a + b*x
))])/b^3 - ((2*I)*d^2*PolyLog[2, E^(I*(a + b*x))])/b^3 + (2*d^2*Sin[a + b*x
])/b^3 - ((c + d*x)^2*Sin[a + b*x])/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
```

```
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2\cos(a+bx)\cot^2(a+bx)\,dx = -\int (c+dx)^2\cos(a+bx)\,dx + \int (c+dx)^2\cot(a+bx)\csc(a+bx)\,dx$$
$$= -\frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(c+dx)^2\sin(a+bx)}{b} + \frac{(2d)\int (c+dx)\csc(a\cdots}{b}$$
$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d(c+dx)\cos(a+bx)}{b^2} - \frac{(c+dx)^2\csc(\cdots}{b}$$
$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d(c+dx)\cos(a+bx)}{b^2} - \frac{(c+dx)^2\csc(\cdots}{b}$$
$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d(c+dx)\cos(a+bx)}{b^2} - \frac{(c+dx)^2\csc(\cdots}{b}$$

**Mathematica [B]** time = 3.92, size = 310, normalized size = 2.23

$$-\frac{2\cos(bx)\left(\sin(a)\left(b^2(c+dx)^2-2d^2\right)+2bd\cos(a)(c+dx)\right)+2\sin(bx)\left(\cos(a)\left(b^2(c+dx)^2-2d^2\right)-2bd\sin(a)(\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -1/2*(8*b*c*d*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + 2*b^2*(c + d*x)^2*Csc
[a] - 4*d^2*(2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + (((b*
x + ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(
b*x + ArcTan[Tan[a]]))]) + I*PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - I*
PolyLog[2, E^(I*(b*x + ArcTan[Tan[a]]))])*Sec[a])/Sqrt[Sec[a]^2]) + 2*Cos[b
*x]*(2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a]) - b^2*(c +
 d*x)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] + b^2*(c + d*x)^2*Sec[a/2]*S
ec[(a + b*x)/2]*Sin[(b*x)/2] + 2*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d
*(c + d*x)*Sin[a])*Sin[b*x])/b^3
```

**fricas [B]** time = 0.50, size = 448, normalized size = 3.22

$$-\frac{2b^2d^2x^2+4b^2cdx+2b^2c^2+id^2\mathrm{Li}_2\left(\cos\left(bx+a\right)+i\sin\left(bx+a\right)\right)\sin\left(bx+a\right)-id^2\mathrm{Li}_2\left(\cos\left(bx+a\right)-i\sin\left(b\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + I*d^2*dilog(cos(b*x + a) + I*si
n(b*x + a))*sin(b*x + a) - I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b
```

```
*x + a) + I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d^2*
dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*s
in(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(
b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x
 + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*s
in(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) +
1/2)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) +
1)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)
*sin(b*x + a) - 2*d^2)/(b^3*sin(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \cos(bx + a) \cot(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*cot(b*x + a)^2, x)
```

**maple [B]** time = 0.13, size = 332, normalized size = 2.39

$$\frac{i\left(d^2x^2b^2 + 2b^2cdx + 2ib\,d^2x + b^2c^2 + 2ibcd - 2d^2\right)e^{i(bx+a)}}{2b^3} - \frac{i\left(d^2x^2b^2 + 2b^2cdx - 2ib\,d^2x + b^2c^2 - 2ibcd - 2d^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x)
```

```
[Out] 1/2*I*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp
(I*(b*x+a))-1/2*I*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*
c*d)/b^3*exp(-I*(b*x+a))-2*I*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*
I*(b*x+a))-1)-4/b^2*d*c*arctanh(exp(I*(b*x+a)))-2/b^2*d^2*ln(exp(I*(b*x+a))
+1)*x-2/b^3*d^2*ln(exp(I*(b*x+a))+1)*a+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b
^3+2/b^2*d^2*ln(1-exp(I*(b*x+a)))*x+2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a-2*I*d^
2*polylog(2,exp(I*(b*x+a)))/b^3+4/b^3*d^2*a*arctanh(exp(I*(b*x+a)))
```

**maxima [B]** time = 1.66, size = 3284, normalized size = 23.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (b^2*d^2*x^2*(-I*cos(a) + sin(a)) + b^2*c^2*(-I*cos(a) + sin(a)) - b*c*d*(2
*cos(a) + 2*I*sin(a)) - 2*d^2*(-I*cos(a) + sin(a)) - (2*b^2*c*d*(I*cos(a) -
 sin(a)) + b*d^2*(2*cos(a) + 2*I*sin(a)))*x - ((4*b*d^2*x*(-I*cos(a) + sin(
a)) + 4*b*c*d*(-I*cos(a) + sin(a)) - (-4*I*b*d^2*x - 4*I*b*c*d)*cos(2*b*x +
 3*a) - 4*(b*d^2*x + b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) - ((4*I*b*d^
2*x + 4*I*b*c*d)*cos(b*x + a) - 4*(b*d^2*x + b*c*d)*sin(b*x + a))*cos(2*b*x
 + 3*a) + 4*(b*d^2*x*(I*cos(a) - sin(a)) + b*c*d*(I*cos(a) - sin(a)))*cos(b
*x + a) + (b*d^2*x*(4*cos(a) + 4*I*sin(a)) + b*c*d*(4*cos(a) + 4*I*sin(a))
- 4*(b*d^2*x + b*c*d)*cos(2*b*x + 3*a) - (4*I*b*d^2*x + 4*I*b*c*d)*sin(2*b*
x + 3*a))*sin(3*b*x + 3*a) + (4*(b*d^2*x + b*c*d)*cos(b*x + a) - (-4*I*b*d^
2*x - 4*I*b*c*d)*sin(b*x + a))*sin(2*b*x + 3*a) - (b*d^2*x*(4*cos(a) + 4*I*
sin(a)) + b*c*d*(4*cos(a) + 4*I*sin(a)))*sin(b*x + a))*arctan2(sin(b*x + a)
, cos(b*x + a) + 1) - (4*b*c*d*(-I*cos(a) + sin(a))*cos(b*x + a) + b*c*d*(4
*cos(a) + 4*I*sin(a))*sin(b*x + a) + (4*b*c*d*(I*cos(a) - sin(a)) - 4*I*b*c
*d*cos(2*b*x + 3*a) + 4*b*c*d*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) - (-4*I*b*
```

```
c*d*cos(b*x + a) + 4*b*c*d*sin(b*x + a))*cos(2*b*x + 3*a) - (b*c*d*(4*cos(a
) + 4*I*sin(a)) - 4*b*c*d*cos(2*b*x + 3*a) - 4*I*b*c*d*sin(2*b*x + 3*a))*si
n(3*b*x + 3*a) - 4*(b*c*d*cos(b*x + a) + I*b*c*d*sin(b*x + a))*sin(2*b*x +
3*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - (4*b*d^2*x*(I*cos(a) - sin(
a))*cos(b*x + a) - b*d^2*x*(4*cos(a) + 4*I*sin(a))*sin(b*x + a) + 4*(b*d^2*
x*(-I*cos(a) + sin(a)) + I*b*d^2*x*cos(2*b*x + 3*a) - b*d^2*x*sin(2*b*x + 3
*a))*cos(3*b*x + 3*a) + 4*(-I*b*d^2*x*cos(b*x + a) + b*d^2*x*sin(b*x + a))*
cos(2*b*x + 3*a) + (b*d^2*x*(4*cos(a) + 4*I*sin(a)) - 4*b*d^2*x*cos(2*b*x +
 3*a) - 4*I*b*d^2*x*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) + (4*b*d^2*x*cos(b*x
 + a) + 4*I*b*d^2*x*sin(b*x + a))*sin(2*b*x + 3*a))*arctan2(sin(b*x + a), -
cos(b*x + a) + 1) + ((I*b^2*d^2*x^2 + I*b^2*c^2 - 2*b*c*d - 2*I*d^2 + (2*I*
b^2*c*d - 2*b*d^2)*x)*cos(3*b*x + 3*a) + (-I*b^2*d^2*x^2 - I*b^2*c^2 + 2*b*
c*d + 2*I*d^2 + (-2*I*b^2*c*d + 2*b*d^2)*x)*cos(b*x + a) - (b^2*d^2*x^2 + b
^2*c^2 + 2*I*b*c*d - 2*d^2 + 2*(b^2*c*d + I*b*d^2)*x)*sin(3*b*x + 3*a) + (b
^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*d - 2*d^2 + 2*(b^2*c*d + I*b*d^2)*x)*sin(b*x
 + a))*cos(3*b*x + 4*a) + ((-6*I*b^2*d^2*x^2 - 12*I*b^2*c*d*x - 6*I*b^2*c^2
 + 4*I*d^2)*cos(b*x + 2*a) + 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2
*d^2)*sin(b*x + 2*a))*cos(3*b*x + 3*a) + (I*b^2*d^2*x^2 + I*b^2*c^2 + 2*b*c
*d - 2*I*d^2 + (2*I*b^2*c*d + 2*b*d^2)*x)*cos(2*b*x + 3*a) + ((6*I*b^2*d^2*
x^2 + 12*I*b^2*c*d*x + 6*I*b^2*c^2 - 4*I*d^2)*cos(b*x + a) - 2*(3*b^2*d^2*x
^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*sin(b*x + a))*cos(b*x + 2*a) - (4*d^2
*(-I*cos(a) + sin(a))*cos(b*x + a) + d^2*(4*cos(a) + 4*I*sin(a))*sin(b*x +
a) + (4*d^2*(I*cos(a) - sin(a)) - 4*I*d^2*cos(2*b*x + 3*a) + 4*d^2*sin(2*b*
x + 3*a))*cos(3*b*x + 3*a) - (-4*I*d^2*cos(b*x + a) + 4*d^2*sin(b*x + a))*c
os(2*b*x + 3*a) - (d^2*(4*cos(a) + 4*I*sin(a)) - 4*d^2*cos(2*b*x + 3*a) - 4
*I*d^2*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) - 4*(d^2*cos(b*x + a) + I*d^2*sin
(b*x + a))*sin(2*b*x + 3*a))*dilog(-e^(I*b*x + I*a)) - (4*d^2*(I*cos(a) - s
in(a))*cos(b*x + a) - d^2*(4*cos(a) + 4*I*sin(a))*sin(b*x + a) + (4*d^2*(-I
*cos(a) + sin(a)) + 4*I*d^2*cos(2*b*x + 3*a) - 4*d^2*sin(2*b*x + 3*a))*cos(
3*b*x + 3*a) - (4*I*d^2*cos(b*x + a) - 4*d^2*sin(b*x + a))*cos(2*b*x + 3*a)
 + (d^2*(4*cos(a) + 4*I*sin(a)) - 4*d^2*cos(2*b*x + 3*a) - 4*I*d^2*sin(2*b*
x + 3*a))*sin(3*b*x + 3*a) + 4*(d^2*cos(b*x + a) + I*d^2*sin(b*x + a))*sin(
2*b*x + 3*a))*dilog(e^(I*b*x + I*a)) + ((b*d^2*x*(2*cos(a) + 2*I*sin(a)) +
b*c*d*(2*cos(a) + 2*I*sin(a)) - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 3*a) + (-2*
I*b*d^2*x - 2*I*b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + (2*(b*d^2*x + b
*c*d)*cos(b*x + a) + (2*I*b*d^2*x + 2*I*b*c*d)*sin(b*x + a))*cos(2*b*x + 3*
a) - (b*d^2*x*(2*cos(a) + 2*I*sin(a)) + b*c*d*(2*cos(a) + 2*I*sin(a)))*cos(
b*x + a) - (2*b*d^2*x*(-I*cos(a) + sin(a)) + 2*b*c*d*(-I*cos(a) + sin(a)) -
 (-2*I*b*d^2*x - 2*I*b*c*d)*cos(2*b*x + 3*a) - 2*(b*d^2*x + b*c*d)*sin(2*b*
x + 3*a))*sin(3*b*x + 3*a) + ((2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a) - 2*(b
*d^2*x + b*c*d)*sin(b*x + a))*sin(2*b*x + 3*a) - 2*(b*d^2*x*(I*cos(a) - sin
(a)) + b*c*d*(I*cos(a) - sin(a)))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) - ((b*d^2*x*(2*cos(a) + 2*I*sin(a)) + b*c*d*
(2*cos(a) + 2*I*sin(a)) - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 3*a) - (2*I*b*d^2
*x + 2*I*b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + (2*(b*d^2*x + b*c*d)*c
os(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*sin(b*x + a))*cos(2*b*x + 3*a) - (
b*d^2*x*(2*cos(a) + 2*I*sin(a)) + b*c*d*(2*cos(a) + 2*I*sin(a)))*cos(b*x +
a) + (2*b*d^2*x*(I*cos(a) - sin(a)) + 2*b*c*d*(I*cos(a) - sin(a)) - (2*I*b*
d^2*x + 2*I*b*c*d)*cos(2*b*x + 3*a) + 2*(b*d^2*x + b*c*d)*sin(2*b*x + 3*a))
*sin(3*b*x + 3*a) - ((-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a) + 2*(b*d^2*x +
 b*c*d)*sin(b*x + a))*sin(2*b*x + 3*a) + 2*(b*d^2*x*(-I*cos(a) + sin(a)) +
b*c*d*(-I*cos(a) + sin(a)))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)
^2 - 2*cos(b*x + a) + 1) - ((b^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*d - 2*d^2 + 2*
(b^2*c*d + I*b*d^2)*x)*cos(3*b*x + 3*a) - (b^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*
d - 2*d^2 + 2*(b^2*c*d + I*b*d^2)*x)*cos(b*x + a) - (-I*b^2*d^2*x^2 - I*b^2
*c^2 + 2*b*c*d + 2*I*d^2 + (-2*I*b^2*c*d + 2*b*d^2)*x)*sin(3*b*x + 3*a) - (
I*b^2*d^2*x^2 + I*b^2*c^2 - 2*b*c*d - 2*I*d^2 + (2*I*b^2*c*d - 2*b*d^2)*x)*
sin(b*x + a))*sin(3*b*x + 4*a) + (2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^
2 - 2*d^2)*cos(b*x + 2*a) + (6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x + 6*I*b^2*c^2
```

```
 - 4*I*d^2)*sin(b*x + 2*a))*sin(3*b*x + 3*a) - (b^2*d^2*x^2 + b^2*c^2 - 2*I
*b*c*d - 2*d^2 + 2*(b^2*c*d - I*b*d^2)*x)*sin(2*b*x + 3*a) - (2*(3*b^2*d^2*
x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*cos(b*x + a) - (-6*I*b^2*d^2*x^2 - 1
2*I*b^2*c*d*x - 6*I*b^2*c^2 + 4*I*d^2)*sin(b*x + a))*sin(b*x + 2*a))/(b^3*(
2*cos(a) + 2*I*sin(a))*cos(b*x + a) + 2*b^3*(I*cos(a) - sin(a))*sin(b*x + a
) - (b^3*(2*cos(a) + 2*I*sin(a)) - 2*b^3*cos(2*b*x + 3*a) - 2*I*b^3*sin(2*b
*x + 3*a))*cos(3*b*x + 3*a) - 2*(b^3*cos(b*x + a) + I*b^3*sin(b*x + a))*cos
(2*b*x + 3*a) + (2*b^3*(-I*cos(a) + sin(a)) + 2*I*b^3*cos(2*b*x + 3*a) - 2*
b^3*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) - (2*I*b^3*cos(b*x + a) - 2*b^3*sin(
b*x + a))*sin(2*b*x + 3*a))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+bx)\cot(a+bx)^2(c+dx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^2,x)
[Out] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2\cos(a+bx)\cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*cot(b*x+a)**2,x)
[Out] Integral((c + d*x)**2*cos(a + b*x)*cot(a + b*x)**2, x)
```

# 3.174 $\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx$

**Optimal**. Leaf size=58

$$-\frac{d\cos(a+bx)}{b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

```
[Out] -d*arctanh(cos(b*x+a))/b^2-d*cos(b*x+a)/b^2-(d*x+c)*csc(b*x+a)/b-(d*x+c)*si
n(b*x+a)/b
```

**Rubi [A]** time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4408, 3296, 2638, 4410, 3770}

$$-\frac{d\cos(a+bx)}{b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -((d*ArcTanh[Cos[a + b*x]])/b^2) - (d*Cos[a + b*x])/b^2 - ((c + d*x)*Csc[a
+ b*x])/b - ((c + d*x)*Sin[a + b*x])/b
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = -\int (c+dx)\cos(a+bx)\,dx + \int (c+dx)\cot(a+bx)\csc(a+bx)\,dx$$
$$= -\frac{(c+dx)\csc(a+bx)}{b} - \frac{(c+dx)\sin(a+bx)}{b} + \frac{d\int \csc(a+bx)\,dx}{b} +$$
$$= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\csc(a+bx)}{b} - \frac{(c$$

**Mathematica** [A] time = 0.68, size = 104, normalized size = 1.79

$$-\frac{2bc\sin(a+bx)+2bc\csc(a+bx)+2bdx\sin(a+bx)+2d\cos(a+bx)+bdx\tan\left(\frac{1}{2}(a+bx)\right)+bdx\cot\left(\frac{1}{2}(a\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -1/2*(2*d*Cos[a + b*x] + b*d*x*Cot[(a + b*x)/2] + 2*b*c*Csc[a + b*x] + 2*d*
Log[Cos[(a + b*x)/2]] - 2*d*Log[Sin[(a + b*x)/2]] + 2*b*c*Sin[a + b*x] + 2*
b*d*x*Sin[a + b*x] + b*d*x*Tan[(a + b*x)/2])/b^2
```

**fricas** [A] time = 0.46, size = 95, normalized size = 1.64

$$-\frac{4\,bdx-2\,(bdx+bc)\cos\left(bx+a\right)^2+2\,d\cos\left(bx+a\right)\sin\left(bx+a\right)+d\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)\sin\left(bx+a\right)-d}{2\,b^2\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + 2*d*cos(b*x + a)*sin(b*x +
 a) + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - d*log(-1/2*cos(b*x + a)
+ 1/2)*sin(b*x + a) + 4*b*c)/(b^2*sin(b*x + a))
```

**giac** [B] time = 5.16, size = 1967, normalized size = 33.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
6*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^3 -
 d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1
/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(t
an(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^3 + d*log(4*(tan(1/2*b*x)^4 + 2
*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 +
 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)
^4*tan(1/2*a)^3 + 6*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - d*log(4*(tan(1/2*b*
x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)
^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*ta
n(1/2*b*x)^3*tan(1/2*a)^4 + d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(
1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(
1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^4 + 6*
b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*b*c*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*d*t
an(1/2*b*x)^4*tan(1/2*a)^3 + 6*b*c*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*d*tan(1/
2*b*x)^3*tan(1/2*a)^4 + b*d*x*tan(1/2*b*x)^4 - 8*b*d*x*tan(1/2*b*x)^3*tan(1
```

```
/2*a) - d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)
+ tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a)
+ 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a) + d*log(4*(tan(1/2*b*x)^
4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x
)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2
*b*x)^4*tan(1/2*a) - 12*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 8*b*d*x*tan(1/2
*b*x)*tan(1/2*a)^3 + b*d*x*tan(1/2*a)^4 - d*log(4*(tan(1/2*b*x)^4*tan(1/2*a
)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b
*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan
(1/2*a)^4 + d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2
*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2
*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4 -
 8*b*c*tan(1/2*b*x)^3*tan(1/2*a) + 2*d*tan(1/2*b*x)^4*tan(1/2*a) - 12*b*c*t
an(1/2*b*x)^2*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 8*b*c*tan(1
/2*b*x)*tan(1/2*a)^3 + 12*d*tan(1/2*b*x)^2*tan(1/2*a)^3 + b*c*tan(1/2*a)^4
+ 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 6*b*d*x*tan(1/2*b*x)^2 + d*log(4*(tan(1/2
*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2
*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))
*tan(1/2*b*x)^3 - d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + t
an(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + t
an(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3 + 8*b*d*x*tan(1/2*b*x)*tan(
1/2*a) + 6*b*d*x*tan(1/2*a)^2 + d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*ta
n(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*
tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a)^3 - d*log(4*(ta
n(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 +
tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 +
1))*tan(1/2*a)^3 + 6*b*c*tan(1/2*b*x)^2 - 2*d*tan(1/2*b*x)^3 + 8*b*c*tan(1/
2*b*x)*tan(1/2*a) - 12*d*tan(1/2*b*x)^2*tan(1/2*a) + 6*b*c*tan(1/2*a)^2 - 1
2*d*tan(1/2*b*x)*tan(1/2*a)^2 - 2*d*tan(1/2*a)^3 + b*d*x + d*log(4*(tan(1/2
*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2
*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))
*tan(1/2*b*x) - d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan
(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan
(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x) + d*log(4*(tan(1/2*b*x)^4*tan(1
/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1
/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a) -
 d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan
(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan
(1/2*a)^2 + 1))*tan(1/2*a) + b*c + 2*d*tan(1/2*b*x) + 2*d*tan(1/2*a))/(b^2*
tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + b^2*tan(1/2
*b*x)^4*tan(1/2*a) + b^2*tan(1/2*b*x)*tan(1/2*a)^4 - b^2*tan(1/2*b*x)^3 - b
^2*tan(1/2*a)^3 - b^2*tan(1/2*b*x) - b^2*tan(1/2*a))
```

**maple** [C] time = 0.14, size = 124, normalized size = 2.14

$$\frac{i\,(bdx+cb+id)\,\mathrm{e}^{i(bx+a)}}{2b^2}-\frac{i\,(bdx+cb-id)\,\mathrm{e}^{-i(bx+a)}}{2b^2}-\frac{2i\mathrm{e}^{i(bx+a)}\,(dx+c)}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)}-\frac{d\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b^2}+\frac{d\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x)
```

```
[Out] 1/2*I*(b*d*x+c*b+I*d)/b^2*exp(I*(b*x+a))-1/2*I*(b*d*x+c*b-I*d)/b^2*exp(-I*(
b*x+a))-2*I*exp(I*(b*x+a))*(d*x+c)/b/(exp(2*I*(b*x+a))-1)-d/b^2*ln(exp(I*(b
*x+a))+1)+d/b^2*ln(exp(I*(b*x+a))-1)
```

**maxima** [B] time = 0.39, size = 2110, normalized size = 36.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*(1/sin(b*x + a) + sin(b*x + a)) - 2*a*d*(1/sin(b*x + a) + sin(b*x
 + a))/b - (((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos(3*b*x +
 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x + 2*a))*sin(3*b
*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x + a)*cos(b*x + a)*sin(
2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a) +
 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)^2 - ((b*x + a)*s
in(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*cos(2*b*x + 2
*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos(
3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 2*a)
 - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b*x + a)*sin(b*x + a) +
 cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a)
 - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^
2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(b*x + a)^2 + (13*(b*x +
a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) + sin(2*b*x
+ 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*sin(b*x +
a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a) + cos(b*x + a))*
cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x
+ 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*
x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)
+ 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a
)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2
- 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos
(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*
b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x +
 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + ((cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b
*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)
^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a
) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos
(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x
 + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x +
 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(
3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(
b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x + 2*a))*
cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2*a)^2 + (b*x + a)*cos(b*x + a)^
2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x + a)*sin(b*x + a)^2 + b*x + 2*((
(b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2*b*x + 2*a) - (b*x + a)*cos(b*x
 + a) - ((b*x + a)*sin(b*x + a) - cos(b*x + a))*sin(2*b*x + 2*a) - sin(b*x
+ a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x +
 a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x
+ a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*sin(3*b*x + 3
*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a)*cos(b*x + a)*sin(b*x + a)^2)*
sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a) - co
s(b*x + a))*d/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*
a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x +
2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2
- 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x
 + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x
+ 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a
)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*b))/b
```

**mupad [B]** time = 2.31, size = 162, normalized size = 2.79

$$e^{a\,1i+b\,x\,1i}\left(\frac{(b\,c+d\,1i)\;1i}{2\,b^2}+\frac{d\,x\,1i}{2\,b}\right)+e^{-a\,1i-b\,x\,1i}\left(\frac{(-b\,c+d\,1i)\;1i}{2\,b^2}-\frac{d\,x\,1i}{2\,b}\right)-\frac{d\,\ln\left(e^{a\,1i+b\,x\,1i}\,1i+1i\right)}{b^2}+\frac{d\,\ln\left(d\,2i-d\,e\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x),x)
```

```
[Out] exp(a*1i + b*x*1i)*(((d*1i + b*c)*1i)/(2*b^2) + (d*x*1i)/(2*b)) + exp(- a*1
i - b*x*1i)*(((d*1i - b*c)*1i)/(2*b^2) - (d*x*1i)/(2*b)) - (d*log(exp(a*1i
+ b*x*1i)*1i + 1i))/b^2 + (d*log(d*2i - d*exp(a*1i)*exp(b*x*1i)*2i))/b^2 +
(2*exp(a*1i + b*x*1i)*(c + d*x))/(b*(exp(a*2i + b*x*2i)*1i - 1i))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*cot(a + b*x)**2, x)
```

# 3.175 $\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=75

$$\text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{c+dx},x\right)-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c),x)-Ci(b*c/d+b*x)*cos(a-b*c/d)
/d+Si(b*c/d+b*x)*sin(a-b*c/d)/d
```

**Rubi** [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d) + (Sin[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][(Cot[a + b*x]*Csc[a + b*x])/(c + d*x
), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx &= -\int \frac{\cos(a+bx)}{c+dx}\,dx+\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx\\ &= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)+\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\int\\ &= -\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\cot(a+bx)\csc(}{c+dx}\end{aligned}$$

**Mathematica** [A] time = 3.72, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x), x]
```

**fricas** [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)\cot(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*cot(b*x + a)^2/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\cot(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*cot(b*x + a)^2/(d*x + c), x)
```

**maple [A]** time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\left(\cot^2(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)
[Out] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="maxima")
[Out] 1/2*(b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x +
I*b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d)
+ (b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*
x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x + I*
b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) +
(b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x
+ I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*
c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*
cos(2*b*x + 2*a)^2 - 4*d*cos(b*x + a)*sin(2*b*x + 2*a) + (b*c*(exp_integral
_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-
(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_inte
gral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(exp_integral_e
(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b
*c - a*d)/d) - b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integr
al_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 +
 (b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x
 + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(1, (I*b*d*x + I*b
*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x
- (2*b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(-2*I*exp_integral_e(1, (I*b*d*x
 + I*b*c)/d) + 2*I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d
)/d) + (2*b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -
(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(-2*I*exp_integral_e(1, (I*
b*d*x + I*b*c)/d) + 2*I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c
- a*d)/d))*x - 4*d*sin(b*x + a))*cos(2*b*x + 2*a) - 2*(b*d^3*x + b*c*d^2 +
(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x + b*c*d^2)*sin(2*b*x + 2*
a)^2 - 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^
2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2
+ (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x
 + b*c^2)*cos(b*x + a)), x) - 2*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*co
```

```
s(2*b*x + 2*a)^2 + (b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x + b*
c*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*
c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d
*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)
), x) - 4*d*sin(b*x + a))/(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x +
2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b
*x + 2*a))
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,\cot(a+b\,x)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*cot(a + b*x)**2/(c + d*x), x)
```

# 3.176 $\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=93

$$\text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2},x\right)+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\cos(a+bx)}{d(c+dx)}$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)+cos(b*x+a)/d/(d*x+c)+b*c
os(a-b*c/d)*Si(b*c/d+b*x)/d^2+b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

**Rubi [A]** time = 0.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Cos[a + b*x]/(d*(c + d*x)) + (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d]
)/d^2 + (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int][(C
ot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx+\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{d}+\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int \text{cot}
\end{aligned}$$

**Mathematica [A]** time = 4.04, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2, x]
```

**fricas [A]** time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)\cot(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*cot(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\cot(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(b*x + a)^2/(d*x + c)^2, x)
```

**maple** [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\left(\cot^2(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,\cot(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x)^2, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*cot(a + b*x)**2/(c + d*x)**2, x)
```

# 3.177 $\int (c+dx)^m \cot^3(a+bx)\,dx$

**Optimal**. Leaf size=19

$$\operatorname{Int}\left(\cot^3(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable((d*x+c)^m*cot(b*x+a)^3,x)
```

**Rubi** [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cot^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]^3,x]
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \int (c+dx)^m \cot^3(a+bx)\,dx$$

**Mathematica** [A] time = 11.48, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]^3,x]
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^3, x]
```

**fricas** [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \cot(bx+a)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="fricas")
[Out] integral((d*x + c)^m*cot(b*x + a)^3, x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cot(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cot(b*x + a)^3, x)
```

**maple** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cot^3(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cot(b*x+a)^3,x)
[Out] int((d*x+c)^m*cot(b*x+a)^3,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cot(bx+a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cot(b*x + a)^3, x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.05

$$\int \cot(a+b\,x)^3\,(c+d\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x)^m,x)
[Out] int(cot(a + b*x)^3*(c + d*x)^m, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cot(b*x+a)**3,x)
[Out] Integral((c + d*x)**m*cot(a + b*x)**3, x)
```

# 3.178 $\int (c+dx)^4 \cot^3(a+bx)\,dx$

**Optimal**. Leaf size=302

$$\frac{3d^4\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^5}+\frac{3d^4\text{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}-\frac{6id^3(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^4}-\frac{3id^3(c+dx)\text{Li}_4\left(e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)^2\text{Li}_3\left(e^{2i(\ldots}\right)}{b^3}$$

```
[Out] -2*I*d*(d*x+c)^3/b^2-1/2*(d*x+c)^4/b+1/5*I*(d*x+c)^5/d-2*d*(d*x+c)^3*cot(b*
x+a)/b^2-1/2*(d*x+c)^4*cot(b*x+a)^2/b+6*d^2*(d*x+c)^2*ln(1-exp(2*I*(b*x+a))
)/b^3-(d*x+c)^4*ln(1-exp(2*I*(b*x+a)))/b-6*I*d^3*(d*x+c)*polylog(2,exp(2*I*
(b*x+a)))/b^4+2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2+3*d^4*polylog
(3,exp(2*I*(b*x+a)))/b^5-3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b^3-3*
I*d^3*(d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,exp(2*I*(b*
x+a)))/b^5
```

**Rubi [A]** time = 0.46, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3720, 3717, 2190, 2531, 2282, 6589, 32, 6609}

$$-\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}-\frac{6id^3(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^4}-\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}+\frac{2id(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]^3,x]
```

```
[Out] ((-2*I)*d*(c + d*x)^3)/b^2 - (c + d*x)^4/(2*b) + ((I/5)*(c + d*x)^5)/d - (2
*d*(c + d*x)^3*Cot[a + b*x])/b^2 - ((c + d*x)^4*Cot[a + b*x]^2)/(2*b) + (6*
d^2*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b^3 - ((c + d*x)^4*Log[1 - E^
((2*I)*(a + b*x))])/b - ((6*I)*d^3*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^4 + ((2*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^4
*PolyLog[3, E^((2*I)*(a + b*x))])/b^5 - (3*d^2*(c + d*x)^2*PolyLog[3, E^((2
*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*(a + b*x))]
)/b^4 + (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \cot^3(a+bx)\,dx = -\frac{(c+dx)^4\cot^2(a+bx)}{2b} + \frac{(2d)\int (c+dx)^3\cot^2(a+bx)\,dx}{b} - \int (c+dx)^4\cot(a+$$

$$= \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c}{1-e^{2i(a}}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\text{co}}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\text{co}}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\text{co}}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\text{co}}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\text{co}}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\text{co}}{2b}$$

**Mathematica [B]** time = 7.12, size = 1534, normalized size = 5.08

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]^3,x]
```

```
[Out] -1/5*(x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)*Cot[a
]) - ((c + d*x)^4*Csc[a + b*x]^2)/(2*b) + (c^2*d^2*E^(I*a)*Csc[a]*((2*b^3*x
^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x
))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1
 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I
)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)
*(a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^3 - (d^4*
E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*
Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((
-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))
] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))
*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E
^((2*I)*a)))/b^5 + (c*d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3
*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2
*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b^2*x^2*Po
lyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a + b*x))] -
 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b
^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^((-I)*(a + b
*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^4 + (d^4*E^(I*a)
*Csc[a]*((2*b^5*x^5)/E^((2*I)*a) + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1 -
 E^((-I)*(a + b*x))] + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1 + E^((-I)*(a
+ b*x))] - (20*(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, -E^((-I)*(a + b*x))]
- (3*I)*b^2*x^2*PolyLog[3, -E^((-I)*(a + b*x))] - 6*b*x*PolyLog[4, -E^((-I)
*(a + b*x))] + (6*I)*PolyLog[5, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (20*(-
1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, E^((-I)*(a + b*x))] - (3*I)*b^2*x^2*Po
lyLog[3, E^((-I)*(a + b*x))] - 6*b*x*PolyLog[4, E^((-I)*(a + b*x))] + (6*I)
*PolyLog[5, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(10*b^5) - (c^4*Csc[a]*(-(b
*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 +
 Sin[a]^2)) + (6*c^2*d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[
a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (2*Csc[a]*Csc[a + b*x]*
(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] + 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin
[b*x]))/b^2 + (2*c^3*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*
x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[T
an[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*Ar
cTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x +
ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^
2 + Sin[a]^2)]) - (6*c*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((
I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcT
an[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] +
2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*
x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^4*Sqrt[Sec[a]^2*(Cos
[a]^2 + Sin[a]^2)])
```

**fricas [C]** time = 0.58, size = 1747, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^4*d^4*x^4 + 16*b^4*c*d^3*x^3 + 24*b^4*c^2*d^2*x^2 + 16*b^4*c^3*d*x
 + 4*b^4*c^4 + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 12*
```

```
I*b*c*d^3 - 12*I*(b^3*c^2*d^2 - b*d^4)*x + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^
3*x^2 + 4*I*b^3*c^3*d - 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 - b*d^4)*x)*cos(2*
b*x + 2*a))*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (4*I*b^3*d^4*x^3
 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d - 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 -
b*d^4)*x + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 12*I*b*
c*d^3 - 12*I*(b^3*c^2*d^2 - b*d^4)*x)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2
*a) - I*sin(2*b*x + 2*a)) + 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^
2*d^2 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*
d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4)*co
s(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) +
 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*c*d
^3 + (a^4 - 6*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2
 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4)*cos(2*b*x + 2*a))*log(-1/2*co
s(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^4*d^4*x^4 + 4*b^4*c*d
^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 -
 6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x
 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(
a^3 - 3*a)*b*c*d^3 - (a^4 - 6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*
(b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*si
n(2*b*x + 2*a) + 1) + 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*
a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 - 6*a^2)*d^4 + 6*(b^4*c^2*d^
2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x - (b^4*d^4*x^4 + 4*b^4*c*d
^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 -
 6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x
)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 3*(d^
4*cos(2*b*x + 2*a) - d^4)*polylog(5, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))
 + 3*(d^4*cos(2*b*x + 2*a) - d^4)*polylog(5, cos(2*b*x + 2*a) - I*sin(2*b*x
 + 2*a)) + (6*I*b*d^4*x + 6*I*b*c*d^3 + (-6*I*b*d^4*x - 6*I*b*c*d^3)*cos(2*
b*x + 2*a))*polylog(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (-6*I*b*d^4
*x - 6*I*b*c*d^3 + (6*I*b*d^4*x + 6*I*b*c*d^3)*cos(2*b*x + 2*a))*polylog(4,
 cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x +
b^2*c^2*d^2 - d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - d^4)*cos(2
*b*x + 2*a))*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + 6*(b^2*d^4
*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b
^2*c^2*d^2 - d^4)*cos(2*b*x + 2*a))*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b
*x + 2*a)) + 8*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d
)*sin(2*b*x + 2*a))/(b^5*cos(2*b*x + 2*a) - b^5)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^4 \cot(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cot(b*x + a)^3, x)
```

**maple [B]** time = 0.20, size = 1868, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cot(b*x+a)^3,x)
```

```
[Out] -1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)+2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))-12/b^3
*c^2*d^2*polylog(3,-exp(I*(b*x+a)))-12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a))
)+1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))-12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x
^2-12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+1/5*I*d^4*x^5+I*c*d^3*x^4-24*I
/b^3*d^3*c*a*x+12*d^4*polylog(3,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(3,exp(I
```

```
*(b*x+a)))/b^5+24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5+24*d^4*polylog(5,exp(I
*(b*x+a)))/b^5+8*I/b*a*c^3*d*x-12*I/b^2*a^2*c^2*d^2*x+8*I/b^3*c*d^3*a^3*x+1
2*I/b^2*polylog(2,exp(I*(b*x+a)))*c^2*d^2*x+12*I/b^2*c*d^3*polylog(2,exp(I*
(b*x+a)))*x^2+6*I/b^4*c*d^3*a^4-24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+4*
I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))-8*I/b^3*a^3*c^2*d^2+4*I/b^2*d^4*polyl
og(2,exp(I*(b*x+a)))*x^3-24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-2*I/b^4*d
^4*a^4*x+4*I/b^2*a^2*c^3*d-I*c^4*x+2/b*c^4*ln(exp(I*(b*x+a)))-1/b*c^4*ln(ex
p(I*(b*x+a))+1)-1/b*c^4*ln(exp(I*(b*x+a))-1)+6/b^5*d^4*a^2*ln(exp(I*(b*x+a)
)-1)-12/b^5*d^4*a^2*ln(exp(I*(b*x+a)))+6/b^3*d^2*c^2*ln(exp(I*(b*x+a))-1)+6
/b^3*d^2*c^2*ln(exp(I*(b*x+a))+1)-12/b^3*d^2*c^2*ln(exp(I*(b*x+a)))-6/b^5*d
^4*a^2*ln(1-exp(I*(b*x+a)))+6/b^3*d^4*ln(1-exp(I*(b*x+a)))*x^2+6/b^3*d^4*ln
(exp(I*(b*x+a))+1)*x^2-4*I/b^2*d^4*x^3+8*I/b^5*d^4*a^3-24*I/b^4*d^4*polylog
(4,-exp(I*(b*x+a)))*x-24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))+4*I/b^2*c^3
*d*polylog(2,-exp(I*(b*x+a)))+4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+2*
(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^2*
exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a))+
b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2*I
*(b*x+a))+2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))+6*I*c*d^3*x^2+6*I*c^2*d^2*
x+2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))-1)^2-4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x-4/
b*c^3*d*ln(1-exp(I*(b*x+a)))*x-4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a-6/b*c^2*d
^2*ln(exp(I*(b*x+a))+1)*x^2-24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3
*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))-6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2-24/b
^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x-8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))+4/b^4
*c*d^3*a^3*ln(exp(I*(b*x+a))-1)-8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))-6/b^3*c^
2*d^2*a^2*ln(exp(I*(b*x+a))-1)+12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))+4/b^2*
c^3*d*a*ln(exp(I*(b*x+a))-1)-1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4-1/b*d^4*ln(ex
p(I*(b*x+a))+1)*x^4-4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3-4/b*c*d^3*ln(1-exp(I
*(b*x+a)))*x^3-4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3+2*I*c^2*d^2*x^3+2*I*c^3
*d*x^2+12/b^3*d^3*c*ln(exp(I*(b*x+a))+1)*x+12/b^3*d^3*c*ln(1-exp(I*(b*x+a))
)*x+12/b^4*d^3*c*ln(1-exp(I*(b*x+a)))*a+24/b^4*d^3*c*a*ln(exp(I*(b*x+a)))+1
2*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+12*I/b^2*c^2*d^2*polylog(2,-ex
p(I*(b*x+a)))*x-8/5*I/b^5*d^4*a^5-12/b^4*d^3*c*a*ln(exp(I*(b*x+a))-1)-12*I/
b^4*d^3*c*polylog(2,-exp(I*(b*x+a)))-12*I/b^4*d^3*c*polylog(2,exp(I*(b*x+a)
))+12*I/b^4*d^4*a^2*x-12*I/b^2*d^3*c*x^2-12*I/b^4*d^3*c*a^2-12*I/b^4*d^4*po
lylog(2,-exp(I*(b*x+a)))*x-12*I/b^4*d^4*polylog(2,exp(I*(b*x+a)))*x
```

**maxima [B]** time = 4.85, size = 7111, normalized size = 23.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^4*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 4*a*c^3*d*(1/sin(b*x +
 a)^2 + log(sin(b*x + a)^2))/b + 6*a^2*c^2*d^2*(1/sin(b*x + a)^2 + log(sin(
b*x + a)^2))/b^2 - 4*a^3*c*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^3
 + a^4*d^4*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^4 - 2*(2*(b*x + a)^5*
d^4 + 40*b^3*c^3*d - 120*a*b^2*c^2*d^2 + 120*a^2*b*c*d^3 - 40*a^3*d^4 + 10*
(b*c*d^3 - a*d^4)*(b*x + a)^4 + 20*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b
*x + a)^3 + 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x
 + a)^2 - (10*(b*x + a)^4*d^4 - 60*b^2*c^2*d^2 + 120*a*b*c*d^3 - 60*a^2*d^4
 + 40*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2
- 1)*d^4)*(b*x + a)^2 + 40*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d
^3 - (a^3 - 3*a)*d^4)*(b*x + a) + 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*
a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 -
2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 20*((
b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*
d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^
2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)
```

```
*(b*x + a))*cos(2*b*x + 2*a) + (10*I*(b*x + a)^4*d^4 - 60*I*b^2*c^2*d^2 + 1
20*I*a*b*c*d^3 - 60*I*a^2*d^4 + (40*I*b*c*d^3 - 40*I*a*d^4)*(b*x + a)^3 + (
60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + (60*I*a^2 - 60*I)*d^4)*(b*x + a)^2 + (
40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + (120*I*a^2 - 120*I)*b*c*d^3 + (-40*I
*a^3 + 120*I*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (-20*I*(b*x + a)^4*d^4 +
 120*I*b^2*c^2*d^2 - 240*I*a*b*c*d^3 + 120*I*a^2*d^4 + (-80*I*b*c*d^3 + 80*
I*a*d^4)*(b*x + a)^3 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 + (-120*I*a^2
+ 120*I)*d^4)*(b*x + a)^2 + (-80*I*b^3*c^3*d + 240*I*a*b^2*c^2*d^2 + (-240*
I*a^2 + 240*I)*b*c*d^3 + (80*I*a^3 - 240*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2
*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (60*b^2*c^2*d^2 - 120*a*b*c*
d^3 + 60*a^2*d^4 + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(4*b*x + 4*a
) - 120*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(2*b*x + 2*a) - (-60*I*b^2
*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4)*sin(4*b*x + 4*a) - (120*I*b^2*c^
2*d^2 - 240*I*a*b*c*d^3 + 120*I*a^2*d^4)*sin(2*b*x + 2*a))*arctan2(sin(b*x
+ a), cos(b*x + a) - 1) + (10*(b*x + a)^4*d^4 + 40*(b*c*d^3 - a*d^4)*(b*x +
 a)^3 + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 40*(b^
3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a
) + 10*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2
- 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2
 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 20*
((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a
^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-10*I*(b*
x + a)^4*d^4 + (-40*I*b*c*d^3 + 40*I*a*d^4)*(b*x + a)^3 + (-60*I*b^2*c^2*d^
2 + 120*I*a*b*c*d^3 + (-60*I*a^2 + 60*I)*d^4)*(b*x + a)^2 + (-40*I*b^3*c^3*
d + 120*I*a*b^2*c^2*d^2 + (-120*I*a^2 + 120*I)*b*c*d^3 + (40*I*a^3 - 120*I*
a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) - (20*I*(b*x + a)^4*d^4 + (80*I*b*c*d^3
 - 80*I*a*d^4)*(b*x + a)^3 + (120*I*b^2*c^2*d^2 - 240*I*a*b*c*d^3 + (120*I*
a^2 - 120*I)*d^4)*(b*x + a)^2 + (80*I*b^3*c^3*d - 240*I*a*b^2*c^2*d^2 + (24
0*I*a^2 - 240*I)*b*c*d^3 + (-80*I*a^3 + 240*I*a)*d^4)*(b*x + a))*sin(2*b*x
+ 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 2*((b*x + a)^5*d^4 + 5*(
b*c*d^3 - a*d^4)*(b*x + a)^4 + 10*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^
4)*(b*x + a)^3 + 10*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a
^3 - 6*a)*d^4)*(b*x + a)^2 - 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x
+ a))*cos(4*b*x + 4*a) - (4*(b*x + a)^5*d^4 + 40*b^3*c^3*d - 120*a*b^2*c^2*
d^2 + 120*a^2*b*c*d^3 - 40*a^3*d^4 + (20*b*c*d^3 - (20*a - 20*I)*d^4)*(b*x
+ a)^4 + (40*b^2*c^2*d^2 - (80*a - 80*I)*b*c*d^3 + 40*(a^2 - 2*I*a - 1)*d^4
)*(b*x + a)^3 + (40*b^3*c^3*d - (120*a - 120*I)*b^2*c^2*d^2 + 120*(a^2 - 2*
I*a - 1)*b*c*d^3 - (40*a^3 - 120*I*a^2 - 120*a)*d^4)*(b*x + a)^2 + (80*I*b^
3*c^3*d - 120*(2*I*a + 1)*b^2*c^2*d^2 + (240*I*a^2 + 240*a)*b*c*d^3 + (-80*
I*a^3 - 120*a^2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (40*b^3*c^3*d - 120*a*b
^2*c^2*d^2 + 40*(b*x + a)^3*d^4 + 120*(a^2 - 1)*b*c*d^3 - 40*(a^3 - 3*a)*d^
4 + 120*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 120*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a
^2 - 1)*d^4)*(b*x + a) + 40*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4
+ 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 +
 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a))*cos(4*b*x + 4*a)
- 80*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 1)*b*c*d^3 -
 (a^3 - 3*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + (a^2 - 1)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-40*I*b^3*c^3*d + 12
0*I*a*b^2*c^2*d^2 - 40*I*(b*x + a)^3*d^4 + (-120*I*a^2 + 120*I)*b*c*d^3 + (
40*I*a^3 - 120*I*a)*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a)^2 + (-12
0*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 + (-120*I*a^2 + 120*I)*d^4)*(b*x + a))*si
n(4*b*x + 4*a) - (80*I*b^3*c^3*d - 240*I*a*b^2*c^2*d^2 + 80*I*(b*x + a)^3*d
^4 + (240*I*a^2 - 240*I)*b*c*d^3 + (-80*I*a^3 + 240*I*a)*d^4 + (240*I*b*c*d
^3 - 240*I*a*d^4)*(b*x + a)^2 + (240*I*b^2*c^2*d^2 - 480*I*a*b*c*d^3 + (240
*I*a^2 - 240*I)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) +
 (40*b^3*c^3*d - 120*a*b^2*c^2*d^2 + 40*(b*x + a)^3*d^4 + 120*(a^2 - 1)*b*c
*d^3 - 40*(a^3 - 3*a)*d^4 + 120*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 120*(b^2*c^
2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a) + 40*(b^3*c^3*d - 3*a*b^2*c^
```

2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 80*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-40*I*b^3*c^3*d + 120*I*a*b^2*c^2*d^2 - 40*I*(b*x + a)^3*d^4 + (-120*I*a^2 + 120*I)*b*c*d^3 + (40*I*a^3 - 120*I*a)*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a)^2 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 + (-120*I*a^2 + 120*I)*d^4)*(b*x + a))*sin(4*b*x + 4*a) - (80*I*b^3*c^3*d - 240*I*a*b^2*c^2*d^2 + 80*I*(b*x + a)^3*d^4 + (240*I*a^2 - 240*I)*b*c*d^3 + (-80*I*a^3 + 240*I*a)*d^4 + (240*I*b*c*d^3 - 240*I*a*d^4)*(b*x + a)^2 + (240*I*b^2*c^2*d^2 - 480*I*a*b*c*d^3 + (240*I*a^2 - 240*I)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-5*I*(b*x + a)^4*d^4 + 30*I*b^2*c^2*d^2 - 60*I*a*b*c*d^3 + 30*I*a^2*d^4 + (-20*I*b*c*d^3 + 20*I*a*d^4)*(b*x + a)^3 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 + (-30*I*a^2 + 30*I)*d^4)*(b*x + a)^2 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^2 + 60*I)*b*c*d^3 + (20*I*a^3 - 60*I*a)*d^4)*(b*x + a) + (-5*I*(b*x + a)^4*d^4 + 30*I*b^2*c^2*d^2 - 60*I*a*b*c*d^3 + 30*I*a^2*d^4 + (-20*I*b*c*d^3 + 20*I*a*d^4)*(b*x + a)^3 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 + (-30*I*a^2 + 30*I)*d^4)*(b*x + a)^2 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^2 + 60*I)*b*c*d^3 + (20*I*a^3 - 60*I*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (10*I*(b*x + a)^4*d^4 - 60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4 + (40*I*b*c*d^3 - 40*I*a*d^4)*(b*x + a)^3 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + (60*I*a^2 - 60*I)*d^4)*(b*x + a)^2 + (40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + (120*I*a^2 - 120*I)*b*c*d^3 + (-40*I*a^3 + 120*I*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 5*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-5*I*(b*x + a)^4*d^4 + 30*I*b^2*c^2*d^2 - 60*I*a*b*c*d^3 + 30*I*a^2*d^4 + (-20*I*b*c*d^3 + 20*I*a*d^4)*(b*x + a)^3 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 + (-30*I*a^2 + 30*I)*d^4)*(b*x + a)^2 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^2 + 60*I)*b*c*d^3 + (20*I*a^3 - 60*I*a)*d^4)*(b*x + a) + (-5*I*(b*x + a)^4*d^4 + 30*I*b^2*c^2*d^2 - 60*I*a*b*c*d^3 + 30*I*a^2*d^4 + (-20*I*b*c*d^3 + 20*I*a*d^4)*(b*x + a)^3 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 + (-30*I*a^2 + 30*I)*d^4)*(b*x + a)^2 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^2 + 60*I)*b*c*d^3 + (20*I*a^3 - 60*I*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (10*I*(b*x + a)^4*d^4 - 60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4 + (40*I*b*c*d^3 - 40*I*a*d^4)*(b*x + a)^3 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + (60*I*a^2 - 60*I)*d^4)*(b*x + a)^2 + (40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + (120*I*a^2 - 120*I)*b*c*d^3 + (-40*I*a^3 + 120*I*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 5*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (240*I*d^4*cos(4*b*x + 4*a) - 480*I*d^4*cos(2*b*x + 2*a) - 240*d^4*sin(4*b*x + 4*a) + 480*d^4*sin(2*b*x + 2*a) + 240*I*d^4)*polylog(5, -e^(I*b*x + I*a)) - (240*I*d^4*cos(4*b*x + 4*a) - 480*I*d^4*cos(2*b*x + 2*a) - 240*d^4*sin(4*b*x + 4*a) + 480*d^4*sin(2*b*x + 2*a) + 240*I*d^4)*polylog(5, e^(I*b*x + I*a)) - (240*b*c*d^3 + 240*(b*x + a)*d^4 - 240*a*d^4 + 240*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) - 480*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*

```
a) + (240*I*b*c*d^3 + 240*I*(b*x + a)*d^4 - 240*I*a*d^4)*sin(4*b*x + 4*a) +
 (-480*I*b*c*d^3 - 480*I*(b*x + a)*d^4 + 480*I*a*d^4)*sin(2*b*x + 2*a))*pol
ylog(4, -e^(I*b*x + I*a)) - (240*b*c*d^3 + 240*(b*x + a)*d^4 - 240*a*d^4 +
240*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) - 480*(b*c*d^3 + (b*
x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) + (240*I*b*c*d^3 + 240*I*(b*x + a)*d^4
 - 240*I*a*d^4)*sin(4*b*x + 4*a) + (-480*I*b*c*d^3 - 480*I*(b*x + a)*d^4 +
480*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, e^(I*b*x + I*a)) - (-120*I*b^2*c^
2*d^2 + 240*I*a*b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (-120*I*a^2 + 120*I)*d^4
+ (-240*I*b*c*d^3 + 240*I*a*d^4)*(b*x + a) + (-120*I*b^2*c^2*d^2 + 240*I*a*
b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (-120*I*a^2 + 120*I)*d^4 + (-240*I*b*c*d^
3 + 240*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (240*I*b^2*c^2*d^2 - 480*I*a
*b*c*d^3 + 240*I*(b*x + a)^2*d^4 + (240*I*a^2 - 240*I)*d^4 + (480*I*b*c*d^3
 - 480*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 120*(b^2*c^2*d^2 - 2*a*b*c*d^
3 + (b*x + a)^2*d^4 + (a^2 - 1)*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*
b*x + 4*a) - 240*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + (a^2 - 1)*d
^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x
+ I*a)) - (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (
-120*I*a^2 + 120*I)*d^4 + (-240*I*b*c*d^3 + 240*I*a*d^4)*(b*x + a) + (-120*
I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (-120*I*a^2 + 120
*I)*d^4 + (-240*I*b*c*d^3 + 240*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (240
*I*b^2*c^2*d^2 - 480*I*a*b*c*d^3 + 240*I*(b*x + a)^2*d^4 + (240*I*a^2 - 240
*I)*d^4 + (480*I*b*c*d^3 - 480*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 120*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + (a^2 - 1)*d^4 + 2*(b*c*d^3 -
a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 240*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x
+ a)^2*d^4 + (a^2 - 1)*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a
))*polylog(3, e^(I*b*x + I*a)) - (-2*I*(b*x + a)^5*d^4 + (-10*I*b*c*d^3 + 1
0*I*a*d^4)*(b*x + a)^4 + (-20*I*b^2*c^2*d^2 + 40*I*a*b*c*d^3 + (-20*I*a^2 +
 40*I)*d^4)*(b*x + a)^3 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^
2 + 120*I)*b*c*d^3 + (20*I*a^3 - 120*I*a)*d^4)*(b*x + a)^2 + (120*I*b^2*c^2
*d^2 - 240*I*a*b*c*d^3 + 120*I*a^2*d^4)*(b*x + a))*sin(4*b*x + 4*a) - (4*I*
(b*x + a)^5*d^4 + 40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + 120*I*a^2*b*c*d^3
- 40*I*a^3*d^4 + (20*I*b*c*d^3 - 20*(I*a + 1)*d^4)*(b*x + a)^4 + (40*I*b^2*
c^2*d^2 - 80*(I*a + 1)*b*c*d^3 + (40*I*a^2 + 80*a - 40*I)*d^4)*(b*x + a)^3
+ (40*I*b^3*c^3*d - 120*(I*a + 1)*b^2*c^2*d^2 + (120*I*a^2 + 240*a - 120*I)
*b*c*d^3 + (-40*I*a^3 - 120*a^2 + 120*I*a)*d^4)*(b*x + a)^2 - (80*b^3*c^3*d
 - (240*a - 120*I)*b^2*c^2*d^2 + 240*(a^2 - I*a)*b*c*d^3 - 40*(2*a^3 - 3*I*
a^2)*d^4)*(b*x + a))*sin(2*b*x + 2*a))/(-10*I*b^4*cos(4*b*x + 4*a) + 20*I*b
^4*cos(2*b*x + 2*a) + 10*b^4*sin(4*b*x + 4*a) - 20*b^4*sin(2*b*x + 2*a) - 1
0*I*b^4))/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(a+b\,x)^3\,(c+d\,x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] int(cot(a + b*x)^3*(c + d*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cot^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**4*cot(a + b*x)**3, x)
```

# 3.179 $\int (c+dx)^3 \cot^3(a+bx)\,dx$

**Optimal**. Leaf size=256

$$-\frac{3id^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{Li}}{2b^2}$$

```
[Out] -3/2*I*d*(d*x+c)^2/b^2-1/2*(d*x+c)^3/b+1/4*I*(d*x+c)^4/d-3/2*d*(d*x+c)^2*co
t(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)^2/b+3*d^2*(d*x+c)*ln(1-exp(2*I*(b*x+a
)))/b^3-(d*x+c)^3*ln(1-exp(2*I*(b*x+a)))/b-3/2*I*d^3*polylog(2,exp(2*I*(b*x
+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)
*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

**Rubi [A]** time = 0.37, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3720, 3717, 2190, 2279, 2391, 32, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{3id^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{PolyLog}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]^3,x]
```

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (c + d*x)^3/(2*b) + ((I/4)*(c + d*x)^4)/d
- (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot[a + b*x]^2)/(2*
b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b^3 - ((c + d*x)^3*Log[
1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))
])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3
*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*d^3*Po
lyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\cot^3(a+bx)\,dx = -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(3d)\int (c+dx)^2\cot^2(a+bx)\,dx}{2b} - \int (c+dx)^3\cot(a+bx$$

$$= \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+d}{1-e^{2i(a+bx}}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(}{2b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(}{2b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(}{2b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(}{2b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(}{2b}$$

**Mathematica [B]** time = 6.89, size = 994, normalized size = 3.88

$$-\frac{\csc(a)(\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a)-bx\cos(a))c^3}{b\left(\cos^2(a)+\sin^2(a)\right)} + \frac{3d\csc(a)\sec(a)\left(b^2e^{i\tan^{-1}(\tan(a))}x^2 + \frac{\left(ibx\left(2\tan^{\cdot}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]^3,x]
```

```
[Out] -1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cot[a]) - ((c + d*x)^3*
Csc[a + b*x]^2)/(2*b) + (c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3
*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 -
 E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x
*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((
2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*Pol
yLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(2*b^3) + (d^3*E^(I*a)*Csc[a]*(
(b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a
 + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] -
(6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*
PolyLog[3, -E^((-I)*(a + b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2
*I)*a) - (6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2
*I)*b*x*PolyLog[3, E^((-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))
/E^((2*I)*a)))/(4*b^4) - (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] +
 Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (3*c*d^2*Csc[a]*(-(b
*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2
 + Sin[a]^2)) + (3*Csc[a]*Csc[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x]
 + d^3*x^2*Sin[b*x]))/(2*b^2) + (3*c^2*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan
[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] -
 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Lo
g[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2
, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sq
rt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan
[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x
)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + P
i*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyL
```

```
og[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^
4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

**fricas [C]** time = 0.55, size = 1135, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 + (-6*I*
b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d + 6*I*d^3 + (6*I*b^2*d^3*x^2
 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d - 6*I*d^3)*cos(2*b*x + 2*a))*dilog(cos(
2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x +
6*I*b^2*c^2*d - 6*I*d^3 + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^
2*d + 6*I*d^3)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a
)) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3 - (
b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3)*cos(2*b*x
+ 2*a))*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3 - (b^3*c^3 - 3*
a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3)*cos(2*b*x + 2*a))*log(
-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^
2*d - b*d^3)*x - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*x)*cos(2*b*x + 2*a))*log(-co
s(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*
x - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 -
 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a) + 1) + (-3*I*d^3*cos(2*b*x + 2*a) + 3*I*d^3)*polylog
(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (3*I*d^3*cos(2*b*x + 2*a) - 3*
I*d^3)*polylog(4, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 6*(b*d^3*x + b*c
*d^2 - (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*polylog(3, cos(2*b*x + 2*a) +
I*sin(2*b*x + 2*a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(2*b*x
+ 2*a))*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 12*(b^2*d^3*x^2
 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(2*b*x + 2*a))/(b^4*cos(2*b*x + 2*a) - b^4
)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cot(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cot(b*x + a)^3, x)
```

**maple [B]** time = 0.14, size = 1194, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cot(b*x+a)^3,x)
```

```
[Out] 1/4*I*d^3*x^4+I*c*d^2*x^3-I*c^3*x+3*I/b^2*a^2*c^2*d+3*I/b^2*c^2*d*polylog(2
,exp(I*(b*x+a)))+3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-4*I/b^3*a^3*c*d^
2+2*I/b^3*d^3*a^3*x+1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)-2/b^4*d^3*a^3*ln(exp
(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,e
xp(I*(b*x+a)))-6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-6/b^3*d^3*polylog(3,-e
```

```
xp(I*(b*x+a)))*x-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-6*I*d^3*polylog(4,-e
xp(I*(b*x+a)))/b^4-1/b*c^3*ln(exp(I*(b*x+a))-1)-1/b*c^3*ln(exp(I*(b*x+a))+1
)+2/b*c^3*ln(exp(I*(b*x+a)))+6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x+(2*
b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2
*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c
^2*d*exp(2*I*(b*x+a))+3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*x+3*I*
c^2*d)/b^2/(exp(2*I*(b*x+a))-1)^2+3/2*I*c^2*d*x^2+3/b^3*d^2*c*ln(exp(I*(b*x
+a))-1)+3/b^3*d^2*c*ln(exp(I*(b*x+a))+1)-6/b^3*d^2*c*ln(exp(I*(b*x+a)))+3/b
^3*d^3*ln(exp(I*(b*x+a))+1)*x+3/b^3*d^3*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln
(1-exp(I*(b*x+a)))*a-3/b^4*d^3*a*ln(exp(I*(b*x+a))-1)+6/b^4*d^3*a*ln(exp(I*
(b*x+a)))-3*I/b^2*d^3*x^2-3*I/b^4*d^3*a^2-3*I/b^4*d^3*polylog(2,-exp(I*(b*x
+a)))+3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+3*I/b^2*c^2*d*polylog(2,-e
xp(I*(b*x+a)))-3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)+6/b^3*c*d^2*a^2*ln(exp(
I*(b*x+a)))-3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x-3/b*c^2*d*ln(1-exp(I*(b*x+a)))
*x-3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))-
3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b^2
*c^2*d*a*ln(exp(I*(b*x+a))-1)-6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))-1/b*d^3*ln(1
-exp(I*(b*x+a)))*x^3-1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-1/b*d^3*ln(exp(I*(b
*x+a))+1)*x^3-6*I/b^2*a^2*c*d^2*x+6*I/b*a*c^2*d*x+6*I/b^2*polylog(2,exp(I*(
b*x+a)))*c*d^2*x-6*I/b^3*d^3*a*x-6*I/b^4*d^3*polylog(4,exp(I*(b*x+a)))+3/2*
I/b^4*d^3*a^4
```

**maxima [B]** time = 1.74, size = 3952, normalized size = 15.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 3*a*c^2*d*(1/sin(b*x +
 a)^2 + log(sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/sin(b*x + a)^2 + log(sin(b*
x + a)^2))/b^2 - a^3*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^3 - 2*(
(b*x + a)^4*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 + 4*(b*c*d^2 - a
*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)^2 - (4*
(b*x + a)^3*d^3 - 12*b*c*d^2 + 12*a*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a)^2
+ 12*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a) + 4*((b*x + a)^3*d
^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d -
 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 8*((b*x + a)^3*
d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d
- 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (4*I*(b*x + a)
^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^
2 + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + (12*I*a^2 - 12*I)*d^3)*(b*x + a))*si
n(4*b*x + 4*a) + (-8*I*(b*x + a)^3*d^3 + 24*I*b*c*d^2 - 24*I*a*d^3 + (-24*I
*b*c*d^2 + 24*I*a*d^3)*(b*x + a)^2 + (-24*I*b^2*c^2*d + 48*I*a*b*c*d^2 + (-
24*I*a^2 + 24*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), co
s(b*x + a) + 1) + (12*b*c*d^2 - 12*a*d^3 + 12*(b*c*d^2 - a*d^3)*cos(4*b*x +
 4*a) - 24*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 + 12*I*a*d^3
)*sin(4*b*x + 4*a) - (24*I*b*c*d^2 - 24*I*a*d^3)*sin(2*b*x + 2*a))*arctan2(
sin(b*x + a), cos(b*x + a) - 1) + (4*(b*x + a)^3*d^3 + 12*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a) + 4*(
(b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*
d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 8*((b*x + a)^3*d^3 + 3*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*
(b*x + a))*cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3*d^3 + (-12*I*b*c*d^2 + 12*I
*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 12*I
)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (8*I*(b*x + a)^3*d^3 + (24*I*b*c*d^2 -
 24*I*a*d^3)*(b*x + a)^2 + (24*I*b^2*c^2*d - 48*I*a*b*c*d^2 + (24*I*a^2 - 2
4*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a)
+ 1) + ((b*x + a)^4*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d -
2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a)^2 - 24*(b*c*d^2 - a*d^3)*(b*x + a))*
```

```
cos(4*b*x + 4*a) - (2*(b*x + a)^4*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^
2*d^3 + (8*b*c*d^2 - (8*a - 8*I)*d^3)*(b*x + a)^3 + (12*b^2*c^2*d - (24*a -
 24*I)*b*c*d^2 + 12*(a^2 - 2*I*a - 1)*d^3)*(b*x + a)^2 + (24*I*b^2*c^2*d -
24*(2*I*a + 1)*b*c*d^2 + (24*I*a^2 + 24*a)*d^3)*(b*x + a))*cos(2*b*x + 2*a)
 + (12*b^2*c^2*d - 24*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 12*(a^2 - 1)*d^3 + 2
4*(b*c*d^2 - a*d^3)*(b*x + a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d
^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 24*(
b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*
d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I
*(b*x + a)^2*d^3 + (-12*I*a^2 + 12*I)*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b
*x + a))*sin(4*b*x + 4*a) - (24*I*b^2*c^2*d - 48*I*a*b*c*d^2 + 24*I*(b*x +
a)^2*d^3 + (24*I*a^2 - 24*I)*d^3 + (48*I*b*c*d^2 - 48*I*a*d^3)*(b*x + a))*s
in(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (12*b^2*c^2*d - 24*a*b*c*d^2 + 1
2*(b*x + a)^2*d^3 + 12*(a^2 - 1)*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) + 12*
(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a
*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 24*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)
^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) -
(-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 + (-12*I*a^2 + 12*
I)*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (24*I*b
^2*c^2*d - 48*I*a*b*c*d^2 + 24*I*(b*x + a)^2*d^3 + (24*I*a^2 - 24*I)*d^3 +
(48*I*b*c*d^2 - 48*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I
*a)) - (-2*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*c*d^2 + 6*
I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 6*I)*
d^3)*(b*x + a) + (-2*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*
c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a
^2 + 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (4*I*(b*x + a)^3*d^3 - 12*I*b*
c*d^2 + 12*I*a*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^2 + (12*I*b^2*c^
2*d - 24*I*a*b*c*d^2 + (12*I*a^2 - 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) +
 2*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2
 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a)
- 4*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^
2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(2*b*x + 2*a)
)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-2*I*(b*x +
a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2
 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 6*I)*d^3)*(b*x + a) + (-2
*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(
b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 6*I)*d^3)*(b*x
+ a))*cos(4*b*x + 4*a) + (4*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 +
 (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2
 + (12*I*a^2 - 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 2*((b*x + a)^3*d^3
- 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*
a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 4*((b*x + a)^3*d^3
 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2
*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (24*d^3*cos(4*b*x + 4*a) - 48*d^3
*cos(2*b*x + 2*a) + 24*I*d^3*sin(4*b*x + 4*a) - 48*I*d^3*sin(2*b*x + 2*a) +
 24*d^3)*polylog(4, -e^(I*b*x + I*a)) - (24*d^3*cos(4*b*x + 4*a) - 48*d^3*c
os(2*b*x + 2*a) + 24*I*d^3*sin(4*b*x + 4*a) - 48*I*d^3*sin(2*b*x + 2*a) + 2
4*d^3)*polylog(4, e^(I*b*x + I*a)) - (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 +
24*I*a*d^3 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*cos(4*b*x +
4*a) + (48*I*b*c*d^2 + 48*I*(b*x + a)*d^3 - 48*I*a*d^3)*cos(2*b*x + 2*a) +
24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 48*(b*c*d^2 + (b*x
+ a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) - (-24*I*b
*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*
d^3 + 24*I*a*d^3)*cos(4*b*x + 4*a) + (48*I*b*c*d^2 + 48*I*(b*x + a)*d^3 - 4
8*I*a*d^3)*cos(2*b*x + 2*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*
x + 4*a) - 48*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3
, e^(I*b*x + I*a)) - (-I*(b*x + a)^4*d^3 + (-4*I*b*c*d^2 + 4*I*a*d^3)*(b*x
+ a)^3 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a
```

```
)^2 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (2*I*(b*x +
 a)^4*d^3 + 12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3 + (8*I*b*c*d^2 -
 8*(I*a + 1)*d^3)*(b*x + a)^3 + (12*I*b^2*c^2*d - 24*(I*a + 1)*b*c*d^2 + (1
2*I*a^2 + 24*a - 12*I)*d^3)*(b*x + a)^2 - (24*b^2*c^2*d - (48*a - 24*I)*b*c
*d^2 + 24*(a^2 - I*a)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(-4*I*b^3*cos(4*b*x
 + 4*a) + 8*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(4*b*x + 4*a) - 8*b^3*sin(2*b
*x + 2*a) - 4*I*b^3))/b
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \cot (a+b\,x)^3\,(c+d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] int(cot(a + b*x)^3*(c + d*x)^3, x)
```

**sympy** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cot^3 (a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*cot(a + b*x)**3, x)
```

# 3.180 $\int (c+dx)^2 \cot^3(a+bx)\,dx$

**Optimal**. Leaf size=168

$$-\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\log(\sin(a+bx))}{b^3}+\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\log\left(1-e^{2i(\cdot}\right.}{b}$$

```
[Out] -c*d*x/b-1/2*d^2*x^2/b+1/3*I*(d*x+c)^3/d-d*(d*x+c)*cot(b*x+a)/b^2-1/2*(d*x+
c)^2*cot(b*x+a)^2/b-(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b+d^2*ln(sin(b*x+a))/b
^3+I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,exp(2*I*(b
*x+a)))/b^3
```

**Rubi [A]** time = 0.27, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3720, 3475, 3717, 2190, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\cot(a+bx)}{b^2}+\frac{d^2\log(\sin(a+bx))}{b^3}-\frac{(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]^3,x]
```

```
[Out] -((c*d*x)/b) - (d^2*x^2)/(2*b) + ((I/3)*(c + d*x)^3)/d - (d*(c + d*x)*Cot[a
 + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]^2)/(2*b) - ((c + d*x)^2*Log[1 - E^
((2*I)*(a + b*x))])/b + (d^2*Log[Sin[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLo
g[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b
^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2\cot^3(a+bx)\,dx &= -\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{d\int (c+dx)\cot^2(a+bx)\,dx}{b}-\int (c+dx)^2\cot(a+bx)\,dx \\
&= \frac{i(c+dx)^3}{3d}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+2i\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}} \\
&= -\frac{cdx}{b}-\frac{d^2x^2}{2b}+\frac{i(c+dx)^3}{3d}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}-\frac{(c+ \\
&= -\frac{cdx}{b}-\frac{d^2x^2}{2b}+\frac{i(c+dx)^3}{3d}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}-\frac{(c+ \\
&= -\frac{cdx}{b}-\frac{d^2x^2}{2b}+\frac{i(c+dx)^3}{3d}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}-\frac{(c+ \\
&= -\frac{cdx}{b}-\frac{d^2x^2}{2b}+\frac{i(c+dx)^3}{3d}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}-\frac{(c+
\end{aligned}$$

**Mathematica [B]** time = 6.68, size = 540, normalized size = 3.21

$$\frac{d^2\csc(a)(\sin(a)\log(\sin(a)\cos(bx)+\cos(a)\sin(bx))-bx\cos(a))}{b^3\left(\sin^2(a)+\cos^2(a)\right)}+\frac{\csc(a)\csc(a+bx)\left(cd\sin(bx)+d^2x\sin(bx)\right)}{b^2}+$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]^3,x]
```

```
[Out] -1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cot[a]) - ((c + d*x)^2*Csc[a + b*x]^2)/
(2*b) + (d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-
2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2
*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-
I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 +
 E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a
 + b*x))]))/E^((2*I)*a)))/(6*b^3) - (c^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*
x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (d^2*Csc[
```

```
a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(C
os[a]^2 + Sin[a]^2)) + (Csc[a]*Csc[a + b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x])
)/b^2 + (c*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2
*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Lo
g[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a
]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan
[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]
^2)])
```

**fricas** [C] time = 0.47, size = 655, normalized size = 3.90

$$\frac{4\,b^2d^2x^2 + 8\,b^2cdx + 4\,b^2c^2 + \left(-2i\,bd^2x - 2i\,bcd + \left(2i\,bd^2x + 2i\,bcd\right)\cos\left(2\,bx + 2\,a\right)\right)\mathrm{Li}_2\left(\cos\left(2\,bx + 2\,a\right) + i\,\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 + (-2*I*b*d^2*x - 2*I*b*c*d +
(2*I*b*d^2*x + 2*I*b*c*d)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) + I*sin(
2*b*x + 2*a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(2
*b*x + 2*a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 2*(b^2*c^2 - 2*
a*b*c*d + (a^2 - 1)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 1)*d^2)*cos(2*b*x +
 2*a))*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^2*c
^2 - 2*a*b*c*d + (a^2 - 1)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 1)*d^2)*cos(
2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2
*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*
d*x + 2*a*b*c*d - a^2*d^2)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*sin(
2*b*x + 2*a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b
^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(2*b*x + 2*a))*log(-cos(
2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) - (d^2*cos(2*b*x + 2*a) - d^2)*polyl
og(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - (d^2*cos(2*b*x + 2*a) - d^2)
*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 4*(b*d^2*x + b*c*d)*si
n(2*b*x + 2*a))/(b^3*cos(2*b*x + 2*a) - b^3)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \cot(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cot(b*x + a)^3, x)
```

**maple** [B] time = 0.11, size = 635, normalized size = 3.78

$$-\frac{d^2a^2\ln\left(\mathrm{e}^{i(bx+a)} - 1\right)}{b^3} + \frac{2d^2a^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^3} - \frac{d^2\ln\left(1 - \mathrm{e}^{i(bx+a)}\right)x^2}{b} + \frac{d^2\ln\left(1 - \mathrm{e}^{i(bx+a)}\right)a^2}{b^3} - \frac{d^2\ln\left(\mathrm{e}^{i(bx+a)} + 1\right)x^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cot(b*x+a)^3,x)
```

```
[Out] -4/3*I/b^3*a^3*d^2-1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+2/b^3*d^2*a^2*ln(exp(
I*(b*x+a)))-1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+1/b^3*d^2*ln(1-exp(I*(b*x+a)))
*a^2-1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+4*I/b*a*c*d*x+1/3*I*d^2*x^3+I*c*d*x^2
-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3-2*d^2*polylog(3,exp(I*(b*x+a)))/b^3+2
/b*c^2*ln(exp(I*(b*x+a)))-1/b*c^2*ln(exp(I*(b*x+a))-1)-1/b*c^2*ln(exp(I*(b*
```

```
x+a))+1)-I*c^2*x-2/b*c*d*ln(1-exp(I*(b*x+a)))*x-2/b^2*c*d*ln(1-exp(I*(b*x+a
)))*a-2/b*c*d*ln(exp(I*(b*x+a))+1)*x-4/b^2*c*d*a*ln(exp(I*(b*x+a)))+2/b^2*c
*d*a*ln(exp(I*(b*x+a))-1)+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(
b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+
a))+I*d^2*x+I*d*c)/b^2/(exp(2*I*(b*x+a))-1)^2+1/b^3*d^2*ln(exp(I*(b*x+a))+1
)-2/b^3*d^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(b*x+a))-1)+2*I/b^2*polyl
og(2,exp(I*(b*x+a)))*d^2*x+2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-2*I/b^2*a^
2*d^2*x+2*I/b^2*a^2*c*d+2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+2*I/b^2*c*
d*polylog(2,-exp(I*(b*x+a)))
```

**maxima** [**B**] time = 0.71, size = 1966, normalized size = 11.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 2*a*c*d*(1/sin(b*x + a
)^2 + log(sin(b*x + a)^2))/b + a^2*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)
^2))/b^2 - 2*(2*(b*x + a)^3*d^2 + 6*(b*c*d - a*d^2)*(b*x + a)^2 + 12*b*c*d
- 12*a*d^2 - (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) - 6*d^2 + 6*
((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(4*b*x + 4*a) - 12
*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) + (
6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a) - 6*I*d^2)*sin(4*
b*x + 4*a) + (-12*I*(b*x + a)^2*d^2 + (-24*I*b*c*d + 24*I*a*d^2)*(b*x + a)
+ 12*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*
d^2*cos(4*b*x + 4*a) - 12*d^2*cos(2*b*x + 2*a) + 6*I*d^2*sin(4*b*x + 4*a) -
 12*I*d^2*sin(2*b*x + 2*a) + 6*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1)
 + (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) + 6*((b*x + a)^2*d^2 +
 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a) - 12*((b*x + a)^2*d^2 + 2*(b
*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^2 + (-12*I*
b*c*d + 12*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - (12*I*(b*x + a)^2*d^2 + (
24*I*b*c*d - 24*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a),
 -cos(b*x + a) + 1) + 2*((b*x + a)^3*d^2 + 3*(b*c*d - a*d^2)*(b*x + a)^2 -
6*(b*x + a)*d^2)*cos(4*b*x + 4*a) - (4*(b*x + a)^3*d^2 + (12*b*c*d - (12*a
- 12*I)*d^2)*(b*x + a)^2 + 12*b*c*d - 12*a*d^2 + (24*I*b*c*d - 12*(2*I*a +
1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d
^2 + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 24*(b*c*d + (b*x
 + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 1
2*I*a*d^2)*sin(4*b*x + 4*a) - (24*I*b*c*d + 24*I*(b*x + a)*d^2 - 24*I*a*d^2
)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (12*b*c*d + 12*(b*x + a)*d^2
- 12*a*d^2 + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 24*(b*c*
d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-12*I*b*c*d - 12*I*(b*x + a)
*d^2 + 12*I*a*d^2)*sin(4*b*x + 4*a) - (24*I*b*c*d + 24*I*(b*x + a)*d^2 - 24
*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-3*I*(b*x + a)^2*d^2
+ (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2 + (-3*I*(b*x + a)^2*d^2 + (-
6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2)*cos(4*b*x + 4*a) + (6*I*(b*x +
a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a) - 6*I*d^2)*cos(2*b*x + 2*a)
+ 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(4*b*x + 4*a)
- 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(2*b*x + 2*a))
*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-3*I*(b*x + a
)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2 + (-3*I*(b*x + a)^2*
d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2)*cos(4*b*x + 4*a) + (6*I
*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a) - 6*I*d^2)*cos(2*b*x
 + 2*a) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(4*b*x
 + 4*a) - 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(2*b*x
 + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-12*I
*d^2*cos(4*b*x + 4*a) + 24*I*d^2*cos(2*b*x + 2*a) + 12*d^2*sin(4*b*x + 4*a)
 - 24*d^2*sin(2*b*x + 2*a) - 12*I*d^2)*polylog(3, -e^(I*b*x + I*a)) - (-12*
I*d^2*cos(4*b*x + 4*a) + 24*I*d^2*cos(2*b*x + 2*a) + 12*d^2*sin(4*b*x + 4*a
```

```
) - 24*d^2*sin(2*b*x + 2*a) - 12*I*d^2)*polylog(3, e^(I*b*x + I*a)) - (-2*I
*(b*x + a)^3*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a)^2 + 12*I*(b*x + a)*d^
2)*sin(4*b*x + 4*a) - (4*I*(b*x + a)^3*d^2 + (12*I*b*c*d - 12*(I*a + 1)*d^2
)*(b*x + a)^2 + 12*I*b*c*d - 12*I*a*d^2 - (24*b*c*d - (24*a - 12*I)*d^2)*(b
*x + a))*sin(2*b*x + 2*a))/(-6*I*b^2*cos(4*b*x + 4*a) + 12*I*b^2*cos(2*b*x
+ 2*a) + 6*b^2*sin(4*b*x + 4*a) - 12*b^2*sin(2*b*x + 2*a) - 6*I*b^2))/b
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] int(cot(a + b*x)^3*(c + d*x)^2, x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \cot^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*cot(a + b*x)**3, x)
```

# 3.181 $\int (c+dx)\cot^3(a+bx)\,dx$

**Optimal**. Leaf size=109

$$\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{dx}{2b}+\frac{i(c+dx)^2}{2d}$$

```
[Out] -1/2*d*x/b+1/2*I*(d*x+c)^2/d-1/2*d*cot(b*x+a)/b^2-1/2*(d*x+c)*cot(b*x+a)^2/
b-(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2
```

**Rubi** [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3720, 3473, 8, 3717, 2190, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{dx}{2b}+\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]^3,x]
```

```
[Out] -(d*x)/(2*b) + ((I/2)*(c + d*x)^2)/d - (d*Cot[a + b*x])/(2*b^2) - ((c + d*x
)*Cot[a + b*x]^2)/(2*b) - ((c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b + ((I/
2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\cot^3(a+bx)\,dx = -\frac{(c+dx)\cot^2(a+bx)}{2b} + \frac{d\int \cot^2(a+bx)\,dx}{2b} - \int (c+dx)\cot(a+bx)\,dx$$

$$= \frac{i(c+dx)^2}{2d} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx - \cdots$$

$$= -\frac{dx}{2b} + \frac{i(c+dx)^2}{2d} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{(c+dx)\log\left(1-e^{2i(\cdots}\right.}{b}$$

$$= -\frac{dx}{2b} + \frac{i(c+dx)^2}{2d} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{(c+dx)\log\left(1-e^{2i(\cdots}\right.}{b}$$

$$= -\frac{dx}{2b} + \frac{i(c+dx)^2}{2d} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{(c+dx)\log\left(1-e^{2i(\cdots}\right.}{b}$$

**Mathematica [B]** time = 6.17, size = 240, normalized size = 2.20

$$\frac{d\csc(a)\sec(a)\left(b^2x^2e^{i\tan^{-1}(\tan(a))} + \frac{\tan(a)\left(i\mathrm{Li}_2\left(e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right)+ibx\left(2\tan^{-1}(\tan(a))-\pi\right)-2\left(\tan^{-1}(\tan(a))+bx\right)\log\left(1-e^{2i\left(\tan^{-1}(\tan\cdots\right.}\right.\right.}{\sqrt{\tan^2(a)+1\cdots}}\right.}{2b^2\sqrt{\sec^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]^3,x]
```

```
[Out] -1/2*(d*x^2*Cot[a]) - (d*x*Csc[a + b*x]^2)/(2*b) - (c*(Cot[a + b*x]^2 + 2*L
og[Cos[a + b*x]] + 2*Log[Tan[a + b*x]]))/(2*b) + (d*Csc[a]*Csc[a + b*x]*Sin
[b*x])/(2*b^2) + (d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(
-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[
a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTa
n[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + Arc
Tan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2
 + Sin[a]^2)])
```

**fricas [B]** time = 0.46, size = 339, normalized size = 3.11

$$\frac{4\,bdx + 4\,bc + (i\,d\cos(2\,bx+2\,a) - i\,d)\mathrm{Li}_2(\cos(2\,bx+2\,a) + i\,\sin(2\,bx+2\,a)) + (-i\,d\cos(2\,bx+2\,a) + i\,d)\mathrm{L}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*d*x + 4*b*c + (I*d*cos(2*b*x + 2*a) - I*d)*dilog(cos(2*b*x + 2*a)
+ I*sin(2*b*x + 2*a)) + (-I*d*cos(2*b*x + 2*a) + I*d)*dilog(cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a)) + 2*(b*c - a*d - (b*c - a*d)*cos(2*b*x + 2*a))*log(
-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b*c - a*d - (b*c
```

```
 - a*d)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a
) + 1/2) + 2*(b*d*x + a*d - (b*d*x + a*d)*cos(2*b*x + 2*a))*log(-cos(2*b*x
+ 2*a) + I*sin(2*b*x + 2*a) + 1) + 2*(b*d*x + a*d - (b*d*x + a*d)*cos(2*b*x
 + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 2*d*sin(2*b*x +
2*a))/(b^2*cos(2*b*x + 2*a) - b^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\cot(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cot(b*x + a)^3, x)
```

**maple [B]** time = 0.09, size = 281, normalized size = 2.58

$$\frac{id\,\mathrm{polylog}\left(2,-e^{i(bx+a)}\right)}{b^2}-icx+\frac{2bdx\,e^{2i(bx+a)}+2bc\,e^{2i(bx+a)}-id\,e^{2i(bx+a)}+id}{b^2\left(e^{2i(bx+a)}-1\right)^2}-\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}-\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cot(b*x+a)^3,x)
```

```
[Out] I*d*polylog(2,-exp(I*(b*x+a)))/b^2-I*c*x+(2*b*d*x*exp(2*I*(b*x+a))+2*b*c*ex
p(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+I*d)/b^2/(exp(2*I*(b*x+a))-1)^2-1/b*c*l
n(exp(I*(b*x+a))-1)-1/b*c*ln(exp(I*(b*x+a))+1)+2/b*c*ln(exp(I*(b*x+a)))+I/b
^2*d*polylog(2,exp(I*(b*x+a)))+1/2*I*d*x^2+I/b^2*d*a^2-1/b*d*ln(exp(I*(b*x+
a))+1)*x+2*I/b*d*a*x-1/b*d*ln(1-exp(I*(b*x+a)))*x-1/b^2*d*ln(1-exp(I*(b*x+a
)))*a+1/b^2*d*a*ln(exp(I*(b*x+a))-1)-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

**maxima [B]** time = 0.53, size = 839, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (b^2*d*x^2 + 2*b^2*c*x - (2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a
) - 4*(b*d*x + b*c)*cos(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*
a) + (-4*I*b*d*x - 4*I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x
 + a) + 1) - (2*b*c*cos(4*b*x + 4*a) - 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin
(4*b*x + 4*a) - 4*I*b*c*sin(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos
(b*x + a) - 1) + (2*b*d*x*cos(4*b*x + 4*a) - 4*b*d*x*cos(2*b*x + 2*a) + 2*I
*b*d*x*sin(4*b*x + 4*a) - 4*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin
(b*x + a), -cos(b*x + a) + 1) + (b^2*d*x^2 + 2*b^2*c*x)*cos(4*b*x + 4*a) -
(2*b^2*d*x^2 + 4*I*b*c + (4*b^2*c + 4*I*b*d)*x + 2*d)*cos(2*b*x + 2*a) + (2
*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a) - 4*I*d
*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*a)) + (2*d*cos(4*b*x + 4*a) -
4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 2*
d)*dilog(e^(I*b*x + I*a)) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b*
x + 4*a) + (2*I*b*d*x + 2*I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x
 + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 + 2*cos(b*x + a) + 1) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b
*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*
x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 - 2*cos(b*x + a) + 1) - (-I*b^2*d*x^2 - 2*I*b^2*c*x)*sin(4*b*x + 4*a)
 - (2*I*b^2*d*x^2 - 4*b*c - 4*(-I*b^2*c + b*d)*x + 2*I*d)*sin(2*b*x + 2*a)
```

```
+ 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*
b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(a+b\,x)^3\,(c+d\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x),x)
```

```
[Out] int(cot(a + b*x)^3*(c + d*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\cot^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*cot(a + b*x)**3, x)
```

# 3.182 $\int \frac{\cot^3(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\frac{\cot^3(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^3/(d*x+c),x)
```

**Rubi [A]** time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^3/(c + d*x),x]
```

```
[Out] Defer[Int][Cot[a + b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx = \int \frac{\cot^3(a+bx)}{c+dx}\,dx$$

**Mathematica [A]** time = 8.08, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^3/(c + d*x),x]
```

```
[Out] Integrate[Cot[a + b*x]^3/(c + d*x), x]
```

**fricas [A]** time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^3/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^3/(d*x + c), x)
```

**maple** [A] time = 2.65, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^3/(d*x+c),x)
[Out] int(cot(b*x+a)^3/(d*x+c),x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cot(a+b\,x)^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3/(c + d*x),x)
[Out] int(cot(a + b*x)^3/(c + d*x), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**3/(d*x+c),x)
[Out] Integral(cot(a + b*x)**3/(c + d*x), x)
```

# 3.183 $\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\frac{\cot^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^3/(d*x+c)^2,x)
```

**Rubi** [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^3/(c + d*x)^2,x]
[Out] Defer[Int][Cot[a + b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx = \int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

**Mathematica** [A] time = 9.29, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^3/(c + d*x)^2,x]
[Out] Integrate[Cot[a + b*x]^3/(c + d*x)^2, x]
```

**fricas** [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cot(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(cot(b*x + a)^3/(d*x + c)^2, x)
```

**maple** [A] time = 4.19, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^3/(d*x+c)^2,x)
[Out] int(cot(b*x+a)^3/(d*x+c)^2,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cot(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3/(c + d*x)^2,x)
[Out] int(cot(a + b*x)^3/(c + d*x)^2, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**3/(d*x+c)**2,x)
[Out] Integral(cot(a + b*x)**3/(c + d*x)**2, x)
```

# 3.184 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=407

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)S\ldots}{4096b^{7/2}}$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/3
2*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2+5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b
^2-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/8192*d^(5/2)*FresnelS(2*b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1
/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/2)*(d*x
+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/128*d^2*co
s(2*b*x+2*a)*(d*x+c)^(1/2)/b^3+15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b^3
```

**Rubi [A]** time = 0.80, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) + (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) - ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$

$$= \frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{(5d)\int(c$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{5d(c+d$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c$$

**Mathematica [A]** time = 12.15, size = 550, normalized size = 1.35

$$-1024b^3c^2\sqrt{c+dx}\cos(2(a+bx))-256b^3c^2\sqrt{c+dx}\cos(4(a+bx))-1024b^3d^2x^2\sqrt{c+dx}\cos(2(a+bx))-2$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] + 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] - 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
rt[c + d*x]*Sin[2*(a + b*x)] + 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] +
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)
```

**fricas [A]** time = 0.57, size = 376, normalized size = 0.92

$$-\frac{15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))
*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
+ 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x +
c)*sqrt(b/(pi*d))) - 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(192*b^3*d^2*x^2 + 384*b^3*c*d*x
+ 192*b^3*c^2 + 360*b*d^2*cos(b*x + a)^2 - 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*
x + 64*b^3*c^2 - 15*b*d^2)*cos(b*x + a)^4 - 225*b*d^2 + 160*(2*(b^2*d^2*x +
 b^2*c*d)*cos(b*x + a)^3 + 3*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x +
a))*sqrt(d*x + c))/b^4
```

**giac [C]** time = 3.32, size = 2418, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16384*(512*(I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)))*c^3 + 24*c*d^2*((I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d
*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*
b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x
 + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*
(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(64*b^2
*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b^2) - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d -
 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2
```

```
+ 16*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x +
 c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/
d)/b^2)/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d
- 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*
b*c + 2*I*a*d)/d)/b^2)/d^2) + d^3*((-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 192*
I*b^2*c^2*d - 72*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^
(3/2)*b^2*c*d + 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 -
72*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(d*x + c)*b + 4
*I*b*c - 4*I*a*d)/d)/b^3)/d^3 + (I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^
2*c^2*d - 72*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^(
3/2)*b^2*c*d + 192*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 7
2*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((4*I*(d*x + c)*b - 4*I
*b*c + 4*I*a*d)/d)/b^3)/d^3 + 32*(-I*sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c^2*d
- 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^
3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*
sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*b*c*d
^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b
^3)/d^3 + 32*(I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d
^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b
*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*
x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*
c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x
 + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 192*(-I*
sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sq
rt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*d)*d*er
f(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 4*sqrt(d*x + c)*d*e^((4
*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*I*(d*x
+ c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b
+ 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*
c - 4*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.04, size = 470, normalized size = 1.15

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{dx+c}\ \cdots}{\sqrt{\pi}\sqrt{\frac{b}{d}}\cdots}\right.\right.}{8b\sqrt{\frac{b}{d}}\cdots}\right.}{4b}\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+5/64/b*d*(1/8/b*d*(d*x+c)^(3/
2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.56, size = 547, normalized size = 1.34

$$\left(1280\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)+10240\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-32\left(\frac{64\,(dx+c)^{\frac{5}{2}}b^4}{d}-15\sqrt{dx+c}\,b^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/65536*(1280*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) + 1024
0*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 32*(64*(d*x + c)
^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d) -
 512*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)*b
 - b*c + a*d)/d) + ((480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^
(1/4)*cos(-2*(b*c - a*d)/d) + (480*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + (
(30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) +
(30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*e
rf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2
/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2
/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + (-(4
80*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*
d)/d) - (480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2
*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

# 3.185 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=351

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)S}{512b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b-3/1
024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelC(2*b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5
/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^
(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2*a)*(d*
x+c)^(1/2)/b^2+3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.56, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]])/
(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$

$$= \frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx + \frac{1}{4}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} - \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{(3d)\int\sqrt{c+}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} - \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} - \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} - \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} - \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\ \mathrm{cc}}{}$$

**Mathematica [A]** time = 2.98, size = 393, normalized size = 1.12

$$\frac{-3\sqrt{2\pi}\,d\sin\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)-48\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-3\sqrt{2\pi}\,d\cos\left(4a-\frac{4bc}{d}\right)S}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] - 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
```

```
x)] - 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
- 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

**fricas [A]** time = 0.50, size = 294, normalized size = 0.84

$$-\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*f
resnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) + 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
 - 6*b^2*d*x - 6*b^2*c - 3*(2*b*d*cos(b*x + a)^3 + 3*b*d*cos(b*x + a))*sin(
b*x + a))*sqrt(d*x + c))/b^3
```

**giac [C]** time = 2.65, size = 1503, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2048*(64*(I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1))
)*c^2 + d^2*((I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4
*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x + c)^(
3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x +
c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 -
16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b^2) - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt
(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2 + 16*(I
*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)
/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sq
rt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(
b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*s
qrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2
*I*a*d)/d)/b^2)/d^2) + 16*(-I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d
```

```
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c -
I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sq
rt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8
*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16
*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*
x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d
*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c)/d
```

**maple** [A] time = 0.04, size = 376, normalized size = 1.07

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{8b}-\frac{d(dx+}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d))))
```

**maxima** [C] time = 0.51, size = 503, normalized size = 1.43

$$-\frac{\left(\frac{256\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{1024\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d}-96\sqrt{dx+c}\,b^2\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-768\sqrt{dx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8192*(256*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 102
4*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 96*sqrt(d*x +
c)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 768*sqrt(d*x + c)*b^2*sin(2*((d
*x + c)*b - b*c + a*d)/d) - (-(48*I + 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2
/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (48*I - 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*
d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) -
 (-(6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (
6*I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*
sqrt(d*x + c)*sqrt(I*b/d)) - ((6*I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*cos(-4*(b*c - a*d)/d) - (6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*si
n(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((48*I - 48)*4^(1/
4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (48*I + 48)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)^3 \sin(a+bx) (c+dx)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a),x)
[Out] Timed out
```

# 3.186 $\int \sqrt{c+dx} \cos^3(a+bx) \sin(a+bx) \, dx$

**Optimal**. Leaf size=299

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{\ldots}}{\sqrt{\ldots}}\right)}{64b^{3/2}}$$

```
[Out] 1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/128*FresnelS(2*b^(1/2)*2^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(
3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2
))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/P
i^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a)*(d*x+
c)^(1/2)/b-1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

**Rubi [A]** time = 0.45, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx)+\frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$

$$= \frac{1}{8}\int \sqrt{c+dx}\sin(4a+4bx)\,dx+\frac{1}{4}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{d\int \frac{\cos(4a+4bx)}{\sqrt{c+dx}}}{64b}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\left(d\cos\left(4a-\right.\right.}{}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\cos\left(4a-\frac{4bc}{d}\right)}{}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(\right.}{}$$

**Mathematica [A]** time = 0.59, size = 264, normalized size = 0.88

$$\frac{\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{2\pi}\sin\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\right.}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] + Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

**fricas** [A] time = 0.51, size = 233, normalized size = 0.78

$$\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)+8\,\pi d\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 8*pi*d*sqr
t(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)
)) - 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(
-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 3*b)*sqrt(d*x + c))/b^2
```

**giac** [C] time = 2.82, size = 818, normalized size = 2.74

$$-\frac{-\frac{i\,\sqrt{2}\,\sqrt{\pi}\,(8\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4i\,bc-4i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{i\,\sqrt{2}\,\sqrt{\pi}\,(8\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4i\,bc+4i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+8\left(i\,\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(-I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*(I*sqrt(2)*sqrt(pi)*d*e
rf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*
c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*
d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4
*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2
*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 8*I*sqrt(pi)*(4*b*c + I*d)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*
d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*
c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 4*sqrt(d*x + c)*
d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*
I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x
+ c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b +
 4*I*b*c - 4*I*a*d)/d)/b)/d
```

**maple** [A] time = 0.04, size = 286, normalized size = 0.96

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c
)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d))-1/64/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima [C]** time = 0.54, size = 425, normalized size = 1.42

$$-\frac{\left(\frac{32\,\sqrt{dx+c}\,b^2\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{128\,\sqrt{dx+c}\,b^2\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d}+\left((8i-8)\cdot 4^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)+(8\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*(32*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 128*sq
rt(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d + ((8*I - 8)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (8*I + 8)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(2*I*b/d)) + ((2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*
c - a*d)/d) + (2*I + 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*
d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(2*I + 2)*sqrt(2)*sqrt(pi)*b*(b
^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2
)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + (-(8*I +
 8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (8*I
 - 8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\sin\left(a+bx\right)\cos^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**3, x)
```

# 3.187 $\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=299

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt{}}\right.}{64b^{3/2}}$$

```
[Out] 1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/128*FresnelS(2*b^(1/2)*2^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(
3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2
))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/P
i^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a)*(d*x+
c)^(1/2)/b-1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

**Rubi [A]** time = 0.45, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\,\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\,\sin(2a+2bx)+\frac{1}{8}\sqrt{c+dx}\,\sin(4a+4bx)\right)dx$$

$$= \frac{1}{8}\int \sqrt{c+dx}\,\sin(4a+4bx)\,dx+\frac{1}{4}\int \sqrt{c+dx}\,\sin(2a+2bx)\,dx$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b}+\frac{d\int \frac{\cos(4a+4bx)}{\sqrt{c+dx}}}{64b}$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b}+\frac{\left(d\cos\left(4a-\right.\right.}{}$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b}+\frac{\cos\left(4a-\frac{4bc}{d}\right.}{}$$

$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(4a+4bx)}{32b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(\right.}{}$$

**Mathematica [A]** time = 0.16, size = 264, normalized size = 0.88

$$\frac{\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{2\pi}\sin\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\right.}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] + Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

**fricas [A]** time = 0.55, size = 233, normalized size = 0.78

$$\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)+8\,\pi d\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 8*pi*d*sqr
t(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)
)) - 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(
-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 3*b)*sqrt(d*x + c))/b^2
```

**giac [C]** time = 0.98, size = 818, normalized size = 2.74

$$-\frac{-\frac{i\sqrt{2}\sqrt{\pi}\,(8\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4i\,bc-4i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{i\sqrt{2}\sqrt{\pi}\,(8\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4i\,bc+4i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+8\left(\frac{i\sqrt{2}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(-I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*(I*sqrt(2)*sqrt(pi)*d*e
rf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*
c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*
d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4
*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2
*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 8*I*sqrt(pi)*(4*b*c + I*d)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*
d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*
c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 4*sqrt(d*x + c)*
d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*
I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x
+ c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b +
 4*I*b*c - 4*I*a*d)/d)/b)/d
```

**maple [A]** time = 0.00, size = 286, normalized size = 0.96

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c
)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d))-1/64/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima** [C] time = 0.51, size = 425, normalized size = 1.42

$$-\frac{\left(\frac{32\sqrt{dx+c}\,b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{128\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}+\left((8i-8)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(8\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*(32*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 128*sq
rt(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d + ((8*I - 8)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (8*I + 8)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(2*I*b/d)) + ((2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*
c - a*d)/d) + (2*I + 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*
d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(2*I + 2)*sqrt(2)*sqrt(pi)*b*(b
^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2
)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + (-(8*I +
 8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (8*I
 - 8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sin(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin(a+bx)\cos^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**3, x)
```

# 3.188 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=351

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2}{\ldots}\right.}{512b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b-3/1
024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelC(2*b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5
/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^
(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2*a)*(d*
x+c)^(1/2)/b^2+3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.57, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$

$$= \frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^{3/2}\sin(2a+2bx)\,d$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{(3d)\int \sqrt{}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{c+}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{c+}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{c+}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}}{}$$

**Mathematica [A]** time = 1.20, size = 393, normalized size = 1.12

$$\frac{-3\sqrt{2\pi}\,d\sin\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)-48\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-3\sqrt{2\pi}\,d\cos\left(4a-\frac{4bc}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] - 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
```

```
x)] - 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
- 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

**fricas [A]** time = 0.51, size = 294, normalized size = 0.84

$$-\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*f
resnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) + 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
 - 6*b^2*d*x - 6*b^2*c - 3*(2*b*d*cos(b*x + a)^3 + 3*b*d*cos(b*x + a))*sin(
b*x + a))*sqrt(d*x + c))/b^3
```

**giac [C]** time = 3.87, size = 1503, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2048*(64*(I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1))
)*c^2 + d^2*((I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4
*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x + c)^(
3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x +
c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 -
16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b^2) - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt
(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2 + 16*(I
*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)
/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sq
rt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(
b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*s
qrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2
*I*a*d)/d)/b^2)/d^2) + 16*(-I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d
```

```
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c -
I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sq
rt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8
*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16
*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*
x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d
*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c)/d
```

**maple** [A] time = 0.00, size = 376, normalized size = 1.07

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{8b}-\frac{d(\cdots}{\cdots}}{d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d))))
```

**maxima** [C] time = 0.53, size = 503, normalized size = 1.43

$$-\frac{\left(\frac{256\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{1024\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}-96\sqrt{dx+c}\,b^2\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right)-768\sqrt{dx\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8192*(256*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 102
4*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 96*sqrt(d*x +
c)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 768*sqrt(d*x + c)*b^2*sin(2*((d
*x + c)*b - b*c + a*d)/d) - (-(48*I + 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2
/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (48*I - 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*
d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) -
 (-(6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (
6*I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*
sqrt(d*x + c)*sqrt(I*b/d)) - ((6*I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*cos(-4*(b*c - a*d)/d) - (6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*si
n(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((48*I - 48)*4^(1/
4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (48*I + 48)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a),x)
[Out] Timed out
```

## 3.189 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=407

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4b}{d}\right.}{4096b}$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/3
2*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2+5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b
^2-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/8192*d^(5/2)*FresnelS(2*b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1
/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/2)*(d*x
+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/128*d^2*co
s(2*b*x+2*a)*(d*x+c)^(1/2)/b^3+15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b^3
```

**Rubi [A]** time = 0.67, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) + (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) - ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$

$$= \frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx + \frac{1}{4}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} + \frac{(5d)\int (c+ \ldots}{\ldots}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} + \frac{5d(c+dx)^{3\ldots}}{\ldots}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{15d^2\sqrt{c+\ldots}}{\ldots}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{15d^2\sqrt{c+\ldots}}{\ldots}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{15d^2\sqrt{c+\ldots}}{\ldots}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{15d^2\sqrt{c+\ldots}}{\ldots}$$

**Mathematica [A]** time = 8.39, size = 550, normalized size = 1.35

$$\frac{-1024b^3c^2\sqrt{c+dx}\,\cos(2(a+bx)) - 256b^3c^2\sqrt{c+dx}\,\cos(4(a+bx)) - 1024b^3d^2x^2\sqrt{c+dx}\,\cos(2(a+bx)) - 256b\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] + 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] - 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
rt[c + d*x]*Sin[2*(a + b*x)] + 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] +
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)
```

**fricas [A]** time = 0.53, size = 376, normalized size = 0.92

$$-\frac{15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-a}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))
*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
+ 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x +
c)*sqrt(b/(pi*d))) - 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(192*b^3*d^2*x^2 + 384*b^3*c*d*x
+ 192*b^3*c^2 + 360*b*d^2*cos(b*x + a)^2 - 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*
x + 64*b^3*c^2 - 15*b*d^2)*cos(b*x + a)^4 - 225*b*d^2 + 160*(2*(b^2*d^2*x +
 b^2*c*d)*cos(b*x + a)^3 + 3*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x +
a))*sqrt(d*x + c))/b^4
```

**giac [C]** time = 3.10, size = 2418, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16384*(512*(I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)))*c^3 + 24*c*d^2*((I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d
*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*
b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x
 + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*
(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(64*b^2
*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b^2) - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d -
 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2
```

```
+ 16*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x +
 c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/
d)/b^2)/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d
- 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*
b*c + 2*I*a*d)/d)/b^2)/d^2) + d^3*((-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 192*
I*b^2*c^2*d - 72*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^
(3/2)*b^2*c*d + 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 -
72*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(d*x + c)*b + 4
*I*b*c - 4*I*a*d)/d)/b^3)/d^3 + (I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^
2*c^2*d - 72*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^(
3/2)*b^2*c*d + 192*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 7
2*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((4*I*(d*x + c)*b - 4*I
*b*c + 4*I*a*d)/d)/b^3)/d^3 + 32*(-I*sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c^2*d
- 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^
3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*
sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*b*c*d
^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b
^3)/d^3 + 32*(I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d
^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b
*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*
x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*
c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x
 + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 192*(-I*
sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sq
rt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*d)*d*er
f(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 4*sqrt(d*x + c)*d*e^((4
*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*I*(d*x
+ c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b
+ 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*
c - 4*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.00, size = 470, normalized size = 1.15

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\cdots\right.}{8b\sqrt{\frac{b}{d}}}\cdots\right)}{4b}\cdots\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+5/64/b*d*(1/8/b*d*(d*x+c)^(3/
2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.52, size = 547, normalized size = 1.34

$$\left(1280\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)+10240\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-32\left(\frac{64\,(dx+c)^{\frac{5}{2}}b^4}{d}-15\,\sqrt{dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/65536*(1280*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) + 1024
0*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 32*(64*(d*x + c)
^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d) -
 512*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)*b
 - b*c + a*d)/d) + ((480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^
(1/4)*cos(-2*(b*c - a*d)/d) + (480*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + (
(30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) +
(30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*e
rf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2
/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2
/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + (-(4
80*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*
d)/d) - (480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2
*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

# 3.190 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=615

$$-\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}\,\sqrt{\frac{10}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\,\sqrt{\frac{6}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a-\frac{bc}{d}\right)C}{32b^{7/2}}$$

```
[Out] 5/16*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/288*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^
2-1/160*d*(d*x+c)^(3/2)*cos(5*b*x+5*a)/b^2+1/8*(d*x+c)^(5/2)*sin(b*x+a)/b-1
/48*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-1/80*(d*x+c)^(5/2)*sin(5*b*x+5*a)/b-3/16
000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/
2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelC(b^(1/2)*10^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(
7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x
+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/3456*d^(5/2)*FresnelC(b^(1/2)
*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/
b^(7/2)+15/64*d^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*FresnelC(b^(1/2)*2^
(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)
-15/32*d^2*sin(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*sin(3*b*x+3*a)*(d*x+c)^(1
/2)/b^3+3/1600*d^2*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b^3
```

**Rubi [A]** time = 1.14, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+15\sqrt{\frac{\pi}{2}}\,d^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(16*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
 + 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(160*b^2) + (15*
d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fr
esnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (3*d^(5
/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*
Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) -
(5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[(Sqrt
[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) - (15
*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(32*b^3) + ((c + d*x)^(5/2)*Sin[a + b*x])/
(8*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5/2)
*Sin[3*a + 3*b*x])/(48*b) + (3*d^2*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(1600*b^
3) - ((c + d*x)^(5/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\cos(a+bx)-\frac{1}{16}(c+dx)^{5/2}\cos(3a+3bx)-\frac{1}{16}(c$$

$$= -\left(\frac{1}{16}\int (c+dx)^{5/2}\cos(3a+3bx)\,dx\right)-\frac{1}{16}\int (c+dx)^{5/2}\cos(5a+$$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{8b}-\frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b}-\frac{(c+dx)^{5/2}\mathrm{si}}{80}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2}-\frac{d(c+d}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2}-\frac{d(c+d}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2}-\frac{d(c+d}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2}-\frac{d(c+d}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2}-\frac{d(c+d}{}$$

**Mathematica [C]** time = 22.42, size = 1795, normalized size = 2.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d]
)/Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x)
)/d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]
*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c
)/d] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sq
rt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2
*b*Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) + ((b/d)^
(3/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^
2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]
*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] +
 (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b
*(c - 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(64*b^5) -
(c^2*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[
c + d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*
a - (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*S
qrt[3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/
Pi]*Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d])
 + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c
*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]
*(Cos[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) - ((b/d)^(3
/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2
*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt
[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3
*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]
*d*Sqrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*S
in[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) - (c^2*(-(Sqrt[2*Pi]*Cos[5*a - (5*b*c
)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*FresnelC[S
qrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt[5]*Sqrt[b
/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (c*d*(Sqrt
[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[5*
a - (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*Fresne
lS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*
Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x)] + 10*
b*x*Sin[5*(a + b*x)])))/(800*Sqrt[5]*b^3) - ((b/d)^(3/2)*d^2*(-(Sqrt[2*Pi]*
FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*((20*b^2*c^2 - 3*d^2)*Cos[5*a
 - (5*b*c)/d] + 12*b*c*d*Sin[5*a - (5*b*c)/d])) - Sqrt[2*Pi]*FresnelC[Sqrt[
b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[5*a - (5*b*c)/d] + (20*b^2*c
^2 - 3*d^2)*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2
*b*(c - 5*d*x)*Cos[5*(a + b*x)] + d*(-3 + 20*b^2*x^2)*Sin[5*(a + b*x)])))/(
3200*Sqrt[5]*b^5)
```

**fricas [A]** time = 0.58, size = 548, normalized size = 0.89

$$-\frac{81\,\sqrt{10}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{5\,(bc-ad)}{d}\right)S\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+625\,\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)S\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_
sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 625*sqrt(6)*pi*d^3*sqrt(b/(pi*
d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
```

```
 - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*f
resnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*
sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-3*(b*c - a*d)/d) + 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(
sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(90*(b^2
*d^2*x + b^2*c*d)*cos(b*x + a)^5 - 50*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 300*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (120*b^3*d^2*x^2 + 240*b^3*c*d*x
 + 120*b^3*c^2 - 9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*c
os(b*x + a)^4 - 428*b*d^2 + (60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 +
11*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

**giac [C]** time = 18.66, size = 3677, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/864000*(1800*(3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sq
rt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c +
3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(10)*sqrt(pi)*d*
erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*
d^2*(9*(sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqr
t(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(10*I*(d*x + c)^(3
/2)*b*d - 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-5*I*(d*x + c
)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 +
4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)*b^2) - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x +
c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 2250*(sqrt(
2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x
+ c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^
2)/d^2 - 2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/
2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b -
 I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d
 - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 6*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^
2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2 + 9*(sqrt(10)*sqrt(
pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqr
t(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I
*a*d)/d)/b^2)/d^2) - d^3*(27*(sqrt(10)*sqrt(pi)*(200*b^3*c^3 + 60*I*b^2*c^2
*d - 18*b*c*d^2 - 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b
```

```
*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^3) + 10*(20*I*(d*x + c)^(5/2)*b^2*d - 60*I*(d*x + c)^(3/2)*b
^2*c*d + 60*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(
d*x + c)*b*c*d^2 - 3*I*sqrt(d*x + c)*d^3)*e^((-5*I*(d*x + c)*b + 5*I*b*c -
5*I*a*d)/d)/b^3)/d^3 + 125*(sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d -
 18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b^3) + 6*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d
 + 36*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x +
c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*
d)/d)/b^3)/d^3 - 6750*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*
c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b
^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*s
qrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^
2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3
 - 6750*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d
^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(4*
I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)
*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqr
t(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + 125*(sqrt(
6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2
*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c
 + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(-12*I*(d*x +
 c)^(5/2)*b^2*d + 36*I*(d*x + c)^(3/2)*b^2*c*d - 36*I*sqrt(d*x + c)*b^2*c^2
*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c
)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^3 + 27*(sqrt(10)*
sqrt(pi)*(200*b^3*c^3 - 60*I*b^2*c^2*d - 18*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*s
qrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c
 + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 10*(-20*I*(d*x +
 c)^(5/2)*b^2*d + 60*I*(d*x + c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^2*c^2
*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c
)*d^3)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^3)/d^3) - 180*(9*sqrt(
10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*(2*b*c + I*
d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)
*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(10)*sqrt(pi)*(10*b*c - I*d)*
d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*I*s
qrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d
*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*I*sqrt(d*x +
c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*I*sqrt(d*x + c)*d*e^((-I
*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d*x + c)*d*e^((-3*I*(d*x +
c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b
 + 5*I*b*c - 5*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.07, size = 716, normalized size = 1.16

$$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{F}\cdots\right.}{4b\sqrt{\frac{b}{d}}}\right.}{2b}\right.}{8b}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/16/b*d*(-1/2/b
*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1
/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1
/96/b*d*(d*x+c)^(5/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/96/b*d*(-1/6/b*d*(
d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2
)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(d*x+c)^(1/2)*b/d))))-1/160/b*d*(d*x+c)^(5/2)*sin(5/d*(d*x+c)*b+5*(a*
d-b*c)/d)+1/32/b*d*(-1/10/b*d*(d*x+c)^(3/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d
)+3/10/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/100/b
*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2
)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(5*(a*d-b*c)/d)*Fresne
lC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.57, size = 820, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3456000*sqrt(2)*(10800*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b -
b*c + a*d)/d)/d + 30000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*
c + a*d)/d)/d - 540000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c +
 a*d)/d)/d + ((162*I + 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*
(b*c - a*d)/d) - (162*I - 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(
-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + ((1250*I + 1250)*9^(1
/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (1250*I - 1250)*
9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x
+ c)*sqrt(3*I*b/d)) + (-(202500*I + 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*
cos(-(b*c - a*d)/d) + (202500*I - 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*si
n(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + ((202500*I - 202500)*sq
rt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (202500*I + 202500)*sqrt
(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b
/d)) + (-(1250*I - 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c
 - a*d)/d) + (1250*I + 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + (-(162*I - 162)*25^(1/4
)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (162*I + 162)*25^(
```

```
1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(-5*I*b/d)) + 1080*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sq
rt(d*x + c)*b^3)*sin(5*((d*x + c)*b - b*c + a*d)/d) + 3000*(12*sqrt(2)*(d*x
 + c)^(5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*sin(3*((d*x + c)*b - b*c
 + a*d)/d) - 54000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x
 + c)*b^3)*sin(((d*x + c)*b - b*c + a*d)/d))*d^2/b^6
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (a + b\,x)^3 \sin (a + b\,x)^2 \,(c + d\,x)^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.191 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=534

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)C}{800b^{5/2}}$$

```
[Out] 1/8*(d*x+c)^(3/2)*sin(b*x+a)/b-1/48*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b-1/80*(d*
x+c)^(3/2)*sin(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2
)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)-3/8000
*d^(3/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-
5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)+1/576*d^(3/2)*cos(3*a-3*b*c/d)*FresnelC(
b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/
576*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*
a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*FresnelC(b^(1
/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/32*d
^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d
)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*cos(b*x+a)*(d*x+c)^(1/2)/b^2-1/96*d*cos(3
*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.84, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*
x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(800*b^2) - (3*d^(3/2)*
Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]])/(16*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sq
rt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi
/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(800*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt
[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*
c)/d])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + ((c + d*x)^(3/2)*Sin
[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(48*b) - ((c + d*x)^(
3/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{3/2}\cos(3a+3bx) - \frac{1}{16}(c+\right.$$

$$= -\left(\frac{1}{16}\int (c+dx)^{3/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{3/2}\cos(5a+5b$$

$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{3/2}\sin(5}{80b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos}{800b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos}{800b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos}{800b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos}{800b}$$

**Mathematica [C]** time = 12.00, size = 1043, normalized size = 1.95

$$-\frac{ice^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b} + \frac{d\left(\sqrt{\frac{b}{d}}\sqrt{2\pi}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(2bc\sin\left(a-\frac{bc}{d}\right) - 3d\,c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/
Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d]
 + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2
/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*S
qrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(32*b^3) - (c*(-(Sqrt[
2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) -
Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d
] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*Sqrt[3]*b*Sqrt
[b/d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*
x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sq
rt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b
*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*
x)] + 2*b*x*Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) - (c*(-(Sqrt[2*Pi]*Cos[5*
a - (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*
FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt
[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) -
(d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*
d*Cos[5*a - (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi
]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d
] + 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x
)] + 10*b*x*Sin[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

**fricas [A]** time = 0.56, size = 446, normalized size = 0.84

$$27\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+125\,\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_co
s(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) - 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(9*b*d*cos(b*x
 + a)^5 - 5*b*d*cos(b*x + a)^3 - 30*b*d*cos(b*x + a) + 10*(3*(b^2*d*x + b^2
*c)*cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2
)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

**giac [C]** time = 7.72, size = 2293, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/144000*(300*(3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
```

```
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqr
t(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3
*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(10)*sqrt(pi)*d*e
rf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(9
*(sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(10*I*(d*x + c)^(3/2)*b*
d - 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-5*I*(d*x + c)*b +
5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*
c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2
)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 2250*(sqrt(2)*sqr
t(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b
*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2
 - 2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d
 - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c
 + I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2
)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(
-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(
(3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2 + 9*(sqrt(10)*sqrt(pi)*(1
00*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x
+ c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/
d)/b^2)/d^2) - 20*(9*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*s
qrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I
*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sq
rt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*
erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(10
)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
(b^2*d^2) + 1)*b) - 90*I*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*
I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d
)/d)/b + 900*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 90
0*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d
*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*x +
c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c)/d
```

**maple [A]** time = 0.06, size = 583, normalized size = 1.09

$$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{8b}-\frac{d(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/16/b*d*(-1/2/b
*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1
/2)*b/d)))-1/96/b*d*(d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d
*(-1/6/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*
3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1/160/b*d*(d*x+c)^(3/2)*s
in(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+3/160/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*
(d*x+c)*b+5*(a*d-b*c)/d)+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(co
s(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d))))
```

**maxima [C]** time = 0.58, size = 754, normalized size = 1.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/576000*sqrt(2)*(3600*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b - b*
c + a*d)/d)/d^2 + 6000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 36000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c +
 a*d)/d)/d^2 + 1080*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c + a*
d)/d)/d + 3000*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)
/d - 54000*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d + (
(54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (
54*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(5*I*b/d)) + ((250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*cos(-3*(b*c - a*d)/d) + (250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (-(13500
*I - 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (13500*I + 1
3500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(I*b/d)) + ((13500*I + 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) + (13500*I - 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)
)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + (-
(54*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (
54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

## 3.192 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\ \ldots}{\ldots}\right)}{8b^{3/2}}$$

```
[Out] 1/800*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelC(b^(1/2)*10^(1/2)/Pi^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(
3/2)+1/288*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/288*FresnelC(b^(1/2)*6^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^
(3/2)-1/16*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/8
*sin(b*x+a)*(d*x+c)^(1/2)/b-1/48*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b-1/80*sin(5*
b*x+5*a)*(d*x+c)^(1/2)/b
```

**Rubi** [A] time = 0.69, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\sqrt{\frac{\pi}{2}}\sqrt{d}\ \ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(8*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x
])/Sqrt[d]])/(80*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) + (Sqrt[d]*S
qrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3
*b*c)/d])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*S
qrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[a
 + b*x])/(8*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(48*b) - (Sqrt[c + d*x]*S
in[5*a + 5*b*x])/(80*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\cos(a+bx)-\frac{1}{16}\sqrt{c+dx}\cos(3a+3bx)-\frac{1}{16}\sqrt{c+dx}\right.$$

$$= -\left(\frac{1}{16}\int \sqrt{c+dx}\cos(3a+3bx)\,dx\right)-\frac{1}{16}\int \sqrt{c+dx}\cos(5a+5bx)\,d$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{48b}-\frac{\sqrt{c+dx}\sin(5a+5b}{80b}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{48b}-\frac{\sqrt{c+dx}\sin(5a+5b}{80b}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{48b}-\frac{\sqrt{c+dx}\sin(5a+5b}{80b}$$

$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\right.}{48b^{3/2}}$$

**Mathematica [C]** time = 6.94, size = 435, normalized size = 0.95

$$-\frac{-\sqrt{2\pi}\sin\left(3a-\frac{3bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin(}{96\sqrt{3}\,b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

[Out] ((-1/16*I)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sq
rt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d]
)/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) - (-(Sqrt[2*Pi]*Cos[3
*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*
FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 2*Sqrt[
3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)])/(96*Sqrt[3]*b*Sqrt[b/d]) - (-(
Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x
]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5
*b*c)/d] + 2*Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)])/(160*Sqrt[5]
*b*Sqrt[b/d])

**fricas [A]** time = 0.54, size = 365, normalized size = 0.80

$$9\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+25\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")

[Out] 1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(sq
rt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-
3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sq
rt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x
+ c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*
sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b/
(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d
)/d) + 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x + c)*sq
rt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^4 - b*cos(b*x +
a)^2 - 2*b)*sqrt(d*x + c)*sin(b*x + a))/b^2

**giac [C]** time = 2.15, size = 1258, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")

[Out] -1/14400*(9*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*
b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqr
t(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b) - 450*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c
+ 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(10)*sqrt(pi
)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b) - 30*(3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)

```
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqr
t(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3
*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(10)*sqrt(pi)*d*e
rf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 90*I*sqr
t(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d*x
 + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*I*sqrt(d*x + c)
*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*I*sqrt(d*x + c)*d*e^((-I*(
d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)
*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b +
5*I*b*c - 5*I*a*d)/d)/b)/d
```

**maple [A]** time = 0.06, size = 444, normalized size = 0.97

$$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{48b}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/32/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/
d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)
/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/160
/b*d*(d*x+c)^(1/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/1600/b*d*2^(1/2)*Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1
/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima [C]** time = 0.52, size = 674, normalized size = 1.47

$$-\frac{\sqrt{2}\left(\frac{360\sqrt{2}\,\sqrt{dx+c}\,b^3\sin\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d^2}+\frac{600\sqrt{2}\,\sqrt{dx+c}\,b^3\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{3600\sqrt{2}\,\sqrt{dx+c}\,b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2}+\left(-\frac{(18i+18)\cdot\cdots}{\cdots}\right.\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/57600*sqrt(2)*(360*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c +
a*d)/d)/d^2 + 600*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)
/d)/d^2 - 3600*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d
^2 + (-(18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)
/d)/d + (18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d
)/d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (-(50*I + 50)*9^(1/4)*sqrt(pi)*b
^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d + (50*I - 50)*9^(1/4)*sqrt(pi)*b
^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)
) + ((900*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (90
0*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x
 + c)*sqrt(I*b/d)) + (-(900*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c
 - a*d)/d)/d + (900*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((50*I - 50)*9^(1/4)*sqrt(pi)*b^2*(
```

```
b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d - (50*I + 50)*9^(1/4)*sqrt(pi)*b^2*(
b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) +
 ((18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d
 - (18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/
d)*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sin(a+b\,x)^2\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^2(a+bx)\cos^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

# 3.193 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt{}}\right.}{8b^{3/2}}$$

```
[Out] 1/800*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelC(b^(1/2)*10^(1/2)/Pi^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(
3/2)+1/288*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/288*FresnelC(b^(1/2)*6^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^
(3/2)-1/16*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/8
*sin(b*x+a)*(d*x+c)^(1/2)/b-1/48*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b-1/80*sin(5*
b*x+5*a)*(d*x+c)^(1/2)/b
```

**Rubi [A]** time = 0.67, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(8*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x
])/Sqrt[d]])/(80*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) + (Sqrt[d]*S
qrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3
*b*c)/d])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*S
qrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[a
 + b*x])/(8*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(48*b) - (Sqrt[c + d*x]*S
in[5*a + 5*b*x])/(80*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\,\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\,\cos(a+bx) - \frac{1}{16}\sqrt{c+dx}\,\cos(3a+3bx) - \frac{1}{16}\sqrt{c+d\right.$$

$$= -\left(\frac{1}{16}\int \sqrt{c+dx}\,\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int \sqrt{c+dx}\,\cos(5a+5b$$

$$= \frac{\sqrt{c+dx}\,\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\,\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\,\sin(5a+}{80b}$$

$$= \frac{\sqrt{c+dx}\,\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\,\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\,\sin(5a+}{80b}$$

$$= \frac{\sqrt{c+dx}\,\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\,\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\,\sin(5a+}{80b}$$

$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\right.}{48b^{3/2}}$$

**Mathematica [C]** time = 6.87, size = 435, normalized size = 0.95

$$-\frac{-\sqrt{2\pi}\sin\left(3a-\frac{3bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right) - \sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right) + 2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\text{si}}{96\sqrt{3}\,b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sq
rt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d]
)/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) - (-(Sqrt[2*Pi]*Cos[3
*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*
FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 2*Sqrt[
3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)])/(96*Sqrt[3]*b*Sqrt[b/d]) - (-(
Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x
]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5
*b*c)/d] + 2*Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)])/(160*Sqrt[5]
*b*Sqrt[b/d])
```

**fricas [A]** time = 0.54, size = 365, normalized size = 0.80

$$9\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+25\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-450$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(sq
rt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-
3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sq
rt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x
+ c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*
sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b/
(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d
)/d) + 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x + c)*sq
rt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^4 - b*cos(b*x +
 a)^2 - 2*b)*sqrt(d*x + c)*sin(b*x + a))/b^2
```

**giac [C]** time = 4.48, size = 1258, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/14400*(9*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*
b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqr
t(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b) - 450*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c
+ 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(10)*sqrt(pi
)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b) - 30*(3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)
```

```
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqr
t(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3
*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(10)*sqrt(pi)*d*e
rf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 90*I*sqr
t(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d*x
 + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*I*sqrt(d*x + c)
*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*I*sqrt(d*x + c)*d*e^((-I*(
d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)
*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b +
5*I*b*c - 5*I*a*d)/d)/b)/d
```

**maple [A]** time = 0.00, size = 444, normalized size = 0.97

$$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{48b}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/32/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/
d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)
/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/160
/b*d*(d*x+c)^(1/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/1600/b*d*2^(1/2)*Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1
/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima [C]** time = 0.51, size = 674, normalized size = 1.47

$$-\frac{\sqrt{2}\left(\frac{360\,\sqrt{2}\,\sqrt{dx+c}\,b^3\sin\left(\frac{5\,((dx+c)b-bc+ad)}{d}\right)}{d^2}+\frac{600\,\sqrt{2}\,\sqrt{dx+c}\,b^3\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{3600\,\sqrt{2}\,\sqrt{dx+c}\,b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2}+\left(-\frac{(18i+\cdots}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/57600*sqrt(2)*(360*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c +
a*d)/d)/d^2 + 600*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)
/d)/d^2 - 3600*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d
^2 + (-(18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)
/d)/d + (18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d
)/d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (-(50*I + 50)*9^(1/4)*sqrt(pi)*b
^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d + (50*I - 50)*9^(1/4)*sqrt(pi)*b
^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)
) + ((900*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (90
0*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x
 + c)*sqrt(I*b/d)) + (-(900*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c
 - a*d)/d)/d + (900*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((50*I - 50)*9^(1/4)*sqrt(pi)*b^2*(
```

```
b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d - (50*I + 50)*9^(1/4)*sqrt(pi)*b^2*(
b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) +
 ((18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d
 - (18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/
d)*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\sin^2\left(a+bx\right)\cos^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

## 3.194 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=534

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)C}{800b^{5/2}}$$

```
[Out] 1/8*(d*x+c)^(3/2)*sin(b*x+a)/b-1/48*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b-1/80*(d*
x+c)^(3/2)*sin(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2
)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)-3/8000
*d^(3/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-
5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)+1/576*d^(3/2)*cos(3*a-3*b*c/d)*FresnelC(
b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/
576*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*
a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*FresnelC(b^(1
/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/32*d
^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d
)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*cos(b*x+a)*(d*x+c)^(1/2)/b^2-1/96*d*cos(3
*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.86, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*
x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(800*b^2) - (3*d^(3/2)*
Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]])/(16*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sq
rt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi
/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(800*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt
[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*
c)/d])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + ((c + d*x)^(3/2)*Sin
[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(48*b) - ((c + d*x)^(
3/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{3/2}\cos(3a+3bx) - \frac{1}{16}(c+$$

$$= -\left(\frac{1}{16}\int (c+dx)^{3/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{3/2}\cos(5a+5b$$

$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{3/2}\sin(5}{80b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos}{800b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos}{800b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos}{800b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos}{800b}$$

**Mathematica [C]** time = 11.71, size = 1043, normalized size = 1.95

$$-\frac{ice^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b} + \frac{d\left(\sqrt{\frac{b}{d}}\sqrt{2\pi}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(2bc\sin\left(a-\frac{bc}{d}\right)-3d\,c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/
Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d]
 + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2
/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*S
qrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(32*b^3) - (c*(-(Sqrt[
2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) -
Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d
] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*Sqrt[3]*b*Sqrt
[b/d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*
x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sq
rt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b
*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*
x)] + 2*b*x*Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) - (c*(-(Sqrt[2*Pi]*Cos[5*
a - (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*
FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt
[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) -
(d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*
d*Cos[5*a - (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi
]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d
] + 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x
)] + 10*b*x*Sin[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

**fricas [A]** time = 0.59, size = 446, normalized size = 0.84

$$27\,\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+125\,\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_co
s(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) - 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(9*b*d*cos(b*x
 + a)^5 - 5*b*d*cos(b*x + a)^3 - 30*b*d*cos(b*x + a) + 10*(3*(b^2*d*x + b^2
*c)*cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2
)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

**giac [C]** time = 7.04, size = 2293, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/144000*(300*(3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
```

```
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqr
t(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3
*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(10)*sqrt(pi)*d*e
rf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(9
*(sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(10*I*(d*x + c)^(3/2)*b*
d - 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-5*I*(d*x + c)*b +
5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*
c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2
)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 2250*(sqrt(2)*sqr
t(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b
*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2
 - 2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d
 - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c
 + I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2
)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(
-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(
(3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2 + 9*(sqrt(10)*sqrt(pi)*(1
00*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x
+ c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/
d)/b^2)/d^2) - 20*(9*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*s
qrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I
*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sq
rt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*
erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(10
)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
(b^2*d^2) + 1)*b) - 90*I*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*
I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d
)/d)/b + 900*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 90
0*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d
*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*x +
c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c)/d
```

**maple** [A] time = 0.00, size = 583, normalized size = 1.09

$$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{8b}-d(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/16/b*d*(-1/2/b
*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1
/2)*b/d)))-1/96/b*d*(d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d
*(-1/6/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*
3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1/160/b*d*(d*x+c)^(3/2)*s
in(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+3/160/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*
(d*x+c)*b+5*(a*d-b*c)/d)+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(co
s(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d))))
```

**maxima** [C] time = 0.53, size = 754, normalized size = 1.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/576000*sqrt(2)*(3600*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b - b*
c + a*d)/d)/d^2 + 6000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 36000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c +
 a*d)/d)/d^2 + 1080*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c + a*
d)/d)/d + 3000*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)
/d - 54000*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d + (
(54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (
54*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(5*I*b/d)) + ((250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*cos(-3*(b*c - a*d)/d) + (250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (-(13500
*I - 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (13500*I + 1
3500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(I*b/d)) + ((13500*I + 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) + (13500*I - 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)
)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + (-
(54*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (
54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^5
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.195 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

**Optimal**. Leaf size=615

$$-\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a-\frac{bc}{d}\right)}{32b^{7/2}}$$

```
[Out] 5/16*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/288*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^
2-1/160*d*(d*x+c)^(3/2)*cos(5*b*x+5*a)/b^2+1/8*(d*x+c)^(5/2)*sin(b*x+a)/b-1
/48*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-1/80*(d*x+c)^(5/2)*sin(5*b*x+5*a)/b-3/16
000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/
2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelC(b^(1/2)*10^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(
7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x
+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/3456*d^(5/2)*FresnelC(b^(1/2)
*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/
b^(7/2)+15/64*d^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*FresnelC(b^(1/2)*2^
(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)
-15/32*d^2*sin(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*sin(3*b*x+3*a)*(d*x+c)^(1
/2)/b^3+3/1600*d^2*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b^3
```

**Rubi** [A] time = 1.02, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+15\sqrt{\frac{\pi}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(16*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
+ 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(160*b^2) + (15*
d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fr
esnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (3*d^(5
/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*
Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) -
(5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[(Sqrt
[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) - (15
*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(32*b^3) + ((c + d*x)^(5/2)*Sin[a + b*x])/
(8*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5/2)
*Sin[3*a + 3*b*x])/(48*b) + (3*d^2*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(1600*b^
3) - ((c + d*x)^(5/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{5/2}\cos(3a+3bx) - \frac{1}{16}(c+$$

$$= -\left(\frac{1}{16}\int (c+dx)^{5/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{5/2}\cos(5a+5b$$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{5/2}\sin(}{80b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{}}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{}}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{}}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{}}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{}}{}$$

**Mathematica [C]** time = 21.75, size = 1795, normalized size = 2.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d]
)/Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x)
)/d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]
*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c
)/d] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sq
rt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2
*b*Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) + ((b/d)^
(3/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^
2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]
*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] +
 (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b
*(c - 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(64*b^5) -
(c^2*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[
c + d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*
a - (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*S
qrt[3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/
Pi]*Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d])
 + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c
*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]
*(Cos[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) - ((b/d)^(3
/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2
*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt
[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3
*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]
*d*Sqrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*S
in[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) - (c^2*(-(Sqrt[2*Pi]*Cos[5*a - (5*b*c
)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*FresnelC[S
qrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt[5]*Sqrt[b
/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (c*d*(Sqrt
[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[5*
a - (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*Fresne
lS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*
Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x)] + 10*
b*x*Sin[5*(a + b*x)])))/(800*Sqrt[5]*b^3) - ((b/d)^(3/2)*d^2*(-(Sqrt[2*Pi]*
FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*((20*b^2*c^2 - 3*d^2)*Cos[5*a
 - (5*b*c)/d] + 12*b*c*d*Sin[5*a - (5*b*c)/d])) - Sqrt[2*Pi]*FresnelC[Sqrt[
b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[5*a - (5*b*c)/d] + (20*b^2*c
^2 - 3*d^2)*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2
*b*(c - 5*d*x)*Cos[5*(a + b*x)] + d*(-3 + 20*b^2*x^2)*Sin[5*(a + b*x)])))/(
3200*Sqrt[5]*b^5)
```

**fricas [A]** time = 0.57, size = 548, normalized size = 0.89

$$-\frac{81\sqrt{10}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+625\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_
sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 625*sqrt(6)*pi*d^3*sqrt(b/(pi*
d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
```

```
 - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*f
resnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*
sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-3*(b*c - a*d)/d) + 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(
sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(90*(b^2
*d^2*x + b^2*c*d)*cos(b*x + a)^5 - 50*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 300*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (120*b^3*d^2*x^2 + 240*b^3*c*d*x
 + 120*b^3*c^2 - 9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*c
os(b*x + a)^4 - 428*b*d^2 + (60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 +
11*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

**giac [C]**    time = 12.74, size = 3677, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/864000*(1800*(3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sq
rt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c +
3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(10)*sqrt(pi)*d*
erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*
d^2*(9*(sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqr
t(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(10*I*(d*x + c)^(3
/2)*b*d - 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-5*I*(d*x + c
)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 +
4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)*b^2) - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x +
c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 2250*(sqrt(
2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x
+ c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^
2)/d^2 - 2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/
2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b -
 I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d
 - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 6*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^
2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2 + 9*(sqrt(10)*sqrt(
pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqr
t(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I
*a*d)/d)/b^2)/d^2) - d^3*(27*(sqrt(10)*sqrt(pi)*(200*b^3*c^3 + 60*I*b^2*c^2
*d - 18*b*c*d^2 - 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b
```

```
*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^3) + 10*(20*I*(d*x + c)^(5/2)*b^2*d - 60*I*(d*x + c)^(3/2)*b
^2*c*d + 60*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(
d*x + c)*b*c*d^2 - 3*I*sqrt(d*x + c)*d^3)*e^((-5*I*(d*x + c)*b + 5*I*b*c -
5*I*a*d)/d)/b^3)/d^3 + 125*(sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d -
 18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b^3) + 6*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d
 + 36*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x +
c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*
d)/d)/b^3)/d^3 - 6750*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*
c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b
^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*s
qrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^
2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3
 - 6750*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d
^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(4*
I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)
*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqr
t(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + 125*(sqrt(
6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2
*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c
 + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(-12*I*(d*x +
 c)^(5/2)*b^2*d + 36*I*(d*x + c)^(3/2)*b^2*c*d - 36*I*sqrt(d*x + c)*b^2*c^2
*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c
)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^3 + 27*(sqrt(10)*
sqrt(pi)*(200*b^3*c^3 - 60*I*b^2*c^2*d - 18*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*s
qrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c
+ 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 10*(-20*I*(d*x +
 c)^(5/2)*b^2*d + 60*I*(d*x + c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^2*c^2
*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c
)*d^3)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^3)/d^3) - 180*(9*sqrt(
10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*(2*b*c + I*
d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)
*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(10)*sqrt(pi)*(10*b*c - I*d)*
d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*I*s
qrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d
*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*I*sqrt(d*x +
c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*I*sqrt(d*x + c)*d*e^((-I
*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d*x + c)*d*e^((-3*I*(d*x +
c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b
+ 5*I*b*c - 5*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.00, size = 716, normalized size = 1.16

$$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)S\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{Fresne}\cdots\right.}{4b\sqrt{\frac{b}{d}}}\right.}{2b}\cdots\right.}{8b}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/16/b*d*(-1/2/b
*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1
/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1
/96/b*d*(d*x+c)^(5/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/96/b*d*(-1/6/b*d*(
d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2
)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(d*x+c)^(1/2)*b/d))))-1/160/b*d*(d*x+c)^(5/2)*sin(5/d*(d*x+c)*b+5*(a*
d-b*c)/d)+1/32/b*d*(-1/10/b*d*(d*x+c)^(3/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d
)+3/10/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/100/b
*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2
)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(5*(a*d-b*c)/d)*Fresne
lC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.58, size = 820, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3456000*sqrt(2)*(10800*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b -
b*c + a*d)/d)/d + 30000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*
c + a*d)/d)/d - 540000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c +
 a*d)/d)/d + ((162*I + 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*
(b*c - a*d)/d) - (162*I - 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(
-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + ((1250*I + 1250)*9^(1
/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (1250*I - 1250)*
9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x
+ c)*sqrt(3*I*b/d)) + (-(202500*I + 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*
cos(-(b*c - a*d)/d) + (202500*I - 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*si
n(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + ((202500*I - 202500)*sq
rt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (202500*I + 202500)*sqrt
(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b
/d)) + (-(1250*I - 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c
 - a*d)/d) + (1250*I + 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + (-(162*I - 162)*25^(1/4
)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (162*I + 162)*25^(
```

```
1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(-5*I*b/d)) + 1080*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sq
rt(d*x + c)*b^3)*sin(5*((d*x + c)*b - b*c + a*d)/d) + 3000*(12*sqrt(2)*(d*x
 + c)^(5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*sin(3*((d*x + c)*b - b*c
 + a*d)/d) - 54000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x
 + c)*b^3)*sin(((d*x + c)*b - b*c + a*d)/d))*d^2/b^6
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (a+b x)^3 \sin (a+b x)^2 (c+d x)^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.196 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=407

$$\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\sin\left(6a-\frac{6bc}{d}\right)S\left(2\ldots\right)}{18432b^{7/2}}$$

```
[Out] -3/64*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(5/2)*cos(6*b*x+6*a)/b+1
5/256*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/2304*d*(d*x+c)^(3/2)*sin(6*b*x+6
*a)/b^2+5/55296*d^(5/2)*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)-5/55296*d^(5/2)*FresnelS(
2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(1/2)*
Pi^(1/2)/b^(7/2)-45/2048*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c
)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+45/2048*d^(5/2)*FresnelS(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+45/102
4*d^2*cos(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-5/9216*d^2*cos(6*b*x+6*a)*(d*x+c)^(1
/2)/b^3
```

**Rubi [A]** time = 0.90, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\sin\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Cos
[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3) +
 ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[6*a
 - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1843
2*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/3]*
FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d]
)/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*(c + d*x)
^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a + 6*b*x])
/(2304*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{5/2}\sin(2a+2bx)-\frac{1}{32}(c+dx)^{5/2}\sin(6a+6bx)\right) d$$

$$= -\left(\frac{1}{32}\int (c+dx)^{5/2}\sin(6a+6bx)\,dx\right)+\frac{3}{32}\int (c+dx)^{5/2}\sin(2a+$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b}-\frac{(5d)\int}{}$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b}+\frac{15d(c}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3}-\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}-\frac{5d^2\sqrt{}}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3}-\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}-\frac{5d^2\sqrt{}}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3}-\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}-\frac{5d^2\sqrt{}}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3}-\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}-\frac{5d^2\sqrt{}}{}$$

**Mathematica [A]** time = 5.07, size = 550, normalized size = 1.35

$$-2592b^3c^2\sqrt{c+dx}\cos(2(a+bx))+288b^3c^2\sqrt{c+dx}\cos(6(a+bx))-2592b^3d^2x^2\sqrt{c+dx}\cos(2(a+bx))+2$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-2592*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 2430*b*d^2*Sqrt[c + d*x]*Co
s[2*(a + b*x)] - 5184*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 2592*b^3*d
^2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 288*b^3*c^2*Sqrt[c + d*x]*Cos[6*(a
+ b*x)] - 30*b*d^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 576*b^3*c*d*x*Sqrt[c +
d*x]*Cos[6*(a + b*x)] + 288*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 5*
Sqrt[b/d]*d^3*Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[3/P
i]*Sqrt[c + d*x]] - 1215*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresne
lC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 5*Sqrt[b/d]*d^3*Sqrt[3*Pi]*Fresn
elS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*c)/d] + 1215*Sqrt[
b/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a -
(2*b*c)/d] + 3240*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 3240*b^2*d^2*x*S
qrt[c + d*x]*Sin[2*(a + b*x)] - 120*b^2*c*d*Sqrt[c + d*x]*Sin[6*(a + b*x)]
- 120*b^2*d^2*x*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(55296*b^4)
```

**fricas [A]** time = 0.59, size = 445, normalized size = 1.09

$$5\sqrt{3}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)C\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-5\sqrt{3}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/55296*(5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*f
resnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) -
1215*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 1215*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(24*b^3*d^2*x^2 + 2*(48*b^3*d^2*
x^2 + 96*b^3*c*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^6 + 48*b^3*c*d*x +
24*b^3*c^2 + 45*b*d^2*cos(b*x + a)^2 - 3*(48*b^3*d^2*x^2 + 96*b^3*c*d*x + 4
8*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^4 - 25*b*d^2 - 20*(2*(b^2*d^2*x + b^2*c*d
)*cos(b*x + a)^5 - 2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3 - 3*(b^2*d^2*x +
b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

**giac [C]** time = 13.77, size = 2417, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/110592*(576*(-I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) + I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) + 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) - 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)))*c^3 + 36*c*d^2*((-I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 + 8*I*b*c*d - d^2)*d
*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*
b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*
x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((-6*I*(d
*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^2)/d^2 + (I*sqrt(3)*sqrt(pi)*(48*b^2*c^
2 - 8*I*b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
```

```
2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b^2) - 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(
d*x + c)*d^2)*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b^2)/d^2 + 9*(I*s
qrt(pi)*(48*b^2*c^2 + 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*I*sqrt(d*x + c)*b*
c*d + 9*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2
)/d^2 + 9*(-I*sqrt(pi)*(48*b^2*c^2 - 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*
I*sqrt(d*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c
+ 2*I*a*d)/d)/b^2)/d^2) + d^3*((I*sqrt(3)*sqrt(pi)*(576*b^3*c^3 + 144*I*b^2
*c^2*d - 36*b*c*d^2 - 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)
*b^2*c*d - 144*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sq
rt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((-6*I*(d*x + c)*b + 6*I*b*c
 - 6*I*a*d)/d)/b^3)/d^3 + (-I*sqrt(3)*sqrt(pi)*(576*b^3*c^3 - 144*I*b^2*c^2
*d - 36*b*c*d^2 + 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)*
b^2*c*d - 144*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqr
t(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((6*I*(d*x + c)*b - 6*I*b*c +
 6*I*a*d)/d)/b^3)/d^3 + 27*(-I*sqrt(pi)*(192*b^3*c^3 + 144*I*b^2*c^2*d - 10
8*b*c*d^2 - 45*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3)
- 2*I*(48*I*(d*x + c)^(5/2)*b^2*d - 144*I*(d*x + c)^(3/2)*b^2*c*d + 144*I*s
qrt(d*x + c)*b^2*c^2*d + 60*(d*x + c)^(3/2)*b*d^2 - 108*sqrt(d*x + c)*b*c*d
^2 - 45*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b
^3)/d^3 + 27*(I*sqrt(pi)*(192*b^3*c^3 - 144*I*b^2*c^2*d - 108*b*c*d^2 + 45*
I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*
I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(48*I*
(d*x + c)^(5/2)*b^2*d - 144*I*(d*x + c)^(3/2)*b^2*c*d + 144*I*sqrt(d*x + c)
*b^2*c^2*d - 60*(d*x + c)^(3/2)*b*d^2 + 108*sqrt(d*x + c)*b*c*d^2 - 45*I*sq
rt(d*x + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 14
4*(I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b) - I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-sqrt(3)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(pi)*(12*b*c + 3*I*d)
*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c -
2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(pi)*(12*b*c
- 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*sqrt(d*x
 + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b + 54*sqrt(d*x + c)*d*
e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^((-2*I
*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^((-6*I*(d*x +
c)*b + 6*I*b*c - 6*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.05, size = 477, normalized size = 1.17

$$-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right.\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}\right.}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+15/128/b*d*(
1/4/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d
*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2
*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/384/b
*d*(d*x+c)^(5/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-5/384/b*d*(1/12/b*d*(d*x+
c)^(3/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4/b*d*(-1/12/b*d*(d*x+c)^(1/2)*
cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.53, size = 557, normalized size = 1.37

$$-\frac{\left(1920\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{6\,((dx+c)b-bc+ad)}{d}\right)-51840\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-96\left(\frac{48\,(dx+c)^{\frac{5}{2}}b^{4}}{d}-5\sqrt{dx+c}\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/884736*(1920*(d*x + c)^(3/2)*b^3*sin(6*((d*x + c)*b - b*c + a*d)/d) - 51
840*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 96*(48*(d*x +
c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(6*((d*x + c)*b - b*c + a*d)/d)
+ 2592*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)
*b - b*c + a*d)/d) + ((10*I - 10)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-6*(b*c - a*d)/d) + (10*I + 10)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*
(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) + (
-(2430*I - 2430)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c
 - a*d)/d) - (2430*I + 2430)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)
*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + ((2430*I + 2430)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (24
30*I - 2430)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a
*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) + (-(10*I + 10)*36^(1/4)*sqrt(2)*
sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (10*I - 10)*36^(1/4)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(-6*I*b/d)))*d/b^5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.197 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=351

$$\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{\frac{3}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{512b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\cos\left(6a-\frac{6bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{\ldots}}{\ldots}\right)}{1536b^{5/2}}$$

```
[Out] -3/64*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(3/2)*cos(6*b*x+6*a)/b+1
/4608*d^(3/2)*cos(6*a-6*b*c/d)*FresnelS(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)+1/4608*d^(3/2)*FresnelC(2*b^(1/2)*3
^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(1/2)*Pi^(1/2)/b^
(5/2)-9/512*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/
2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-9/512*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+9/256*d*sin(2*b*x+2*a
)*(d*x+c)^(1/2)/b^2-1/768*d*sin(6*b*x+6*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.63, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{512b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\cos\left(6\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a + 6
*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])
])/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[P
i]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)
/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2) - (d*Sq
rt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{3/2}\sin(6a+6bx)\right) d$$

$$= -\left(\frac{1}{32}\int (c+dx)^{3/2}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^{3/2}\sin(2a+$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} - \frac{d\int\sqrt{}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{d^{3/2}\sqrt{}}{}$$

**Mathematica [A]** time = 2.83, size = 391, normalized size = 1.11

$$\sqrt{3\pi}\,d\sin\left(6a-\frac{6bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right) - 81\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi}\,d\cos\left(6a-\frac{6bc}{d}\right)S\Big($$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-216*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 216*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 24*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[6*(a + b*
x)] + 24*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[6*(a + b*x)] + d*Sqrt[3*Pi]*Cos[
6*a - (6*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]] - 81*d*Sqrt
[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] +
d*Sqrt[3*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*
c)/d] - 81*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*
a - (2*b*c)/d] + 162*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 6*Sqrt[b/
d]*d*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(4608*b^2*Sqrt[b/d])
```

**fricas [A]** time = 0.56, size = 326, normalized size = 0.93

$$\sqrt{3}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{6\,(bc-ad)}{d}\right)S\left(2\,\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+\sqrt{3}\,\pi d^2\sqrt{\frac{b}{\pi d}}\;C\left(2\,\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-81\,\pi$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4608*(sqrt(3)*pi*d^2*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_sin(2*s
qrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(3)*pi*d^2*sqrt(b/(pi*d))*fresne
l_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 81*pi
*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(
b/(pi*d))) - 81*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) + 96*(8*(b^2*d*x + b^2*c)*cos(b*x + a)^6 - 12*
(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 2*b^2*d*x + 2*b^2*c - (2*b*d*cos(b*x + a
)^5 - 2*b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c
))/b^3
```

**giac [C]** time = 12.68, size = 1502, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/9216*(48*(-I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) + 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)) - 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^2 + d^2*((-I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 + 8*I*b*c*d - d^2)*d*erf(-sq
rt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*x + c)^(
3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((-6*I*(d*x + c)*
b + 6*I*b*c - 6*I*a*d)/d)/b^2)/d^2 + (I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 - 8*I*
b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2
) - 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)
*d^2)*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b^2)/d^2 + 9*(I*sqrt(pi)*
(48*b^2*c^2 + 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*I*sqrt(d*x + c)*b*c*d + 9*
sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)/d^2 +
9*(-I*sqrt(pi)*(48*b^2*c^2 - 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*I*sqrt(d
```

```
*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*
d)/d)/b^2)/d^2) + 8*(I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqrt(3)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*
d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6
*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(pi
)*(12*b*c + 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I
*sqrt(pi)*(12*b*c - 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b) - 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b + 5
4*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 54*sqrt(d
*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 6*sqrt(d*x + c)*
d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)*c)/d
```

**maple [A]** time = 0.04, size = 383, normalized size = 1.09

$$-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)S\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{64b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+9/128/b*d*(1
/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b
/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
+1/384/b*d*(d*x+c)^(3/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/128/b*d*(1/12/b
*d*(d*x+c)^(1/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/144/b*d*2^(1/2)*Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)
/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

**maxima [C]** time = 0.51, size = 513, normalized size = 1.46

$$\left(\frac{384\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{6\,((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{3456\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d}-96\sqrt{dx+c}\,b^2\sin\left(\frac{6\,((dx+c)b-bc+ad)}{d}\right)+2592\sqrt{dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/73728*(384*(d*x + c)^(3/2)*b^3*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 345
6*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 96*sqrt(d*x +
c)*b^2*sin(6*((d*x + c)*b - b*c + a*d)/d) + 2592*sqrt(d*x + c)*b^2*sin(2*((
d*x + c)*b - b*c + a*d)/d) - (-(2*I + 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2
/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) + (2*I - 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) -
((162*I + 162)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a
*d)/d) - (162*I - 162)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (-(162*I - 162)*4^(1/4)*
sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (162*I + 162)*
4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
```

```
t(d*x + c)*sqrt(-2*I*b/d)) - ((2*I - 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/
d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (2*I + 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d
/b^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)^3 \sin(a+bx)^3 (c+dx)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
[Out] Timed out
```

# 3.198 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=299

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(6a-\frac{6bc}{d}\right)S\left(\right.}{384b^{3/2}}$$

```
[Out] -1/1152*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*d^(1/2)*3^(1/2)*Pi^(1/2)/b^(3/2)+1/1152*FresnelS(2*b^(1/2)*3^(1/2)
/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*d^(1/2)*3^(1/2)*Pi^(1/2)/
b^(3/2)+3/128*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^
(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-3/128*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1
/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-3/64*cos(2*b*x+2*a)
*(d*x+c)^(1/2)/b+1/192*cos(6*b*x+6*a)*(d*x+c)^(1/2)/b
```

**Rubi [A]** time = 0.46, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\,\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b) + (Sqrt[c + d*x]*Cos[6*a + 6*b*x
])/(192*b) - (Sqrt[d]*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*S
qrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(384*b^(3/2)) + (3*Sqrt[d]*Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(1
28*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d
*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(384*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi]*Fre
snelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(
128*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}\sqrt{c+dx}\sin(2a+2bx) - \frac{1}{32}\sqrt{c+dx}\sin(6a+6bx)\right)dx$$

$$= -\left(\frac{1}{32}\int \sqrt{c+dx}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int \sqrt{c+dx}\sin(2a+2bx)\,d$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{d\int \frac{\cos(6a+6bx)}{\sqrt{c+dx}}}{384b}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\left(d\cos\left(6a-\right.\right.}{}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\cos\left(6a-\frac{6bc}{d}\right)}{}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\cos\left(6\right.}{}$$

**Mathematica [A]** time = 1.25, size = 264, normalized size = 0.88

$$\frac{-\sqrt{3\pi}\cos\left(6a-\frac{6bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right)+27\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+\sqrt{3\pi}\sin\left(6a-\frac{6bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\right.}{115}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-54*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 6*Sqrt[b/d]*Sqrt[c + d*x]*C
os[6*(a + b*x)] - Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[3/Pi]*Sqrt[c + d*x]] + 27*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b
/d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[3*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*S
qrt[c + d*x]]*Sin[6*a - (6*b*c)/d] - 27*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt
[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(1152*b*Sqrt[b/d])
```

**fricas [A]** time = 0.51, size = 242, normalized size = 0.81

$$-\frac{\sqrt{3}\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{3}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-27\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/1152*(sqrt(3)*pi*d*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(2*sq
rt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(3)*pi*d*sqrt(b/(pi*d))*fresnel_s
in(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 27*pi*d*
sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi
*d))) + 27*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-2*(b*c - a*d)/d) - 48*(4*b*cos(b*x + a)^6 - 6*b*cos(b*x + a)^4 + b)*sq
rt(d*x + c))/b^2
```

**giac [C]** time = 7.83, size = 818, normalized size = 2.74

$$-\frac{\frac{i\,\sqrt{3}\,\sqrt{\pi}\,(12\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{3}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{6i\,bc-6i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{i\,\sqrt{3}\,\sqrt{\pi}\,(12\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{3}\,\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-6i\,bc+6i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+12\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2304*(I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-sqrt(
3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 12*(-I*sqrt(3)*sqrt(pi)
*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*
I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(3)*sqrt(
pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 9*I*sqrt(
pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c
 - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(pi)*d*erf(-
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 9*I*sqrt(pi)*(12*b*c + 3*
I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*
c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(pi)*(12*
b*c - 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*sqrt
(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b + 54*sqrt(d*x + c
)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^((
-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^((-6*I*(d*
x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)/d
```

**maple [A]** time = 0.04, size = 293, normalized size = 0.98

$$\frac{-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{dx+c}\,\cos\left(\frac{6(dx+c)b}{d}+\frac{6da-6c\cdots}{d}\right)}{192b}\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/256/b*d*Pi
^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d))+1/384/b*d*(d*x+c)^(1/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4608/
b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima [C]** time = 0.52, size = 435, normalized size = 1.45

$$\frac{\left(\frac{96\sqrt{dx+c}\,b^2\cos\left(\frac{6((dx+c)b-bc+ad)}{d}\right)}{d} - \frac{864\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d} + \left((2i-2)\cdot 36^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{6(bc-ad)}{d}\right) + (2i + \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18432*(96*sqrt(d*x + c)*b^2*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 864*sq
rt(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d + ((2*I - 2)*36^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) + (2*I + 2)*36^(1/
4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(6*I*b/d)) + (-(54*I - 54)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/
4)*cos(-2*(b*c - a*d)/d) - (54*I + 54)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)
^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + ((54*I + 5
4)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (54*I
 - 54)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*er
f(sqrt(d*x + c)*sqrt(-2*I*b/d)) + (-(2*I + 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(
b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (2*I - 2)*36^(1/4)*sqrt(2)*sqrt(pi)*
b*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))
*d/b^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a + b\,x)^3 \sin(a + b\,x)^3 \sqrt{c + d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.199 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=299

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(6a-\frac{6bc}{d}\right)S\left(\cdots\right.}{384b^{3/2}}$$

```
[Out] -1/1152*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*d^(1/2)*3^(1/2)*Pi^(1/2)/b^(3/2)+1/1152*FresnelS(2*b^(1/2)*3^(1/2)
/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*d^(1/2)*3^(1/2)*Pi^(1/2)/
b^(3/2)+3/128*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^
(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-3/128*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1
/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-3/64*cos(2*b*x+2*a)
*(d*x+c)^(1/2)/b+1/192*cos(6*b*x+6*a)*(d*x+c)^(1/2)/b
```

**Rubi** [A] time = 0.45, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\,\text{si}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b) + (Sqrt[c + d*x]*Cos[6*a + 6*b*x
])/(192*b) - (Sqrt[d]*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*S
qrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(384*b^(3/2)) + (3*Sqrt[d]*Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(1
28*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d
*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(384*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi]*Fre
snelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(
128*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}\sqrt{c+dx}\sin(2a+2bx) - \frac{1}{32}\sqrt{c+dx}\sin(6a+6bx)\right)dx$$

$$= -\left(\frac{1}{32}\int \sqrt{c+dx}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int \sqrt{c+dx}\sin(2a+2bx)\,d$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{d\int \frac{\cos(6a+6bx)}{\sqrt{c+dx}}}{384b}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\left(d\cos\left(6a-\frac{6}{}\right.\right.}{}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\cos\left(6a-\frac{6bc}{d}\right)}{}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\cos\left(6\right.}{}$$

**Mathematica [A]** time = 0.52, size = 264, normalized size = 0.88

$$\frac{-\sqrt{3\pi}\cos\left(6a-\frac{6bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right) + 27\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi}\sin\left(6a-\frac{6bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\right.}{115}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-54*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 6*Sqrt[b/d]*Sqrt[c + d*x]*C
os[6*(a + b*x)] - Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[3/Pi]*Sqrt[c + d*x]] + 27*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b
/d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[3*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*S
qrt[c + d*x]]*Sin[6*a - (6*b*c)/d] - 27*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt
[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(1152*b*Sqrt[b/d])
```

**fricas [A]** time = 0.56, size = 242, normalized size = 0.81

$$-\frac{\sqrt{3}\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{3}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-27}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/1152*(sqrt(3)*pi*d*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(2*sq
rt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(3)*pi*d*sqrt(b/(pi*d))*fresnel_s
in(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 27*pi*d*
sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi
*d))) + 27*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-2*(b*c - a*d)/d) - 48*(4*b*cos(b*x + a)^6 - 6*b*cos(b*x + a)^4 + b)*sq
rt(d*x + c))/b^2
```

**giac [C]** time = 3.74, size = 818, normalized size = 2.74

$$-\frac{\frac{i\,\sqrt{3}\,\sqrt{\pi}\,(12\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{3}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{6i\,bc-6i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{i\,\sqrt{3}\,\sqrt{\pi}\,(12\,bc-i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{3}\,\sqrt{bd}\,\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-6i\,bc+6i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2304*(I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-sqrt(
3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 12*(-I*sqrt(3)*sqrt(pi)
*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*
I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(3)*sqrt(
pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 9*I*sqrt(
pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c
 - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(pi)*d*erf(-
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 9*I*sqrt(pi)*(12*b*c + 3*
I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*
c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(pi)*(12*
b*c - 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*sqrt
(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b + 54*sqrt(d*x + c
)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^((
-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^((-6*I*(d*
x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)/d
```

**maple [A]** time = 0.00, size = 293, normalized size = 0.98

$$\frac{-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{dx+c}\,\cos\left(\frac{6(dx+c)b}{d}+\frac{6da-6c}{d}\right)}{192b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/256/b*d*Pi
^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d))+1/384/b*d*(d*x+c)^(1/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4608/
b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

**maxima [C]** time = 0.51, size = 435, normalized size = 1.45

$$\frac{\left(\frac{96\sqrt{dx+c}\,b^2\cos\left(\frac{6((dx+c)b-bc+ad)}{d}\right)}{d} - \frac{864\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d} + \left((2i-2)\cdot 36^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{6(bc-ad)}{d}\right) + (2i+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18432*(96*sqrt(d*x + c)*b^2*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 864*sq
rt(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d + ((2*I - 2)*36^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) + (2*I + 2)*36^(1/
4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(6*I*b/d)) + (-(54*I - 54)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/
4)*cos(-2*(b*c - a*d)/d) - (54*I + 54)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)
^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + ((54*I + 5
4)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (54*I
 - 54)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*er
f(sqrt(d*x + c)*sqrt(-2*I*b/d)) + (-(2*I + 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(
b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (2*I - 2)*36^(1/4)*sqrt(2)*sqrt(pi)*
b*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))
*d/b^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.200 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=351

$$\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\cos\left(6a-\frac{6bc}{d}\right)S\left(\frac{2\sqrt{}}{}\right.}{1536b^{5/2}}$$

```
[Out] -3/64*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(3/2)*cos(6*b*x+6*a)/b+1
/4608*d^(3/2)*cos(6*a-6*b*c/d)*FresnelS(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)+1/4608*d^(3/2)*FresnelC(2*b^(1/2)*3
^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(1/2)*Pi^(1/2)/b^
(5/2)-9/512*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/
2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-9/512*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+9/256*d*sin(2*b*x+2*a
)*(d*x+c)^(1/2)/b^2-1/768*d*sin(6*b*x+6*a)*(d*x+c)^(1/2)/b^2
```

**Rubi [A]** time = 0.56, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{512b^{5/2}}+\sqrt{\frac{\pi}{3}}\,d^{3/2}\cos$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a + 6
*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])
])/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[P
i]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)
/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2) - (d*Sq
rt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{3/2}\sin(6a+6bx)\right)dx$$

$$= -\left(\frac{1}{32}\int (c+dx)^{3/2}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^{3/2}\sin(2a+2b$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} - \frac{d\int\sqrt{c+}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+d}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+d}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+d}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{d^{3/2}\sqrt{\frac{\pi}{3}}\,c}{}$$

**Mathematica [A]** time = 0.20, size = 391, normalized size = 1.11

$$\sqrt{3\pi}\,d\sin\left(6a-\frac{6bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right) - 81\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi}\,d\cos\left(6a-\frac{6bc}{d}\right)S\left(2\sqrt{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

[Out] (-216*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 216*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 24*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[6*(a + b*
x)] + 24*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[6*(a + b*x)] + d*Sqrt[3*Pi]*Cos[
6*a - (6*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]] - 81*d*Sqrt
[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] +
d*Sqrt[3*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*
c)/d] - 81*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*
a - (2*b*c)/d] + 162*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 6*Sqrt[b/
d]*d*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(4608*b^2*Sqrt[b/d])

**fricas [A]** time = 0.58, size = 326, normalized size = 0.93

$$\frac{\sqrt{3}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{6\,(bc-ad)}{d}\right)S\left(2\,\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+\sqrt{3}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,C\left(2\,\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")

[Out] 1/4608*(sqrt(3)*pi*d^2*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_sin(2*s
qrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(3)*pi*d^2*sqrt(b/(pi*d))*fresne
l_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 81*pi
*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(
b/(pi*d))) - 81*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) + 96*(8*(b^2*d*x + b^2*c)*cos(b*x + a)^6 - 12*
(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 2*b^2*d*x + 2*b^2*c - (2*b*d*cos(b*x + a
)^5 - 2*b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c
))/b^3

**giac [C]** time = 13.12, size = 1502, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")

[Out] -1/9216*(48*(-I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) + 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)) - 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^2 + d^2*((-I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 + 8*I*b*c*d - d^2)*d*erf(-sq
rt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*x + c)^(
3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((-6*I*(d*x + c)*
b + 6*I*b*c - 6*I*a*d)/d)/b^2)/d^2 + (I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 - 8*I*
b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2
) - 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)
*d^2)*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b^2)/d^2 + 9*(I*sqrt(pi)*
(48*b^2*c^2 + 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*I*sqrt(d*x + c)*b*c*d + 9*
sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)/d^2 +
9*(-I*sqrt(pi)*(48*b^2*c^2 - 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*I*sqrt(d

```
*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*
d)/d)/b^2)/d^2) + 8*(I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqrt(3)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*
d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6
*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(pi
)*(12*b*c + 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I
*sqrt(pi)*(12*b*c - 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b) - 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b + 5
4*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 54*sqrt(d
*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 6*sqrt(d*x + c)*
d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)*c)/d
```

**maple** [A] time = 0.00, size = 383, normalized size = 1.09

$$\frac{-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)S\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{64b}+\frac{d(dx}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+9/128/b*d*(1
/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b
/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
+1/384/b*d*(d*x+c)^(3/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/128/b*d*(1/12/b
*d*(d*x+c)^(1/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/144/b*d*2^(1/2)*Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)
/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

**maxima** [C] time = 0.54, size = 513, normalized size = 1.46

$$\left(\frac{384\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{6\,((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{3456\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d}-96\,\sqrt{dx+c}\,b^2\sin\left(\frac{6\,((dx+c)b-bc+ad)}{d}\right)+2592\,\sqrt{dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/73728*(384*(d*x + c)^(3/2)*b^3*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 345
6*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 96*sqrt(d*x +
c)*b^2*sin(6*((d*x + c)*b - b*c + a*d)/d) + 2592*sqrt(d*x + c)*b^2*sin(2*((
d*x + c)*b - b*c + a*d)/d) - (-(2*I + 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2
/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) + (2*I - 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) -
((162*I + 162)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a
*d)/d) - (162*I - 162)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (-(162*I - 162)*4^(1/4)*
sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (162*I + 162)*
4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
```

```
t(d*x + c)*sqrt(-2*I*b/d)) - ((2*I - 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/
d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (2*I + 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d
/b^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)^3 \sin(a+bx)^3 (c+dx)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
[Out] Timed out
```

# 3.201 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

**Optimal**. Leaf size=407

$$\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\sin\left(6a-\frac{6bc}{d}\right)S\left(2\ldots\right.}{18432b^{7/2}}$$

```
[Out] -3/64*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(5/2)*cos(6*b*x+6*a)/b+1
5/256*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/2304*d*(d*x+c)^(3/2)*sin(6*b*x+6
*a)/b^2+5/55296*d^(5/2)*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)-5/55296*d^(5/2)*FresnelS(
2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(1/2)*
Pi^(1/2)/b^(7/2)-45/2048*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c
)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+45/2048*d^(5/2)*FresnelS(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+45/102
4*d^2*cos(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-5/9216*d^2*cos(6*b*x+6*a)*(d*x+c)^(1
/2)/b^3
```

**Rubi [A]** time = 0.67, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2048b^{7/2}}-5\sqrt{\frac{\pi}{3}}\,d^{5/2}\sin\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Cos
[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3) +
 ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[6*a
 - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1843
2*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/3]*
FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d]
)/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*(c + d*x)
^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a + 6*b*x])
/(2304*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{5/2}\sin(2a+2bx)-\frac{1}{32}(c+dx)^{5/2}\sin(6a+6bx)\right) d$$

$$= -\left(\frac{1}{32}\int (c+dx)^{5/2}\sin(6a+6bx)\,dx\right)+\frac{3}{32}\int (c+dx)^{5/2}\sin(2a+$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b}-\frac{(5d)\int}{}$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b}+\frac{15d(c}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3}-\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}-\frac{5d^2\sqrt{}}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3}-\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}-\frac{5d^2\sqrt{}}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3}-\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}-\frac{5d^2\sqrt{}}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3}-\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}-\frac{5d^2\sqrt{}}{}$$

**Mathematica [A]** time = 2.74, size = 550, normalized size = 1.35

$$-2592b^3c^2\sqrt{c+dx}\cos(2(a+bx))+288b^3c^2\sqrt{c+dx}\cos(6(a+bx))-2592b^3d^2x^2\sqrt{c+dx}\cos(2(a+bx))+2$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-2592*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 2430*b*d^2*Sqrt[c + d*x]*Co
s[2*(a + b*x)] - 5184*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 2592*b^3*d
^2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 288*b^3*c^2*Sqrt[c + d*x]*Cos[6*(a
+ b*x)] - 30*b*d^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 576*b^3*c*d*x*Sqrt[c +
d*x]*Cos[6*(a + b*x)] + 288*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 5*
Sqrt[b/d]*d^3*Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[3/P
i]*Sqrt[c + d*x]] - 1215*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresne
lC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 5*Sqrt[b/d]*d^3*Sqrt[3*Pi]*Fresn
elS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*c)/d] + 1215*Sqrt[
b/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a -
(2*b*c)/d] + 3240*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 3240*b^2*d^2*x*S
qrt[c + d*x]*Sin[2*(a + b*x)] - 120*b^2*c*d*Sqrt[c + d*x]*Sin[6*(a + b*x)]
- 120*b^2*d^2*x*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(55296*b^4)
```

**fricas [A]** time = 0.61, size = 445, normalized size = 1.09

$$5\sqrt{3}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-5\sqrt{3}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/55296*(5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*f
resnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) -
1215*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 1215*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(24*b^3*d^2*x^2 + 2*(48*b^3*d^2*
x^2 + 96*b^3*c*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^6 + 48*b^3*c*d*x +
24*b^3*c^2 + 45*b*d^2*cos(b*x + a)^2 - 3*(48*b^3*d^2*x^2 + 96*b^3*c*d*x + 4
8*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^4 - 25*b*d^2 - 20*(2*(b^2*d^2*x + b^2*c*d
)*cos(b*x + a)^5 - 2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3 - 3*(b^2*d^2*x +
b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

**giac [C]** time = 16.88, size = 2417, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/110592*(576*(-I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) + I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) + 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) - 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)))*c^3 + 36*c*d^2*((-I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 + 8*I*b*c*d - d^2)*d
*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*
b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*
x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((-6*I*(d
*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^2)/d^2 + (I*sqrt(3)*sqrt(pi)*(48*b^2*c^
2 - 8*I*b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
```

```
2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b^2) - 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(
d*x + c)*d^2)*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b^2)/d^2 + 9*(I*s
qrt(pi)*(48*b^2*c^2 + 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*I*sqrt(d*x + c)*b*
c*d + 9*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2
)/d^2 + 9*(-I*sqrt(pi)*(48*b^2*c^2 - 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*
I*sqrt(d*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c
+ 2*I*a*d)/d)/b^2)/d^2) + d^3*((I*sqrt(3)*sqrt(pi)*(576*b^3*c^3 + 144*I*b^2
*c^2*d - 36*b*c*d^2 - 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)
*b^2*c*d - 144*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sq
rt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((-6*I*(d*x + c)*b + 6*I*b*c
 - 6*I*a*d)/d)/b^3)/d^3 + (-I*sqrt(3)*sqrt(pi)*(576*b^3*c^3 - 144*I*b^2*c^2
*d - 36*b*c*d^2 + 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)*
b^2*c*d - 144*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqr
t(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((6*I*(d*x + c)*b - 6*I*b*c +
 6*I*a*d)/d)/b^3)/d^3 + 27*(-I*sqrt(pi)*(192*b^3*c^3 + 144*I*b^2*c^2*d - 10
8*b*c*d^2 - 45*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3)
- 2*I*(48*I*(d*x + c)^(5/2)*b^2*d - 144*I*(d*x + c)^(3/2)*b^2*c*d + 144*I*s
qrt(d*x + c)*b^2*c^2*d + 60*(d*x + c)^(3/2)*b*d^2 - 108*sqrt(d*x + c)*b*c*d
^2 - 45*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b
^3)/d^3 + 27*(I*sqrt(pi)*(192*b^3*c^3 - 144*I*b^2*c^2*d - 108*b*c*d^2 + 45*
I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*
I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(48*I*
(d*x + c)^(5/2)*b^2*d - 144*I*(d*x + c)^(3/2)*b^2*c*d + 144*I*sqrt(d*x + c)
*b^2*c^2*d - 60*(d*x + c)^(3/2)*b*d^2 + 108*sqrt(d*x + c)*b*c*d^2 - 45*I*sq
rt(d*x + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 14
4*(I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b) - I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-sqrt(3)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(pi)*(12*b*c + 3*I*d)
*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c -
2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(pi)*(12*b*c
- 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*sqrt(d*x
 + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b + 54*sqrt(d*x + c)*d*
e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^((-2*I
*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^((-6*I*(d*x +
c)*b + 6*I*b*c - 6*I*a*d)/d)/b)*c^2)/d
```

**maple [A]** time = 0.00, size = 477, normalized size = 1.17

$$-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right.\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}\right.}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+15/128/b*d*(
1/4/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d
*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2
*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/384/b
*d*(d*x+c)^(5/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-5/384/b*d*(1/12/b*d*(d*x+
c)^(3/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4/b*d*(-1/12/b*d*(d*x+c)^(1/2)*
cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))))
```

**maxima [C]** time = 0.56, size = 557, normalized size = 1.37

$$-\frac{\left(1920\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{6\,((dx+c)b-bc+ad)}{d}\right)-51840\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-96\left(\frac{48\,(dx+c)^{\frac{5}{2}}b^{4}}{d}-5\sqrt{dx+c}\,b^{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/884736*(1920*(d*x + c)^(3/2)*b^3*sin(6*((d*x + c)*b - b*c + a*d)/d) - 51
840*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 96*(48*(d*x +
c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(6*((d*x + c)*b - b*c + a*d)/d)
+ 2592*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)
*b - b*c + a*d)/d) + ((10*I - 10)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-6*(b*c - a*d)/d) + (10*I + 10)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*
(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) + (
-(2430*I - 2430)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c
 - a*d)/d) - (2430*I + 2430)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)
*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + ((2430*I + 2430)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (24
30*I - 2430)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a
*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) + (-(10*I + 10)*36^(1/4)*sqrt(2)*
sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (10*I - 10)*36^(1/4)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(-6*I*b/d)))*d/b^5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.202 $\int x^3 \cos^2(x) \cot^2(x) \, dx$

**Optimal**. Leaf size=112

$$-3ix\text{Li}_2\left(e^{2ix}\right)+\frac{3}{2}\text{Li}_3\left(e^{2ix}\right)-\frac{3x^4}{8}-ix^3-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)+\frac{3x^2}{8}+3x^2\log\left(1-e^{2ix}\right)-\frac{3}{4}x^2\cos^2(x)+\frac{3\cos^2(x)}{8}$$

```
[Out] 3/8*x^2-I*x^3-3/8*x^4+3/8*cos(x)^2-3/4*x^2*cos(x)^2-x^3*cot(x)+3*x^2*ln(1-e
xp(2*I*x))-3*I*x*polylog(2,exp(2*I*x))+3/2*polylog(3,exp(2*I*x))+3/4*x*cos(
x)*sin(x)-1/2*x^3*cos(x)*sin(x)
```

**Rubi [A]** time = 0.19, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4408, 3311, 30, 3310, 3720, 3717, 2190, 2531, 2282, 6589}

$$-3ix\text{PolyLog}\left(2,e^{2ix}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{2ix}\right)-\frac{3x^4}{8}-ix^3+\frac{3x^2}{8}+3x^2\log\left(1-e^{2ix}\right)-\frac{3}{4}x^2\cos^2(x)-x^3\cot(x)-\frac{1}{2}x^3\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] (3*x^2)/8 - I*x^3 - (3*x^4)/8 + (3*Cos[x]^2)/8 - (3*x^2*Cos[x]^2)/4 - x^3*C
ot[x] + 3*x^2*Log[1 - E^((2*I)*x)] - (3*I)*x*PolyLog[2, E^((2*I)*x)] + (3*P
olyLog[3, E^((2*I)*x)])/2 + (3*x*Cos[x]*Sin[x])/4 - (x^3*Cos[x]*Sin[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
```

```
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\cos^2(x)\cot^2(x)\,dx = -\int x^3\cos^2(x)\,dx + \int x^3\cot^2(x)\,dx$$
$$= -\frac{3}{4}x^2\cos^2(x) - x^3\cot(x) - \frac{1}{2}x^3\cos(x)\sin(x) - \frac{\int x^3\,dx}{2} + \frac{3}{2}\int x\cos^2(x)\,dx + 3\int x$$
$$= -ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4}x^2\cos^2(x) - x^3\cot(x) + \frac{3}{4}x\cos(x)\sin(x) - \frac{1}{2}x^3\cos(x)\mathrm{s}$$
$$= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4}x^2\cos^2(x) - x^3\cot(x) + 3x^2\log\left(1-e^{2ix}\right) + \frac{3}{4}x\cos$$
$$= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4}x^2\cos^2(x) - x^3\cot(x) + 3x^2\log\left(1-e^{2ix}\right) - 3ix\mathrm{Li}_2$$
$$= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4}x^2\cos^2(x) - x^3\cot(x) + 3x^2\log\left(1-e^{2ix}\right) - 3ix\mathrm{Li}_2$$
$$= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4}x^2\cos^2(x) - x^3\cot(x) + 3x^2\log\left(1-e^{2ix}\right) - 3ix\mathrm{Li}_2$$

**Mathematica [A]** time = 0.17, size = 104, normalized size = 0.93

$$\frac{1}{16}\left(48ix\text{Li}_2\left(e^{-2ix}\right)+24\text{Li}_3\left(e^{-2ix}\right)-6x^4+16ix^3-4x^3\sin(2x)-16x^3\cot(x)+48x^2\log\left(1-e^{-2ix}\right)-6x^2\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] ((-2*I)*Pi^3 + (16*I)*x^3 - 6*x^4 + 3*Cos[2*x] - 6*x^2*Cos[2*x] - 16*x^3*Co
t[x] + 48*x^2*Log[1 - E^((-2*I)*x)] + (48*I)*x*PolyLog[2, E^((-2*I)*x)] + 2
4*PolyLog[3, E^((-2*I)*x)] + 6*x*Sin[2*x] - 4*x^3*Sin[2*x])/16
```

**fricas [C]** time = 0.51, size = 244, normalized size = 2.18

$$\frac{4\left(2x^3-3x\right)\cos(x)^3+24x^2\log\left(\cos(x)+i\sin(x)+1\right)\sin(x)+24x^2\log\left(\cos(x)-i\sin(x)+1\right)\sin(x)+24x^2\text{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(2*x^3 - 3*x)*cos(x)^3 + 24*x^2*log(cos(x) + I*sin(x) + 1)*sin(x) +
 24*x^2*log(cos(x) - I*sin(x) + 1)*sin(x) + 24*x^2*log(-cos(x) + I*sin(x) +
 1)*sin(x) + 24*x^2*log(-cos(x) - I*sin(x) + 1)*sin(x) - 48*I*x*dilog(cos(x
) + I*sin(x))*sin(x) + 48*I*x*dilog(cos(x) - I*sin(x))*sin(x) + 48*I*x*dilo
g(-cos(x) + I*sin(x))*sin(x) - 48*I*x*dilog(-cos(x) - I*sin(x))*sin(x) - 12
*(2*x^3 - x)*cos(x) - 3*(2*x^4 + 2*(2*x^2 - 1)*cos(x)^2 - 2*x^2 + 1)*sin(x)
 + 48*polylog(3, cos(x) + I*sin(x))*sin(x) + 48*polylog(3, cos(x) - I*sin(x
))*sin(x) + 48*polylog(3, -cos(x) + I*sin(x))*sin(x) + 48*polylog(3, -cos(x
) - I*sin(x))*sin(x))/sin(x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cos(x)^2\cot(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(x)^2*cot(x)^2, x)
```

**maple [A]** time = 0.12, size = 150, normalized size = 1.34

$$-\frac{3x^4}{8}+\frac{i\left(4x^3+6ix^2-6x-3i\right)e^{2ix}}{32}-\frac{i\left(4x^3-6ix^2-6x+3i\right)e^{-2ix}}{32}-\frac{2ix^3}{e^{2ix}-1}-2ix^3+3x^2\ln\left(1+e^{ix}\right)-6ix\,\text{polylog}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x)^2*cot(x)^2,x)
```

```
[Out] -3/8*x^4+1/32*I*(6*I*x^2+4*x^3-3*I-6*x)*exp(2*I*x)-1/32*I*(-6*I*x^2+4*x^3+3
*I-6*x)*exp(-2*I*x)-2*I*x^3/(exp(2*I*x)-1)-2*I*x^3+3*x^2*ln(1+exp(I*x))-6*I
*x*polylog(2,-exp(I*x))+6*polylog(3,-exp(I*x))+3*x^2*ln(1-exp(I*x))-6*I*x*p
olylog(2,exp(I*x))+6*polylog(3,exp(I*x))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \cos(x)^2 \cot(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x)^2*cot(x)^2,x)
```

```
[Out] int(x^3*cos(x)^2*cot(x)^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cos^2(x) \cot^2(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x)**2*cot(x)**2,x)
```

```
[Out] Integral(x**3*cos(x)**2*cot(x)**2, x)
```

# 3.203 $\int x^2 \cos^2(x) \cot^2(x) \, dx$

**Optimal**. Leaf size=83

$$-i\text{Li}_2\left(e^{2ix}\right)-\frac{x^3}{2}-ix^2-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{x}{4}+2x\log\left(1-e^{2ix}\right)-\frac{1}{2}x\cos^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] 1/4*x-I*x^2-1/2*x^3-1/2*x*cos(x)^2-x^2*cot(x)+2*x*ln(1-exp(2*I*x))-I*polylo
g(2,exp(2*I*x))+1/4*cos(x)*sin(x)-1/2*x^2*cos(x)*sin(x)
```

**Rubi [A]** time = 0.17, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4408, 3311, 30, 2635, 8, 3720, 3717, 2190, 2279, 2391}

$$-i\text{PolyLog}\left(2,e^{2ix}\right)-\frac{x^3}{2}-ix^2-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{x}{4}+2x\log\left(1-e^{2ix}\right)-\frac{1}{2}x\cos^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] x/4 - I*x^2 - x^3/2 - (x*Cos[x]^2)/2 - x^2*Cot[x] + 2*x*Log[1 - E^((2*I)*x)
] - I*PolyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2\cos^2(x)\cot^2(x)\,dx &= -\int x^2\cos^2(x)\,dx + \int x^2\cot^2(x)\,dx\\
&= -\frac{1}{2}x\cos^2(x) - x^2\cot(x) - \frac{1}{2}x^2\cos(x)\sin(x) - \frac{\int x^2\,dx}{2} + \frac{1}{2}\int\cos^2(x)\,dx + 2\int x\,\mathrm{co}\\
&= -ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x) - 4i\int\frac{e}{1-}\\
&= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + 2x\log\left(1-e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\,\mathrm{co}\\
&= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + 2x\log\left(1-e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\,\mathrm{co}\\
&= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + 2x\log\left(1-e^{2ix}\right) - i\mathrm{Li}_2\left(e^{2ix}\right) + \frac{1}{4}\cos(x)\,\mathrm{sin}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 72, normalized size = 0.87

$$\frac{1}{8}\left(-8i\mathrm{Li}_2\left(e^{2ix}\right) - 4x^3 - 8ix^2 - 2x^2\sin(2x) - 8x^2\cot(x) + 16x\log\left(1-e^{2ix}\right) + \sin(2x) - 2x\cos(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] ((-8*I)*x^2 - 4*x^3 - 2*x*Cos[2*x] - 8*x^2*Cot[x] + 16*x*Log[1 - E^((2*I)*x
)] - (8*I)*PolyLog[2, E^((2*I)*x)] + Sin[2*x] - 2*x^2*Sin[2*x])/8
```

**fricas [B]** time = 0.48, size = 162, normalized size = 1.95

$$\frac{\left(2x^2-1\right)\cos(x)^3+4x\log\left(\cos(x)+i\sin(x)+1\right)\sin(x)+4x\log\left(\cos(x)-i\sin(x)+1\right)\sin(x)+4x\log\left(-\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*x^2 - 1)*cos(x)^3 + 4*x*log(cos(x) + I*sin(x) + 1)*sin(x) + 4*x*log
(cos(x) - I*sin(x) + 1)*sin(x) + 4*x*log(-cos(x) + I*sin(x) + 1)*sin(x) + 4
*x*log(-cos(x) - I*sin(x) + 1)*sin(x) - (6*x^2 - 1)*cos(x) - (2*x^3 + 2*x*c
os(x)^2 - x)*sin(x) - 4*I*dilog(cos(x) + I*sin(x))*sin(x) + 4*I*dilog(cos(x
) - I*sin(x))*sin(x) + 4*I*dilog(-cos(x) + I*sin(x))*sin(x) - 4*I*dilog(-co
s(x) - I*sin(x))*sin(x))/sin(x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cos(x)^2\cot(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(x)^2*cot(x)^2, x)
```

**maple [A]** time = 0.11, size = 112, normalized size = 1.35

$$-\frac{x^3}{2}+\frac{i\left(2x^2+2ix-1\right)e^{2ix}}{16}-\frac{i\left(2x^2-2ix-1\right)e^{-2ix}}{16}-\frac{2ix^2}{e^{2ix}-1}+2x\ln\left(1+e^{ix}\right)+2x\ln\left(1-e^{ix}\right)-2ix^2-2i\operatorname{polylog}\left(2,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2*cot(x)^2,x)
```

```
[Out] -1/2*x^3+1/16*I*(2*I*x+2*x^2-1)*exp(2*I*x)-1/16*I*(-2*I*x+2*x^2-1)*exp(-2*I
*x)-2*I*x^2/(exp(2*I*x)-1)+2*x*ln(1+exp(I*x))+2*x*ln(1-exp(I*x))-2*I*x^2-2*
I*polylog(2,-exp(I*x))-2*I*polylog(2,exp(I*x))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\cos(x)^2\cot(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2*cot(x)^2,x)
```

```
[Out] int(x^2*cos(x)^2*cot(x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cos^2(x)\cot^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)**2*cot(x)**2,x)
```

```
[Out] Integral(x**2*cos(x)**2*cot(x)**2, x)
```

# 3.204 $\int x\cos^2(x)\cot^2(x)\,dx$

**Optimal**. Leaf size=33

$$-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\sin(x)\cos(x)$$

[Out] -3/4*x^2-1/4*cos(x)^2-x*cot(x)+ln(sin(x))-1/2*x*cos(x)*sin(x)

**Rubi [A]** time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4408, 3310, 30, 3720, 3475}

$$-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

[In] Int[x*Cos[x]^2*Cot[x]^2,x]

[Out] (-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[x])/2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\cos^2(x)\cot^2(x)\,dx = -\int x\cos^2(x)\,dx + \int x\cot^2(x)\,dx$$
$$= -\frac{1}{4}\cos^2(x) - x\cot(x) - \frac{1}{2}x\cos(x)\sin(x) - \frac{\int x\,dx}{2} - \int x\,dx + \int \cot(x)\,dx$$
$$= -\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) + \log(\sin(x)) - \frac{1}{2}x\cos(x)\sin(x)$$

**Mathematica** [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{3x^2}{4} - \frac{1}{4}x\sin(2x) - \frac{1}{8}\cos(2x) - x\cot(x) + \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^2*Cot[x]^2,x]
[Out] (-3*x^2)/4 - Cos[2*x]/8 - x*Cot[x] + Log[Sin[x]] - (x*Sin[2*x])/4
```

**fricas** [A] time = 0.46, size = 45, normalized size = 1.36

$$\frac{4\,x\cos(x)^3 - 12\,x\cos(x) - \left(6\,x^2 + 2\,\cos(x)^2 - 1\right)\sin(x) + 8\,\log\left(\frac{1}{2}\,\sin(x)\right)\sin(x)}{8\,\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^2,x, algorithm="fricas")
[Out] 1/8*(4*x*cos(x)^3 - 12*x*cos(x) - (6*x^2 + 2*cos(x)^2 - 1)*sin(x) + 8*log(1
/2*sin(x))*sin(x))/sin(x)
```

**giac** [B] time = 0.22, size = 206, normalized size = 6.24

$$-\frac{6\,x^2\tan\left(\frac{1}{2}\,x\right)^5 - 4\,x\tan\left(\frac{1}{2}\,x\right)^6 - 4\,\log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^2}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,x\right)^5 + 12\,x^2\tan\left(\frac{1}{2}\,x\right)^3 - 12\,x\tan\left(\frac{1}{2}\,x\right)^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^2,x, algorithm="giac")
[Out] -1/8*(6*x^2*tan(1/2*x)^5 - 4*x*tan(1/2*x)^6 - 4*log(16*tan(1/2*x)^2/(tan(1/
2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^5 + 12*x^2*tan(1/2*x)^3 - 12*x*tan
(1/2*x)^4 + tan(1/2*x)^5 - 8*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*
x)^2 + 1))*tan(1/2*x)^3 + 6*x^2*tan(1/2*x) + 12*x*tan(1/2*x)^2 - 6*tan(1/2*
x)^3 - 4*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x
) + 4*x + tan(1/2*x))/(tan(1/2*x)^5 + 2*tan(1/2*x)^3 + tan(1/2*x))
```

**maple** [B] time = 0.12, size = 76, normalized size = 2.30

$$\frac{-x - \frac{x^2\tan(x)}{2}}{2\tan(x)} - \frac{\ln\left(1+\tan^2(x)\right)}{2} + \ln\left(\tan(x)\right) + \frac{-\frac{\tan(x)}{2} - x - 2x\left(\tan^2(x)\right) - x^2\tan(x) - x^2\left(\tan^3(x)\right)}{2\tan(x)\left(1+\tan^2(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2*cot(x)^2,x)
```

```
[Out] 1/2*(-x-1/2*x^2*tan(x))/tan(x)-1/2*ln(1+tan(x)^2)+ln(tan(x))+1/2*(-1/2*tan(
x)-x-2*x*tan(x)^2-x^2*tan(x)-x^2*tan(x)^3)/tan(x)/(1+tan(x)^2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**mupad [B]** time = 1.23, size = 56, normalized size = 1.70

$$\ln\left(e^{x\,2\mathrm{i}}-1\right)-e^{-x\,2\mathrm{i}}\,\left(\frac{1}{16}+\frac{x\,1\mathrm{i}}{8}\right)+e^{x\,2\mathrm{i}}\,\left(-\frac{1}{16}+\frac{x\,1\mathrm{i}}{8}\right)-\frac{3\,x^2}{4}-x\,2\mathrm{i}-\frac{x\,2\mathrm{i}}{e^{x\,2\mathrm{i}}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2*cot(x)^2,x)
```

```
[Out] log(exp(x*2i) - 1) - x*2i - exp(-x*2i)*((x*1i)/8 + 1/16) + exp(x*2i)*((x*1i
)/8 - 1/16) - (x*2i)/(exp(x*2i) - 1) - (3*x^2)/4
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos^2(x)\cot^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**2*cot(x)**2,x)
```

```
[Out] Integral(x*cos(x)**2*cot(x)**2, x)
```

# 3.205 $\int x^3 \cos^2(x) \cot^3(x) \, dx$

**Optimal**. Leaf size=180

$$3ix^2\text{Li}_2\left(e^{2ix}\right)-3x\text{Li}_3\left(e^{2ix}\right)-\frac{3}{2}i\text{Li}_2\left(e^{2ix}\right)-\frac{3}{2}i\text{Li}_4\left(e^{2ix}\right)+\frac{ix^4}{2}-\frac{3x^3}{4}-2x^3\log\left(1-e^{2ix}\right)+\frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)-$$

```
[Out] 3/8*x+3*I*x^2*polylog(2,exp(2*I*x))-3/4*x^3+1/2*I*x^4-3/2*x^2*cot(x)-1/2*x^
3*cot(x)^2+3*x*ln(1-exp(2*I*x))-2*x^3*ln(1-exp(2*I*x))-3/2*I*polylog(2,exp(
2*I*x))-3/2*I*x^2-3*x*polylog(3,exp(2*I*x))-3/2*I*polylog(4,exp(2*I*x))-3/8
*cos(x)*sin(x)+3/4*x^2*cos(x)*sin(x)-3/4*x*sin(x)^2+1/2*x^3*sin(x)^2
```

**Rubi [A]** time = 0.40, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {4408, 3443, 3311, 30, 2635, 8, 3717, 2190, 2531, 6609, 2282, 6589, 3720, 2279, 2391}

$$3ix^2\text{PolyLog}\left(2,e^{2ix}\right)-3x\text{PolyLog}\left(3,e^{2ix}\right)-\frac{3}{2}i\text{PolyLog}\left(2,e^{2ix}\right)-\frac{3}{2}i\text{PolyLog}\left(4,e^{2ix}\right)+\frac{ix^4}{2}-\frac{3x^3}{4}-\frac{3ix^2}{2}-2x^3\log$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (3*x)/8 - ((3*I)/2)*x^2 - (3*x^3)/4 + (I/2)*x^4 - (3*x^2*Cot[x])/2 - (x^3*C
ot[x]^2)/2 + 3*x*Log[1 - E^((2*I)*x)] - 2*x^3*Log[1 - E^((2*I)*x)] - ((3*I)
/2)*PolyLog[2, E^((2*I)*x)] + (3*I)*x^2*PolyLog[2, E^((2*I)*x)] - 3*x*PolyL
og[3, E^((2*I)*x)] - ((3*I)/2)*PolyLog[4, E^((2*I)*x)] - (3*Cos[x]*Sin[x])/
8 + (3*x^2*Cos[x]*Sin[x])/4 - (3*x*Sin[x]^2)/4 + (x^3*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\cos^2(x)\cot^3(x)\,dx = -\int x^3\cos^2(x)\cot(x)\,dx + \int x^3\cot^3(x)\,dx$$
$$= -\frac{1}{2}x^3\cot^2(x) + \frac{3}{2}\int x^2\cot^2(x)\,dx - 2\int x^3\cot(x)\,dx + \int x^3\cos(x)\sin(x)\,dx$$
$$= -\frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + \frac{1}{2}x^3\sin^2(x) - 2\left(-\frac{ix^4}{4} - 2i\int\frac{e^{2ix}x^3}{1-e^{2ix}}\,dx\right) - \frac{3\int x^2\,dx}{2}$$
$$= -\frac{3ix^2}{2} - \frac{x^3}{2} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + \frac{3}{4}x^2\cos(x)\sin(x) - \frac{3}{4}x\sin^2(x) + \frac{1}{2}x^3\sin$$
$$= -\frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{8}\cos(x)\sin(x) + \frac{3}{4}x^2$$
$$= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{8}\cos(x)\sin(x) +$$
$$= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{2}i\text{Li}_2\left(e^{2ix}\right) - \frac{3}{8}c$$
$$= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{2}i\text{Li}_2\left(e^{2ix}\right) - 2\Big($$

**Mathematica [A]** time = 0.41, size = 159, normalized size = 0.88

$$\frac{1}{32}\left(-96ix^2\text{Li}_2\left(e^{-2ix}\right) - 96x\text{Li}_3\left(e^{-2ix}\right) - 48i\text{Li}_2\left(e^{2ix}\right) + 48i\text{Li}_4\left(e^{-2ix}\right) - 16ix^4 - 64x^3\log\left(1-e^{-2ix}\right) - 8x^3\cos(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (I*Pi^4 - (48*I)*x^2 - (16*I)*x^4 + 12*x*Cos[2*x] - 8*x^3*Cos[2*x] - 48*x^2
*Cot[x] - 16*x^3*Csc[x]^2 - 64*x^3*Log[1 - E^((-2*I)*x)] + 96*x*Log[1 - E^(
(2*I)*x)] - (96*I)*x^2*PolyLog[2, E^((-2*I)*x)] - (48*I)*PolyLog[2, E^((2*I
)*x)] - 96*x*PolyLog[3, E^((-2*I)*x)] + (48*I)*PolyLog[4, E^((-2*I)*x)] - 6
*Sin[2*x] + 12*x^2*Sin[2*x])/32
```

**fricas [C]** time = 0.51, size = 508, normalized size = 2.82

$$-\frac{2\left(2x^3-3x\right)\cos(x)^4 - 2x^3 - 3\left(2x^3-3x\right)\cos(x)^2 - \left(\left(24ix^2-12i\right)\cos(x)^2 - 24ix^2 + 12i\right)\text{Li}_2\left(\cos(x) + i\,\text{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(2*x^3 - 3*x)*cos(x)^4 - 2*x^3 - 3*(2*x^3 - 3*x)*cos(x)^2 - ((24*I*
x^2 - 12*I)*cos(x)^2 - 24*I*x^2 + 12*I)*dilog(cos(x) + I*sin(x)) - ((-24*I*
x^2 + 12*I)*cos(x)^2 + 24*I*x^2 - 12*I)*dilog(cos(x) - I*sin(x)) - ((-24*I*
x^2 + 12*I)*cos(x)^2 + 24*I*x^2 - 12*I)*dilog(-cos(x) + I*sin(x)) - ((24*I*
x^2 - 12*I)*cos(x)^2 - 24*I*x^2 + 12*I)*dilog(-cos(x) - I*sin(x)) - 4*(2*x^
3 - (2*x^3 - 3*x)*cos(x)^2 - 3*x)*log(cos(x) + I*sin(x) + 1) - 4*(2*x^3 - (
2*x^3 - 3*x)*cos(x)^2 - 3*x)*log(cos(x) - I*sin(x) + 1) - 4*(2*x^3 - (2*x^3
 - 3*x)*cos(x)^2 - 3*x)*log(-cos(x) + I*sin(x) + 1) - 4*(2*x^3 - (2*x^3 - 3
*x)*cos(x)^2 - 3*x)*log(-cos(x) - I*sin(x) + 1) - (-48*I*cos(x)^2 + 48*I)*p
olylog(4, cos(x) + I*sin(x)) - (48*I*cos(x)^2 - 48*I)*polylog(4, cos(x) - I
*sin(x)) - (48*I*cos(x)^2 - 48*I)*polylog(4, -cos(x) + I*sin(x)) - (-48*I*c
os(x)^2 + 48*I)*polylog(4, -cos(x) - I*sin(x)) + 48*(x*cos(x)^2 - x)*polylo
g(3, cos(x) + I*sin(x)) + 48*(x*cos(x)^2 - x)*polylog(3, cos(x) - I*sin(x))
 + 48*(x*cos(x)^2 - x)*polylog(3, -cos(x) + I*sin(x)) + 48*(x*cos(x)^2 - x)
*polylog(3, -cos(x) - I*sin(x)) - 3*((2*x^2 - 1)*cos(x)^3 + (2*x^2 + 1)*cos
(x))*sin(x) - 3*x)/(cos(x)^2 - 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cos(x)^2 \cot(x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(x)^2*cot(x)^3, x)
```

**maple [A]** time = 0.16, size = 240, normalized size = 1.33

$$6ix^2 \operatorname{polylog}\left(2, e^{ix}\right) - \frac{\left(4x^3 + 6ix^2 - 6x - 3i\right) e^{2ix}}{32} - \frac{\left(4x^3 - 6ix^2 - 6x + 3i\right) e^{-2ix}}{32} + \frac{x^2 \left(2x e^{2ix} - 3i e^{2ix} + 3i\right)}{\left(e^{2ix} - 1\right)^2} - 2x^3 \ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x)^2*cot(x)^3,x)
```

```
[Out] -3*I*polylog(2,-exp(I*x))-1/32*(6*I*x^2+4*x^3-3*I-6*x)*exp(2*I*x)-1/32*(-6*
I*x^2+4*x^3+3*I-6*x)*exp(-2*I*x)+x^2*(2*x*exp(2*I*x)-3*I*exp(2*I*x)+3*I)/(e
xp(2*I*x)-1)^2-2*x^3*ln(1+exp(I*x))-2*x^3*ln(1-exp(I*x))-12*I*polylog(4,-ex
p(I*x))-12*x*polylog(3,exp(I*x))+3*x*ln(1+exp(I*x))+3*x*ln(1-exp(I*x))-12*x
*polylog(3,-exp(I*x))+6*I*x^2*polylog(2,exp(I*x))-3*I*x^2+6*I*x^2*polylog(2
,-exp(I*x))-3*I*polylog(2,exp(I*x))+1/2*I*x^4-12*I*polylog(4,exp(I*x))
```

**maxima [B]** time = 0.95, size = 3719, normalized size = 20.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] -(4*x^3 + (4*x^3 + 6*I*x^2 - 6*x - 3*I)*cos(6*x)^2 - (-32*I*x^4 - 16*x^3 +
168*I*x^2 + 24*x + 12*I)*cos(4*x)^2 - (-32*I*x^4 + 56*x^3 + 96*I*x^2 + 12*x
)*cos(2*x)^2 - (4*x^3 + 6*I*x^2 - 6*x - 3*I)*sin(6*x)^2 - (32*I*x^4 + 16*x^
3 - 168*I*x^2 - 24*x - 12*I)*sin(4*x)^2 - (32*I*x^4 - 56*x^3 - 96*I*x^2 - 1
2*x)*sin(2*x)^2 - 6*I*x^2 - ((128*I*x^3 - 192*I*x)*cos(4*x)^2 + (128*I*x^3
- 192*I*x)*cos(2*x)^2 + (-128*I*x^3 + 192*I*x)*sin(4*x)^2 + (-128*I*x^3 + 1
92*I*x)*sin(2*x)^2 + (-64*I*x^3 + (-64*I*x^3 + 96*I*x)*cos(4*x) + (128*I*x^
3 - 192*I*x)*cos(2*x) + 32*(2*x^3 - 3*x)*sin(4*x) - 64*(2*x^3 - 3*x)*sin(2*
x) + 96*I*x)*cos(6*x) + (128*I*x^3 + (-320*I*x^3 + 480*I*x)*cos(2*x) + 160*
(2*x^3 - 3*x)*sin(2*x) - 192*I*x)*cos(4*x) + (-64*I*x^3 + 96*I*x)*cos(2*x)
```

```
+ (64*x^3 + 32*(2*x^3 - 3*x)*cos(4*x) - 64*(2*x^3 - 3*x)*cos(2*x) + (64*I*x
^3 - 96*I*x)*sin(4*x) + (-128*I*x^3 + 192*I*x)*sin(2*x) - 96*x)*sin(6*x) -
(128*x^3 + 128*(2*x^3 - 3*x)*cos(4*x) - 160*(2*x^3 - 3*x)*cos(2*x) - (320*I
*x^3 - 480*I*x)*sin(2*x) - 192*x)*sin(4*x) + 32*(2*x^3 - 4*(2*x^3 - 3*x)*co
s(2*x) - 3*x)*sin(2*x))*arctan2(sin(x), cos(x) + 1) - ((-128*I*x^3 + 192*I*
x)*cos(4*x)^2 + (-128*I*x^3 + 192*I*x)*cos(2*x)^2 + (128*I*x^3 - 192*I*x)*s
in(4*x)^2 + (128*I*x^3 - 192*I*x)*sin(2*x)^2 + (64*I*x^3 + (64*I*x^3 - 96*I
*x)*cos(4*x) + (-128*I*x^3 + 192*I*x)*cos(2*x) - 32*(2*x^3 - 3*x)*sin(4*x)
+ 64*(2*x^3 - 3*x)*sin(2*x) - 96*I*x)*cos(6*x) + (-128*I*x^3 + (320*I*x^3 -
 480*I*x)*cos(2*x) - 160*(2*x^3 - 3*x)*sin(2*x) + 192*I*x)*cos(4*x) + (64*I
*x^3 - 96*I*x)*cos(2*x) - (64*x^3 + 32*(2*x^3 - 3*x)*cos(4*x) - 64*(2*x^3 -
 3*x)*cos(2*x) - (-64*I*x^3 + 96*I*x)*sin(4*x) - (128*I*x^3 - 192*I*x)*sin(
2*x) - 96*x)*sin(6*x) + (128*x^3 + 128*(2*x^3 - 3*x)*cos(4*x) - 160*(2*x^3
- 3*x)*cos(2*x) + (-320*I*x^3 + 480*I*x)*sin(2*x) - 192*x)*sin(4*x) - 32*(2
*x^3 - 4*(2*x^3 - 3*x)*cos(2*x) - 3*x)*sin(2*x))*arctan2(sin(x), -cos(x) +
1) - (16*I*x^4 + 8*x^3 - 12*I*x^2 + (16*I*x^4 + 16*x^3 - 72*I*x^2 - 24*x -
12*I)*cos(4*x) + (-32*I*x^4 + 52*x^3 + 90*I*x^2 + 18*x + 3*I)*cos(2*x) - (1
6*x^4 - 16*I*x^3 - 72*x^2 + 24*I*x - 12)*sin(4*x) + (32*x^4 + 52*I*x^3 - 90
*x^2 + 18*I*x - 3)*sin(2*x) - 12*x + 6*I)*cos(6*x) - (-32*I*x^4 - 20*x^3 +
30*I*x^2 + (80*I*x^4 - 104*x^3 - 276*I*x^2 - 36*x - 6*I)*cos(2*x) - (80*x^4
 + 104*I*x^3 - 276*x^2 + 36*I*x - 6)*sin(2*x) + 30*x - 15*I)*cos(4*x) - (16
*I*x^4 + 16*x^3 - 24*I*x^2 - 24*x + 12*I)*cos(2*x) - ((-384*I*x^2 + 192*I)*
cos(4*x)^2 + (-384*I*x^2 + 192*I)*cos(2*x)^2 + (384*I*x^2 - 192*I)*sin(4*x)
^2 + (384*I*x^2 - 192*I)*sin(2*x)^2 + (192*I*x^2 + (192*I*x^2 - 96*I)*cos(4
*x) + (-384*I*x^2 + 192*I)*cos(2*x) - 96*(2*x^2 - 1)*sin(4*x) + 192*(2*x^2
- 1)*sin(2*x) - 96*I)*cos(6*x) + (-384*I*x^2 + (960*I*x^2 - 480*I)*cos(2*x)
 - 480*(2*x^2 - 1)*sin(2*x) + 192*I)*cos(4*x) + (192*I*x^2 - 96*I)*cos(2*x)
 - (192*x^2 + 96*(2*x^2 - 1)*cos(4*x) - 192*(2*x^2 - 1)*cos(2*x) - (-192*I*
x^2 + 96*I)*sin(4*x) - (384*I*x^2 - 192*I)*sin(2*x) - 96)*sin(6*x) + (384*x
^2 + 384*(2*x^2 - 1)*cos(4*x) - 480*(2*x^2 - 1)*cos(2*x) + (-960*I*x^2 + 48
0*I)*sin(2*x) - 192)*sin(4*x) - 96*(2*x^2 - 4*(2*x^2 - 1)*cos(2*x) - 1)*sin
(2*x))*dilog(-e^(I*x)) - ((-384*I*x^2 + 192*I)*cos(4*x)^2 + (-384*I*x^2 + 1
92*I)*cos(2*x)^2 + (384*I*x^2 - 192*I)*sin(4*x)^2 + (384*I*x^2 - 192*I)*sin
(2*x)^2 + (192*I*x^2 + (192*I*x^2 - 96*I)*cos(4*x) + (-384*I*x^2 + 192*I)*c
os(2*x) - 96*(2*x^2 - 1)*sin(4*x) + 192*(2*x^2 - 1)*sin(2*x) - 96*I)*cos(6*
x) + (-384*I*x^2 + (960*I*x^2 - 480*I)*cos(2*x) - 480*(2*x^2 - 1)*sin(2*x)
+ 192*I)*cos(4*x) + (192*I*x^2 - 96*I)*cos(2*x) - (192*x^2 + 96*(2*x^2 - 1)
*cos(4*x) - 192*(2*x^2 - 1)*cos(2*x) - (-192*I*x^2 + 96*I)*sin(4*x) - (384*
I*x^2 - 192*I)*sin(2*x) - 96)*sin(6*x) + (384*x^2 + 384*(2*x^2 - 1)*cos(4*x
) - 480*(2*x^2 - 1)*cos(2*x) + (-960*I*x^2 + 480*I)*sin(2*x) - 192)*sin(4*x
) - 96*(2*x^2 - 4*(2*x^2 - 1)*cos(2*x) - 1)*sin(2*x))*dilog(e^(I*x)) - (32*
(2*x^3 - 3*x)*cos(4*x)^2 + 32*(2*x^3 - 3*x)*cos(2*x)^2 - 32*(2*x^3 - 3*x)*s
in(4*x)^2 - 32*(2*x^3 - 3*x)*sin(2*x)^2 - (32*x^3 + 16*(2*x^3 - 3*x)*cos(4*
x) - 32*(2*x^3 - 3*x)*cos(2*x) - (-32*I*x^3 + 48*I*x)*sin(4*x) - (64*I*x^3
- 96*I*x)*sin(2*x) - 48*x)*cos(6*x) + (64*x^3 - 80*(2*x^3 - 3*x)*cos(2*x) +
 (-160*I*x^3 + 240*I*x)*sin(2*x) - 96*x)*cos(4*x) - 16*(2*x^3 - 3*x)*cos(2*
x) + (-32*I*x^3 + (-32*I*x^3 + 48*I*x)*cos(4*x) + (64*I*x^3 - 96*I*x)*cos(2
*x) + 16*(2*x^3 - 3*x)*sin(4*x) - 32*(2*x^3 - 3*x)*sin(2*x) + 48*I*x)*sin(6
*x) + (64*I*x^3 + (128*I*x^3 - 192*I*x)*cos(4*x) + (-160*I*x^3 + 240*I*x)*c
os(2*x) + 80*(2*x^3 - 3*x)*sin(2*x) - 96*I*x)*sin(4*x) + (-32*I*x^3 + (128*
I*x^3 - 192*I*x)*cos(2*x) + 48*I*x)*sin(2*x))*log(cos(x)^2 + sin(x)^2 + 2*c
os(x) + 1) - (32*(2*x^3 - 3*x)*cos(4*x)^2 + 32*(2*x^3 - 3*x)*cos(2*x)^2 - 3
2*(2*x^3 - 3*x)*sin(4*x)^2 - 32*(2*x^3 - 3*x)*sin(2*x)^2 - (32*x^3 + 16*(2*
x^3 - 3*x)*cos(4*x) - 32*(2*x^3 - 3*x)*cos(2*x) - (-32*I*x^3 + 48*I*x)*sin(
4*x) - (64*I*x^3 - 96*I*x)*sin(2*x) - 48*x)*cos(6*x) + (64*x^3 - 80*(2*x^3
- 3*x)*cos(2*x) + (-160*I*x^3 + 240*I*x)*sin(2*x) - 96*x)*cos(4*x) - 16*(2*
x^3 - 3*x)*cos(2*x) + (-32*I*x^3 + (-32*I*x^3 + 48*I*x)*cos(4*x) + (64*I*x^
3 - 96*I*x)*cos(2*x) + 16*(2*x^3 - 3*x)*sin(4*x) - 32*(2*x^3 - 3*x)*sin(2*x
) + 48*I*x)*sin(6*x) + (64*I*x^3 + (128*I*x^3 - 192*I*x)*cos(4*x) + (-160*I
```

```
*x^3 + 240*I*x)*cos(2*x) + 80*(2*x^3 - 3*x)*sin(2*x) - 96*I*x)*sin(4*x) + (
-32*I*x^3 + (128*I*x^3 - 192*I*x)*cos(2*x) + 48*I*x)*sin(2*x))*log(cos(x)^2
 + sin(x)^2 - 2*cos(x) + 1) - ((-384*I*cos(4*x) + 768*I*cos(2*x) + 384*sin(
4*x) - 768*sin(2*x) - 384*I)*cos(6*x) + (-1920*I*cos(2*x) + 1920*sin(2*x) +
 768*I)*cos(4*x) + 768*I*cos(4*x)^2 + 768*I*cos(2*x)^2 + (384*cos(4*x) - 76
8*cos(2*x) + 384*I*sin(4*x) - 768*I*sin(2*x) + 384)*sin(6*x) - (1536*cos(4*
x) - 1920*cos(2*x) - 1920*I*sin(2*x) + 768)*sin(4*x) - 768*I*sin(4*x)^2 - 3
84*(4*cos(2*x) - 1)*sin(2*x) - 768*I*sin(2*x)^2 - 384*I*cos(2*x))*polylog(4
, -e^(I*x)) - ((-384*I*cos(4*x) + 768*I*cos(2*x) + 384*sin(4*x) - 768*sin(2
*x) - 384*I)*cos(6*x) + (-1920*I*cos(2*x) + 1920*sin(2*x) + 768*I)*cos(4*x)
 + 768*I*cos(4*x)^2 + 768*I*cos(2*x)^2 + (384*cos(4*x) - 768*cos(2*x) + 384
*I*sin(4*x) - 768*I*sin(2*x) + 384)*sin(6*x) - (1536*cos(4*x) - 1920*cos(2*
x) - 1920*I*sin(2*x) + 768)*sin(4*x) - 768*I*sin(4*x)^2 - 384*(4*cos(2*x) -
 1)*sin(2*x) - 768*I*sin(2*x)^2 - 384*I*cos(2*x))*polylog(4, e^(I*x)) - (76
8*x*cos(4*x)^2 + 768*x*cos(2*x)^2 - 768*x*sin(4*x)^2 - 768*x*sin(2*x)^2 - (
384*x*cos(4*x) - 768*x*cos(2*x) + 384*I*x*sin(4*x) - 768*I*x*sin(2*x) + 384
*x)*cos(6*x) - 384*(5*x*cos(2*x) + 5*I*x*sin(2*x) - 2*x)*cos(4*x) - 384*x*c
os(2*x) + (-384*I*x*cos(4*x) + 768*I*x*cos(2*x) + 384*x*sin(4*x) - 768*x*si
n(2*x) - 384*I*x)*sin(6*x) + (1536*I*x*cos(4*x) - 1920*I*x*cos(2*x) + 1920*
x*sin(2*x) + 768*I*x)*sin(4*x) + (1536*I*x*cos(2*x) - 384*I*x)*sin(2*x))*po
lylog(3, -e^(I*x)) - (768*x*cos(4*x)^2 + 768*x*cos(2*x)^2 - 768*x*sin(4*x)^
2 - 768*x*sin(2*x)^2 - (384*x*cos(4*x) - 768*x*cos(2*x) + 384*I*x*sin(4*x)
- 768*I*x*sin(2*x) + 384*x)*cos(6*x) - 384*(5*x*cos(2*x) + 5*I*x*sin(2*x) -
 2*x)*cos(4*x) - 384*x*cos(2*x) + (-384*I*x*cos(4*x) + 768*I*x*cos(2*x) + 3
84*x*sin(4*x) - 768*x*sin(2*x) - 384*I*x)*sin(6*x) + (1536*I*x*cos(4*x) - 1
920*I*x*cos(2*x) + 1920*x*sin(2*x) + 768*I*x)*sin(4*x) + (1536*I*x*cos(2*x)
 - 384*I*x)*sin(2*x))*polylog(3, e^(I*x)) + (16*x^4 - 8*I*x^3 - 12*x^2 - (-
8*I*x^3 + 12*x^2 + 12*I*x - 6)*cos(6*x) + (16*x^4 - 16*I*x^3 - 72*x^2 + 24*
I*x - 12)*cos(4*x) - (32*x^4 + 52*I*x^3 - 90*x^2 + 18*I*x - 3)*cos(2*x) - (
-16*I*x^4 - 16*x^3 + 72*I*x^2 + 24*x + 12*I)*sin(4*x) - (32*I*x^4 - 52*x^3
- 90*I*x^2 - 18*x - 3*I)*sin(2*x) + 12*I*x + 6)*sin(6*x) - (32*x^4 - 20*I*x
^3 - 30*x^2 + (64*x^4 - 32*I*x^3 - 336*x^2 + 48*I*x - 24)*cos(4*x) - (80*x^
4 + 104*I*x^3 - 276*x^2 + 36*I*x - 6)*cos(2*x) + (-80*I*x^4 + 104*x^3 + 276
*I*x^2 + 36*x + 6*I)*sin(2*x) + 30*I*x + 15)*sin(4*x) + (16*x^4 - 16*I*x^3
- 24*x^2 - (64*x^4 + 112*I*x^3 - 192*x^2 + 24*I*x)*cos(2*x) + 24*I*x + 12)*
sin(2*x) - 6*x + 3*I)/((32*cos(4*x) - 64*cos(2*x) + 32*I*sin(4*x) - 64*I*si
n(2*x) + 32)*cos(6*x) + (160*cos(2*x) + 160*I*sin(2*x) - 64)*cos(4*x) - 64*
cos(4*x)^2 - 64*cos(2*x)^2 - (-32*I*cos(4*x) + 64*I*cos(2*x) + 32*sin(4*x)
- 64*sin(2*x) - 32*I)*sin(6*x) - (128*I*cos(4*x) - 160*I*cos(2*x) + 160*sin
(2*x) + 64*I)*sin(4*x) + 64*sin(4*x)^2 - (128*I*cos(2*x) - 32*I)*sin(2*x) +
 64*sin(2*x)^2 + 32*cos(2*x))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \cos(x)^2 \cot(x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x)^2*cot(x)^3,x)
[Out] int(x^3*cos(x)^2*cot(x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cos^2(x) \cot^3(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x)**2*cot(x)**3,x)
[Out] Integral(x**3*cos(x)**2*cot(x)**3, x)
```

## 3.206 $\int x^2 \cos^2(x) \cot^3(x) \, dx$

**Optimal**. Leaf size=106

$$2ix\text{Li}_2\left(e^{2ix}\right)-\text{Li}_3\left(e^{2ix}\right)+\frac{2ix^3}{3}-\frac{3x^2}{4}-2x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+\log(\sin(x)$$

```
[Out] -3/4*x^2+2/3*I*x^3-x*cot(x)-1/2*x^2*cot(x)^2-2*x^2*ln(1-exp(2*I*x))+ln(sin(
x))+2*I*x*polylog(2,exp(2*I*x))-polylog(3,exp(2*I*x))+1/2*x*cos(x)*sin(x)-1
/4*sin(x)^2+1/2*x^2*sin(x)^2
```

**Rubi [A]** time = 0.28, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {4408, 3443, 3310, 30, 3717, 2190, 2531, 2282, 6589, 3720, 3475}

$$2ix\text{PolyLog}\left(2,e^{2ix}\right)-\text{PolyLog}\left(3,e^{2ix}\right)+\frac{2ix^3}{3}-\frac{3x^2}{4}-2x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\text{ co}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (-3*x^2)/4 + ((2*I)/3)*x^3 - x*Cot[x] - (x^2*Cot[x]^2)/2 - 2*x^2*Log[1 - E^
((2*I)*x)] + Log[Sin[x]] + (2*I)*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3, E^(
(2*I)*x)] + (x*Cos[x]*Sin[x])/2 - Sin[x]^2/4 + (x^2*Sin[x]^2)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
```

```
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\cos^2(x)\cot^3(x)\,dx = -\int x^2\cos^2(x)\cot(x)\,dx + \int x^2\cot^3(x)\,dx$$
$$= -\frac{1}{2}x^2\cot^2(x) - 2\int x^2\cot(x)\,dx + \int x\cot^2(x)\,dx + \int x^2\cos(x)\sin(x)\,dx$$
$$= -x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \frac{1}{2}x^2\sin^2(x) - 2\left(-\frac{ix^3}{3} - 2i\int \frac{e^{2ix}x^2}{1-e^{2ix}}\,dx\right) - \int x\,dx + \int$$
$$= -\frac{x^2}{2} - x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \log(\sin(x)) + \frac{1}{2}x\cos(x)\sin(x) - \frac{\sin^2(x)}{4} + \frac{1}{2}x^2\sin^2$$
$$= -\frac{3x^2}{4} - x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \log(\sin(x)) + \frac{1}{2}x\cos(x)\sin(x) - \frac{\sin^2(x)}{4} + \frac{1}{2}x^2\sin$$
$$= -\frac{3x^2}{4} - x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \log(\sin(x)) + \frac{1}{2}x\cos(x)\sin(x) - \frac{\sin^2(x)}{4} + \frac{1}{2}x^2\sin$$
$$= -\frac{3x^2}{4} - x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \log(\sin(x)) - 2\left(-\frac{ix^3}{3} + x^2\log\left(1-e^{2ix}\right) - ix\text{Li}_2\left(e\right.\right.$$

**Mathematica** [A] time = 0.33, size = 108, normalized size = 1.02

$$-2ix\text{Li}_2\left(e^{-2ix}\right) - \text{Li}_3\left(e^{-2ix}\right) - \frac{2ix^3}{3} - 2x^2\log\left(1-e^{-2ix}\right) - \frac{1}{4}x^2\cos(2x) - \frac{1}{2}x^2\csc^2(x) + \frac{1}{4}x\sin(2x) + \frac{1}{8}\cos(2x) - x\cot($$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (I/12)*Pi^3 - ((2*I)/3)*x^3 + Cos[2*x]/8 - (x^2*Cos[2*x])/4 - x*Cot[x] - (x
^2*Csc[x]^2)/2 - 2*x^2*Log[1 - E^((-2*I)*x)] + Log[Sin[x]] - (2*I)*x*PolyLo
g[2, E^((-2*I)*x)] - PolyLog[3, E^((-2*I)*x)] + (x*Sin[2*x])/4
```

**fricas** [C] time = 0.54, size = 370, normalized size = 3.49

$$-\frac{2\left(2x^2-1\right)\cos(x)^4 - 3\left(2x^2-1\right)\cos(x)^2 - 2x^2 - \left(16ix\cos(x)^2 - 16ix\right)\text{Li}_2\left(\cos(x) + i\sin(x)\right) - \left(-16ix\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(2*x^2 - 1)*cos(x)^4 - 3*(2*x^2 - 1)*cos(x)^2 - 2*x^2 - (16*I*x*cos
(x)^2 - 16*I*x)*dilog(cos(x) + I*sin(x)) - (-16*I*x*cos(x)^2 + 16*I*x)*dilo
g(cos(x) - I*sin(x)) - (-16*I*x*cos(x)^2 + 16*I*x)*dilog(-cos(x) + I*sin(x)
) - (16*I*x*cos(x)^2 - 16*I*x)*dilog(-cos(x) - I*sin(x)) + 4*((2*x^2 - 1)*c
os(x)^2 - 2*x^2 + 1)*log(cos(x) + I*sin(x) + 1) + 4*((2*x^2 - 1)*cos(x)^2 -
 2*x^2 + 1)*log(cos(x) - I*sin(x) + 1) - 4*(cos(x)^2 - 1)*log(-1/2*cos(x) +
 1/2*I*sin(x) + 1/2) - 4*(cos(x)^2 - 1)*log(-1/2*cos(x) - 1/2*I*sin(x) + 1/
2) + 8*(x^2*cos(x)^2 - x^2)*log(-cos(x) + I*sin(x) + 1) + 8*(x^2*cos(x)^2 -
 x^2)*log(-cos(x) - I*sin(x) + 1) + 16*(cos(x)^2 - 1)*polylog(3, cos(x) + I
*sin(x)) + 16*(cos(x)^2 - 1)*polylog(3, cos(x) - I*sin(x)) + 16*(cos(x)^2 -
 1)*polylog(3, -cos(x) + I*sin(x)) + 16*(cos(x)^2 - 1)*polylog(3, -cos(x) -
 I*sin(x)) - 4*(x*cos(x)^3 + x*cos(x))*sin(x) - 1)/(cos(x)^2 - 1)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cos(x)^2\cot(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(x)^2*cot(x)^3, x)
```

**maple** [A] time = 0.16, size = 170, normalized size = 1.60

$$\frac{2ix^3}{3}-\frac{\left(2x^2+2ix-1\right)\mathrm{e}^{2ix}}{16}-\frac{\left(2x^2-2ix-1\right)\mathrm{e}^{-2ix}}{16}+\frac{2x\left(x\,\mathrm{e}^{2ix}-i\mathrm{e}^{2ix}+i\right)}{\left(\mathrm{e}^{2ix}-1\right)^2}+\ln\left(1+\mathrm{e}^{ix}\right)+\ln\left(\mathrm{e}^{ix}-1\right)-2\ln\left(\mathrm{e}^{ix}\right)-2x^2\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2*cot(x)^3,x)
```

```
[Out] 2/3*I*x^3-1/16*(2*I*x+2*x^2-1)*exp(2*I*x)-1/16*(-2*I*x+2*x^2-1)*exp(-2*I*x)
+2*x*(x*exp(2*I*x)-I*exp(2*I*x)+I)/(exp(2*I*x)-1)^2+ln(1+exp(I*x))+ln(exp(I
*x)-1)-2*ln(exp(I*x))-2*x^2*ln(1+exp(I*x))+4*I*x*polylog(2,-exp(I*x))-4*pol
ylog(3,-exp(I*x))-2*x^2*ln(1-exp(I*x))+4*I*x*polylog(2,exp(I*x))-4*polylog(
3,exp(I*x))
```

**maxima** [B] time = 0.75, size = 2855, normalized size = 26.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] -(3*(2*x^2 + 2*I*x - 1)*cos(6*x)^2 - (-64*I*x^3 - 24*x^2 + 168*I*x + 12)*co
s(4*x)^2 - (-64*I*x^3 + 84*x^2 + 96*I*x + 6)*cos(2*x)^2 - 3*(2*x^2 + 2*I*x
- 1)*sin(6*x)^2 - (64*I*x^3 + 24*x^2 - 168*I*x - 12)*sin(4*x)^2 - (64*I*x^3
 - 84*x^2 - 96*I*x - 6)*sin(2*x)^2 + 6*x^2 - ((192*I*x^2 - 96*I)*cos(4*x)^2
 + (192*I*x^2 - 96*I)*cos(2*x)^2 + (-192*I*x^2 + 96*I)*sin(4*x)^2 + (-192*I
*x^2 + 96*I)*sin(2*x)^2 + (-96*I*x^2 + (-96*I*x^2 + 48*I)*cos(4*x) + (192*I
*x^2 - 96*I)*cos(2*x) + 48*(2*x^2 - 1)*sin(4*x) - 96*(2*x^2 - 1)*sin(2*x) +
 48*I)*cos(6*x) + (192*I*x^2 + (-480*I*x^2 + 240*I)*cos(2*x) + 240*(2*x^2 -
 1)*sin(2*x) - 96*I)*cos(4*x) + (-96*I*x^2 + 48*I)*cos(2*x) + (96*x^2 + 48*
(2*x^2 - 1)*cos(4*x) - 96*(2*x^2 - 1)*cos(2*x) + (96*I*x^2 - 48*I)*sin(4*x)
 + (-192*I*x^2 + 96*I)*sin(2*x) - 48)*sin(6*x) - (192*x^2 + 192*(2*x^2 - 1)
*cos(4*x) - 240*(2*x^2 - 1)*cos(2*x) - (480*I*x^2 - 240*I)*sin(2*x) - 96)*s
in(4*x) + 48*(2*x^2 - 4*(2*x^2 - 1)*cos(2*x) - 1)*sin(2*x))*arctan2(sin(x),
 cos(x) + 1) - ((48*I*cos(4*x) - 96*I*cos(2*x) - 48*sin(4*x) + 96*sin(2*x)
+ 48*I)*cos(6*x) + (240*I*cos(2*x) - 240*sin(2*x) - 96*I)*cos(4*x) - 96*I*c
os(4*x)^2 - 96*I*cos(2*x)^2 - (48*cos(4*x) - 96*cos(2*x) + 48*I*sin(4*x) -
96*I*sin(2*x) + 48)*sin(6*x) + (192*cos(4*x) - 240*cos(2*x) - 240*I*sin(2*x
) + 96)*sin(4*x) + 96*I*sin(4*x)^2 + 48*(4*cos(2*x) - 1)*sin(2*x) + 96*I*si
n(2*x)^2 + 48*I*cos(2*x))*arctan2(sin(x), cos(x) - 1) - (-192*I*x^2*cos(4*x
)^2 - 192*I*x^2*cos(2*x)^2 + 192*I*x^2*sin(4*x)^2 + 192*I*x^2*sin(2*x)^2 +
96*I*x^2*cos(2*x) + (96*I*x^2*cos(4*x) - 192*I*x^2*cos(2*x) - 96*x^2*sin(4*
x) + 192*x^2*sin(2*x) + 96*I*x^2)*cos(6*x) + (480*I*x^2*cos(2*x) - 480*x^2*
sin(2*x) - 192*I*x^2)*cos(4*x) - (96*x^2*cos(4*x) - 192*x^2*cos(2*x) + 96*I
*x^2*sin(4*x) - 192*I*x^2*sin(2*x) + 96*x^2)*sin(6*x) + 96*(4*x^2*cos(4*x)
- 5*x^2*cos(2*x) - 5*I*x^2*sin(2*x) + 2*x^2)*sin(4*x) + 96*(4*x^2*cos(2*x)
- x^2)*sin(2*x))*arctan2(sin(x), -cos(x) + 1) - (32*I*x^3 + 12*x^2 + (32*I*
x^3 + 24*x^2 - 72*I*x - 12)*cos(4*x) + (-64*I*x^3 + 78*x^2 + 90*I*x + 9)*co
s(2*x) - (32*x^3 - 24*I*x^2 - 72*x + 12*I)*sin(4*x) + (64*x^3 + 78*I*x^2 -
90*x + 9*I)*sin(2*x) - 12*I*x - 6)*cos(6*x) - (-64*I*x^3 - 30*x^2 + (160*I*
x^3 - 156*x^2 - 276*I*x - 18)*cos(2*x) - (160*x^3 + 156*I*x^2 - 276*x + 18*
I)*sin(2*x) + 30*I*x + 15)*cos(4*x) - (32*I*x^3 + 24*x^2 - 24*I*x - 12)*cos
(2*x) - (-384*I*x*cos(4*x)^2 - 384*I*x*cos(2*x)^2 + 384*I*x*sin(4*x)^2 + 38
4*I*x*sin(2*x)^2 + (192*I*x*cos(4*x) - 384*I*x*cos(2*x) - 192*x*sin(4*x) +
384*x*sin(2*x) + 192*I*x)*cos(6*x) + (960*I*x*cos(2*x) - 960*x*sin(2*x) - 3
```

```
84*I*x)*cos(4*x) + 192*I*x*cos(2*x) - (192*x*cos(4*x) - 384*x*cos(2*x) + 19
2*I*x*sin(4*x) - 384*I*x*sin(2*x) + 192*x)*sin(6*x) + 192*(4*x*cos(4*x) - 5
*x*cos(2*x) - 5*I*x*sin(2*x) + 2*x)*sin(4*x) + 192*(4*x*cos(2*x) - x)*sin(2
*x))*dilog(-e^(I*x)) - (-384*I*x*cos(4*x)^2 - 384*I*x*cos(2*x)^2 + 384*I*x*
sin(4*x)^2 + 384*I*x*sin(2*x)^2 + (192*I*x*cos(4*x) - 384*I*x*cos(2*x) - 19
2*x*sin(4*x) + 384*x*sin(2*x) + 192*I*x)*cos(6*x) + (960*I*x*cos(2*x) - 960
*x*sin(2*x) - 384*I*x)*cos(4*x) + 192*I*x*cos(2*x) - (192*x*cos(4*x) - 384*
x*cos(2*x) + 192*I*x*sin(4*x) - 384*I*x*sin(2*x) + 192*x)*sin(6*x) + 192*(4
*x*cos(4*x) - 5*x*cos(2*x) - 5*I*x*sin(2*x) + 2*x)*sin(4*x) + 192*(4*x*cos(
2*x) - x)*sin(2*x))*dilog(e^(I*x)) - (48*(2*x^2 - 1)*cos(4*x)^2 + 48*(2*x^2
 - 1)*cos(2*x)^2 - 48*(2*x^2 - 1)*sin(4*x)^2 - 48*(2*x^2 - 1)*sin(2*x)^2 -
(48*x^2 + 24*(2*x^2 - 1)*cos(4*x) - 48*(2*x^2 - 1)*cos(2*x) - (-48*I*x^2 +
24*I)*sin(4*x) - (96*I*x^2 - 48*I)*sin(2*x) - 24)*cos(6*x) + (96*x^2 - 120*
(2*x^2 - 1)*cos(2*x) + (-240*I*x^2 + 120*I)*sin(2*x) - 48)*cos(4*x) - 24*(2
*x^2 - 1)*cos(2*x) + (-48*I*x^2 + (-48*I*x^2 + 24*I)*cos(4*x) + (96*I*x^2 -
 48*I)*cos(2*x) + 24*(2*x^2 - 1)*sin(4*x) - 48*(2*x^2 - 1)*sin(2*x) + 24*I)
*sin(6*x) + (96*I*x^2 + (192*I*x^2 - 96*I)*cos(4*x) + (-240*I*x^2 + 120*I)*
cos(2*x) + 120*(2*x^2 - 1)*sin(2*x) - 48*I)*sin(4*x) + (-48*I*x^2 + (192*I*
x^2 - 96*I)*cos(2*x) + 24*I)*sin(2*x))*log(cos(x)^2 + sin(x)^2 + 2*cos(x) +
 1) - (48*(2*x^2 - 1)*cos(4*x)^2 + 48*(2*x^2 - 1)*cos(2*x)^2 - 48*(2*x^2 -
1)*sin(4*x)^2 - 48*(2*x^2 - 1)*sin(2*x)^2 - (48*x^2 + 24*(2*x^2 - 1)*cos(4*
x) - 48*(2*x^2 - 1)*cos(2*x) - (-48*I*x^2 + 24*I)*sin(4*x) - (96*I*x^2 - 48
*I)*sin(2*x) - 24)*cos(6*x) + (96*x^2 - 120*(2*x^2 - 1)*cos(2*x) + (-240*I*
x^2 + 120*I)*sin(2*x) - 48)*cos(4*x) - 24*(2*x^2 - 1)*cos(2*x) + (-48*I*x^2
 + (-48*I*x^2 + 24*I)*cos(4*x) + (96*I*x^2 - 48*I)*cos(2*x) + 24*(2*x^2 - 1
)*sin(4*x) - 48*(2*x^2 - 1)*sin(2*x) + 24*I)*sin(6*x) + (96*I*x^2 + (192*I*
x^2 - 96*I)*cos(4*x) + (-240*I*x^2 + 120*I)*cos(2*x) + 120*(2*x^2 - 1)*sin(
2*x) - 48*I)*sin(4*x) + (-48*I*x^2 + (192*I*x^2 - 96*I)*cos(2*x) + 24*I)*si
n(2*x))*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + ((192*cos(4*x) - 384*cos(
2*x) + 192*I*sin(4*x) - 384*I*sin(2*x) + 192)*cos(6*x) + (960*cos(2*x) + 96
0*I*sin(2*x) - 384)*cos(4*x) - 384*cos(4*x)^2 - 384*cos(2*x)^2 - (-192*I*co
s(4*x) + 384*I*cos(2*x) + 192*sin(4*x) - 384*sin(2*x) - 192*I)*sin(6*x) - (
768*I*cos(4*x) - 960*I*cos(2*x) + 960*sin(2*x) + 384*I)*sin(4*x) + 384*sin(
4*x)^2 - (768*I*cos(2*x) - 192*I)*sin(2*x) + 384*sin(2*x)^2 + 192*cos(2*x))
*polylog(3, -e^(I*x)) + ((192*cos(4*x) - 384*cos(2*x) + 192*I*sin(4*x) - 38
4*I*sin(2*x) + 192)*cos(6*x) + (960*cos(2*x) + 960*I*sin(2*x) - 384)*cos(4*
x) - 384*cos(4*x)^2 - 384*cos(2*x)^2 - (-192*I*cos(4*x) + 384*I*cos(2*x) +
192*sin(4*x) - 384*sin(2*x) - 192*I)*sin(6*x) - (768*I*cos(4*x) - 960*I*cos
(2*x) + 960*sin(2*x) + 384*I)*sin(4*x) + 384*sin(4*x)^2 - (768*I*cos(2*x) -
 192*I)*sin(2*x) + 384*sin(2*x)^2 + 192*cos(2*x))*polylog(3, e^(I*x)) + (32
*x^3 - 12*I*x^2 - (-12*I*x^2 + 12*x + 6*I)*cos(6*x) + (32*x^3 - 24*I*x^2 -
72*x + 12*I)*cos(4*x) - (64*x^3 + 78*I*x^2 - 90*x + 9*I)*cos(2*x) - (-32*I*
x^3 - 24*x^2 + 72*I*x + 12)*sin(4*x) - (64*I*x^3 - 78*x^2 - 90*I*x - 9)*sin
(2*x) - 12*x + 6*I)*sin(6*x) - (64*x^3 - 30*I*x^2 + (128*x^3 - 48*I*x^2 - 3
36*x + 24*I)*cos(4*x) - (160*x^3 + 156*I*x^2 - 276*x + 18*I)*cos(2*x) + (-1
60*I*x^3 + 156*x^2 + 276*I*x + 18)*sin(2*x) - 30*x + 15*I)*sin(4*x) + (32*x
^3 - 24*I*x^2 - (128*x^3 + 168*I*x^2 - 192*x + 12*I)*cos(2*x) - 24*x + 12*I
)*sin(2*x) - 6*I*x - 3)/((48*cos(4*x) - 96*cos(2*x) + 48*I*sin(4*x) - 96*I*
sin(2*x) + 48)*cos(6*x) + (240*cos(2*x) + 240*I*sin(2*x) - 96)*cos(4*x) - 9
6*cos(4*x)^2 - 96*cos(2*x)^2 - (-48*I*cos(4*x) + 96*I*cos(2*x) + 48*sin(4*x
) - 96*sin(2*x) - 48*I)*sin(6*x) - (192*I*cos(4*x) - 240*I*cos(2*x) + 240*s
in(2*x) + 96*I)*sin(4*x) + 96*sin(4*x)^2 - (192*I*cos(2*x) - 48*I)*sin(2*x)
 + 96*sin(2*x)^2 + 48*cos(2*x))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cos(x)^2 \cot(x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2*cot(x)^3,x)
```

```
[Out] int(x^2*cos(x)^2*cot(x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cos^2(x) \cot^3(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)**2*cot(x)**3,x)
```

```
[Out] Integral(x**2*cos(x)**2*cot(x)**3, x)
```

# 3.207 $\int x\cos^2(x)\cot^3(x)\,dx$

**Optimal**. Leaf size=73

$$i\mathrm{Li}_2\left(e^{2ix}\right)+ix^2-\frac{3x}{4}-2x\log\left(1-e^{2ix}\right)+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)-\frac{\cot(x)}{2}+\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] -3/4*x+I*x^2-1/2*cot(x)-1/2*x*cot(x)^2-2*x*ln(1-exp(2*I*x))+I*polylog(2,exp
(2*I*x))+1/4*cos(x)*sin(x)+1/2*x*sin(x)^2
```

**Rubi [A]** time = 0.16, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4408, 3443, 2635, 8, 3717, 2190, 2279, 2391, 3720, 3473}

$$i\mathrm{PolyLog}\left(2,e^{2ix}\right)+ix^2-\frac{3x}{4}-2x\log\left(1-e^{2ix}\right)+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)-\frac{\cot(x)}{2}+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (-3*x)/4 + I*x^2 - Cot[x]/2 - (x*Cot[x]^2)/2 - 2*x*Log[1 - E^((2*I)*x)] + I
*PolyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 + (x*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\cos^2(x)\cot^3(x)\,dx &= -\int x\cos^2(x)\cot(x)\,dx + \int x\cot^3(x)\,dx \\
&= -\frac{1}{2}x\cot^2(x) + \frac{1}{2}\int \cot^2(x)\,dx - 2\int x\cot(x)\,dx + \int x\cos(x)\sin(x)\,dx \\
&= -\frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) + \frac{1}{2}x\sin^2(x) - 2\left(-\frac{ix^2}{2} - 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\right) - \frac{\int 1\,dx}{2} - \frac{1}{2}\int \sin^2(x \\
&= -\frac{x}{2} - \frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) + \frac{1}{4}\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) - \frac{\int 1\,dx}{4} - 2\left(-\frac{ix^2}{2} + x\log\left(1\right.\right. \\
&= -\frac{3x}{4} - \frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) + \frac{1}{4}\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) - 2\left(-\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right)\right. - \\
&= -\frac{3x}{4} - \frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) - 2\left(-\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) - \frac{1}{2}i\mathrm{Li}_2\left(e^{2ix}\right)\right) + \frac{1}{4}\cos(x)\sin(x
\end{aligned}
$$

**Mathematica [A]** time = 0.11, size = 62, normalized size = 0.85

$$\frac{1}{8}\left(8i\mathrm{Li}_2\left(e^{2ix}\right) + 8ix^2 - 16x\log\left(1-e^{2ix}\right) + \sin(2x) - 2x\cos(2x) - 4\cot(x) - 4x\csc^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] ((8*I)*x^2 - 2*x*Cos[2*x] - 4*Cot[x] - 4*x*Csc[x]^2 - 16*x*Log[1 - E^((2*I)
*x)] + (8*I)*PolyLog[2, E^((2*I)*x)] + Sin[2*x])/8
```

**fricas [B]** time = 0.48, size = 203, normalized size = 2.78

$$-\frac{2\,x\cos(x)^4-3\,x\cos(x)^2-\left(4i\,\cos(x)^2-4i\right)\mathrm{Li}_2\left(\cos(x)+i\,\sin(x)\right)-\left(-4i\,\cos(x)^2+4i\right)\mathrm{Li}_2\left(\cos(x)-i\,\sin(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*x*cos(x)^4 - 3*x*cos(x)^2 - (4*I*cos(x)^2 - 4*I)*dilog(cos(x) + I*s
in(x)) - (-4*I*cos(x)^2 + 4*I)*dilog(cos(x) - I*sin(x)) - (-4*I*cos(x)^2 +
4*I)*dilog(-cos(x) + I*sin(x)) - (4*I*cos(x)^2 - 4*I)*dilog(-cos(x) - I*sin
(x)) + 4*(x*cos(x)^2 - x)*log(cos(x) + I*sin(x) + 1) + 4*(x*cos(x)^2 - x)*l
og(cos(x) - I*sin(x) + 1) + 4*(x*cos(x)^2 - x)*log(-cos(x) + I*sin(x) + 1)
+ 4*(x*cos(x)^2 - x)*log(-cos(x) - I*sin(x) + 1) - (cos(x)^3 + cos(x))*sin(
x) - x)/(cos(x)^2 - 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(x)^2\cot(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cos(x)^2*cot(x)^3, x)
```

**maple [A]** time = 0.14, size = 109, normalized size = 1.49

$$ix^2-\frac{(i+2x)\,\mathrm{e}^{2ix}}{16}-\frac{(2x-i)\,\mathrm{e}^{-2ix}}{16}+\frac{2x\,\mathrm{e}^{2ix}-i\mathrm{e}^{2ix}+i}{\left(\mathrm{e}^{2ix}-1\right)^2}-2x\ln\left(1+\mathrm{e}^{ix}\right)-2x\ln\left(1-\mathrm{e}^{ix}\right)+2i\,\mathrm{polylog}\left(2,-\mathrm{e}^{ix}\right)+2i\,\mathrm{po}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2*cot(x)^3,x)
```

```
[Out] I*x^2-1/16*(I+2*x)*exp(2*I*x)-1/16*(2*x-I)*exp(-2*I*x)+(2*x*exp(2*I*x)-I*ex
p(2*I*x)+I)/(exp(2*I*x)-1)^2-2*x*ln(1+exp(I*x))-2*x*ln(1-exp(I*x))+2*I*poly
log(2,-exp(I*x))+2*I*polylog(2,exp(I*x))
```

**maxima [B]** time = 0.59, size = 1739, normalized size = 23.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] -((2*x + I)*cos(6*x)^2 - (-32*I*x^2 - 8*x - 4*I)*cos(4*x)^2 - (-32*I*x^2 +
28*x - 16*I)*cos(2*x)^2 - (2*x + I)*sin(6*x)^2 - (32*I*x^2 + 8*x + 4*I)*sin
(4*x)^2 - (32*I*x^2 - 28*x + 16*I)*sin(2*x)^2 - (64*I*x*cos(4*x)^2 + 64*I*x
*cos(2*x)^2 - 64*I*x*sin(4*x)^2 - 64*I*x*sin(2*x)^2 + (-32*I*x*cos(4*x) + 6
4*I*x*cos(2*x) + 32*x*sin(4*x) - 64*x*sin(2*x) - 32*I*x)*cos(6*x) + (-160*I
*x*cos(2*x) + 160*x*sin(2*x) + 64*I*x)*cos(4*x) - 32*I*x*cos(2*x) + (32*x*c
os(4*x) - 64*x*cos(2*x) + 32*I*x*sin(4*x) - 64*I*x*sin(2*x) + 32*x)*sin(6*x
) - 32*(4*x*cos(4*x) - 5*x*cos(2*x) - 5*I*x*sin(2*x) + 2*x)*sin(4*x) - 32*(
4*x*cos(2*x) - x)*sin(2*x))*arctan2(sin(x), cos(x) + 1) - (-64*I*x*cos(4*x)
^2 - 64*I*x*cos(2*x)^2 + 64*I*x*sin(4*x)^2 + 64*I*x*sin(2*x)^2 + (32*I*x*co
s(4*x) - 64*I*x*cos(2*x) - 32*x*sin(4*x) + 64*x*sin(2*x) + 32*I*x)*cos(6*x)
 + (160*I*x*cos(2*x) - 160*x*sin(2*x) - 64*I*x)*cos(4*x) + 32*I*x*cos(2*x)
- (32*x*cos(4*x) - 64*x*cos(2*x) + 32*I*x*sin(4*x) - 64*I*x*sin(2*x) + 32*x
```

```
)*sin(6*x) + 32*(4*x*cos(4*x) - 5*x*cos(2*x) - 5*I*x*sin(2*x) + 2*x)*sin(4*
x) + 32*(4*x*cos(2*x) - x)*sin(2*x))*arctan2(sin(x), -cos(x) + 1) - (16*I*x
^2 + (16*I*x^2 + 8*x + 4*I)*cos(4*x) + (-32*I*x^2 + 26*x - 17*I)*cos(2*x) -
 4*(4*x^2 - 2*I*x + 1)*sin(4*x) + (32*x^2 + 26*I*x + 17)*sin(2*x) + 4*x + 1
4*I)*cos(6*x) - (-32*I*x^2 + (80*I*x^2 - 52*x + 34*I)*cos(2*x) - 2*(40*x^2
+ 26*I*x + 17)*sin(2*x) - 10*x - 27*I)*cos(4*x) - (16*I*x^2 + 8*x + 12*I)*c
os(2*x) - ((32*I*cos(4*x) - 64*I*cos(2*x) - 32*sin(4*x) + 64*sin(2*x) + 32*
I)*cos(6*x) + (160*I*cos(2*x) - 160*sin(2*x) - 64*I)*cos(4*x) - 64*I*cos(4*
x)^2 - 64*I*cos(2*x)^2 - (32*cos(4*x) - 64*cos(2*x) + 32*I*sin(4*x) - 64*I*
sin(2*x) + 32)*sin(6*x) + (128*cos(4*x) - 160*cos(2*x) - 160*I*sin(2*x) + 6
4)*sin(4*x) + 64*I*sin(4*x)^2 + 32*(4*cos(2*x) - 1)*sin(2*x) + 64*I*sin(2*x
)^2 + 32*I*cos(2*x))*dilog(-e^(I*x)) - ((32*I*cos(4*x) - 64*I*cos(2*x) - 32
*sin(4*x) + 64*sin(2*x) + 32*I)*cos(6*x) + (160*I*cos(2*x) - 160*sin(2*x) -
 64*I)*cos(4*x) - 64*I*cos(4*x)^2 - 64*I*cos(2*x)^2 - (32*cos(4*x) - 64*cos
(2*x) + 32*I*sin(4*x) - 64*I*sin(2*x) + 32)*sin(6*x) + (128*cos(4*x) - 160*
cos(2*x) - 160*I*sin(2*x) + 64)*sin(4*x) + 64*I*sin(4*x)^2 + 32*(4*cos(2*x)
 - 1)*sin(2*x) + 64*I*sin(2*x)^2 + 32*I*cos(2*x))*dilog(e^(I*x)) - (32*x*co
s(4*x)^2 + 32*x*cos(2*x)^2 - 32*x*sin(4*x)^2 - 32*x*sin(2*x)^2 - (16*x*cos(
4*x) - 32*x*cos(2*x) + 16*I*x*sin(4*x) - 32*I*x*sin(2*x) + 16*x)*cos(6*x) -
 16*(5*x*cos(2*x) + 5*I*x*sin(2*x) - 2*x)*cos(4*x) - 16*x*cos(2*x) + (-16*I
*x*cos(4*x) + 32*I*x*cos(2*x) + 16*x*sin(4*x) - 32*x*sin(2*x) - 16*I*x)*sin
(6*x) + (64*I*x*cos(4*x) - 80*I*x*cos(2*x) + 80*x*sin(2*x) + 32*I*x)*sin(4*
x) + (64*I*x*cos(2*x) - 16*I*x)*sin(2*x))*log(cos(x)^2 + sin(x)^2 + 2*cos(x
) + 1) - (32*x*cos(4*x)^2 + 32*x*cos(2*x)^2 - 32*x*sin(4*x)^2 - 32*x*sin(2*
x)^2 - (16*x*cos(4*x) - 32*x*cos(2*x) + 16*I*x*sin(4*x) - 32*I*x*sin(2*x) +
 16*x)*cos(6*x) - 16*(5*x*cos(2*x) + 5*I*x*sin(2*x) - 2*x)*cos(4*x) - 16*x*
cos(2*x) + (-16*I*x*cos(4*x) + 32*I*x*cos(2*x) + 16*x*sin(4*x) - 32*x*sin(2
*x) - 16*I*x)*sin(6*x) + (64*I*x*cos(4*x) - 80*I*x*cos(2*x) + 80*x*sin(2*x)
 + 32*I*x)*sin(4*x) + (64*I*x*cos(2*x) - 16*I*x)*sin(2*x))*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1) + (16*x^2 + 2*(2*I*x - 1)*cos(6*x) + 4*(4*x^2 - 2*I
*x + 1)*cos(4*x) - (32*x^2 + 26*I*x + 17)*cos(2*x) - (-16*I*x^2 - 8*x - 4*I
)*sin(4*x) - (32*I*x^2 - 26*x + 17*I)*sin(2*x) - 4*I*x + 14)*sin(6*x) - (32
*x^2 + 8*(8*x^2 - 2*I*x + 1)*cos(4*x) - 2*(40*x^2 + 26*I*x + 17)*cos(2*x) +
 (-80*I*x^2 + 52*x - 34*I)*sin(2*x) - 10*I*x + 27)*sin(4*x) + 4*(4*x^2 - 2*
(8*x^2 + 7*I*x + 4)*cos(2*x) - 2*I*x + 3)*sin(2*x) + 2*x - I)/((16*cos(4*x)
 - 32*cos(2*x) + 16*I*sin(4*x) - 32*I*sin(2*x) + 16)*cos(6*x) + (80*cos(2*x
) + 80*I*sin(2*x) - 32)*cos(4*x) - 32*cos(4*x)^2 - 32*cos(2*x)^2 - (-16*I*c
os(4*x) + 32*I*cos(2*x) + 16*sin(4*x) - 32*sin(2*x) - 16*I)*sin(6*x) - (64*
I*cos(4*x) - 80*I*cos(2*x) + 80*sin(2*x) + 32*I)*sin(4*x) + 32*sin(4*x)^2 -
 (64*I*cos(2*x) - 16*I)*sin(2*x) + 32*sin(2*x)^2 + 16*cos(2*x))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x\cos(x)^2\cot(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2*cot(x)^3,x)
[Out] int(x*cos(x)^2*cot(x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos^2(x)\cot^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**2*cot(x)**3,x)
[Out] Integral(x*cos(x)**2*cot(x)**3, x)
```

## 3.208 $\int (c+dx)^m \tan(a+bx)\,dx$

**Optimal**. Leaf size=17

$$\text{Int}\left(\tan(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable((d*x+c)^m*tan(b*x+a),x)
```

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Tan[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Tan[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \tan(a+bx)\,dx = \int (c+dx)^m \tan(a+bx)\,dx$$

**Mathematica [A]** time = 2.55, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Tan[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Tan[a + b*x], x]
```

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \sec(bx+a)\sin(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

**maple [A]** time = 0.13, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x)
[Out] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec (bx+a) \sin (bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\sin (a+b\,x)\,\left(c+d\,x\right)^m}{\cos (a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x)^m)/cos(a + b*x),x)
[Out] int((sin(a + b*x)*(c + d*x)^m)/cos(a + b*x), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin (a+bx) \sec (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a),x)
[Out] Integral((c + d*x)**m*sin(a + b*x)*sec(a + b*x), x)
```

## 3.209 $\int (c+dx)^4 \tan(a+bx)\,dx$

**Optimal**. Leaf size=158

$$\frac{3d^4\text{Li}_5\left(-e^{2i(a+bx)}\right)}{2b^5}-\frac{3id^3(c+dx)\text{Li}_4\left(-e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{(c\cdots}{}$$

```
[Out] 1/5*I*(d*x+c)^5/d-(d*x+c)^4*ln(1+exp(2*I*(b*x+a)))/b+2*I*d*(d*x+c)^3*polylo
g(2,-exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3
-3*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,-exp(2*
I*(b*x+a)))/b^5
```

**Rubi** [A] time = 0.21, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3719, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}-\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}+\frac{2id(c+dx)^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Tan[a + b*x],x]
```

```
[Out] ((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b + ((2*
I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)^2
*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLog[4, -E
^((2*I)*(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4\tan(a+bx)\,dx = \frac{i(c+dx)^5}{5d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1+e^{2i(a+bx)}}\,dx$$

$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{(4d)\int (c+dx)^3\log\left(1+e^{2i(a+bx)}\right)dx}{b}$$

$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{\left(6id^2\right)\int (c$$

$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx$$

$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx$$

$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx$$

$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx$$

**Mathematica [A]** time = 0.07, size = 157, normalized size = 0.99

$$\frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2\left(2b^2(c+dx)^2\text{Li}_3\left(-e^{2i(a+bx)}\right)+d\left(2ib(c+dx)\text{Li}_4\left(-e^{2i(a+bx)}\right)-d\text{Li}_5\left(-e^{2i(a+bx)}\right)\right)\right)}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Tan[a + b*x],x]
```

```
[Out] ((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b + ((2*
I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(2*b^2*(c +
 d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))] + d*((2*I)*b*(c + d*x)*PolyLog[4,
-E^((2*I)*(a + b*x))] - d*PolyLog[5, -E^((2*I)*(a + b*x))])))/(2*b^5)
```

**fricas [C]** time = 0.57, size = 1402, normalized size = 8.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(24*d^4*polylog(5, I*cos(b*x + a) + sin(b*x + a)) + 24*d^4*polylog(5, I
*cos(b*x + a) - sin(b*x + a)) + 24*d^4*polylog(5, -I*cos(b*x + a) + sin(b*x
 + a)) + 24*d^4*polylog(5, -I*cos(b*x + a) - sin(b*x + a)) + (-4*I*b^3*d^4*
x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(I*cos(
```

```
b*x + a) + sin(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3
*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (4*I*b^3
*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-
I*cos(b*x + a) + sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 1
2*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) -
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log
(cos(b*x + a) + I*sin(b*x + a) + I) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x + a) + I) -
 (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b
^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x + a)
+ sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 +
4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 +
 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*
a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^4*d^4*x
^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d -
6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) - sin(b*x
+ a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) - I*sin(b
*x + a) + I) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) + si
n(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) - si
n(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) + s
in(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) - s
in(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*
cos(b*x + a) + sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^
2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^
3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*(b^2*d^4
*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x +
a)))/b^5
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \sec(bx+a)\sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)*sin(b*x + a), x)
```

**maple [B]** time = 0.11, size = 616, normalized size = 3.90

$$-\frac{3c^2d^2 \operatorname{polylog}\left(3,-e^{2i(bx+a)}\right)}{b^3}-\frac{3d^4 \operatorname{polylog}\left(3,-e^{2i(bx+a)}\right)x^2}{b^3}-\frac{d^4\ln\left(1+e^{2i(bx+a)}\right)x^4}{b}-ic^4x+\frac{2c^4\ln\left(e^{i(bx+a)}\right)}{b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] 3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5+2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))-3
/b^3*c^2*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/b^3*d^4*polylog(3,-exp(2*I*(b*x
+a)))*x^2+1/5*I*d^4*x^5+I*c*d^3*x^4+8*I/b*a*c^3*d*x-12*I/b^2*a^2*c^2*d^2*x+
8*I/b^3*c*d^3*a^3*x+6*I/b^4*c*d^3*a^4-8*I/b^3*a^3*c^2*d^2-2*I/b^4*d^4*a^4*x
+4*I/b^2*a^2*c^3*d-4/b*c*d^3*ln(1+exp(2*I*(b*x+a)))*x^3+6*I/b^2*c*d^3*polyl
og(2,-exp(2*I*(b*x+a)))*x^2+6*I/b^2*c^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-
I*c^4*x-1/b*c^4*ln(1+exp(2*I*(b*x+a)))+2/b*c^4*ln(exp(I*(b*x+a)))-1/b*d^4*l
n(1+exp(2*I*(b*x+a)))*x^4-8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))-8/b^4*c*d^3*a^3*
ln(exp(I*(b*x+a)))+12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))+2*I*c^2*d^2*x^3+2*
I*c^3*d*x^2-4/b*c^3*d*ln(1+exp(2*I*(b*x+a)))*x-6/b^3*c*d^3*polylog(3,-exp(2
```

```
*I*(b*x+a))*x-6/b*c^2*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-8/5*I/b^5*d^4*a^5-3*I
/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x+2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x
+a)))*x^3-3*I/b^4*c*d^3*polylog(4,-exp(2*I*(b*x+a)))+2*I/b^2*c^3*d*polylog(
2,-exp(2*I*(b*x+a)))
```

**maxima** [B] time = 0.60, size = 792, normalized size = 5.01

$$-\frac{15\,c^4\log\left(-\sin\left(bx+a\right)^2+1\right)-\frac{60\,ac^3d\log\left(-\sin(bx+a)^2+1\right)}{b}+\frac{90\,a^2c^2d^2\log\left(-\sin(bx+a)^2+1\right)}{b^2}-\frac{60\,a^3cd^3\log\left(-\sin(bx+a)^2+1\right)}{b^3}+\frac{15}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/30*(15*c^4*log(-sin(b*x + a)^2 + 1) - 60*a*c^3*d*log(-sin(b*x + a)^2 + 1
)/b + 90*a^2*c^2*d^2*log(-sin(b*x + a)^2 + 1)/b^2 - 60*a^3*c*d^3*log(-sin(b
*x + a)^2 + 1)/b^3 + 15*a^4*d^4*log(-sin(b*x + a)^2 + 1)/b^4 + 2*(-3*I*(b*x
 + a)^5*d^4 + (-15*I*b*c*d^3 + 15*I*a*d^4)*(b*x + a)^4 - 45*d^4*polylog(5,
-e^(2*I*b*x + 2*I*a)) + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 - 30*I*a^2*d^4)
*(b*x + a)^3 + (-30*I*b^3*c^3*d + 90*I*a*b^2*c^2*d^2 - 90*I*a^2*b*c*d^3 + 3
0*I*a^3*d^4)*(b*x + a)^2 + (30*I*(b*x + a)^4*d^4 + (80*I*b*c*d^3 - 80*I*a*d
^4)*(b*x + a)^3 + (90*I*b^2*c^2*d^2 - 180*I*a*b*c*d^3 + 90*I*a^2*d^4)*(b*x
+ a)^2 + (60*I*b^3*c^3*d - 180*I*a*b^2*c^2*d^2 + 180*I*a^2*b*c*d^3 - 60*I*a
^3*d^4)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (-30*I
*b^3*c^3*d + 90*I*a*b^2*c^2*d^2 - 90*I*a^2*b*c*d^3 - 60*I*(b*x + a)^3*d^4 +
 30*I*a^3*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a)^2 + (-90*I*b^2*c^2
*d^2 + 180*I*a*b*c*d^3 - 90*I*a^2*d^4)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a
)) + 5*(3*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3
*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2 + 2*cos(2*b*x + 2*a) + 1) + (60*I*b*c*d^3 + 90*I*(b*x + a)*d^4 - 60*I*a
*d^4)*polylog(4, -e^(2*I*b*x + 2*I*a)) + 15*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 +
6*(b*x + a)^2*d^4 + 3*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -
e^(2*I*b*x + 2*I*a)))/b^4)/b
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^4}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x)^4)/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)*(c + d*x)^4)/cos(a + b*x), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \sin\left(a+bx\right)\sec\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*sin(a + b*x)*sec(a + b*x), x)
```

# 3.210 $\int (c+dx)^3 \tan(a+bx)\,dx$

**Optimal**. Leaf size=132

$$-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(}{}$$

```
[Out] 1/4*I*(d*x+c)^4/d-(d*x+c)^3*ln(1+exp(2*I*(b*x+a)))/b+3/2*I*d*(d*x+c)^2*poly
log(2,-exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b
^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4
```

**Rubi [A]** time = 0.18, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3719, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id^3\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}-\frac{(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tan[a + b*x],x]
```

```
[Out] ((I/4)*(c + d*x)^4)/d - ((c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3
*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*
x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E
^((2*I)*(a + b*x))])/b^4
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\tan(a+bx)\,dx = \frac{i(c+dx)^4}{4d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}\,dx$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{(3d)\int (c+dx)^2\log\left(1+e^{2i(a+bx)}\right)dx}{b}$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{\left(3id^2\right)\int (c}{}$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3d^2(c+dx}{}$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3d^2(c+dx}{}$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3d^2(c+dx}{}$$

**Mathematica [A]** time = 0.08, size = 126, normalized size = 0.95

$$\frac{1}{4}i\left(\frac{3d\left(2b^2(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)+d\left(2ib(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)-d\text{Li}_4\left(-e^{2i(a+bx)}\right)\right)\right)}{b^4}+\frac{4i(c+dx)^3\log\left(1+e^{2i}\right.}{b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Tan[a + b*x],x]
```

```
[Out] (I/4)*((c + d*x)^4/d + ((4*I)*(c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b +
 (3*d*(2*b^2*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))] + d*((2*I)*b*(c +
 d*x)*PolyLog[3, -E^((2*I)*(a + b*x))] - d*PolyLog[4, -E^((2*I)*(a + b*x))]
)))/b^4)
```

**fricas [C]** time = 0.54, size = 970, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*polylog(4,
 I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + sin(
b*x + a)) + 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*
d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x +
 a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(I*cos(b*x
+ a) - sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*
```

```
dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x
 - 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I) -
 (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*s
in(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) -
(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(
-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) -
sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log
(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b*d^3*x + b*c*
d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*poly
log(3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I
*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x
 + a) - sin(b*x + a)))/b^4
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \sec(bx + a) \sin(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a), x)
```

**maple [B]** time = 0.08, size = 423, normalized size = 3.20

$$\frac{3ic^2d\,a^2}{b^2}+\frac{3ic^2d\,x^2}{2}+\frac{2ia^3d^3x}{b^3}-ic^3x-\frac{4ic\,d^2a^3}{b^3}-\frac{3id^3\,\mathrm{polylog}\left(4,-\mathrm{e}^{2i(bx+a)}\right)}{4b^4}-\frac{c^3\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)}{b}-\frac{2d^3a^3\ln\left(\mathrm{e}^{i(bx+}\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/4*I*d^3*x^4+I*c*d^2*x^3-I*c^3*x+3*I/b^2*a^2*c^2*d-4*I/b^3*a^3*c*d^2+2*I/b
^3*d^3*a^3*x-1/b*c^3*ln(1+exp(2*I*(b*x+a)))-2/b^4*d^3*a^3*ln(exp(I*(b*x+a))
)-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/2/b^3*d^3*polylog(3,-exp(2*I
*(b*x+a)))*x-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+2/b*c^3*ln(exp(I*(b
*x+a)))+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+3/2*I*c^2*d*x^2+6/b^3*
c*d^2*a^2*ln(exp(I*(b*x+a)))-6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))-1/b*d^3*ln(1+
exp(2*I*(b*x+a)))*x^3+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2-6*I/b^
2*a^2*c*d^2*x+6*I/b*a*c^2*d*x+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))+
3/2*I/b^4*d^3*a^4-3/b*c^2*d*ln(1+exp(2*I*(b*x+a)))*x-3/b*c*d^2*ln(1+exp(2*I
*(b*x+a)))*x^2
```

**maxima [B]** time = 0.57, size = 490, normalized size = 3.71

$$-\frac{6\,c^3\log\left(-\sin(bx+a)^2+1\right)-\frac{18\,ac^2d\log\left(-\sin(bx+a)^2+1\right)}{b}+\frac{18\,a^2cd^2\log\left(-\sin(bx+a)^2+1\right)}{b^2}-\frac{6\,a^3d^3\log\left(-\sin(bx+a)^2+1\right)}{b^3}+\frac{-3i}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/12*(6*c^3*log(-sin(b*x + a)^2 + 1) - 18*a*c^2*d*log(-sin(b*x + a)^2 + 1)
/b + 18*a^2*c*d^2*log(-sin(b*x + a)^2 + 1)/b^2 - 6*a^3*d^3*log(-sin(b*x + a
)^2 + 1)/b^3 + (-3*I*(b*x + a)^4*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x +
```

```
a)^3 + 12*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) + (-18*I*b^2*c^2*d + 36*I*
a*b*c*d^2 - 18*I*a^2*d^3)*(b*x + a)^2 + (16*I*(b*x + a)^3*d^3 + (36*I*b*c*d
^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*a^2*
d^3)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (-18*I*b^
2*c^2*d + 36*I*a*b*c*d^2 - 24*I*(b*x + a)^2*d^3 - 18*I*a^2*d^3 + (-36*I*b*c
*d^2 + 36*I*a*d^3)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + 2*(4*(b*x + a)^
3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
 2*a) + 1) + 6*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*polylog(3, -e^(2*I*b
*x + 2*I*a)))/b^3)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (a+b\,x)\,\left(c+d\,x\right)^3}{\cos (a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x)^3)/cos(a + b*x),x)
[Out] int((sin(a + b*x)*(c + d*x)^3)/cos(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \sin (a+bx) \sec (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a),x)
[Out] Integral((c + d*x)**3*sin(a + b*x)*sec(a + b*x), x)
```

# 3.211 $\int (c+dx)^2 \tan(a+bx)\,dx$

**Optimal**. Leaf size=96

$$-\frac{d^2 \text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^3}{3d}$$

```
[Out] 1/3*I*(d*x+c)^3/d-(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b+I*d*(d*x+c)*polylog(2,
-exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3
```

**Rubi** [A] time = 0.15, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3719, 2190, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Tan[a + b*x],x]
```

```
[Out] ((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b + (I*d
*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d^2*PolyLog[3, -E^((2*I
)*(a + b*x))])/(2*b^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\tan(a+bx)\,dx = \frac{i(c+dx)^3}{3d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx$$
$$= \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{(2d)\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b}$$
$$= \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{\left(id^2\right)\int \text{Li}_2\left(\right.}{b}$$
$$= \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\text{Li}_3\left(-e^{2i(a\right.}}{2b^3}$$

**Mathematica [A]** time = 0.04, size = 100, normalized size = 1.04

$$\frac{2ib^2(c+dx)^2\left(b(c+dx)+3id\log\left(1+e^{2i(a+bx)}\right)\right)+6ibd^2(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)-3d^3\text{Li}_3\left(-e^{2i(a+bx)}\right)}{6b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Tan[a + b*x],x]
```

```
[Out] ((2*I)*b^2*(c + d*x)^2*(b*(c + d*x) + (3*I)*d*Log[1 + E^((2*I)*(a + b*x))])
 + (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] - 3*d^3*PolyLog[3
, -E^((2*I)*(a + b*x))])/(6*b^3*d)
```

**fricas [C]** time = 0.50, size = 594, normalized size = 6.19

$$-\frac{2\,d^2\text{polylog}\left(3, i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)+2\,d^2\text{polylog}\left(3, i\,\cos\left(bx+a\right)-\sin\left(bx+a\right)\right)+2\,d^2\text{polylog}\left(3,-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 2*d^2*polylog(3, I*
cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x +
 a)) + 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - (-2*I*b*d^2*x - 2
*I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) - (2*I*b*d^2*x + 2*I*b*c*d)*
dilog(I*cos(b*x + a) - sin(b*x + a)) - (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*c
os(b*x + a) + sin(b*x + a)) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-I*cos(b*x +
 a) - sin(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*
sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*si
n(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*c
os(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d -
a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2
*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a
) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a)
 + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a)
+ I))/b^3
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\sec\left(bx+a\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a), x)
```

**maple [B]** time = 0.06, size = 257, normalized size = 2.68

$$\frac{id^2x^3}{3}-\frac{4id^2a^3}{3b^3}+\frac{id^2\operatorname{polylog}\left(2,-\mathrm{e}^{2i(bx+a)}\right)x}{b^2}-\frac{c^2\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)}{b}+\frac{2c^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b}+\frac{2d^2a^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^3}+icd\,x^2-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/3*I*d^2*x^3+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-2*I/b^2*a^2*d^2*x-1/
b*c^2*ln(1+exp(2*I*(b*x+a)))+2/b*c^2*ln(exp(I*(b*x+a)))+2/b^3*d^2*a^2*ln(ex
p(I*(b*x+a)))+2*I/b^2*a^2*c*d-I*c^2*x-4/3*I/b^3*a^3*d^2-1/b*d^2*ln(1+exp(2*
I*(b*x+a)))*x^2+I*c*d*x^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-4/b^2*c*
d*a*ln(exp(I*(b*x+a)))+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))+4*I/b*a*c*d*x
-2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x
```

**maxima [B]** time = 0.53, size = 280, normalized size = 2.92

$$-\frac{3\,c^2\log\left(-\sin\left(bx+a\right)^2+1\right)-\frac{6\,acd\log\left(-\sin(bx+a)^2+1\right)}{b}+\frac{3\,a^2d^2\log\left(-\sin(bx+a)^2+1\right)}{b^2}+\frac{-2i\,(bx+a)^3d^2+\left(-6i\,bcd+6i\,ad^2\right)(bx+a)^2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^2*log(-sin(b*x + a)^2 + 1) - 6*a*c*d*log(-sin(b*x + a)^2 + 1)/b +
 3*a^2*d^2*log(-sin(b*x + a)^2 + 1)/b^2 + (-2*I*(b*x + a)^3*d^2 + (-6*I*b*c
*d + 6*I*a*d^2)*(b*x + a)^2 + 3*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) + (6*I
*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x +
 2*a), cos(2*b*x + 2*a) + 1) + (-6*I*b*c*d - 6*I*(b*x + a)*d^2 + 6*I*a*d^2)
*dilog(-e^(2*I*b*x + 2*I*a)) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)
)/b^2)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x)^2)/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)*(c + d*x)^2)/cos(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2\sin\left(a+bx\right)\sec\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*sin(a + b*x)*sec(a + b*x), x)
```

# 3.212 $\int (c+dx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=66

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^2}{2d}$$

```
[Out] 1/2*I*(d*x+c)^2/d-(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,-exp(2
*I*(b*x+a)))/b^2
```

**Rubi [A]** time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3719, 2190, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tan[a + b*x],x]
```

```
[Out] ((I/2)*(c + d*x)^2)/d - ((c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b + ((I/2)
*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\int (c+dx)\tan(a+bx)\,dx = \frac{i(c+dx)^2}{2d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx$$
$$= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{d\int \log\left(1+e^{2i(a+bx)}\right)\,dx}{b}$$
$$= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{(id)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i(a+bx)}\right)}{2b^2}$$
$$= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}$$

**Mathematica [A]** time = 0.01, size = 70, normalized size = 1.06

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{c\log(\cos(a+bx))}{b} - \frac{dx\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{1}{2}idx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Tan[a + b*x],x]
```

```
[Out] (I/2)*d*x^2 - (d*x*Log[1 + E^((2*I)*(a + b*x))])/b - (c*Log[Cos[a + b*x]])/
b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2
```

**fricas [B]** time = 0.49, size = 310, normalized size = 4.70

$$-i\,d\text{Li}_2\left(i\cos\left(bx+a\right)+\sin\left(bx+a\right)\right) + i\,d\text{Li}_2\left(i\cos\left(bx+a\right)-\sin\left(bx+a\right)\right) + i\,d\text{Li}_2\left(-i\cos\left(bx+a\right)+\sin\left(bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(I*cos(b*x + a) -
 sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(-I*c
os(b*x + a) - sin(b*x + a)) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a)
 + I) - (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d*x + a*d)*
log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x + a) -
 sin(b*x + a) + 1) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)
- (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c - a*d)*log(-
cos(b*x + a) + I*sin(b*x + a) + I) - (b*c - a*d)*log(-cos(b*x + a) - I*sin(
b*x + a) + I))/b^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\sec\left(bx+a\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a), x)
```

**maple [B]** time = 0.07, size = 123, normalized size = 1.86

$$\frac{id\,x^2}{2} - icx - \frac{c\ln\left(1+e^{2i(bx+a)}\right)}{b} + \frac{2c\ln\left(e^{i(bx+a)}\right)}{b} + \frac{2idax}{b} + \frac{id\,a^2}{b^2} - \frac{d\ln\left(1+e^{2i(bx+a)}\right)x}{b} + \frac{id\,\text{polylog}\left(2,-e^{2i(bx+a)}\right)}{2b^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/2*I*d*x^2-I*c*x-1/b*c*ln(1+exp(2*I*(b*x+a)))+2/b*c*ln(exp(I*(b*x+a)))+2*I
/b*d*a*x+I/b^2*d*a^2-1/b*d*ln(1+exp(2*I*(b*x+a)))*x+1/2*I*d*polylog(2,-exp(
2*I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

**maxima [B]** time = 0.53, size = 114, normalized size = 1.73

$$-\frac{-i\,b^2dx^2-2i\,b^2cx+(2i\,bdx+2i\,bc)\arctan\left(\sin\left(2\,bx+2\,a\right),\cos\left(2\,bx+2\,a\right)+1\right)-i\,d\mathrm{Li}_2\left(-e^{(2i\,bx+2i\,a)}\right)+(bdx+}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*b^2*d*x^2 - 2*I*b^2*c*x + (2*I*b*d*x + 2*I*b*c)*arctan2(sin(2*b*x
+ 2*a), cos(2*b*x + 2*a) + 1) - I*d*dilog(-e^(2*I*b*x + 2*I*a)) + (b*d*x +
b*c)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1))
/b^2
```

**mupad [B]** time = 1.57, size = 148, normalized size = 2.24

$$\frac{c\,\ln\left(\tan\left(a+b\,x\right)^2+1\right)}{2\,b}-\frac{d\,\left(\pi\,\ln\left(\cos\left(b\,x\right)\right)+\mathrm{polylog}\left(2,-\mathrm{e}^{-a\,2\mathrm{i}}\,\mathrm{e}^{-b\,x\,2\mathrm{i}}\right)\,1\mathrm{i}-\pi\,\ln\left(\mathrm{e}^{-a\,2\mathrm{i}}\,\mathrm{e}^{-b\,x\,2\mathrm{i}}+1\right)+2\,a\,\ln\left(\mathrm{e}^{-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x))/cos(a + b*x),x)
```

```
[Out] (c*log(tan(a + b*x)^2 + 1))/(2*b) - (d*(polylog(2, -exp(-a*2i)*exp(-b*x*2i)
)*1i - pi*log(exp(b*x*2i) + 1) - pi*log(exp(-a*2i)*exp(-b*x*2i) + 1) + 2*a*
log(exp(-a*2i)*exp(-b*x*2i) + 1) + pi*log(cos(b*x)) + b^2*x^2*1i - log(cos(
a + b*x))*(2*a - pi) + 2*b*x*log(exp(-a*2i)*exp(-b*x*2i) + 1) + a*b*x*2i))/
(2*b^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\sin(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)*sin(a + b*x)*sec(a + b*x), x)
```

# 3.213 $\int \frac{\tan(a+bx)}{c+dx} dx$

**Optimal**. Leaf size=17

$$\operatorname{Int}\left(\frac{\tan(a+bx)}{c+dx}, x\right)$$

```
[Out] Unintegrable(tan(b*x+a)/(d*x+c),x)
```

**Rubi** [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]/(c + d*x),x]
[Out] Defer[Int][Tan[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\tan(a+bx)}{c+dx} dx = \int \frac{\tan(a+bx)}{c+dx} dx$$

**Mathematica** [A] time = 3.53, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]/(c + d*x),x]
[Out] Integrate[Tan[a + b*x]/(c + d*x), x]
```

**fricas** [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\sin(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

**maple** [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\sin(a+b\,x)}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)),x)
```

```
[Out] int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

## 3.214 $\int \frac{\tan(a+bx)}{(c+dx)^2} dx$

**Optimal**. Leaf size=17

$$\text{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable(tan(b*x+a)/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]/(c + d*x)^2,x]
[Out] Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tan(a+bx)}{(c+dx)^2} dx = \int \frac{\tan(a+bx)}{(c+dx)^2} dx$$

**Mathematica [A]** time = 5.30, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]/(c + d*x)^2,x]
[Out] Integrate[Tan[a + b*x]/(c + d*x)^2, x]
```

**fricas [A]** time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\sin(bx+a)}{d^2x^2 + 2\,cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sec(b*x + a)*sin(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c)^2, x)
```

**maple** [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c)^2, x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\sin(a+b\,x)}{\cos(a+b\,x)\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)^2), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

# 3.215 $\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=148

$$\text{Int}\left(\sec(a+bx)(c+dx)^m,x\right)+\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)}{2b}$$

```
[Out] 1/2*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)
/d)^m)-1/2*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d
*x+c)/d)^m)+Unintegrable((d*x+c)^m*sec(b*x+a),x)
```

**Rubi** [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/
(b*(((-I)*b*(c + d*x))/d)^m) - ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*
x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x
)^m*Sec[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx &= -\int (c+dx)^m\cos(a+bx)\,dx+\int (c+dx)^m\sec(a+bx)\,dx\\ &= -\left(\frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m\,dx\right)-\frac{1}{2}\int e^{i(a+bx)}(c+dx)^m\,dx+\int (c+dx\\ &= \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+d}{}\end{aligned}$$

**Mathematica** [A] time = 6.62, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x], x]
```

**fricas** [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos(bx+a)^2-1\right)(dx+c)^m\sec(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*(d*x + c)^m*sec(b*x + a), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\sin(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^2, x)
```

**maple** [A] time = 0.27, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\left(\sin^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x)
[Out] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\sin(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^2, x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^2\,(c+d\,x)^m}{\cos(a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^2*(c + d*x)^m)/cos(a + b*x),x)
[Out] int((sin(a + b*x)^2*(c + d*x)^m)/cos(a + b*x), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a)**2,x)
[Out] Timed out
```

# 3.216 $\int (c+dx)^3 \sin(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=275

$$-\frac{6id^3\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{6id^3\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^4}+\frac{6d^3\cos(a+bx)}{b^4}-\frac{6d^2(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{Li}_3\left(ie^{i(a+b}\right.}{b^3}$$

```
[Out] -2*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b+6*d^3*cos(b*x+a)/b^4-3*d*(d*x+c)^2*
cos(b*x+a)/b^2+3*I*d*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^2-3*I*d*(d*x+
c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x+
a)))/b^3+6*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3-6*I*d^3*polylog(4,-I
*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4+6*d^2*(d*x+c)*
sin(b*x+a)/b^3-(d*x+c)^3*sin(b*x+a)/b
```

**Rubi** [A] time = 0.21, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4407, 3296, 2638, 4181, 2531, 6609, 2282, 6589}

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (6*d^3*Cos[a + b*x])/b^4 -
 (3*d*(c + d*x)^2*Cos[a + b*x])/b^2 + ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*
E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])
/b^2 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (6*d^2*(c +
 d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, (-I)*E^(I*
(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 + (6*d^2*(
c + d*x)*Sin[a + b*x])/b^3 - ((c + d*x)^3*Sin[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \sin(a+bx)\tan(a+bx)\,dx = -\int (c+dx)^3\cos(a+bx)\,dx + \int (c+dx)^3\sec(a+bx)\,dx$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(c+dx)^3\sin(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\,\mathrm{lo}}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} + \frac{3id(c+dx)^2\mathrm{Li}}{b^2}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} + \frac{3id(c+dx)^2\mathrm{Li}}{b^2}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{6d^3\cos(a+bx)}{b^4} - \frac{3d(c+dx)^2\cos(a+bx}{b^2}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{6d^3\cos(a+bx)}{b^4} - \frac{3d(c+dx)^2\cos(a+bx}{b^2}$$

**Mathematica [B]** time = 1.47, size = 557, normalized size = 2.03

$$-\frac{b^3c^3\sin(a+bx) + 2ib^3c^3\tan^{-1}\left(e^{i(a+bx)}\right) - 3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right) + 3b^3c^2dx\log\left(1+ie^{i(a+bx)}\right) + 3b^3c^2dx\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] -(((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^2*c^2*d*Cos[a + b*x] - 6*d^3
*Cos[a + b*x] + 6*b^2*c*d^2*x*Cos[a + b*x] + 3*b^2*d^3*x^2*Cos[a + b*x] - 3
```

```
*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a
 + b*x))] - b^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 +
I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + b^3*d^3*x
^3*Log[1 + I*E^(I*(a + b*x))] - (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(
I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] + 6*b
*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a
 + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3,
I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] - (6*I)*d^3
*PolyLog[4, I*E^(I*(a + b*x))] + b^3*c^3*Sin[a + b*x] - 6*b*c*d^2*Sin[a + b
*x] + 3*b^3*c^2*d*x*Sin[a + b*x] - 6*b*d^3*x*Sin[a + b*x] + 3*b^3*c*d^2*x^2
*Sin[a + b*x] + b^3*d^3*x^3*Sin[a + b*x])/b^4)
```

**fricas [C]** time = 0.56, size = 1071, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4,
 I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + sin(
b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b^2*d^3
*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a) + (-3*I*b^2*d^3*x^2
- 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (
-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*x + a) -
sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-
I*cos(b*x + a) + sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b
^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x +
 a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^
3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*
x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b
*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x
+ a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*
x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b*d^3*x + b*c*d^2)*pol
ylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I
*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x
 + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) -
sin(b*x + a)) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*
(b^3*c^2*d - 2*b*d^3)*x)*sin(b*x + a))/b^4
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sec(bx+a)\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a)^2, x)
```

**maple [B]** time = 0.36, size = 901, normalized size = 3.28

$$-\frac{6id^2c\,\mathrm{polylog}\left(2, i e^{i(bx+a)}\right)x}{b^2}+\frac{6id^2c\,\mathrm{polylog}\left(2, -i e^{i(bx+a)}\right)x}{b^2}-\frac{3id^3\,\mathrm{polylog}\left(2, i e^{i(bx+a)}\right)x^2}{b^2}+\frac{3id^3\,\mathrm{polylog}\left(2,\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*
b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))
+6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)
))/b^4-1/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,I*exp(I*(b*
x+a)))*x+1/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3-1/b*d^3*ln(1+I*exp(I*(b*x+a)))*
x^3-6/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x+6/b^3*d^2*c*polylog(3,I*exp(I*
(b*x+a)))-6/b^3*d^2*c*polylog(3,-I*exp(I*(b*x+a)))+1/b^4*a^3*d^3*ln(1-I*exp
(I*(b*x+a)))-2*I/b*c^3*arctan(exp(I*(b*x+a)))+3/b*d^2*c*ln(1-I*exp(I*(b*x+a
)))*x^2-3/b*d^2*c*ln(1+I*exp(I*(b*x+a)))*x^2+3/b*c^2*d*ln(1-I*exp(I*(b*x+a)
))*x+3/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a+3/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x
+a)))-3/b*c^2*d*ln(1+I*exp(I*(b*x+a)))*x-3/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))
*a-3/b^3*a^2*c*d^2*ln(1-I*exp(I*(b*x+a)))+3*I/b^2*c^2*d*polylog(2,-I*exp(I*
(b*x+a)))+2*I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,I*
exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+3*I/b^2*d^3*pol
ylog(2,-I*exp(I*(b*x+a)))*x^2-1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*
d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2
*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-6*I/b^2*d^2*c*polylog(2,I*exp(I*(b*x+a)))*x
-6*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))+6*I/b^2*c^2*d*a*arctan(exp(I*(b*x
+a)))+6*I/b^2*d^2*c*polylog(2,-I*exp(I*(b*x+a)))*x
```

**maxima [B]** time = 0.67, size = 924, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^3*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a)) -
 3*a*c^2*d*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))
/b + 3*a^2*c*d^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x
 + a))/b^2 - a^3*d^3*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin
(b*x + a))/b^3 + (12*I*d^3*polylog(4, I*e^(I*b*x + I*a)) - 12*I*d^3*polylog
(4, -I*e^(I*b*x + I*a)) + (-2*I*(b*x + a)^3*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3
)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a))*
arctan2(cos(b*x + a), sin(b*x + a) + 1) + (-2*I*(b*x + a)^3*d^3 + (-6*I*b*c
*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*a^2*
d^3)*(b*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 6*(b^2*c^2*d - 2
*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a
))*cos(b*x + a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 -
6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*dilog(I*e^(I*b*x + I*
a)) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 +
 (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*dilog(-I*e^(I*b*x + I*a)) + ((b*x +
 a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) +
1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin
(b*x + a) + 1) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, I*e^(I*b*x
 + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -I*e^(I*b*x + I*
a)) - 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x +
 a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(b*x + a)
)/b^3)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(a+b\,x)^2\,(c+d\,x)^3}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^2*(c + d*x)^3)/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)^2*(c + d*x)^3)/cos(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \sin^2(a + bx) \sec(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*sin(a + b*x)**2*sec(a + b*x), x)
```

# 3.217 $\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=186

$$-\frac{2d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\sin(a+bx)}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{2}{}$$

```
[Out] -2*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b-2*d*(d*x+c)*cos(b*x+a)/b^2+2*I*d*(d
*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,I*exp(I*(b*x
+a)))/b^2-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,I*exp(I*(b
*x+a)))/b^3+2*d^2*sin(b*x+a)/b^3-(d*x+c)^2*sin(b*x+a)/b
```

**Rubi [A]** time = 0.15, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {4407, 3296, 2637, 4181, 2531, 2282, 6589}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{2d^2\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{PolyLog}(}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b - (2*d*(c + d*x)*Cos[a + b*x
])/b^2 + ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((2*I)*
d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (2*d^2*PolyLog[3, (-I)*E^(
I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (2*d^2*Sin
[a + b*x])/b^3 - ((c + d*x)^2*Sin[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
```

```
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\sin(a+bx)\tan(a+bx)\,dx=-\int (c+dx)^2\cos(a+bx)\,dx+\int (c+dx)^2\sec(a+bx)\,dx$$

$$=-\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{(2d)\int(c+dx)}{}$$

$$=-\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)\cos(a+bx)}{b^2}+\frac{2id(c+dx)\mathrm{L}}{b}$$

$$=-\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)\cos(a+bx)}{b^2}+\frac{2id(c+dx)\mathrm{L}}{b}$$

$$=-\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)\cos(a+bx)}{b^2}+\frac{2id(c+dx)\mathrm{L}}{b}$$

**Mathematica [A]** time = 0.83, size = 315, normalized size = 1.69

$$-\frac{b^2c^2\sin(a+bx)+2ib^2c^2\tan^{-1}\left(e^{i(a+bx)}\right)-2b^2cdx\log\left(1-ie^{i(a+bx)}\right)+2b^2cdx\log\left(1+ie^{i(a+bx)}\right)+2b^2cdx\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] -(((2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b*c*d*Cos[a + b*x] + 2*b*d^2*x
*Cos[a + b*x] - 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1
- I*E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] + b^2*d^2*x^2
*Log[1 + I*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a +
 b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + 2*d^2*PolyLog
[3, (-I)*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, I*E^(I*(a + b*x))] + b^2*c^2*S
in[a + b*x] - 2*d^2*Sin[a + b*x] + 2*b^2*c*d*x*Sin[a + b*x] + b^2*d^2*x^2*S
in[a + b*x])/b^3)
```

**fricas [C]** time = 0.52, size = 656, normalized size = 3.53

$$-\frac{2\,d^2\mathrm{polylog}\,(3,i\,\cos\,(bx+a)+\sin\,(bx+a))-2\,d^2\mathrm{polylog}\,(3,i\,\cos\,(bx+a)-\sin\,(bx+a))+2\,d^2\mathrm{polylog}\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, I*
cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x +
 a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 4*(b*d^2*x + b*c*
d)*cos(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x
 + a)) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) -
(2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - (2*I*b*d^
2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c
*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d
^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x +
a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2
)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2 - 2*d^2)*sin(b*x + a))/b^3
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \sec(bx+a)\sin(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a)^2, x)
```

**maple [B]** time = 0.26, size = 512, normalized size = 2.75

$$\frac{i\left(d^2x^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)e^{i(bx+a)}}{2b^3}+\frac{4icda\arctan\left(e^{i(bx+a)}\right)}{b^2}-\frac{2cd\ln\left(1+ie^{i(bx+a)}\right)x}{b}-\frac{2d^2\,\mathrm{p}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/2*I*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp
(I*(b*x+a))+4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-2/b*c*d*ln(1+I*exp(I*(b*x+
a)))*x-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3-1/2*I*(d^2*x^2*b^2+2*b^2*c*d*
x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/b^3*exp(-I*(b*x+a))-2*I/b^2*c*d*poly
log(2,I*exp(I*(b*x+a)))-2*I/b*c^2*arctan(exp(I*(b*x+a)))-1/b*d^2*ln(1+I*exp
(I*(b*x+a)))*x^2+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-2*I/b^2*d^2*polylog(
2,I*exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a+2/b*c*d*ln(1-I*exp
(I*(b*x+a)))*x+2*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-2/b^2*c*d*ln(1+I*
exp(I*(b*x+a)))*a+1/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))-2*I/b^3*d^2*a^2*arct
an(exp(I*(b*x+a)))+2*I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))+1/b*d^2*ln(1-I*
exp(I*(b*x+a)))*x^2-1/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))
```

**maxima [B]** time = 0.60, size = 510, normalized size = 2.74

$$\frac{c^2\left(\log(\sin(bx+a)+1)-\log(\sin(bx+a)-1)-2\sin(bx+a)\right)-\frac{2\,acd\left(\log(\sin(bx+a)+1)-\log(\sin(bx+a)-1)-2\sin(bx+a)\right)}{b}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a)) -
 2*a*c*d*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))/b
 + a^2*d^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))
```

```
/b^2 + (4*d^2*polylog(3, I*e^(I*b*x + I*a)) - 4*d^2*polylog(3, -I*e^(I*b*x
+ I*a)) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*arcta
n2(cos(b*x + a), sin(b*x + a) + 1) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d +
4*I*a*d^2)*(b*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 4*(b*c*d +
 (b*x + a)*d^2 - a*d^2)*cos(b*x + a) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*
I*a*d^2)*dilog(I*e^(I*b*x + I*a)) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*
d^2)*dilog(-I*e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x + a
)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
- 2*sin(b*x + a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) -
2*d^2)*sin(b*x + a))/b^2)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^2\,(c+d\,x)^2}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^2*(c + d*x)^2)/cos(a + b*x),x)
[Out] int((sin(a + b*x)^2*(c + d*x)^2)/cos(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2\sin^2(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a)**2,x)
[Out] Integral((c + d*x)**2*sin(a + b*x)**2*sec(a + b*x), x)
```

# 3.218 $\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=103

$$\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}$$

```
[Out] -2*I*(d*x+c)*arctan(exp(I*(b*x+a)))/b-d*cos(b*x+a)/b^2+I*d*polylog(2,-I*exp
(I*(b*x+a)))/b^2-I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-(d*x+c)*sin(b*x+a)/b
```

**Rubi [A]** time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4407, 3296, 2638, 4181, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - (d*Cos[a + b*x])/b^2 + (I*d*
PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])
/b^2 - ((c + d*x)*Sin[a + b*x])/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
```

```
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx = -\int (c+dx)\cos(a+bx)\,dx + \int (c+dx)\sec(a+bx)\,dx$$
$$= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(c+dx)\sin(a+bx)}{b} - \frac{d\int \log\left(1-ie^{i(a+\right.}}{b}$$
$$= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\sin(a+bx)}{b} + \cdots$$
$$= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)}{b^2} + \frac{id\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{id\mathrm{Li}}{}$$

**Mathematica [B]** time = 0.40, size = 213, normalized size = 2.07

$$\frac{d\left(i\left(\mathrm{Li}_2\left(-e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)-\mathrm{Li}_2\left(e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)\right)+\left(-a-bx+\frac{\pi}{2}\right)\left(\log\left(1-e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)-\log\left(1+e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)\right)-(\cdots}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Sin[a + b*x]*Tan[a + b*x],x]

[Out] (c*ArcTanh[Sin[a + b*x]])/b + (d*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a + Pi/2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[(-a + Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[2, E^(I*(-a + Pi/2 - b*x))])))/b^2 - (d*Cos[b*x]*(Cos[a] + b*x*Sin[a]))/b^2 - (d*(b*x*Cos[a] - Sin[a])*Sin[b*x])/b^2 - (c*Sin[a + b*x])/b

**fricas [B]** time = 1.04, size = 331, normalized size = 3.21

$$-\frac{2\,d\cos(bx+a)+i\,d\mathrm{Li}_2(i\cos(bx+a)+\sin(bx+a))+i\,d\mathrm{Li}_2(i\cos(bx+a)-\sin(bx+a))-i\,d\mathrm{Li}_2(-i\cos\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")

[Out] -1/2*(2*d*cos(b*x + a) + I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(I*cos(b*x + a) - sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d*x + a*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b*d*x + b*c)*sin(b*x + a))/b^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\sec(bx+a)\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a)^2, x)

**maple** [B] time = 0.06, size = 209, normalized size = 2.03

$$-\frac{d\sin(bx+a)\,x}{b}-\frac{d\cos(bx+a)}{b^2}-\frac{c\sin(bx+a)}{b}-\frac{d\ln\left(1+i\mathrm{e}^{i(bx+a)}\right)x}{b}+\frac{d\ln\left(1-i\mathrm{e}^{i(bx+a)}\right)x}{b}-\frac{id\,\mathrm{dilog}\left(1-i\mathrm{e}^{i(bx+a)}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] -1/b*d*sin(b*x+a)*x-d*cos(b*x+a)/b^2-1/b*c*sin(b*x+a)-1/b*d*ln(1+I*exp(I*(b
*x+a)))*x+1/b*d*ln(1-I*exp(I*(b*x+a)))*x-I/b^2*d*dilog(1-I*exp(I*(b*x+a)))-
1/b^2*d*ln(1+I*exp(I*(b*x+a)))*a+1/b^2*d*ln(1-I*exp(I*(b*x+a)))*a+I/b^2*d*d
ilog(1+I*exp(I*(b*x+a)))-1/b^2*d*a*ln(sec(b*x+a)+tan(b*x+a))+1/b*c*ln(sec(b
*x+a)+tan(b*x+a))
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^2\,(c+d\,x)}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^2*(c + d*x))/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)^2*(c + d*x))/cos(a + b*x), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\sin^2(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*sin(a + b*x)**2*sec(a + b*x), x)
```

# 3.219 $\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=69

$$\text{Int}\left(\frac{\sec(a+bx)}{c+dx},x\right)-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] -Ci(b*c/d+b*x)*cos(a-b*c/d)/d+Si(b*c/d+b*x)*sin(a-b*c/d)/d+Unintegrable(sec
(b*x+a)/(d*x+c),x)
```

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] -((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d) + (Sin[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][Sec[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx &= -\int \frac{\cos(a+bx)}{c+dx}\,dx+\int \frac{\sec(a+bx)}{c+dx}\,dx\\ &= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)+\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\int\\ &= -\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\sec(a+bx)}{c+dx}\,dx\end{aligned}$$

**Mathematica [A]** time = 6.04, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

**fricas [A]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(bx+a\right)^2-1\right)\sec\left(bx+a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)/(d*x + c), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)^2/(d*x + c), x)
```

**maple** [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\left(\sin^2(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^2}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)),x)
```

```
[Out] int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

# 3.220 $\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=87

$$\operatorname{Int}\left(\frac{\sec(a+bx)}{(c+dx)^2},x\right)+\frac{b\sin\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\cos(a+bx)}{d(c+dx)}$$

```
[Out] cos(b*x+a)/d/(d*x+c)+b*cos(a-b*c/d)*Si(b*c/d+b*x)/d^2+b*Ci(b*c/d+b*x)*sin(a
-b*c/d)/d^2+Unintegrable(sec(b*x+a)/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Cos[a + b*x]/(d*(c + d*x)) + (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d]
)/d^2 + (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int][Se
c[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx+\int \frac{\sec(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{d}+\int \frac{\sec(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,d}{d}\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\operatorname{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int \frac{\mathrm{se}}{}
\end{aligned}$$

**Mathematica [A]** time = 7.08, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

**fricas [A]** time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\left(\cos(bx+a)^2-1\right)\sec(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sec(b*x + a)*sin(b*x + a)^2/(d*x + c)^2, x)
```

**maple [A]** time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\left(\sin^2(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)
[Out] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^2}{\cos(a+b\,x)\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2),x)
[Out] int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

# 3.221 $\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=152

$$\text{Int}\left(\tan(a+bx)(c+dx)^m,x\right)+\frac{2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+
$$

```
[Out] 2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b*
(d*x+c)/d)^m)+2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b*
c/d))/((I*b*(d*x+c)/d)^m)+Unintegrable((d*x+c)^m*tan(b*x+a),x)
```

**Rubi [A]** time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c +
 d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-3 - m)*(c + d*x)^m*Gamma[1 +
 m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^
m) + Defer[Int][(c + d*x)^m*Tan[a + b*x], x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx &= -\int (c+dx)^m\cos(a+bx)\sin(a+bx)\,dx+\int (c+dx)^m\tan(a+bx) \\
&= -\int \frac{1}{2}(c+dx)^m\sin(2a+2bx)\,dx+\int (c+dx)^m\tan(a+bx)\,dx \\
&= -\left(\frac{1}{2}\int (c+dx)^m\sin(2a+2bx)\,dx\right)+\int (c+dx)^m\tan(a+bx)\,dx \\
&= -\left(\frac{1}{4}i\int e^{-i(2a+2bx)}(c+dx)^m\,dx\right)+\frac{1}{4}i\int e^{i(2a+2bx)}(c+dx)^m\,dx+\int ( \\
&= \frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-3-m}e
\end{aligned}$$

**Mathematica [A]** time = 7.81, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x], x]
```

**fricas [A]** time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(bx+a\right)^2-1\right)(dx+c)^m\sec\left(bx+a\right)\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*(d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\sin(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^3, x)
```

**maple** [A] time = 0.25, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\left(\sin^3(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x)
[Out] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\sin(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^3, x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^3\,(c+d\,x)^m}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^3*(c + d*x)^m)/cos(a + b*x),x)
[Out] int((sin(a + b*x)^3*(c + d*x)^m)/cos(a + b*x), x)
```

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a)**3,x)
[Out] Exception raised: HeuristicGCDFailed
```

## 3.222 $\int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=251

$$-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^3\sin(a+bx)\cos(a+bx)}{8b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}+\frac{3id}{}$$

```
[Out] -3/8*d^3*x/b^3+1/4*(d*x+c)^3/b+1/4*I*(d*x+c)^4/d-(d*x+c)^3*ln(1+exp(2*I*(b*
x+a)))/b+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)
*polylog(3,-exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^
4+3/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4-3/4*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b
^2+3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3-1/2*(d*x+c)^3*sin(b*x+a)^2/b
```

**Rubi [A]** time = 0.30, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4407, 4404, 3311, 32, 2635, 8, 3719, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id^3\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^2(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (-3*d^3*x)/(8*b^3) + (c + d*x)^3/(4*b) + ((I/4)*(c + d*x)^4)/d - ((c + d*x)
^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E
^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))
])/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (3*d^3*
Cos[a + b*x]*Sin[a + b*x])/(8*b^4) - (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a +
b*x])/(4*b^2) + (3*d^2*(c + d*x)*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^3*Sin
[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx = -\int (c+dx)^3 \cos(a+bx)\sin(a+bx)\,dx + \int (c+dx)^3 \tan(a+bx)\,d$$

$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}\,dx + \frac{(3}{}$$

$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cos(a+bx)\,\mathrm{s}}{4b^2}$$

$$= \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2}{2}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id}{}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id}{}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id}{}$$

**Mathematica [B]** time = 7.12, size = 1720, normalized size = 6.85

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((-1/4*I)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*
I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
 - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*
E^(I*a)) - (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a
))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*P
olyLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x
))] - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] - (c^3*
Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Co
s[a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Co
t[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x
- ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*
x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*
I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a
]^2*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(64*b^4) - ((I/64)*S
in[2*a + 2*b*x])/b^4)*(8*b^3*c^3*Cos[a] - (12*I)*b^2*c^2*d*Cos[a] - 12*b*c*
d^2*Cos[a] + (6*I)*d^3*Cos[a] + 24*b^3*c^2*d*x*Cos[a] - (24*I)*b^2*c*d^2*x*
Cos[a] - 12*b*d^3*x*Cos[a] + 24*b^3*c*d^2*x^2*Cos[a] - (12*I)*b^2*d^3*x^2*C
os[a] + 8*b^3*d^3*x^3*Cos[a] + (32*I)*b^4*c^3*x*Cos[a + 2*b*x] + (48*I)*b^4
*c^2*d*x^2*Cos[a + 2*b*x] + (32*I)*b^4*c*d^2*x^3*Cos[a + 2*b*x] + (8*I)*b^4
*d^3*x^4*Cos[a + 2*b*x] - (32*I)*b^4*c^3*x*Cos[3*a + 2*b*x] - (48*I)*b^4*c^
2*d*x^2*Cos[3*a + 2*b*x] - (32*I)*b^4*c*d^2*x^3*Cos[3*a + 2*b*x] - (8*I)*b^
4*d^3*x^4*Cos[3*a + 2*b*x] + 4*b^3*c^3*Cos[3*a + 4*b*x] + (6*I)*b^2*c^2*d*C
os[3*a + 4*b*x] - 6*b*c*d^2*Cos[3*a + 4*b*x] - (3*I)*d^3*Cos[3*a + 4*b*x] +
 12*b^3*c^2*d*x*Cos[3*a + 4*b*x] + (12*I)*b^2*c*d^2*x*Cos[3*a + 4*b*x] - 6*
b*d^3*x*Cos[3*a + 4*b*x] + 12*b^3*c*d^2*x^2*Cos[3*a + 4*b*x] + (6*I)*b^2*d^
3*x^2*Cos[3*a + 4*b*x] + 4*b^3*d^3*x^3*Cos[3*a + 4*b*x] + 4*b^3*c^3*Cos[5*a
 + 4*b*x] + (6*I)*b^2*c^2*d*Cos[5*a + 4*b*x] - 6*b*c*d^2*Cos[5*a + 4*b*x] -
 (3*I)*d^3*Cos[5*a + 4*b*x] + 12*b^3*c^2*d*x*Cos[5*a + 4*b*x] + (12*I)*b^2*
c*d^2*x*Cos[5*a + 4*b*x] - 6*b*d^3*x*Cos[5*a + 4*b*x] + 12*b^3*c*d^2*x^2*Co
s[5*a + 4*b*x] + (6*I)*b^2*d^3*x^2*Cos[5*a + 4*b*x] + 4*b^3*d^3*x^3*Cos[5*a
 + 4*b*x] - 32*b^4*c^3*x*Sin[a + 2*b*x] - 48*b^4*c^2*d*x^2*Sin[a + 2*b*x] -
 32*b^4*c*d^2*x^3*Sin[a + 2*b*x] - 8*b^4*d^3*x^4*Sin[a + 2*b*x] + 32*b^4*c^
```

```
3*x*Sin[3*a + 2*b*x] + 48*b^4*c^2*d*x^2*Sin[3*a + 2*b*x] + 32*b^4*c*d^2*x^3
*Sin[3*a + 2*b*x] + 8*b^4*d^3*x^4*Sin[3*a + 2*b*x] + (4*I)*b^3*c^3*Sin[3*a
+ 4*b*x] - 6*b^2*c^2*d*Sin[3*a + 4*b*x] - (6*I)*b*c*d^2*Sin[3*a + 4*b*x] +
3*d^3*Sin[3*a + 4*b*x] + (12*I)*b^3*c^2*d*x*Sin[3*a + 4*b*x] - 12*b^2*c*d^2
*x*Sin[3*a + 4*b*x] - (6*I)*b*d^3*x*Sin[3*a + 4*b*x] + (12*I)*b^3*c*d^2*x^2
*Sin[3*a + 4*b*x] - 6*b^2*d^3*x^2*Sin[3*a + 4*b*x] + (4*I)*b^3*d^3*x^3*Sin[
3*a + 4*b*x] + (4*I)*b^3*c^3*Sin[5*a + 4*b*x] - 6*b^2*c^2*d*Sin[5*a + 4*b*x
] - (6*I)*b*c*d^2*Sin[5*a + 4*b*x] + 3*d^3*Sin[5*a + 4*b*x] + (12*I)*b^3*c^
2*d*x*Sin[5*a + 4*b*x] - 12*b^2*c*d^2*x*Sin[5*a + 4*b*x] - (6*I)*b*d^3*x*Si
n[5*a + 4*b*x] + (12*I)*b^3*c*d^2*x^2*Sin[5*a + 4*b*x] - 6*b^2*d^3*x^2*Sin[
5*a + 4*b*x] + (4*I)*b^3*d^3*x^3*Sin[5*a + 4*b*x])
```

**fricas [C]** time = 0.61, size = 1134, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 24*I*d^3*polylog(4, I*cos(b*x + a)
+ sin(b*x + a)) + 24*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 24*I
*d^3*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 24*I*d^3*polylog(4, -I*co
s(b*x + a) - sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3
 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2
 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3*
c^2*d - b*d^3)*x - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*
dilog(I*cos(b*x + a) + sin(b*x + a)) - (12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x
 + 12*I*b^2*c^2*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - (12*I*b^2*d^3*x^2
 + 24*I*b^2*c*d^2*x + 12*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a))
 - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*dilog(-I*cos(b*x
 + a) - sin(b*x + a)) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 4*(b^3*d^3*
x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3
*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b
*x + a) - sin(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*
d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*
x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + 4*
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) + I*s
in(b*x + a) + I) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*lo
g(-cos(b*x + a) - I*sin(b*x + a) + I) + 24*(b*d^3*x + b*c*d^2)*polylog(3, I
*cos(b*x + a) + sin(b*x + a)) + 24*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x
 + a) - sin(b*x + a)) + 24*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) +
 sin(b*x + a)) + 24*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*
x + a)))/b^4
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sec(bx+a)\sin(bx+a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a)^3, x)
```

**maple [B]** time = 0.22, size = 641, normalized size = 2.55

$$-\frac{3d^3\,\mathrm{polylog}\left(3,-e^{2i(bx+a)}\right)x}{2b^3}-\frac{d^3\ln\left(1+e^{2i(bx+a)}\right)x^3}{b}-\frac{6icd^2a^2x}{b^2}+\frac{6ic^2dax}{b}-\frac{2d^3a^3\ln\left(e^{i(bx+a)}\right)}{b^4}+\frac{3ia^4d^3}{2b^4}+icd^2x^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/4*I*d^3*x^4+I*c*d^2*x^3-I*c^3*x+3*I/b^2*a^2*c^2*d-4*I/b^3*a^3*c*d^2+2*I/b
^3*d^3*a^3*x-1/b*c^3*ln(1+exp(2*I*(b*x+a)))-2/b^4*d^3*a^3*ln(exp(I*(b*x+a))
)+1/32*(4*d^3*x^3*b^3+6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2+12*I*b^2*c*d^2*x+12*
b^3*c^2*d*x+6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x-3*I*d^3-6*c*d^2*b)/b^4*exp(2*
I*(b*x+a))+1/32*(4*d^3*x^3*b^3-6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2-12*I*b^2*c*
d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x+3*I*d^3-6*c*d^2*b)/b
^4*exp(-2*I*(b*x+a))-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/2/b^3*d^3
*polylog(3,-exp(2*I*(b*x+a)))*x-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+
2/b*c^3*ln(exp(I*(b*x+a)))+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+3/2
*I*c^2*d*x^2+6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-6/b^2*c^2*d*a*ln(exp(I*(b*x
+a)))-1/b*d^3*ln(1+exp(2*I*(b*x+a)))*x^3+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(
b*x+a)))*x^2-6*I/b^2*a^2*c*d^2*x+6*I/b*a*c^2*d*x+3/2*I/b^2*c^2*d*polylog(2,
-exp(2*I*(b*x+a)))+3/2*I/b^4*d^3*a^4-3/b*c^2*d*ln(1+exp(2*I*(b*x+a)))*x-3/b
*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2
```

**maxima [B]** time = 0.56, size = 685, normalized size = 2.73

$$-\frac{24\left(\sin\left(bx+a\right)^2+\log\left(\sin\left(bx+a\right)^2-1\right)\right)c^3-\frac{72\left(\sin(bx+a)^2+\log\left(\sin(bx+a)^2-1\right)\right)ac^2d}{b}+\frac{72\left(\sin(bx+a)^2+\log\left(\sin(bx+a)^2-1\right)\right.}{b^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(24*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*c^3 - 72*(sin(b*x + a)
^2 + log(sin(b*x + a)^2 - 1))*a*c^2*d/b + 72*(sin(b*x + a)^2 + log(sin(b*x
+ a)^2 - 1))*a^2*c*d^2/b^2 - 24*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*
a^3*d^3/b^3 + (-12*I*(b*x + a)^4*d^3 + (-48*I*b*c*d^2 + 48*I*a*d^3)*(b*x +
a)^3 + 48*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) + (-72*I*b^2*c^2*d + 144*I
*a*b*c*d^2 - 72*I*a^2*d^3)*(b*x + a)^2 + (64*I*(b*x + a)^3*d^3 + (144*I*b*c
*d^2 - 144*I*a*d^3)*(b*x + a)^2 + (144*I*b^2*c^2*d - 288*I*a*b*c*d^2 + 144*
I*a^2*d^3)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(
2*(b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a
) + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2 - 96*I*(b*x + a)^2*d^3 - 72*I*a^2*d^
3 + (-144*I*b*c*d^2 + 144*I*a*d^3)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) +
 8*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1) + 24*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*pol
ylog(3, -e^(2*I*b*x + 2*I*a)) + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^
2*d^3 + (2*a^2 - 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))/
b^3)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^3}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^3*(c + d*x)^3)/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)^3*(c + d*x)^3)/cos(a + b*x), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.223 $\int (c+dx)^2 \sin^2(a+bx) \tan(a+bx)\, dx$

**Optimal**. Leaf size=184

$$-\frac{d^2 \text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2 \sin^2(a+bx)}{4b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}-\frac{(c+dx)^2 \text{l}}{}$$

```
[Out] 1/2*c*d*x/b+1/4*d^2*x^2/b+1/3*I*(d*x+c)^3/d-(d*x+c)^2*ln(1+exp(2*I*(b*x+a))
)/b+I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2*I
*(b*x+a)))/b^3-1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2+1/4*d^2*sin(b*x+a)^2
/b^3-1/2*(d*x+c)^2*sin(b*x+a)^2/b
```

**Rubi [A]** time = 0.23, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4407, 4404, 3310, 3719, 2190, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}+\frac{d^2 \sin^2(a+b}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (c*d*x)/(2*b) + (d^2*x^2)/(4*b) + ((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*Log[
1 + E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x)
)])/b^2 - (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Cos
[a + b*x]*Sin[a + b*x])/(2*b^2) + (d^2*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)
^2*Sin[a + b*x]^2)/(2*b)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\sin^2(a+bx)\tan(a+bx)\,dx = -\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx + \int (c+dx)^2\tan(a+bx)\,dx$$

$$= \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx + \frac{d\int (c$$

$$= \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cos(a+bx)\sin(a}{2b^2}$$

$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\mathrm{Li}_2}{b^2}$$

$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\mathrm{Li}_2}{b^2}$$

$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\mathrm{Li}_2}{b^2}$$

**Mathematica [B]** time = 6.47, size = 518, normalized size = 2.82

$$-\frac{cd\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))} - \frac{\cot(a)\left(i\mathrm{Li}_2\left(e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan^{-1}\right.}\right.\right.}{\sqrt{\cot^2(a)+1}}\right.}{b^2\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((-1/12*I)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I
)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
```

```
- (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E
^(I*a)) - (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*
Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot
[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)
] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi
*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLo
g[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*S
qrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Cos[2*b*x]*(2*b^2*c^2*Cos[2*a] - d^
2*Cos[2*a] + 4*b^2*c*d*x*Cos[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b*c*d*Sin[2*
a] - 2*b*d^2*x*Sin[2*a]))/(8*b^3) - ((2*b*c*d*Cos[2*a] + 2*b*d^2*x*Cos[2*a]
 + 2*b^2*c^2*Sin[2*a] - d^2*Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] + 2*b^2*d^2*x^2
*Sin[2*a])*Sin[2*b*x])/(8*b^3) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3
```

**fricas [C]** time = 0.54, size = 688, normalized size = 3.74

$$-\frac{b^2d^2x^2 + 2\,b^2cdx - \left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(bx+a\right)^2 + 4\,d^2\text{polylog}\left(3, i\,\cos\left(bx+a\right) + \sin\left(bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2
- d^2)*cos(b*x + a)^2 + 4*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 4
*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 4*d^2*polylog(3, -I*cos(b*
x + a) + sin(b*x + a)) + 4*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) +
 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - (-4*I*b*d^2*x - 4*I*b*c*d)
*dilog(I*cos(b*x + a) + sin(b*x + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(I*c
os(b*x + a) - sin(b*x + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(-I*cos(b*x +
a) + sin(b*x + a)) - (-4*I*b*d^2*x - 4*I*b*c*d)*dilog(-I*cos(b*x + a) - sin
(b*x + a)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*x
 + a) + I) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x
 + a) + I) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a
^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d
*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b^2*
d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x
+ a) + 1) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x
 + a) + I) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*
x + a) + I))/b^3
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \sec\left(bx+a\right)\sin\left(bx+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a)^3, x)
```

**maple [B]** time = 0.33, size = 379, normalized size = 2.06

$$\frac{id^2x^3}{3} - \frac{2id^2a^2x}{b^2} + \frac{2icd\,a^2}{b^2} + \frac{\left(2d^2x^2b^2 + 4b^2cdx + 2ib\,d^2x + 2b^2c^2 + 2ibcd - d^2\right)e^{2i(bx+a)}}{16b^3} + \frac{\left(2d^2x^2b^2 + 4b^2cdx - 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))+1/3*I*d^2*x^3-2*I/b^2*a^2*d^2*x+1/16
*(2*d^2*x^2*b^2+2*I*b*d^2*x+4*b^2*c*d*x+2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(2*
```

```
I*(b*x+a))+1/16*(2*d^2*x^2*b^2-2*I*b*d^2*x+4*b^2*c*d*x-2*I*b*c*d+2*b^2*c^2-
d^2)/b^3*exp(-2*I*(b*x+a))-1/b*c^2*ln(1+exp(2*I*(b*x+a)))+2/b*c^2*ln(exp(I*
(b*x+a)))+2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+I/b^2*d^2*polylog(2,-exp(2*I*(b*
x+a)))*x+2*I/b^2*a^2*c*d+4*I/b*a*c*d*x-1/b*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-I
*c^2*x-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-4/b^2*c*d*a*ln(exp(I*(b*x+a
)))+I*c*d*x^2-4/3*I/b^3*a^3*d^2-2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x
```

**maxima [B]** time = 0.53, size = 379, normalized size = 2.06

$$-\frac{12\left(\sin\left(bx+a\right)^{2}+\log\left(\sin\left(bx+a\right)^{2}-1\right)\right)c^{2}-\frac{24\left(\sin(bx+a)^{2}+\log\left(\sin(bx+a)^{2}-1\right)\right)acd}{b}+\frac{12\left(\sin(bx+a)^{2}+\log\left(\sin(bx+a)^{2}-1\right)\right)a^{2}d^{2}}{b^{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(12*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*c^2 - 24*(sin(b*x + a)
^2 + log(sin(b*x + a)^2 - 1))*a*c*d/b + 12*(sin(b*x + a)^2 + log(sin(b*x +
a)^2 - 1))*a^2*d^2/b^2 + (-8*I*(b*x + a)^3*d^2 + (-24*I*b*c*d + 24*I*a*d^2)
*(b*x + a)^2 + 12*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) + (24*I*(b*x + a)^2*
d^2 + (48*I*b*c*d - 48*I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*
b*x + 2*a) + 1) - 3*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - d^2)
*cos(2*b*x + 2*a) + (-24*I*b*c*d - 24*I*(b*x + a)*d^2 + 24*I*a*d^2)*dilog(-
e^(2*I*b*x + 2*I*a)) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*l
og(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 6*(b
*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))/b^2)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)^2}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^3*(c + d*x)^2)/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)^3*(c + d*x)^2)/cos(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)^{2}\sin^{3}\left(a+bx\right)\sec\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*sin(a + b*x)**3*sec(a + b*x), x)
```

## 3.224 $\int (c + dx)\sin^2(a + bx)\tan(a + bx)\,dx$

**Optimal**. Leaf size=115

$$\frac{id\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{dx}{4b}+\frac{i(c+dx)^2}{2d}$$

```
[Out] 1/4*d*x/b+1/2*I*(d*x+c)^2/d-(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b+1/2*I*d*polylo
g(2,-exp(2*I*(b*x+a)))/b^2-1/4*d*cos(b*x+a)*sin(b*x+a)/b^2-1/2*(d*x+c)*sin(
b*x+a)^2/b
```

**Rubi [A]** time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4407, 4404, 2635, 8, 3719, 2190, 2279, 2391}

$$\frac{id\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{dx}{4b}+\frac{i}{-}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (d*x)/(4*b) + ((I/2)*(c + d*x)^2)/d - ((c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d*Cos[a + b*x]*Si
n[a + b*x])/(4*b^2) - ((c + d*x)*Sin[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
```

```
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx = -\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx + \int (c+dx)\tan(a+bx)\,dx$$
$$= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx + \frac{d\int \sin^2$$
$$= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} - (c$$
$$= \frac{dx}{4b} + \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)\sin(a+bx}{4b^2}$$
$$= \frac{dx}{4b} + \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - d\,\mathrm{co}$$

**Mathematica [A]** time = 0.30, size = 134, normalized size = 1.17

$$\frac{d\left(\frac{1}{2}i\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)+\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)\right)}{b^2}-\frac{d\sin(2(a+bx))}{8b^2}+\frac{ad\log(\cos(a+bx))}{b^2}-\frac{c\left(\log(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (d*x*Cos[2*(a + b*x)])/(4*b) + (a*d*Log[Cos[a + b*x]])/b^2 - (c*(-1/2*Cos[a
 + b*x]^2 + Log[Cos[a + b*x]]))/b + (d*((I/2)*(a + b*x)^2 - (a + b*x)*Log[1
 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I)*(a + b*x))]))/b^2 - (d
*Sin[2*(a + b*x)])/(8*b^2)
```

**fricas [B]** time = 0.49, size = 346, normalized size = 3.01

$$-\frac{bdx-2\left(bdx+bc\right)\cos\left(bx+a\right)^2+d\cos\left(bx+a\right)\sin\left(bx+a\right)+2i\,d\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)-2i\,d\mathrm{Li}_2\left(i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a)
+ 2*I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*I*d*dilog(I*cos(b*x + a) -
```

```
sin(b*x + a)) - 2*I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*I*d*dilog(
-I*cos(b*x + a) - sin(b*x + a)) + 2*(b*c - a*d)*log(cos(b*x + a) + I*sin(b*
x + a) + I) + 2*(b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b*d
*x + a*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*d*x + a*d)*log(I*co
s(b*x + a) - sin(b*x + a) + 1) + 2*(b*d*x + a*d)*log(-I*cos(b*x + a) + sin(
b*x + a) + 1) + 2*(b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + 2
*(b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b*c - a*d)*log(-c
os(b*x + a) - I*sin(b*x + a) + I))/b^2
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c) \sec(bx + a) \sin(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a)^3, x)
```

**maple** [A] time = 0.34, size = 179, normalized size = 1.56

$$\frac{id\,x^2}{2} - icx + \frac{(2bdx + 2cb + id)\,e^{2i(bx+a)}}{16b^2} + \frac{(2bdx + 2cb - id)\,e^{-2i(bx+a)}}{16b^2} - \frac{c\ln\left(1 + e^{2i(bx+a)}\right)}{b} + \frac{2c\ln\left(e^{i(bx+a)}\right)}{b} + \frac{2idax}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/2*I*d*x^2-I*c*x+1/16*(2*b*d*x+I*d+2*c*b)/b^2*exp(2*I*(b*x+a))+1/16*(2*b*d
*x-I*d+2*c*b)/b^2*exp(-2*I*(b*x+a))-1/b*c*ln(1+exp(2*I*(b*x+a)))+2/b*c*ln(e
xp(I*(b*x+a)))+2*I/b*d*a*x+I/b^2*d*a^2-1/b*d*ln(1+exp(2*I*(b*x+a)))*x+1/2*I
*d*polylog(2,-exp(2*I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

**maxima** [A] time = 0.52, size = 145, normalized size = 1.26

$$-\frac{-4i\,b^2dx^2 - 8i\,b^2cx + (8i\,bdx + 8i\,bc)\arctan\left(\sin(2\,bx + 2\,a), \cos(2\,bx + 2\,a) + 1\right) - 2\,(bdx + bc)\cos(2\,bx +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(-4*I*b^2*d*x^2 - 8*I*b^2*c*x + (8*I*b*d*x + 8*I*b*c)*arctan2(sin(2*b*
x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - 4*I*d*
dilog(-e^(2*I*b*x + 2*I*a)) + 4*(b*d*x + b*c)*log(cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + d*sin(2*b*x + 2*a))/b^2
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a + b\,x)^3\,(c + d\,x)}{\cos(a + b\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^3*(c + d*x))/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)^3*(c + d*x))/cos(a + b*x), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \sin^3(a + bx) \sec(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*sin(a + b*x)**3*sec(a + b*x), x)
```

# 3.225 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=82

$$\mathrm{Int}\left(\frac{\tan(a+bx)}{c+dx},x\right)-\frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] -1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b
*c/d)/d+Unintegrable(tan(b*x+a)/(d*x+c),x)
```

**Rubi** [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] -(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) - (Cos[2*a - (
2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Defer[Int][Tan[a + b*x]/(
c + d*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx &= -\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx+\int \frac{\tan(a+bx)}{c+dx}\,dx\\
&= -\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx+\int \frac{\tan(a+bx)}{c+dx}\,dx\\
&= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\right)+\int \frac{\tan(a+bx)}{c+dx}\,dx\\
&= -\left(\frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx\right)-\frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+\right.}{c+d}\\
&= -\frac{\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\int \frac{\tan(a+}{c+d}
\end{aligned}$$

**Mathematica** [A] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

**fricas** [A] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(\cos(bx+a)^2-1\right)\sec(bx+a)\sin(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)\sin\left(bx+a\right)^{3}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)^3/(d*x + c), x)
```

**maple [A]** time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)\left(\sin^{3}\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(i\,E_{1}\left(\frac{2i\,bdx+2i\,bc}{d}\right)-i\,E_{1}\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+8\,d\int\frac{\sin(2\,bx+2\,a)}{(dx+c)\left(\cos(2\,bx+2\,a)^{2}+\sin(2\,bx+2\,a)^{2}+2\,\cos(2\,bx+2\,a)+1\right)}\,dx+\left(\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*((I*exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(1, -(
2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 8*d*integrate(sin(2*b*x +
2*a)/((d*x + c)*cos(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2
*(d*x + c)*cos(2*b*x + 2*a) + c), x) + (exp_integral_e(1, (2*I*b*d*x + 2*I*
b*c)/d) + exp_integral_e(1, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d
))/d
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(a+b\,x\right)}^3}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)),x)
```

```
[Out] int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)), x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

## 3.226 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=102

$$\mathrm{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2},x\right)-\frac{b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a+2bx)}{2d(c+dx)}$$

```
[Out] -b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2+b*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c
/d)/d^2+1/2*sin(2*b*x+2*a)/d/(d*x+c)+Unintegrable(tan(b*x+a)/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2) + Sin[2*a +
2*b*x]/(2*d*(c + d*x)) + (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*
b*x])/d^2 + Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx+\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx\\
&= -\int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx+\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx\\
&= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\right)+\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d}+\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\mathrm{si}}{}}{d}\\
&= -\frac{b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a+2bx)}{2d(c+dx)}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}\right.}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 2.40, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(\cos(bx+a)^2-1\right)\sec(bx+a)\sin(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)*sin(b*x + a)/(d^2*x^2 + 2*c*d*x
 + c^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)^3/(d*x + c)^2, x)
```

**maple [A]** time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\left(\sin^3(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(i E_2\left(\frac{2ibdx+2ibc}{d}\right) - i E_2\left(-\frac{2ibdx+2ibc}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right) + 8\left(d^2x+cd\right)\int \frac{\sin(2bx+2a)}{(dx+c)^2\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)\right)}}{4\left(d^2x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((I*exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(2, -(
2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 8*(d^2*x + c*d)*integrate(
sin(2*b*x + 2*a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x +
 2*a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 +
 2*c*d*x + c^2)*cos(2*b*x + 2*a)), x) + (exp_integral_e(2, (2*I*b*d*x + 2*I
*b*c)/d) + exp_integral_e(2, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/
d))/(d^2*x + c*d)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^3}{\cos(a+b\,x)\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)^2), x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.227 $\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\csc(a+bx)\sec(a+bx)(c+dx)^m, x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

**Rubi [A]** time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

**Mathematica [A]** time = 5.89, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]
```

**fricas [A]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \csc(bx+a)\sec(bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

**maple [A]** time = 0.06, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)\,\sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)),x)
```

```
[Out] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*csc(a + b*x)*sec(a + b*x), x)
```

# 3.228 $\int (c+dx)^4 \csc(a+bx) \sec(a+bx)\, dx$

**Optimal**. Leaf size=247

$$\frac{3d^4\text{Li}_5\left(-e^{2i(a+bx)}\right)}{2b^5}-\frac{3d^4\text{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}-\frac{3id^3(c+dx)\text{Li}_4\left(-e^{2i(a+bx)}\right)}{b^4}+\frac{3id^3(c+dx)\text{Li}_4\left(e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)^2\text{L}}{b^3}$$

```
[Out] -2*(d*x+c)^4*arctanh(exp(2*I*(b*x+a)))/b+2*I*d*(d*x+c)^3*polylog(2,-exp(2*I
*(b*x+a)))/b^2-2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x+c
)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x
+a)))/b^3-3*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a)))/b^4+3*I*d^3*(d*x+c)*
polylog(4,exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-3/
2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5
```

**Rubi [A]** time = 0.23, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4419, 4183, 2531, 6609, 2282, 6589}

$$-\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^4*ArcTanh[E^((2*I)*(a + b*x))])/b + ((2*I)*d*(c + d*x)^3*Poly
Log[2, -E^((2*I)*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I
)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b
^3 + (3*d^2*(c + d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(
c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d^3*(c + d*x)*PolyL
og[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/
(2*b^5) - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
```

```
x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)^4 \csc(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{(4d)\int (c+dx)^3 \log\left(1-e^{i(2a+2bx)}\right)dx}{b} +$$
$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+d}{}$$
$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+d}{}$$
$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+d}{}$$
$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+d}{}$$
$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+d}{}$$

**Mathematica [B]** time = 1.29, size = 578, normalized size = 2.34

$$\frac{-4b^4c^4\tanh^{-1}\left(e^{2i(a+bx)}\right)+8b^4c^3dx\log\left(1-e^{2i(a+bx)}\right)-8b^4c^3dx\log\left(1+e^{2i(a+bx)}\right)+12b^4c^2d^2x^2\log\left(1-e^{2i(a+bx)}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-4*b^4*c^4*ArcTanh[E^((2*I)*(a + b*x))] + 8*b^4*c^3*d*x*Log[1 - E^((2*I)*(
a + b*x))] + 12*b^4*c^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 8*b^4*c*d^3*
x^3*Log[1 - E^((2*I)*(a + b*x))] + 2*b^4*d^4*x^4*Log[1 - E^((2*I)*(a + b*x)
)] - 8*b^4*c^3*d*x*Log[1 + E^((2*I)*(a + b*x))] - 12*b^4*c^2*d^2*x^2*Log[1
+ E^((2*I)*(a + b*x))] - 8*b^4*c*d^3*x^3*Log[1 + E^((2*I)*(a + b*x))] - 2*b
^4*d^4*x^4*Log[1 + E^((2*I)*(a + b*x))] + (4*I)*b^3*d*(c + d*x)^3*PolyLog[2
, -E^((2*I)*(a + b*x))] - (4*I)*b^3*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a +
b*x))] - 6*b^2*c^2*d^2*PolyLog[3, -E^((2*I)*(a + b*x))] - 12*b^2*c*d^3*x*Po
lyLog[3, -E^((2*I)*(a + b*x))] - 6*b^2*d^4*x^2*PolyLog[3, -E^((2*I)*(a + b*
x))] + 6*b^2*c^2*d^2*PolyLog[3, E^((2*I)*(a + b*x))] + 12*b^2*c*d^3*x*PolyL
og[3, E^((2*I)*(a + b*x))] + 6*b^2*d^4*x^2*PolyLog[3, E^((2*I)*(a + b*x))]
- (6*I)*b*c*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] - (6*I)*b*d^4*x*PolyLog[4,
```

```
-E^((2*I)*(a + b*x))] + (6*I)*b*c*d^3*PolyLog[4, E^((2*I)*(a + b*x))] + (6
*I)*b*d^4*x*PolyLog[4, E^((2*I)*(a + b*x))] + 3*d^4*PolyLog[5, -E^((2*I)*(a
 + b*x))] - 3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

**fricas [C]** time = 0.66, size = 2600, normalized size = 10.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
cos(b*x + a) - I*sin(b*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) + sin(b*x
 + a)) - 24*d^4*polylog(5, I*cos(b*x + a) - sin(b*x + a)) - 24*d^4*polylog(
5, -I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x + a) - si
n(b*x + a)) + 24*d^4*polylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*po
lylog(5, -cos(b*x + a) - I*sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d
^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(cos(b*x + a) + I*sin(b*x
 + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b
^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b
^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) + s
in(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x +
 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 1
2*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-I*cos(b*x +
a) + sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*
d^2*x - 4*I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (4*I*b^3*d^4
*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-cos(
b*x + a) + I*sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*
b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^4
*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*l
og(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^
2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) + I*sin(b*x + a) + I)
 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4
*c^4)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x +
a) + I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*
x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d
^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3
- a^4*d^4)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*
d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2
*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (
b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3
*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) -
 sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*
c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2
*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3
+ 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4
*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x
 + a) + I*sin(b*x + a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^
2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 -
 a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) - I*sin(
b*x + a) + I) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, cos(b*x + a) + I*s
in(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, cos(b*x + a) - I*s
in(b*x + a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) + si
n(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) - si
n(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) + s
```

```
in(b*x + a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) - s
in(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) + I*
sin(b*x + a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) - I*
sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, c
os(b*x + a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d
^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d
^3*x + b^2*c^2*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 12*(b^2*d^4
*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a
)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x +
 a) + sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylo
g(3, -I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^
2*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2
*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/b^5
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \csc(bx+a)\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a), x)
```

**maple [B]** time = 0.17, size = 1242, normalized size = 5.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] 3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)
+12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,exp(I*(
b*x+a)))-1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))+12/b^3*d^4*polylog(3,exp(I*(b*x
+a)))*x^2+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2-3/b^3*c^2*d^2*polylog(3
,-exp(2*I*(b*x+a)))-3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2-24*d^4*polyl
og(5,-exp(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5-4/b*c*d^3*ln
(1+exp(2*I*(b*x+a)))*x^3+6*I/b^2*c*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+6*I
/b^2*c^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-1/b*c^4*ln(1+exp(2*I*(b*x+a)))+
1/b*c^4*ln(exp(I*(b*x+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)-1/b*d^4*ln(1+exp(
2*I*(b*x+a)))*x^4+4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+4/b*c^3*d*ln(1-exp(I*(b*
x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a+6/b*c^2*d^2*ln(exp(I*(b*x+a))+1
)*x^2+24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x-6/b^3*c^2*d^2*a^2*ln(1-exp(
I*(b*x+a)))+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2+24/b^3*c*d^3*polylog(3,exp
(I*(b*x+a)))*x+24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))+24*I/b^4*c*d^3*pol
ylog(4,exp(I*(b*x+a)))-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3+24*I/b^4*d
^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+2
4*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x-4*I/b^2*c^3*d*polylog(2,-exp(I*(b*
x+a)))-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(I*(b*
x+a))-1)+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a))-1)-4/b^2*c^3*d*a*ln(exp(I*(b*x
+a))-1)+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-1
2*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(2,-ex
p(I*(b*x+a)))*x-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I/b^2*c*d^3
*polylog(2,exp(I*(b*x+a)))*x^2+4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+4/b*c*d^3
*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3-4/b*c^3*d*ln
(1+exp(2*I*(b*x+a)))*x-6/b^3*c*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-6/b*c^2*d
^2*ln(1+exp(2*I*(b*x+a)))*x^2-3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x+2*
I/b^2*d^4*polylog(2,-exp(2*I*(b*x+a)))*x^3-3*I/b^4*c*d^3*polylog(4,-exp(2*I
*(b*x+a)))+2*I/b^2*c^3*d*polylog(2,-exp(2*I*(b*x+a)))
```

**maxima [B]** time = 0.71, size = 1779, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^4*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 12*a*c^3*d*(l
og(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + 18*a^2*c^2*d^2*(log(sin(b
*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 12*a^3*c*d^3*(log(sin(b*x + a)^
2 - 1) - log(sin(b*x + a)^2))/b^3 + 3*a^4*d^4*(log(sin(b*x + a)^2 - 1) - lo
g(sin(b*x + a)^2))/b^4 - (18*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) - 144*d^4
*polylog(5, -e^(I*b*x + I*a)) - 144*d^4*polylog(5, e^(I*b*x + I*a)) - (12*I
*(b*x + a)^4*d^4 + (32*I*b*c*d^3 - 32*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*
d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a
*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a))*arctan2(sin(2*b*
x + 2*a), cos(2*b*x + 2*a) + 1) - (-6*I*(b*x + a)^4*d^4 + (-24*I*b*c*d^3 +
24*I*a*d^4)*(b*x + a)^3 + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^
4)*(b*x + a)^2 + (-24*I*b^3*c^3*d + 72*I*a*b^2*c^2*d^2 - 72*I*a^2*b*c*d^3 +
 24*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (6*I*(b
*x + a)^4*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*d^2
 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a*b^
2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a
), -cos(b*x + a) + 1) - (-12*I*b^3*c^3*d + 36*I*a*b^2*c^2*d^2 - 36*I*a^2*b*
c*d^3 - 24*I*(b*x + a)^3*d^4 + 12*I*a^3*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*
(b*x + a)^2 + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^4)*(b*x + a)
)*dilog(-e^(2*I*b*x + 2*I*a)) - (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I
*a^2*b*c*d^3 + 24*I*(b*x + a)^3*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*I*a
*d^4)*(b*x + a)^2 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*(b*
x + a))*dilog(-e^(I*b*x + I*a)) - (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72
*I*a^2*b*c*d^3 + 24*I*(b*x + a)^3*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*I
*a*d^4)*(b*x + a)^2 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*(
b*x + a))*dilog(e^(I*b*x + I*a)) - 2*(3*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^
4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 6*(b
^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(2*
b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 3*((b*x + a)^
4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d
^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) +
3*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a
*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*
c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x
 + a) + 1) - (24*I*b*c*d^3 + 36*I*(b*x + a)*d^4 - 24*I*a*d^4)*polylog(4, -e
^(2*I*b*x + 2*I*a)) - (-144*I*b*c*d^3 - 144*I*(b*x + a)*d^4 + 144*I*a*d^4)*
polylog(4, -e^(I*b*x + I*a)) - (-144*I*b*c*d^3 - 144*I*(b*x + a)*d^4 + 144*
I*a*d^4)*polylog(4, e^(I*b*x + I*a)) - 6*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(
b*x + a)^2*d^4 + 3*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(
2*I*b*x + 2*I*a)) + 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d
^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(I*b*x + I*a)) + 72*(b^2*
c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*
x + a))*polylog(3, e^(I*b*x + I*a)))/b^4)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^4}{\cos(a+b\,x)\,\sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(cos(a + b*x)*sin(a + b*x)),x)
```

```
[Out] int((c + d*x)^4/(cos(a + b*x)*sin(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*csc(a + b*x)*sec(a + b*x), x)
```

# 3.229 $\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx$

**Optimal**. Leaf size=197

$$-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\cdots}{2\cdots}$$

```
[Out] -2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a)))/b+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2
*I*(b*x+a)))/b^2-3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*
(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^3+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*
(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/4*I*d^3*polylog(
4,exp(2*I*(b*x+a)))/b^4
```

**Rubi [A]** time = 0.17, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4419, 4183, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d*(c + d*x)^2*
PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2,
 E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x
))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) -
(((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/4)*d^3*PolyL
og[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)^3 \csc(2a+2bx)\,dx$$

$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{(3d)\int (c+dx)^2 \log\left(1-e^{i(2a+2bx)}\right)dx}{b} +$$

$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+d}{}$$

$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+d}{}$$

$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+d}{}$$

$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+d}{}$$

**Mathematica [A]** time = 0.95, size = 350, normalized size = 1.78

$$\frac{-8b^3c^3 \tanh^{-1}\left(e^{2i(a+bx)}\right) + 12b^3c^2dx \log\left(1-e^{2i(a+bx)}\right) - 12b^3c^2dx \log\left(1+e^{2i(a+bx)}\right) + 12b^3cd^2x^2 \log\left(1-e^{2i(a+bx)}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-8*b^3*c^3*ArcTanh[E^((2*I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 - E^((2*I)*
(a + b*x))] + 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 4*b^3*d^3*x^3
*Log[1 - E^((2*I)*(a + b*x))] - 12*b^3*c^2*d*x*Log[1 + E^((2*I)*(a + b*x))]
 - 12*b^3*c*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] - 4*b^3*d^3*x^3*Log[1 + E^
((2*I)*(a + b*x))] + (6*I)*b^2*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x)
)] - (6*I)*b^2*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))] - 6*b*d^2*(c +
 d*x)*PolyLog[3, -E^((2*I)*(a + b*x))] + 6*b*c*d^2*PolyLog[3, E^((2*I)*(a +
 b*x))] + 6*b*d^3*x*PolyLog[3, E^((2*I)*(a + b*x))] - (3*I)*d^3*PolyLog[4,
-E^((2*I)*(a + b*x))] + (3*I)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/(4*b^4)
```

**fricas [C]** time = 0.60, size = 1778, normalized size = 9.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4,
 cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b
*x + a)) - 6*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*poly
log(4, -I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4, -I*cos(b*x + a)
 - sin(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + 6*I
*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b
^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2
*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x
+ a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*
x + a) + sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d
)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x
 + 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2
 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) +
 (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) +
I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilo
g(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*
c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I)
 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x +
a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a)
+ 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d
^3)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) +
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) -
 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x +
 a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x
 + a) + I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x
+ a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*
x + a) - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a
) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin
(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)
) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*
d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x +
b*c*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)
*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog
(3, -cos(b*x + a) - I*sin(b*x + a)))/b^4
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \csc(bx + a) \sec(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a), x)
```

**maple [B]** time = 0.12, size = 816, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] -6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,-exp(I*(
b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-1/b*c^3*ln(1+exp(2*I*(b*x+
a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a
)))+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)
))*x+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6*I/b^4*d^3*polylog(4,-exp(I*(b
*x+a)))-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/2/b^3*d^3*polylog(3,-e
xp(2*I*(b*x+a)))*x-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+1/b*c^3*ln(ex
p(I*(b*x+a))-1)+1/b*c^3*ln(exp(I*(b*x+a))+1)+3*I/b^2*c*d^2*polylog(2,-exp(2
*I*(b*x+a)))*x+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-3*I/b^2*c^2*d*polylog(2
,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-3*I/b^2*d^3*polyl
og(2,exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+3/b*c^2
*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1
-exp(I*(b*x+a)))*a-3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+3/b*c*d^2*ln(1-exp(
I*(b*x+a)))*x^2+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/b^2*c^2*d*a*ln(exp(I*(
b*x+a))-1)+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*
a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-1/b*d^3*ln(1+exp(2*I*(b*x+a)))*x^3+3/2
*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+3/2*I/b^2*c^2*d*polylog(2,-exp(
2*I*(b*x+a)))-3/b*c^2*d*ln(1+exp(2*I*(b*x+a)))*x-3/b*c*d^2*ln(1+exp(2*I*(b*
x+a)))*x^2
```

**maxima** [B] time = 0.61, size = 1063, normalized size = 5.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^3*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 9*a*c^2*d*(lo
g(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + 9*a^2*c*d^2*(log(sin(b*x +
 a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 3*a^3*d^3*(log(sin(b*x + a)^2 - 1)
- log(sin(b*x + a)^2))/b^3 + (6*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) - 36
*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 36*I*d^3*polylog(4, e^(I*b*x + I*a))
+ (8*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^
2*c^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(b*x + a))*arctan2(sin(2*b*x + 2*a
), cos(2*b*x + 2*a) + 1) + (-6*I*(b*x + a)^3*d^3 + (-18*I*b*c*d^2 + 18*I*a*
d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3)*(b*x +
 a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*I*(b*x + a)^3*d^3 + (18*I
*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*
I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (-9*I*b^2*
c^2*d + 18*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 9*I*a^2*d^3 + (-18*I*b*c*d^
2 + 18*I*a*d^3)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + (18*I*b^2*c^2*d -
36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I
*a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) + (18*I*b^2*c^2*d - 36*I*a*b*c*d
^2 + 18*I*(b*x + a)^2*d^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x
 + a))*dilog(e^(I*b*x + I*a)) + (4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b
*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(2*b*x
+ 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 3*((b*x + a)^3*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*(
(b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x +
 a) + 1) + 3*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*polylog(3, -e^(2*I*b*x
 + 2*I*a)) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -e^(I*b*x + I*
a)) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, e^(I*b*x + I*a)))/b^3
)/b
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{\cos(a+b\,x)\,\sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)),x)
[Out] int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \csc(a + bx) \sec(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a),x)
[Out] Integral((c + d*x)**3*csc(a + b*x)*sec(a + b*x), x)
```

# 3.230 $\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx$

**Optimal**. Leaf size=127

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}$$

```
[Out] -2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b+I*d*(d*x+c)*polylog(2,-exp(2*I*(b*
x+a)))/b^2-I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-e
xp(2*I*(b*x+a)))/b^3+1/2*d^2*polylog(3,exp(2*I*(b*x+a)))/b^3
```

**Rubi [A]** time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4419, 4183, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*PolyLog[2,
 -E^((2*I)*(a + b*x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^2 - (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E
^((2*I)*(a + b*x))])/(2*b^3)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)^2 \csc(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{(2d)\int (c+dx)\log\left(1-e^{i(2a+2bx)}\right)dx}{b}$$
$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c+dx}{}$$
$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c+dx}{}$$
$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c+dx}{}$$

**Mathematica [A]** time = 0.59, size = 213, normalized size = 1.68

$$\frac{-4b^2c^2\tanh^{-1}\left(e^{2i(a+bx)}\right) + 4b^2cdx\log\left(1-e^{2i(a+bx)}\right) - 4b^2cdx\log\left(1+e^{2i(a+bx)}\right) + 2b^2d^2x^2\log\left(1-e^{2i(a+bx)}\right) -}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-4*b^2*c^2*ArcTanh[E^((2*I)*(a + b*x))] + 4*b^2*c*d*x*Log[1 - E^((2*I)*(a
+ b*x))] + 2*b^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] - 4*b^2*c*d*x*Log[1 +
 E^((2*I)*(a + b*x))] - 2*b^2*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] + (2*I)*
b*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLo
g[2, E^((2*I)*(a + b*x))] - d^2*PolyLog[3, -E^((2*I)*(a + b*x))] + d^2*Poly
Log[3, E^((2*I)*(a + b*x))])/(2*b^3)
```

**fricas [C]** time = 0.56, size = 1090, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a
)) - 2*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*d^2*polylog(3, -I*
cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x +
 a)) + 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3,
-cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(cos(b*x
+ a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(cos(b*x + a) - I*s
in(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x +
a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (2*I
*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x
 - 2*I*b*c*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*
c*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dil
og(-cos(b*x + a) - I*sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*
log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b
```

```
*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b
^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b
^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b
*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos
(b*x + a) - sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*co
s(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*
x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a)
 + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a)
+ I))/b^3
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc(bx+a)\sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a), x)
```

**maple [B]** time = 0.09, size = 469, normalized size = 3.69

$$-\frac{d^2 \operatorname{polylog}\left(3, -e^{2i(bx+a)}\right)}{2b^3}+\frac{d^2a^2 \ln\left(e^{i(bx+a)}-1\right)}{b^3}+\frac{d^2 \ln\left(1-e^{i(bx+a)}\right)x^2}{b}-\frac{d^2 \ln\left(1-e^{i(bx+a)}\right)a^2}{b^3}+\frac{d^2 \ln\left(e^{i(bx+a)}+1\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] -1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1
)+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d
^2*ln(exp(I*(b*x+a))+1)*x^2+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*poly
log(3,exp(I*(b*x+a)))/b^3-1/b*c^2*ln(1+exp(2*I*(b*x+a)))+1/b*c^2*ln(exp(I*(
b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1)-1/b*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-
2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a
)))*x-2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2
*c*d*ln(1-exp(I*(b*x+a)))*a+2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*a*ln(e
xp(I*(b*x+a))-1)-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+I/b^2*d^2*polylog(2,
-exp(2*I*(b*x+a)))*x+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))-2*I/b^2*c*d*pol
ylog(2,-exp(I*(b*x+a)))
```

**maxima [B]** time = 0.60, size = 590, normalized size = 4.65

$$-\frac{c^2\left(\log\left(\sin(bx+a)^2-1\right)-\log\left(\sin(bx+a)^2\right)\right)-\frac{2\,acd\left(\log\left(\sin(bx+a)^2-1\right)-\log\left(\sin(bx+a)^2\right)\right)}{b}+\frac{a^2d^2\left(\log\left(\sin(bx+a)^2-1\right)-\log\left(\sin\right.\right.}{b^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 2*a*c*d*(log(si
n(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + a^2*d^2*(log(sin(b*x + a)^2 -
1) - log(sin(b*x + a)^2))/b^2 + (d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) - 4*d
^2*polylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x + I*a)) + (2*I*
(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x + 2*
a), cos(2*b*x + 2*a) + 1) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2
)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (2*I*(b*x + a)^2*d^2
 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) +
```

```
 1) + (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*dilog(-e^(2*I*b*x + 2*I*
a)) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*dilog(-e^(I*b*x + I*a)) +
 (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*dilog(e^(I*b*x + I*a)) + ((b*x
 + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b
*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2
)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((
b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 - 2*cos(b*x + a) + 1))/b^2)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)),x)
```

```
[Out] int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \csc\left(a+bx\right)\sec\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x), x)
```

# 3.231 $\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$

**Optimal**. Leaf size=71

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}$$

```
[Out] -2*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))
/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2
```

**Rubi** [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4419, 4183, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-2*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b + ((I/2)*d*PolyLog[2, -E^((2*
I)*(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)\csc(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\int \log\left(1-e^{i(2a+2bx)}\right)dx}{b} + \frac{d\int \log\left(1\right.}{}$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{(id)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{i(2a+2bx)}\right)}{2b^2} -$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}$$

**Mathematica** [A] time = 0.11, size = 141, normalized size = 1.99

$$\frac{d\left(i\left(\text{Li}_2\left(-e^{i(2a+2bx)}\right) - \text{Li}_2\left(e^{i(2a+2bx)}\right)\right) + (2a+2bx)\left(\log\left(1-e^{i(2a+2bx)}\right) - \log\left(1+e^{i(2a+2bx)}\right)\right) - 2a\log\left(\tan\left(\frac{1}{2}\right.\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] -((c*Log[Cos[a + b*x]])/b) + (c*Log[Sin[a + b*x]])/b + (d*((2*a + 2*b*x)*(L
og[1 - E^(I*(2*a + 2*b*x))] - Log[1 + E^(I*(2*a + 2*b*x))]) - 2*a*Log[Tan[(
2*a + 2*b*x)/2]] + I*(PolyLog[2, -E^(I*(2*a + 2*b*x))] - PolyLog[2, E^(I*(2
*a + 2*b*x))])))/(2*b^2)
```

**fricas** [B] time = 0.52, size = 554, normalized size = 7.80

$$\frac{-i\,d\text{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right) + i\,d\text{Li}_2\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right) - i\,d\text{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(cos(b*x + a) - I
*sin(b*x + a)) - I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(I*cos
(b*x + a) - sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d
*dilog(-I*cos(b*x + a) - sin(b*x + a)) + I*d*dilog(-cos(b*x + a) + I*sin(b*
x + a)) - I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b*d*x + b*c)*log(cos
(b*x + a) + I*sin(b*x + a) + 1) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x
+ a) + I) + (b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a
*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d*x + a*d)*log(I*cos(b*x +
a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x + a) - sin(b*x + a) +
1) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*
log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*log(-1/2*cos(b*x + a)
 + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*si
n(b*x + a) + 1/2) + (b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) -
 (b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*d*x + a*d)*log(-c
os(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a*d)*log(-cos(b*x + a) - I*sin(b
*x + a) + I))/b^2
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\csc\left(bx+a\right)\sec\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a), x)
```

**maple [B]** time = 0.08, size = 208, normalized size = 2.93

$$\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}-\frac{c\ln\left(1+e^{2i(bx+a)}\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}-\frac{d\ln\left(1+e^{2i(bx+a)}\right)x}{b}+\frac{id\operatorname{polylog}\left(2,-e^{2i(bx+a)}\right)}{2b^2}+\frac{d\ln\left(e^{i}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] 1/b*c*ln(exp(I*(b*x+a))-1)-1/b*c*ln(1+exp(2*I*(b*x+a)))+1/b*c*ln(exp(I*(b*x
+a))+1)-1/b*d*ln(1+exp(2*I*(b*x+a)))*x+1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))
/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2+1/b*d*
ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(
b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

**maxima [B]** time = 0.70, size = 267, normalized size = 3.76

$$-\frac{2i\,bdx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right)-2i\,bc\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)-1\right)+\left(2i\,bdx+2i\,bc\right)a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2(
sin(b*x + a), cos(b*x + a) - 1) + (2*I*b*d*x + 2*I*b*c)*arctan2(sin(2*b*x +
 2*a), cos(2*b*x + 2*a) + 1) + (-2*I*b*d*x - 2*I*b*c)*arctan2(sin(b*x + a),
 cos(b*x + a) + 1) - I*d*dilog(-e^(2*I*b*x + 2*I*a)) + 2*I*d*dilog(-e^(I*b*
x + I*a)) + 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(2*b*x + 2*
a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (b*d*x + b*c)*log(cos
(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (b*d*x + b*c)*log(cos(
b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)*sin(a + b*x)),x)
```

```
[Out] int((c + d*x)/(cos(a + b*x)*sin(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\csc\left(a+bx\right)\sec\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)*sec(a + b*x), x)
```

## 3.232 $\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=22

$$2\text{Int}\left(\frac{\csc(2a+2bx)}{c+dx},x\right)$$

```
[Out] 2*Unintegrable(csc(2*b*x+2*a)/(d*x+c),x)
```

**Rubi** [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x),x]
[Out] 2*Defer[Int][Csc[2*a + 2*b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = 2\int \frac{\csc(2a+2bx)}{c+dx}\,dx$$

**Mathematica** [A] time = 4.53, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x), x]
```

**fricas** [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)\sec(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

**maple** [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

# 3.233 $\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=22

$$2\text{Int}\left(\frac{\csc(2a+2bx)}{(c+dx)^2},x\right)$$

```
[Out] 2*Unintegrable(csc(2*b*x+2*a)/(d*x+c)^2,x)
```

**Rubi** [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2,x]
[Out] 2*Defer[Int][Csc[2*a + 2*b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = 2\int \frac{\csc(2a+2bx)}{(c+dx)^2}\,dx$$

**Mathematica** [A] time = 5.72, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2, x]
```

**fricas** [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)\sec(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\sec(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c)^2, x)
```

**maple** [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\csc \left(bx+a\right) \sec \left(bx+a\right)}{\left(dx+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc \left(bx+a\right) \sec \left(bx+a\right)}{\left(dx+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c)^2, x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cos \left(a+b\,x\right)\,\sin \left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc \left(a+bx\right) \sec \left(a+bx\right)}{\left(c+dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

## 3.234 $\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$

**Optimal**. Leaf size=25

$$\operatorname{Int}\left(\csc^2(a+bx)\sec(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

**Rubi [A]** time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx=\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$$

**Mathematica [A]** time = 17.79, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]
```

**fricas [A]** time = 0.46, size = 0, normalized size = 0.00

$$\operatorname{integral}\left((dx+c)^m \csc(bx+a)^2 \sec(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

**maple [A]** time = 0.08, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^2(bx+a)\right)\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \csc (bx + a)^2 \sec (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c + d\,x)^m}{\cos (a + b\,x)\,\sin (a + b\,x)^2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^2),x)
```

```
[Out] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a),x)
```

```
[Out] Timed out
```

# 3.235 $\int (c+dx)^3 \csc^2(a+bx) \sec(a+bx) \, dx$

**Optimal**. Leaf size=350

$$-\frac{6d^3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}-\frac{6id^3\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{6id^3\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^4}+\frac{6id^2(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{6ic}{}$$

```
[Out] -2*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b-6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)
))/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^
3+3*I*d*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^2-3*I*d*(d*x+c)^2*polylog(
2,I*exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^3-6*d^3
*polylog(3,-exp(I*(b*x+a)))/b^4-6*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/
b^3+6*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,exp(I*(b*
x+a)))/b^4-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,I*exp
(I*(b*x+a)))/b^4
```

**Rubi [A]** time = 0.64, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {2621, 321, 207, 4420, 6741, 12, 6742, 6273, 4181, 2531, 6609, 2282, 6589, 4183}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b - (6*d*(c + d*x)^2*ArcTanh[E
^(I*(a + b*x))])/b^2 - ((c + d*x)^3*Csc[a + b*x])/b + ((6*I)*d^2*(c + d*x)*
PolyLog[2, -E^(I*(a + b*x))])/b^3 + ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*E^
(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b
^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog
[3, -E^(I*(a + b*x))])/b^4 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x
))])/b^3 + (6*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (6*d^3*Pol
yLog[3, E^(I*(a + b*x))])/b^4 - ((6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))]
)/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3\csc^2(a+bx)\sec(a+bx)\,dx = \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - (3d)\int (c+$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - (3d)\int \frac{(c+}{}$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(3d)\int (c+}{}$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(3d)\int ((c+}{}$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(3d)\int (c+}{}$$

$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3\csc(a+bx)}{b} + \frac{\int b(c+dx)}{}$$

$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3\csc(a+bx)}{b} + \frac{6id^2(c+dx}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+d}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+d}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+d}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+d}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+d}{}$$

**Mathematica [B]** time = 6.41, size = 739, normalized size = 2.11

$$\frac{3d\left(\frac{2id\left(b(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)+id\text{Li}_3\left(-e^{i(a+bx)}\right)\right)}{b^2}+\frac{2d\left(d\text{Li}_3\left(e^{i(a+bx)}\right)-ib(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)\right)}{b^2}+(c+dx)^2\log\left(1-e^{i(a+bx)}\right)-(c+dx)\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] -(((c + d*x)^3*Csc[a])/b) + (3*d*((c + d*x)^2*Log[1 - E^(I*(a + b*x))] - (c
 + d*x)^2*Log[1 + E^(I*(a + b*x))] + ((2*I)*d*(b*(c + d*x)*PolyLog[2, -E^(I
*(a + b*x))] + I*d*PolyLog[3, -E^(I*(a + b*x))]))/b^2 + (2*d*((-I)*b*(c + d
*x)*PolyLog[2, E^(I*(a + b*x))] + d*PolyLog[3, E^(I*(a + b*x))]))/b^2))/b^2
 + ((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - I*E^(I*(
a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1
 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 3*b^3*c*
d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x))]
 + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] - (3*I)*b^2*d*(
c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(
a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog
[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] - (6*I)*d^
3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))
])/b^4 + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^3*Sin[(b*x)/2]) - 3*c^2*d*x*Sin[
(b*x)/2] - 3*c*d^2*x^2*Sin[(b*x)/2] - d^3*x^3*Sin[(b*x)/2]))/(2*b) + (Csc[a
/2]*Csc[a/2 + (b*x)/2]*(c^3*Sin[(b*x)/2] + 3*c^2*d*x*Sin[(b*x)/2] + 3*c*d^2
*x^2*Sin[(b*x)/2] + d^3*x^3*Sin[(b*x)/2]))/(2*b)
```

**fricas [C]** time = 0.65, size = 1753, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 6*I*d^3
*polylog(4, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*I*d^3*polylog(4
, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 6*I*d^3*polylog(4, -I*cos(b
*x + a) + sin(b*x + a))*sin(b*x + a) + 6*I*d^3*polylog(4, -I*cos(b*x + a) -
 sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) + I*sin(b*x + a
))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x +
 a) + 6*d^3*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3
*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (-6*I*b*d^3*x -
6*I*b*c*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b*d^3
*x + 6*I*b*c*d^2)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (-3*I
*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(
b*x + a))*sin(b*x + a) - (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*
d)*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - (3*I*b^2*d^3*x^2 + 6
*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b
*x + a) - (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-I*cos(
b*x + a) - sin(b*x + a))*sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(
-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*
dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2
*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) -
 (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*s
in(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)
*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(
b*x + a) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x +
a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) - (
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2
)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a
```

```
) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
 + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x +
 a) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a)
 + I*sin(b*x + a) + I)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*
b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*si
n(b*x + a) + I)*sin(b*x + a) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x +
 a) + sin(b*x + a))*sin(b*x + a) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b
*x + a) - sin(b*x + a))*sin(b*x + a) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*
cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*(b*d^3*x + b*c*d^2)*polylog(3
, -I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a))/(b^4*sin(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \csc (bx+a)^2 \sec (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a), x)
```

**maple [B]** time = 0.50, size = 1158, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x)
```

```
[Out] 6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^
4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+3/b^2*c^2*d*ln(exp(I*(b*x+a))-1)-3/b^
2*c^2*d*ln(exp(I*(b*x+a))+1)+3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-1)-3/b^2*d^3*l
n(exp(I*(b*x+a))+1)*x^2+3/b^2*d^3*ln(1-exp(I*(b*x+a)))*x^2+3/b^4*d^3*ln(1-e
xp(I*(b*x+a)))*a^2-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4-1/b^4*a^3*d^3*l
n(1+I*exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x+1/b*d^3*ln(1-
I*exp(I*(b*x+a)))*x^3-1/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3-6/b^3*d^3*polylog(
3,-I*exp(I*(b*x+a)))*x+6/b^3*d^2*c*polylog(3,I*exp(I*(b*x+a)))-6/b^3*d^2*c*
polylog(3,-I*exp(I*(b*x+a)))+1/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))-2*I/b*c^3
*arctan(exp(I*(b*x+a)))+3/b*d^2*c*ln(1-I*exp(I*(b*x+a)))*x^2-3/b*d^2*c*ln(1
+I*exp(I*(b*x+a)))*x^2+3/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-
I*exp(I*(b*x+a)))*a+3/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x+a)))-3/b*c^2*d*ln(1+I
*exp(I*(b*x+a)))*x-3/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a-3/b^3*a^2*c*d^2*ln(
1-I*exp(I*(b*x+a)))+3*I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))+2*I/b^4*d^3*
a^3*arctan(exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))-3*I/b^
2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+3*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a
)))*x^2-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(
b*x+a))-1)-6/b^3*c*d^2*a*ln(exp(I*(b*x+a))-1)+6/b^3*d^3*ln(1-exp(I*(b*x+a))
)*a*x+6*I/b^3*d^2*c*dilog(exp(I*(b*x+a))+1)-6*I/b^4*d^3*a*dilog(exp(I*(b*x+
a))+1)+6*I/b^4*d^3*polylog(2,-exp(I*(b*x+a)))*a-6*I/b^4*d^3*polylog(2,exp(I
*(b*x+a)))*a+6*I/b^3*dilog(exp(I*(b*x+a)))*c*d^2-6*I/b^4*dilog(exp(I*(b*x+a
)))*d^3*a-6*I/b^3*d^3*polylog(2,exp(I*(b*x+a)))*x-6/b^2*d^2*c*ln(exp(I*(b*x
+a))+1)*x+6*I/b^3*d^3*polylog(2,-exp(I*(b*x+a)))*x-6*I/b^2*d^2*c*polylog(2,
I*exp(I*(b*x+a)))*x-6*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))+6*I/b^2*c^2*d*
a*arctan(exp(I*(b*x+a)))+6*I/b^2*d^2*c*polylog(2,-I*exp(I*(b*x+a)))*x
```

**maxima [B]** time = 1.41, size = 3240, normalized size = 9.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))
- 3*a*c^2*d*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1)
)/b + 3*a^2*c*d^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a
) - 1))/b^2 - a^3*d^3*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x
 + a) - 1))/b^3 - 2*((2*(b*x + a)^3*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) - 2*((b*x + a)^3*d^3 + 3*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x +
 a))*cos(2*b*x + 2*a) + (-2*I*(b*x + a)^3*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*
(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a))*si
n(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (2*(b*x + a)^3*d^
3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
*(b*x + a) - 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-2*I*(b*x + a
)^3*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a
*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a),
-sin(b*x + a) + 1) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^
2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x
+ a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-
6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I
*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), c
os(b*x + a) + 1) - (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*a^2*d^3 - 6*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*cos(2*b*x + 2*a) - (6*I*b^2*c^2*d - 12*I*a*b*c*d^2
+ 6*I*a^2*d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) +
(6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 +
2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-6*I*(b*x + a)^2*d^3 + (
-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x +
a), -cos(b*x + a) + 1) - 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)
^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(b*x + a) + (6*b^2
*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d^3 + 12*(b*c*d^2 - a*d^3
)*(b*x + a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b
*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*
d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x
 + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (6*b^2*c^2*d - 12*a*b*c
*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*(
b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(
b*x + a))*cos(2*b*x + 2*a) - (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a
)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(2*b*x +
2*a))*dilog(-I*e^(I*b*x + I*a)) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3
 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (12*I*b*c*d^2 +
12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a))
+ (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 -
 a*d^3)*cos(2*b*x + 2*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3
)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^
2 - 3*I*(b*x + a)^2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a
) + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + (6
*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*(b^2*c^2*d - 2*a*b*
c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*
x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (3*I*
b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + (6*I*b*c*d^
2 - 6*I*a*d^3)*(b*x + a) + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*(b*x + a)^
2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a
) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a
*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*
cos(b*x + a) + 1) + (I*(b*x + a)^3*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a
)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*a^2*d^3)*(b*x + a) + (-I*(b*x +
a)^3*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a
*b*c*d^2 - 3*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 + 3*
(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x
```

```
+ a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a
) + 1) + (-I*(b*x + a)^3*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3
*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*a^2*d^3)*(b*x + a) + (I*(b*x + a)^3*d^3
+ (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 +
3*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 -
a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 12
*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) - d^3)*polylog(4, I*e^(I*b*
x + I*a)) - 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) - d^3)*polylo
g(4, -I*e^(I*b*x + I*a)) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3
+ (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(2*b*x + 2*a) + 12*(
b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, I*e^(I*b*x +
I*a)) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (12*I*b*c*d^2 +
12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)
*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -I*e^(I*b*x + I*a)) + (12*I*d^3*
cos(2*b*x + 2*a) - 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*polylog(3, -e^(I*b*x
 + I*a)) + (-12*I*d^3*cos(2*b*x + 2*a) + 12*d^3*sin(2*b*x + 2*a) + 12*I*d^3
)*polylog(3, e^(I*b*x + I*a)) + (-4*I*(b*x + a)^3*d^3 + (-12*I*b*c*d^2 + 12
*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3)*(
b*x + a))*sin(b*x + a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a)
 + 2*I*b^3))/b
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)^2),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \csc^2(a + bx) \sec(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*csc(a + b*x)**2*sec(a + b*x), x)
```

# 3.236 $\int (c+dx)^2 \csc^2(a+bx)\sec(a+bx)\,dx$

**Optimal**. Leaf size=226

$$\frac{2id^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{2d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+$$

```
[Out] -2*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b-4*d*(d*x+c)*arctanh(exp(I*(b*x+a)))
/b^2-(d*x+c)^2*csc(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3+2*I*d*(d
*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,I*exp(I*(b*x
+a)))/b^2-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3-2*d^2*polylog(3,-I*exp(I*(b
*x+a)))/b^3+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3
```

**Rubi [A]** time = 0.38, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {2621, 321, 207, 4420, 6741, 12, 6742, 6273, 4181, 2531, 2282, 6589, 4183, 2279, 2391}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}+\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}(}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b - (4*d*(c + d*x)*ArcTanh[E^(
I*(a + b*x))])/b^2 - ((c + d*x)^2*Csc[a + b*x])/b + ((2*I)*d^2*PolyLog[2, -
E^(I*(a + b*x))])/b^3 + ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))]
)/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*
PolyLog[2, E^(I*(a + b*x))])/b^3 - (2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])
/b^3 + (2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc^2(a+bx)\sec(a+bx)\,dx = \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - (2d)\int (c+dx$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - (2d)\int \frac{(c+dx}{}$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(2d)\int (c+dx)}{}$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(2d)\int ((c+dx}{}$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(2d)\int (c+dx)}{}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2\csc(a+bx)}{b} + \frac{\int b(c+dx)^2\operatorname{se}}{b}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2\csc(a+bx)}{b} + \frac{(2id^2)\operatorname{Subst}(\int}{}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2 c}{}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2 c}{}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2 c}{}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2 c}{}$$

**Mathematica [B]** time = 6.27, size = 593, normalized size = 2.62

$$\frac{2d^2\left(-\frac{2\tan^{-1}(\tan(a))\tanh^{-1}\left(\frac{\sin(a)\tan\left(\frac{bx}{2}\right)-\cos(a)}{\sqrt{\sin^2(a)+\cos^2(a)}}\right)}{\sqrt{\sin^2(a)+\cos^2(a)}} + \frac{\sec(a)\left(i\left(\operatorname{Li}_2\left(-e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)-\operatorname{Li}_2\left(e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)\right)+\left(\tan^{-1}(\tan(a))+bx\right)\left(\log\left(1-e^{i(\mathrm{ta}}\right.\right.\right.}{\sqrt{\tan^2(a)+1}}\right.}{b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] -(((c + d*x)^2*Csc[a])/b) + ((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2
*c*d*x*Log[1 - I*E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))]
- 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a +
b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(
c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*
x))] + 2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + ((4*I)*c*d*ArcTan[(I*Cos[
a] - I*Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/(b^2*Sqrt[Cos[a]^2
+ Sin[a]^2]) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^2*Sin[(b*x)/2]) - 2*c*d*x*
Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2]))/(2*b) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*
(c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2] + d^2*x^2*Sin[(b*x)/2]))/(2*b) + (
2*d^2*((-2*ArcTan[Tan[a]]*ArcTanh[(-Cos[a] + Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[
a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^2 + Sin[a]^2] + (((b*x + ArcTan[Tan[a]])*(Lo
g[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b*x + ArcTan[Tan[a]]))]
) + I*(PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - PolyLog[2, E^(I*(b*x + A
rcTan[Tan[a]]))]))*Sec[a])/Sqrt[1 + Tan[a]^2]))/b^3
```

**fricas [C]** time = 0.56, size = 1067, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*I*d^2*dilog(cos(b*x + a)
+ I*sin(b*x + a))*sin(b*x + a) - 2*I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a
))*sin(b*x + a) + 2*I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a
) - 2*I*d^2*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 2*d^2*poly
log(3, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 2*d^2*polylog(3, I*cos
(b*x + a) - sin(b*x + a))*sin(b*x + a) + 2*d^2*polylog(3, -I*cos(b*x + a) +
 sin(b*x + a))*sin(b*x + a) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x +
a))*sin(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*
x + a))*sin(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) - si
n(b*x + a))*sin(b*x + a) - (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a)
+ sin(b*x + a))*sin(b*x + a) - (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x +
 a) - sin(b*x + a))*sin(b*x + a) + 2*(b*d^2*x + b*c*d)*log(cos(b*x + a) + I
*sin(b*x + a) + 1)*sin(b*x + a) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b
*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 2*(b*d^2*x + b*c*d)*log(cos(b*
x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2
*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*
x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x +
a) - sin(b*x + a) + 1)*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*
d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b^2*d^
2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x +
a) + 1)*sin(b*x + a) - 2*(b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(
b*x + a) + 1/2)*sin(b*x + a) - 2*(b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/
2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b*d^2*x + a*d^2)*log(-cos(b*x + a
) + I*sin(b*x + a) + 1)*sin(b*x + a) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(
-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) - 2*(b*d^2*x + a*d^2)*log(
-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a))/(b^3*sin(b*x
+ a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \csc (bx + a)^2 \sec (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a), x)

**maple [B]** time = 0.28, size = 556, normalized size = 2.46

$$\frac{4icda\arctan\left(e^{i(bx+a)}\right)}{b^2}+\frac{2dc\ln\left(e^{i(bx+a)}-1\right)}{b^2}-\frac{2dc\ln\left(e^{i(bx+a)}+1\right)}{b^2}+\frac{2i\operatorname{dilog}\left(e^{i(bx+a)}\right)d^2}{b^3}-\frac{2ie^{i(bx+a)}\left(d^2x^2+2cdx+\right.}{b\left(e^{2i(bx+a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x)

[Out] 4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))+2/b^2*d*c*ln(exp(I*(b*x+a))-1)-2/b^2*d*c*ln(exp(I*(b*x+a))+1)-2*I*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))-1)+2*I/b^3*dilog(exp(I*(b*x+a)))*d^2-2*I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x+2*I/b^3*d^2*dilog(exp(I*(b*x+a))+1)-2*I/b*c^2*arctan(exp(I*(b*x+a)))-2*I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))+2*I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))+2*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-1/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))-1/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2-2/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a+1/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-2/b^3*d^2*a*ln(exp(I*(b*x+a))-1)-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3-2*I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))-2/b^2*d^2*ln(exp(I*(b*x+a))+1)*x+2/b*c*d*ln(1-I*exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a-2/b*c*d*ln(1+I*exp(I*(b*x+a)))*x+1/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2

**maxima [B]** time = 0.77, size = 1632, normalized size = 7.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")

[Out] -1/2*(c^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1)) - 2*a*c*d*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b + a^2*d^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b^2 - 2*((2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (4*b*c*d - 4*a*d^2 - 4*(b*c*d - a*d^2)*cos(2*b*x + 2*a) - (4*I*b*c*d - 4*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - (4*(b*x + a)*d^2*cos(2*b*x + 2*a) + 4*I*(b*x + a)*d^2*sin(2*b*x + 2*a) - 4*(b*x + a)*d^2)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(b*x + a) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + 4*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) - 4*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x + I*a)) + (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2 + (2*I*b*c*d + 2*I*(b*x + a)*d^2 - 2*I*a*d^2)*cos(2*b*x + 2*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*I*b*c*d + 2*I*(b*x + a)*d^2 - 2*I*a*d^2 + (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*cos(2*b*x + 2*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x

```
+ a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b
*c*d - 2*I*a*d^2)*(b*x + a) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2
)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x +
 a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a)
 + 1) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + (I*(b*x
+ a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x +
a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (-4*I*d^2*cos(2*b*x + 2*a) + 4*d
^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, I*e^(I*b*x + I*a)) + (4*I*d^2*cos
(2*b*x + 2*a) - 4*d^2*sin(2*b*x + 2*a) - 4*I*d^2)*polylog(3, -I*e^(I*b*x +
I*a)) + (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*sin(b*x
 + a))/(-2*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(2*b*x + 2*a) + 2*I*b^2))/b
```

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)^2),x)
[Out] \text{Hanged}
```

**sympy** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \csc^2(a+bx) \sec(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a),x)
[Out] Integral((c + d*x)**2*csc(a + b*x)**2*sec(a + b*x), x)
```

# 3.237 $\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$

**Optimal**. Leaf size=131

$$\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}+\frac{(c+dx)\tanh^{-1}(\sin(a+bx))}{b}-\frac{2}{}$$

```
[Out] -2*I*d*x*arctan(exp(I*(b*x+a)))/b-d*arctanh(cos(b*x+a))/b^2-d*x*arctanh(sin
(b*x+a))/b+(d*x+c)*arctanh(sin(b*x+a))/b-(d*x+c)*csc(b*x+a)/b+I*d*polylog(2
,-I*exp(I*(b*x+a)))/b^2-I*d*polylog(2,I*exp(I*(b*x+a)))/b^2
```

**Rubi** **[A]** time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2621, 321, 207, 4420, 6271, 12, 4181, 2279, 2391, 3770}

$$\frac{id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}+\frac{(c+dx)\tanh^{-1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*d*x*ArcTan[E^(I*(a + b*x))])/b - (d*ArcTanh[Cos[a + b*x]])/b^2 - (d
*x*ArcTanh[Sin[a + b*x]])/b + ((c + d*x)*ArcTanh[Sin[a + b*x]])/b - ((c + d
*x)*Csc[a + b*x])/b + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*Pol
yLog[2, I*E^(I*(a + b*x))])/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
```

```
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = \frac{(c+dx)\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)\csc(a+bx)}{b} - d\int \Bigg( \frac{\tanh^{-1}(\text{s}$$

$$= \frac{(c+dx)\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)\csc(a+bx)}{b} - \frac{d\int \tanh^{-1}(\text{sin}}{b}$$

$$= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+bx))}{b} + \frac{(c+dx)\tanh^{-1}(}{b}$$

$$= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+bx))}{b} + \frac{(c+dx)\tanh^{-1}(}{b}$$

$$= -\frac{2idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+b}{b}$$

$$= -\frac{2idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+b}{b}$$

$$= -\frac{2idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+b}{b}$$

**Mathematica [C]** time = 2.91, size = 517, normalized size = 3.95

$$\frac{d\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} - \frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} + \frac{d\left(a\cos\left(\frac{1}{2}(a+bx)\right) - (a+bx)\cos\left(\frac{1}{2}(a+bx)\right)\right)\csc\left(\frac{1}{2}(a+b\right.}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] (d*(a*Cos[(a + b*x)/2] - (a + b*x)*Cos[(a + b*x)/2])*Csc[(a + b*x)/2])/(2*b
^2) - (c*Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b -
(d*Log[Cos[(a + b*x)/2]])/b^2 + (d*Log[Sin[(a + b*x)/2]])/b^2 - (d*x*(a*Log
[1 - Tan[(a + b*x)/2]] - a*Log[1 + Tan[(a + b*x)/2]] - I*(Log[1 + I*Tan[(a
+ b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, ((1 + I) -
(1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(1/2 + I
/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/2]
)]) - I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)/2
])] + PolyLog[2, ((1 + I) + (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 + I*Ta
n[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] + PolyLog[2, ((1
- I) + (1 + I)*Tan[(a + b*x)/2])/2])))/(b*(a - I*Log[1 - I*Tan[(a + b*x)/2]
] + I*Log[1 + I*Tan[(a + b*x)/2]])) + (d*Sec[(a + b*x)/2]*(a*Sin[(a + b*x)/
2] - (a + b*x)*Sin[(a + b*x)/2]))/(2*b^2)
```

**fricas [B]** time = 0.50, size = 434, normalized size = 3.31

$$-\frac{2\,bdx + i\,d\mathrm{Li}_2\left(i\,\cos\left(bx+a\right) + \sin\left(bx+a\right)\right)\sin\left(bx+a\right) + i\,d\mathrm{Li}_2\left(i\,\cos\left(bx+a\right) - \sin\left(bx+a\right)\right)\sin\left(bx+a\right) - i\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + I*d*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + I*d
*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - I*d*dilog(-I*cos(b*x +
 a) + sin(b*x + a))*sin(b*x + a) - I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)
)*sin(b*x + a) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x
 + a) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + d
*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - (b*d*x + a*d)*log(I*cos(b*x + a
) + sin(b*x + a) + 1)*sin(b*x + a) + (b*d*x + a*d)*log(I*cos(b*x + a) - sin
(b*x + a) + 1)*sin(b*x + a) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x +
 a) + 1)*sin(b*x + a) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) +
1)*sin(b*x + a) - d*log(-1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - (b*c - a*d)
*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + (b*c - a*d)*log(-co
s(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b*c)/(b^2*sin(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\csc\left(bx + a\right)^2 \sec\left(bx + a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^2*sec(b*x + a), x)
```

**maple [A]** time = 0.13, size = 235, normalized size = 1.79

$$-\frac{2ie^{i(bx+a)}\left(dx+c\right)}{b\left(e^{2i(bx+a)}-1\right)}+\frac{2ida\arctan\left(e^{i(bx+a)}\right)}{b^2}+\frac{d\ln\left(e^{i(bx+a)}-1\right)}{b^2}-\frac{d\ln\left(e^{i(bx+a)}+1\right)}{b^2}-\frac{2ic\arctan\left(e^{i(bx+a)}\right)}{b}-\frac{id\,\mathrm{dilog}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x)
```

```
[Out] -2*I*exp(I*(b*x+a))*(d*x+c)/b/(exp(2*I*(b*x+a))-1)+2*I/b^2*d*a*arctan(exp(I
*(b*x+a)))+d/b^2*ln(exp(I*(b*x+a))-1)-d/b^2*ln(exp(I*(b*x+a))+1)-2*I/b*c*ar
ctan(exp(I*(b*x+a)))-I/b^2*d*dilog(1-I*exp(I*(b*x+a)))+I/b^2*d*dilog(1+I*ex
```

```
p(I*(b*x+a)))-1/b*d*ln(1+I*exp(I*(b*x+a)))*x-1/b^2*d*ln(1+I*exp(I*(b*x+a)))
*a+1/b*d*ln(1-I*exp(I*(b*x+a)))*x+1/b^2*d*ln(1-I*exp(I*(b*x+a)))*a
```

**maxima** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
[Out] Timed out
```

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)*sin(a + b*x)^2),x)
[Out] \text{Hanged}
```

**sympy** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)\csc^2(a + bx)\sec(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a),x)
[Out] Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x), x)
```

## 3.238 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\frac{\csc^2(a+bx)\sec(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

**Rubi** [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x),x]
[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

**Mathematica** [A] time = 11.55, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]
```

**fricas** [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^2*sec(b*x + a)/(d*x + c), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2*sec(b*x + a)/(d*x + c), x)
```

**maple** [**A**] time = 2.63, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2(bx+a)\right)\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
[Out] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

**maxima** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

**mupad** [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)), x)
```

**sympy** [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)/(d*x+c),x)
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

## 3.239 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=25

$$\mathrm{Int}\left(\frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2,x]
[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx=\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

**Mathematica [A]** time = 11.08, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]
```

**fricas [A]** time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\csc(bx+a)^2\sec(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^2*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] sage0*x
```

**maple [A]** time = 3.12, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2(bx+a)\right)\sec(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
[Out] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)/(d*x+c)**2,x)
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

# 3.240 $\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\csc^3(a+bx)\sec(a+bx)(c+dx)^m,x\right)$$

[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)

**Rubi** [A] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x],x]

[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]

Rubi steps

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$$

**Mathematica** [A] time = 18.29, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x],x]

[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]

**fricas** [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \csc(bx+a)^3 \sec(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")

[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)

**maple** [A] time = 0.11, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^3(bx+a)\right)\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)\,\sin(a+b\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^3),x)
```

```
[Out] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^3), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Timed out
```

# 3.241 $\int (c+dx)^3 \csc^3(a+bx) \sec(a+bx)\, dx$

**Optimal**. Leaf size=325

$$-\frac{3id^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right.}{2b^3}$$

```
[Out] -3/2*I*d*(d*x+c)^2/b^2-1/2*(d*x+c)^3/b-2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a))
)/b-3/2*d*(d*x+c)^2*cot(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)^2/b+3*d^2*(d*x+
c)*ln(1-exp(2*I*(b*x+a)))/b^3+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a))
)/b^2-3/2*I*d^3*polylog(2,exp(2*I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2
,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^3+3/2
*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(
b*x+a)))/b^4+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

**Rubi [A]** time = 0.82, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {2620, 14, 4420, 6741, 12, 6742, 3720, 3717, 2190, 2279, 2391, 32, 2551, 4183, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (c + d*x)^3/(2*b) - (2*(c + d*x)^3*ArcTanh
[E^((2*I)*(a + b*x))])/b - (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - ((c + d
*x)^3*Cot[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))]
)/b^3 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((
(3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^
2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2
*I)*(a + b*x))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))]
)/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/
4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - (3d)\int (c + \ldots$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - (3d)\int \frac{(c+\ldots}{}$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - \frac{(3d)\int (c+d\ldots}{}$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - \frac{(3d)\int \left(-(c\ldots\right.}{}$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(3d)\int (c+d\ldots}{}$$

$$= -\frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^3 c\ldots}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot\ldots}{2b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+d\ldots}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+d\ldots}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+d\ldots}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+d\ldots}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+d\ldots}{}$$

**Mathematica [B]** time = 6.97, size = 1477, normalized size = 4.54

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -1/2*((c + d*x)^3*Csc[a + b*x]^2)/b - (c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^
((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] +
(3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^(
(2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a +
 b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a +
b*x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(2*b^3) - (d^3*E^
(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[
1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*
(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))
] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a + b*x))] - 2*PolyLog[4, -E^((-I)*(a +
b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a
 + b*x))] - (2*I)*b*x*PolyLog[3, E^((-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)
*(a + b*x))]))/E^((2*I)*a)))/(4*b^4) + (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2
+ d^3*x^3)*Csc[a]*Sec[a])/4 - ((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E
^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLo
g[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)
*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) - (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a
) - ((4*I)*(1 + E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1
```

```
+ E^((2*I)*a))*(2*b^2*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*Pol
yLog[3, -E^((-2*I)*(a + b*x))] - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E
^((2*I)*a)))*Sec[a] - (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[
b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (c^3*Csc[a]*(-(b*x*Cos[a])
 + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)
) + (3*c*d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]
*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*Arc
Tan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*
I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])
)] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I
*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])
/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (3*Csc[a]*Csc[a + b*x]*(c^2
*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) - (3*c^2*d*Cs
c[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]
) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*
(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x
+ ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a]
)/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^
3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan
[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2
*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[
b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Ta
n[a])/Sqrt[1 + Tan[a]^2]))/(2*b^4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

**fricas [C]** time = 0.77, size = 3459, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 3*(b^2*d^3*x
^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) + (3*I*b^2*d^3*x^
2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2
*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*
sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d
^3 + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3)*cos(b*x
+ a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c
*d^2*x + 3*I*b^2*c^2*d + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*
d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2
 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3
*I*b^2*c^2*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-3*I*
b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + (3*I*b^2*d^3*x^2 + 6*I*b^2*
c*d^2*x + 3*I*b^2*c^2*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*x +
a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + (-3*I*b^2*d^3*x^
2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a)
- sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I
*d^3 + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3)*cos(b*
x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^
2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x -
 3*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x +
 a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 - (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x
+ a)^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(cos(b*x + a) + I*sin(b*x + a) + 1) +
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x
+ a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 - (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 + 3*(b^3*c^2*d + b*d^3)*x)*cos
(b*x + a)^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(cos(b*x + a) - I*sin(b*x + a) +
1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^
```

```
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin
(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c
^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2
*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)^2)*log(I*cos(b
*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^
2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)^2
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b^3*d^3*x^3 + 3
*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*c
os(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b
^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2 + a^3*d^3)*cos(b*x + a)^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3
*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3 - (b^3*c^3 - 3
*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2)*log(-
1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^3*c^3 - 3*a*b^2*c^2*d + 3
*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 +
1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I
*sin(b*x + a) + 1/2) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2 + (a^3 + 3*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a
)^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x +
 a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
 (a^3 + 3*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3
*c^2*d + b*d^3)*x)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + (
6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*polylog(4, cos(b*x + a) + I*sin(b*x + a))
 + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*polylog(4, cos(b*x + a) - I*sin(b*x
+ a)) + (6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*polylog(4, I*cos(b*x + a) + sin(
b*x + a)) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*polylog(4, I*cos(b*x + a) -
 sin(b*x + a)) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*polylog(4, -I*cos(b*x
+ a) + sin(b*x + a)) + (6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*polylog(4, -I*cos
(b*x + a) - sin(b*x + a)) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*polylog(4,
-cos(b*x + a) + I*sin(b*x + a)) + (6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*polylo
g(4, -cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*
c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3
*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a)
- I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)
^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d
^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) +
 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -I*c
os(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*co
s(b*x + a)^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c
*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) + I*sin
(b*x + a)) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*pol
ylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^4*cos(b*x + a)^2 - b^4)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \csc(bx+a)^3 \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a), x)
```

**maple [B]** time = 0.19, size = 1223, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x)
```

```
[Out] -6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,-exp(I*(
b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-1/b*c^3*ln(1+exp(2*I*(b*x+
a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a
)))+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)
))*x+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6*I/b^4*d^3*polylog(4,-exp(I*(b
*x+a)))-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-3/2/b^3*c*d^2*polylog(3,-exp(
2*I*(b*x+a)))-3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-3/4*I*d^3*polylog(
4,-exp(2*I*(b*x+a)))/b^4+1/b*c^3*ln(exp(I*(b*x+a))-1)+1/b*c^3*ln(exp(I*(b*x
+a))+1)+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+(2*b*d^3*x^3*exp(2*I*(
b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d
^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a
))+3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*x+3*I*c^2*d)/b^2/(exp(2*I
*(b*x+a))-1)^2+3/b^3*d^2*c*ln(exp(I*(b*x+a))-1)+3/b^3*d^2*c*ln(exp(I*(b*x+a
))+1)-6/b^3*d^2*c*ln(exp(I*(b*x+a)))+3/b^3*d^3*ln(exp(I*(b*x+a))+1)*x+3/b^3
*d^3*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a-3/b^4*d^3*a*ln
(exp(I*(b*x+a))-1)+6/b^4*d^3*a*ln(exp(I*(b*x+a)))-3*I/b^2*d^3*x^2-3*I/b^4*d
^3*a^2-3*I/b^4*d^3*polylog(2,-exp(I*(b*x+a)))+3/b^3*c*d^2*a^2*ln(exp(I*(b*x
+a))-1)-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-ex
p(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog
(2,-exp(I*(b*x+a)))*x^2+3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp
(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-3/b^3*c*d^2*a^2*ln(1-exp(
I*(b*x+a)))+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+3/b*c*d^2*ln(exp(I*(b*x+a))+
1)*x^2-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+
1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-1/b*d^3
*ln(1+exp(2*I*(b*x+a)))*x^3+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2-
6*I/b^3*d^3*a*x+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-3/b*c^2*d*ln(1
+exp(2*I*(b*x+a)))*x-3/b*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2
```

**maxima [B]** time = 2.14, size = 5140, normalized size = 15.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)
) - 3*a*c^2*d*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a
)^2))/b + 3*a^2*c*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin
(b*x + a)^2))/b^2 - a^3*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - l
og(sin(b*x + a)^2))/b^3 - 2*(18*b^2*c^2*d - 36*a*b*c*d^2 + 18*a^2*d^3 - (8*
(b*x + a)^3*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*(b*x + a) + 2*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*
x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a
) - 4*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (8*I*(b*x + a)^3*d^3
+ (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^
2 + 18*I*a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-16*I*(b*x + a)^3*d^3 + (-
36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2
- 36*I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(
2*b*x + 2*a) + 1) + (6*(b*x + a)^3*d^3 + 18*b*c*d^2 - 18*a*d^3 + 18*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x
```

```
+ a) + 6*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
+ a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x +
 4*a) - 12*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*
x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(2*b*x
 + 2*a) - (-6*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-18*I*b*c*d^
2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^
2 - 18*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (12*I*(b*x + a)^3*d^3 + 36*I*b
*c*d^2 - 36*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c
^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 36*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))
*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (18*b*c*d^2 - 18*a*d^3 + 18*(b*c
*d^2 - a*d^3)*cos(4*b*x + 4*a) - 36*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) - (-
18*I*b*c*d^2 + 18*I*a*d^3)*sin(4*b*x + 4*a) - (36*I*b*c*d^2 - 36*I*a*d^3)*s
in(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - (6*(b*x + a)^3*d
^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2
+ 1)*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2
+ 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) -
 12*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a
*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^
3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*
d^2 + (18*I*a^2 + 18*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-12*I*(b*x + a)
^3*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I
*a*b*c*d^2 + (-36*I*a^2 - 36*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(s
in(b*x + a), -cos(b*x + a) + 1) - 18*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)
*(b*x + a))*cos(4*b*x + 4*a) - (12*I*(b*x + a)^3*d^3 + 18*b^2*c^2*d - 36*a*
b*c*d^2 + 18*a^2*d^3 + (36*I*b*c*d^2 - 18*(2*I*a + 1)*d^3)*(b*x + a)^2 + (3
6*I*b^2*c^2*d - 36*(2*I*a + 1)*b*c*d^2 + (36*I*a^2 + 36*a)*d^3)*(b*x + a))*
cos(2*b*x + 2*a) + (9*b^2*c^2*d - 18*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 9*a^2
*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a) + 3*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b
*x + a)^2*d^3 + 3*a^2*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a)
 - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*a^2*d^3 + 6*(b*c*d^
2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 -
 12*I*(b*x + a)^2*d^3 - 9*I*a^2*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a
))*sin(4*b*x + 4*a) - (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 24*I*(b*x + a)^2*d
^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a)
)*dilog(-e^(2*I*b*x + 2*I*a)) - (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)
^2*d^3 + 18*(a^2 + 1)*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
+ a))*cos(4*b*x + 4*a) - 36*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a
^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (18*I*b^2*c
^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 18*I)*d^3 + (36*
I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-36*I*b^2*c^2*d + 72
*I*a*b*c*d^2 - 36*I*(b*x + a)^2*d^3 + (-36*I*a^2 - 36*I)*d^3 + (-72*I*b*c*d
^2 + 72*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - (18
*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 18*(a^2 + 1)*d^3 + 36*(b*c
*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (
a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 36*(b^2*c^
2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(
b*x + a))*cos(2*b*x + 2*a) + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x +
 a)^2*d^3 + (18*I*a^2 + 18*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*
sin(4*b*x + 4*a) + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 - 36*I*(b*x + a)^2*d^3
 + (-36*I*a^2 - 36*I)*d^3 + (-72*I*b*c*d^2 + 72*I*a*d^3)*(b*x + a))*sin(2*b
*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-4*I*(b*x + a)^3*d^3 + (-9*I*b*c*d^2 +
 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 - 9*I*a^2*d^3)*(
b*x + a) + (-4*I*(b*x + a)^3*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 +
 (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 - 9*I*a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a
) + (8*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*
b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (4
*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(4*(b*x + a)^3*d^3 + 9*(b*c
```

```
*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)
)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b
*x + 2*a) + 1) - (3*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b*c*
d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 +
 9*I)*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*
I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I
*a^2 + 9*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-6*I*(b*x + a)^3*d^3 - 18*I
*b*c*d^2 + 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b
^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 18*I)*d^3)*(b*x + a))*cos(2*b*x +
2*a) - 3*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
+ a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*sin(4*b*x +
 4*a) + 6*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
 + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*sin(2*b*x
+ 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (3*I*(b
*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x +
a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 9*I)*d^3)*(b*x + a) + (
3*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(
b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 9*I)*d^3)*(b*x +
a))*cos(4*b*x + 4*a) + (-6*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 +
(-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^
2 + (-18*I*a^2 - 18*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^
3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d -
2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 6*((b*x + a)^3*d
^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d -
 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (6*d^3*cos(4*b*x + 4*a) - 12*d^
3*cos(2*b*x + 2*a) + 6*I*d^3*sin(4*b*x + 4*a) - 12*I*d^3*sin(2*b*x + 2*a) +
 6*d^3)*polylog(4, -e^(2*I*b*x + 2*I*a)) + (36*d^3*cos(4*b*x + 4*a) - 72*d^
3*cos(2*b*x + 2*a) + 36*I*d^3*sin(4*b*x + 4*a) - 72*I*d^3*sin(2*b*x + 2*a)
+ 36*d^3)*polylog(4, -e^(I*b*x + I*a)) + (36*d^3*cos(4*b*x + 4*a) - 72*d^3*
cos(2*b*x + 2*a) + 36*I*d^3*sin(4*b*x + 4*a) - 72*I*d^3*sin(2*b*x + 2*a) +
36*d^3)*polylog(4, e^(I*b*x + I*a)) - (-9*I*b*c*d^2 - 12*I*(b*x + a)*d^3 +
9*I*a*d^3 + (-9*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 9*I*a*d^3)*cos(4*b*x + 4*a
) + (18*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 18*I*a*d^3)*cos(2*b*x + 2*a) + 3*(
3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*sin(4*b*x + 4*a) - 6*(3*b*c*d^2 + 4*
(b*x + a)*d^3 - 3*a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(2*I*b*x + 2*I*a))
 - (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I*(
b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + (-72*I*b*c*d^2 - 72*I*(b*x +
a)*d^3 + 72*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3
)*sin(4*b*x + 4*a) + 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))
*polylog(3, -e^(I*b*x + I*a)) - (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a
*d^3 + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) +
(-72*I*b*c*d^2 - 72*I*(b*x + a)*d^3 + 72*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*
c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 72*(b*c*d^2 + (b*x + a)*d
^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) - (18*I*(b*x + a)
^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (12*(b*x
 + a)^3*d^3 - 18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3 + (36*b*c*d^2
- (36*a - 18*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a - 36*I)*b*c*d^2 +
36*(a^2 - I*a)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(-6*I*b^3*cos(4*b*x + 4*a)
 + 12*I*b^3*cos(2*b*x + 2*a) + 6*b^3*sin(4*b*x + 4*a) - 12*b^3*sin(2*b*x +
2*a) - 6*I*b^3))/b
```

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Timed out
```

# 3.242 $\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx$

**Optimal**. Leaf size=201

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\log(\sin(a+bx))}{b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{d}{}$$

```
[Out] -c*d*x/b-1/2*d^2*x^2/b-2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d*(d*x+c)*co
t(b*x+a)/b^2-1/2*(d*x+c)^2*cot(b*x+a)^2/b+d^2*ln(sin(b*x+a))/b^3+I*d*(d*x+c
)*polylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/
b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/2*d^2*polylog(3,exp(2*I*(b*x
+a)))/b^3
```

**Rubi [A]** time = 0.44, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {2620, 14, 4420, 6741, 12, 6742, 3720, 3475, 2551, 4183, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{PolyLog}\left(3,e^{2i}\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -((c*d*x)/b) - (d^2*x^2)/(2*b) - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))
])/b - (d*(c + d*x)*Cot[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]^2)/(2*b)
+ (d^2*Log[Sin[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*
x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*Poly
Log[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E^((2*I)*(a + b*x))
])/(2*b^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} - (2d)\int (c+dx)$$

$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} - (2d)\int \frac{(c+dx}{}$$

$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} - \frac{d\int (c+dx)\left(-\right.}{}$$

$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} - \frac{d\int \left(-(c+dx)\right.}{}$$

$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{d\int (c+dx)\cot}{b}$$

$$= -\frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^2\csc(2a}{b}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{d^2\log}{}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cot(a+bx)}{b^2} -$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cot(a+bx)}{b^2} -$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cot(a+bx)}{b^2} -$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cot(a+bx)}{b^2} -$$

**Mathematica [B]** time = 6.76, size = 872, normalized size = 4.34

$$-\frac{\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))c^2}{b\left(\cos^2(a)+\sin^2(a)\right)} + \frac{\csc(a)(\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\,\mathrm{s}}{b\left(\cos^2(a)+\sin^2(a)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -1/2*((c + d*x)^2*Csc[a + b*x]^2)/b - (d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((
2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3
*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2
*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b
*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*
x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(6*b^3) + (x*(3*c^2
 + 3*c*d*x + d^2*x^2)*Csc[a]*Sec[a])/3 - ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3
*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a)
)*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3,
-E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) - (c^2*Sec[a]*(Cos[a]*Log[Cos
[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) +
(c^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a])
)/(b*(Cos[a]^2 + Sin[a]^2)) + (d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin
[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*(
(b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) -
Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x
 - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - Ar
```

```
cTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 +
Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Csc[a]*Csc
[a + b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x]))/b^2 - (c*d*Csc[a]*Sec[a]*(b^2*E^
(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((
-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a
]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]
+ I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2
]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

**fricas** [C] time = 0.64, size = 1987, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)
*sin(b*x + a) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b
*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d
 + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x
 + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)
^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d + (2*I
*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a))
+ (-2*I*b*d^2*x - 2*I*b*c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dil
og(-I*cos(b*x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2
*x - 2*I*b*c*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (-2
*I*b*d^2*x - 2*I*b*c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(-c
os(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x -
2*I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + d^2)*
cos(b*x + a)^2 + d^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(co
s(b*x + a) + I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + d^2)*cos(b*x + a)^2 + d^2)*log(cos(b*
x + a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 -
2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I)
 + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*
c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(I*cos(b*x + a) + sin(b*x +
 a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2
+ 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(I*cos(b*x + a) - s
in(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*
d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-I*cos(b*x
 + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^
2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-
I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2 -
 (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2 - (b^2
*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1
/2*I*sin(b*x + a) + 1/2) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2
 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-c
os(b*x + a) + I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a
) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 +
2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*si
n(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(d^2*co
s(b*x + a)^2 - d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*(d^2*cos(
b*x + a)^2 - d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 2*(d^2*cos(b*
x + a)^2 - d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*(d^2*cos(b*x
+ a)^2 - d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*(d^2*cos(b*x +
```

```
a)^2 - d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*(d^2*cos(b*x + a
)^2 - d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*(d^2*cos(b*x + a)
^2 - d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^
2 - d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^3*cos(b*x + a)^2 -
b^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc(bx+a)^3 \sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a), x)
```

**maple [B]** time = 0.16, size = 632, normalized size = 3.14

$$\frac{d^2a^2\ln\left(e^{i(bx+a)}-1\right)}{b^3}+\frac{d^2\ln\left(1-e^{i(bx+a)}\right)x^2}{b}-\frac{d^2\ln\left(1-e^{i(bx+a)}\right)a^2}{b^3}+\frac{d^2\ln\left(e^{i(bx+a)}+1\right)x^2}{b}-\frac{c^2\ln\left(1+e^{2i(bx+a)}\right)}{b}-\frac{d^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x)
```

```
[Out] -1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1
)+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d
^2*ln(exp(I*(b*x+a))+1)*x^2+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*poly
log(3,exp(I*(b*x+a)))/b^3-1/b*c^2*ln(1+exp(2*I*(b*x+a)))+1/b*c^2*ln(exp(I*(
b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1)-1/b*d^2*ln(1+exp(2*I*(b*x+a)))*x^2+
I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+
a)))-2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*
c*d*ln(1-exp(I*(b*x+a)))*a+2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*a*ln(ex
p(I*(b*x+a))-1)-2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*d^2*polylo
g(2,exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-2*I/b^2*c*d*po
lylog(2,exp(I*(b*x+a)))+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*
x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+a)
)+I*d^2*x+I*d*c)/b^2/(exp(2*I*(b*x+a))-1)^2+1/b^3*d^2*ln(exp(I*(b*x+a))+1)-
2/b^3*d^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(b*x+a))-1)
```

**maxima [B]** time = 0.82, size = 2522, normalized size = 12.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)
) - 2*a*c*d*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^
2))/b + a^2*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2))/b^2 + 2*(4*(b*x + a)*d^2*cos(4*b*x + 4*a) + 4*I*(b*x + a)*d^2*sin(4
*b*x + 4*a) - 4*b*c*d + 4*a*d^2 + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b
*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a
) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (2
*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) +
(-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a
))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (2*(b*x + a)^2*d^2 + 4
*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)
*(b*x + a) + d^2)*cos(4*b*x + 4*a) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)
*(b*x + a) + d^2)*cos(2*b*x + 2*a) - (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d +
```

```
4*I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(4*b*x + 4*a) - (4*I*(b*x + a)^2*d^2 + (
8*I*b*c*d - 8*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b
*x + a), cos(b*x + a) + 1) - (2*d^2*cos(4*b*x + 4*a) - 4*d^2*cos(2*b*x + 2*
a) + 2*I*d^2*sin(4*b*x + 4*a) - 4*I*d^2*sin(2*b*x + 2*a) + 2*d^2)*arctan2(s
in(b*x + a), cos(b*x + a) - 1) + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*
x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a)
 - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (2*
I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) + (
-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a)
)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (4*I*(b*x + a)^2*d^2 + 4*b*c*d
 - 4*a*d^2 + (8*I*b*c*d - 4*(2*I*a + 1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) -
(2*b*c*d + 2*(b*x + a)*d^2 - 2*a*d^2 + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*co
s(4*b*x + 4*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-2*I
*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*sin(4*b*x + 4*a) - (4*I*b*c*d + 4*I
*(b*x + a)*d^2 - 4*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) +
 (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*c
os(4*b*x + 4*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (4*I
*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (-8*I*b*c*d - 8*
I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (4
*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(
4*b*x + 4*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (4*I*b*
c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (-8*I*b*c*d - 8*I*(
b*x + a)*d^2 + 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b
*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + (-I*(b*x + a)^2*d^2 +
(-2*I*b*c*d + 2*I*a*d^2)*(b*x + a))*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2
 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 +
 2*(b*c*d - a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*
c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*
x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*
I*a*d^2)*(b*x + a) + I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(
b*x + a) + I*d^2)*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d +
4*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*
c*d - a*d^2)*(b*x + a) + d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*
c*d - a*d^2)*(b*x + a) + d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^
2)*(b*x + a) + I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x +
a) + I*d^2)*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*
d^2)*(b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d -
a*d^2)*(b*x + a) + d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d -
a*d^2)*(b*x + a) + d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)
^2 - 2*cos(b*x + a) + 1) + (-I*d^2*cos(4*b*x + 4*a) + 2*I*d^2*cos(2*b*x + 2
*a) + d^2*sin(4*b*x + 4*a) - 2*d^2*sin(2*b*x + 2*a) - I*d^2)*polylog(3, -e^
(2*I*b*x + 2*I*a)) + (4*I*d^2*cos(4*b*x + 4*a) - 8*I*d^2*cos(2*b*x + 2*a) -
 4*d^2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, -e^(
I*b*x + I*a)) + (4*I*d^2*cos(4*b*x + 4*a) - 8*I*d^2*cos(2*b*x + 2*a) - 4*d^
2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, e^(I*b*x
+ I*a)) - (4*(b*x + a)^2*d^2 - 4*I*b*c*d + 4*I*a*d^2 + (8*b*c*d - (8*a - 4*
I)*d^2)*(b*x + a))*sin(2*b*x + 2*a))/(-2*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*c
os(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2
))/b
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)**3*sec(a + b*x), x)
```

## 3.243 $\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx$

**Optimal**. Leaf size=141

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}-\frac{2dx\tanh}{}$$

```
[Out] -1/2*d*x/b-2*d*x*arctanh(exp(2*I*(b*x+a)))/b-1/2*d*cot(b*x+a)/b^2-1/2*(d*x+
c)*cot(b*x+a)^2/b-d*x*ln(tan(b*x+a))/b+(d*x+c)*ln(tan(b*x+a))/b+1/2*I*d*pol
ylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2
```

**Rubi [A]** time = 0.14, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2620, 14, 4420, 3473, 8, 2548, 12, 4183, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -(d*x)/(2*b) - (2*d*x*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*Cot[a + b*x])/(2
*b^2) - ((c + d*x)*Cot[a + b*x]^2)/(2*b) - (d*x*Log[Tan[a + b*x]])/b + ((c
+ d*x)*Log[Tan[a + b*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^
2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rubi steps

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx = -\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}-d\int\left(-\frac{\cot^2(a+bx)}{2b}\right.$$
$$= -\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}+\frac{d\int\cot^2(a+bx)\,d}{2b}$$
$$= -\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx}{}$$
$$= -\frac{dx}{2b}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c}{}$$
$$= -\frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}-$$
$$= -\frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}-$$
$$= -\frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}-$$

**Mathematica [A]** time = 0.90, size = 210, normalized size = 1.49

$$\frac{d\left(\frac{1}{2}i\text{Li}_2\left(-e^{2i(a+bx)}\right)+\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)\right)}{b^2}+\frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2+\text{L}\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -1/2*(d*Cot[a + b*x])/b^2 - (d*x*Csc[a + b*x]^2)/(2*b) + (a*d*Log[Cos[a + b
*x]])/b^2 - (c*(Csc[a + b*x]^2 + 2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]])
)/(2*b) - (a*d*Log[Sin[a + b*x]])/b^2 + (d*((I/2)*(a + b*x)^2 - (a + b*x)*L
```

```
og[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I)*(a + b*x))]))/b^2
+ (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((a + b*x)^2 + PolyLog
[2, E^((2*I)*(a + b*x))])))/b^2
```

**fricas [B]** time = 0.54, size = 942, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x + d*cos(b*x + a)*sin(b*x + a) + b*c + (-I*d*cos(b*x + a)^2 + I*d
)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(c
os(b*x + a) - I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(I*cos(b*x
 + a) + sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(I*cos(b*x + a) - s
in(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-I*cos(b*x + a) + sin(b*x +
 a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) +
(I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-I*d*co
s(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b*d*x - (b*d*x
 + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - ((b*
c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(cos(b*x + a) + I*sin(b*x + a) + I)
 - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) - I*sin(b*
x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(cos(b*x + a) - I
*sin(b*x + a) + I) + (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(I*cos
(b*x + a) + sin(b*x + a) + 1) + (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b*d*x - (b*d*x + a*d)*cos(b*x +
 a)^2 + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x - (b*d*x + a*
d)*cos(b*x + a)^2 + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + ((b*c -
a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
 + 1/2) + ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2*cos(b*x + a) -
1/2*I*sin(b*x + a) + 1/2) - (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*lo
g(-cos(b*x + a) + I*sin(b*x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c +
 a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b*d*x - (b*d*x + a*d)*cos(
b*x + a)^2 + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - ((b*c - a*d)*co
s(b*x + a)^2 - b*c + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/(b^2*cos
(b*x + a)^2 - b^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\csc(bx+a)^3\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a), x)
```

**maple [B]** time = 0.12, size = 270, normalized size = 1.91

$$\frac{2bdx\,e^{2i(bx+a)}+2bc\,e^{2i(bx+a)}-id\,e^{2i(bx+a)}+id}{b^2\left(e^{2i(bx+a)}-1\right)^2}+\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}-\frac{c\ln\left(1+e^{2i(bx+a)}\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}-d\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x)
```

```
[Out] (2*b*d*x*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+I*d)/
b^2/(exp(2*I*(b*x+a))-1)^2+1/b*c*ln(exp(I*(b*x+a))-1)-1/b*c*ln(1+exp(2*I*(b
*x+a)))+1/b*c*ln(exp(I*(b*x+a))+1)-1/b*d*ln(1+exp(2*I*(b*x+a)))*x+1/2*I*d*p
olylog(2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,
```

```
-exp(I*(b*x+a)))/b^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a
)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

**maxima [B]** time = 0.62, size = 1035, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*cos
(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*a) + (-4*I*b*d*x - 4*I*
b*c)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (2
*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*cos(2*b
*x + 2*a) - (-2*I*b*d*x - 2*I*b*c)*sin(4*b*x + 4*a) - (4*I*b*d*x + 4*I*b*c)
*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos(4*b
*x + 4*a) - 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin(4*b*x + 4*a) - 4*I*b*c*sin
(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*c
os(4*b*x + 4*a) - 4*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*sin(4*b*x + 4*a) - 4
*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) +
1) + (4*I*b*d*x + 4*I*b*c + 2*d)*cos(2*b*x + 2*a) - (d*cos(4*b*x + 4*a) - 2
*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*di
log(-e^(2*I*b*x + 2*I*a)) + (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) +
2*I*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*
a)) + (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a)
 - 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c
 + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*cos(2*b*x +
2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*l
og(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*b
*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*
cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b
*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*
cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (4*b
*d*x + 4*b*c - 2*I*d)*sin(2*b*x + 2*a) - 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) +
4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a)
- 2*I*b^2)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)**3*sec(a + b*x), x)
```

## 3.244 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\frac{\csc^3(a+bx)\sec(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

**Rubi** [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x),x]
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

**Mathematica** [A] time = 14.07, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]
```

**fricas** [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^3\sec(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)/(d*x + c), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^3\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)/(d*x + c), x)
```

**maple** [A] time = 3.38, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^3(bx+a)\right)\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
[Out] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)/(d*x+c),x)
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)/(c + d*x), x)
```

## 3.245 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2,x]
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

**Mathematica [A]** time = 16.23, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]
```

**fricas [A]** time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^3\sec(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] Timed out
```

**maple** [A]   time = 5.59, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{3}\left(bx+a\right)\right)\sec\left(bx+a\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
[Out] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
```

**maxima** [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A]   time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^3\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2), x)
```

**sympy** [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}\left(a+bx\right)\sec\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)/(d*x+c)**2,x)
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)/(c + d*x)**2, x)
```

# 3.246 $\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\tan(a+bx)\sec(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

**Rubi [A]** time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$$

**Mathematica [A]** time = 2.70, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]
```

**fricas [A]** time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \sec(bx+a)\tan(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\tan(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

**maple [A]** time = 0.08, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\tan(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \sec(bx + a)\tan(bx + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a + b\,x)\,(c + d\,x)^m}{\cos(a + b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x),x)
```

```
[Out] int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \tan(a + bx)\sec(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*tan(a + b*x)*sec(a + b*x), x)
```

# 3.247 $\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=227

$$\frac{24id^4\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^5}-\frac{24id^4\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^5}+\frac{24d^3(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{12id^2(c+\cdots}{\cdots}$$

```
[Out] 8*I*d*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b^2-12*I*d^2*(d*x+c)^2*polylog(2,-I*
exp(I*(b*x+a)))/b^3+12*I*d^2*(d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^3+24*d
^3*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^4-24*d^3*(d*x+c)*polylog(3,I*exp(
I*(b*x+a)))/b^4+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5-24*I*d^4*polylog(
4,I*exp(I*(b*x+a)))/b^5+(d*x+c)^4*sec(b*x+a)/b
```

**Rubi [A]** time = 0.19, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4409, 4181, 2531, 6609, 2282, 6589}

$$\frac{24d^3(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4}-\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((8*I)*d*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b^2 - ((12*I)*d^2*(c + d*x)^2
*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((12*I)*d^2*(c + d*x)^2*PolyLog[2,
 I*E^(I*(a + b*x))])/b^3 + (24*d^3*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x)
)])/b^4 - (24*d^3*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + ((24*I)*d^
4*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^5 - ((24*I)*d^4*PolyLog[4, I*E^(I*(a
+ b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
```

```
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4\sec(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^4\sec(a+bx)}{b} - \frac{(4d)\int (c+dx)^3\sec(a+bx)\,dx}{b}$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^4\sec(a+bx)}{b} + \frac{\left(12d^2\right)\int (c+dx)$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c+$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c+$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c+$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c+$$

**Mathematica [A]** time = 1.17, size = 428, normalized size = 1.89

$$\frac{(c+dx)^4\sec(a+bx)}{b} - \frac{4d\left(-2ib^3c^3\tan^{-1}\left(e^{i(a+bx)}\right) + 3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right) - 3b^3c^2dx\log\left(1+ie^{i(a+bx)}\right) + 3b^3c$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (-4*d*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - I*E^(
I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*d^3*x^3*Lo
g[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 3*b^3
*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x
))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] - (3*I)*b^2*
d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, (-I)*E^(
I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*c*d^2*Poly
Log[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] - (6*I)
*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*
x))]))/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

**fricas [C]** time = 0.57, size = 1186, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^
4 - 12*I*d^4*cos(b*x + a)*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 12*I*
d^4*cos(b*x + a)*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 12*I*d^4*cos(b
*x + a)*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 12*I*d^4*cos(b*x + a)*
polylog(4, -I*cos(b*x + a) - sin(b*x + a)) + (6*I*b^2*d^4*x^2 + 12*I*b^2*c*
d^3*x + 6*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a))
+ (6*I*b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*cos(b*x + a)*dilog
(I*cos(b*x + a) - sin(b*x + a)) + (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*x - 6*
I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-6*I*b
^2*d^4*x^2 - 12*I*b^2*c*d^3*x - 6*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(
b*x + a) - sin(b*x + a)) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b^3*c^3*
d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(cos(b*x + a
) - I*sin(b*x + a) + I) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*
x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*
x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(b*x +
 a) - sin(b*x + a) + 1) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*
x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2
*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*cos(b*x
 + a) - sin(b*x + a) + 1) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3
- a^3*d^4)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b^3*c^
3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(-cos(b*x
+ a) - I*sin(b*x + a) + I) + 12*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polylog(3,
 I*cos(b*x + a) + sin(b*x + a)) - 12*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polyl
og(3, I*cos(b*x + a) - sin(b*x + a)) + 12*(b*d^4*x + b*c*d^3)*cos(b*x + a)*
polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*(b*d^4*x + b*c*d^3)*cos(b*x
 + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)))/(b^5*cos(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \sec(bx+a)\tan(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)*tan(b*x + a), x)
```

**maple [B]** time = 0.20, size = 767, normalized size = 3.38

$$\frac{24id^3c\,a^2\arctan\left(e^{i(bx+a)}\right)}{b^4}-\frac{24id^2c^2a\arctan\left(e^{i(bx+a)}\right)}{b^3}-\frac{24id^3c\,\mathrm{polylog}\left(2,-ie^{i(bx+a)}\right)x}{b^3}+\frac{24id^3c\,\mathrm{polylog}\left(2,ie^{i(\right.}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] 24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5-24*I*d^4*polylog(4,I*exp(I*(b*x+a
)))/b^5+12/b^2*d^3*c*ln(1+I*exp(I*(b*x+a)))*x^2-12/b^2*d^3*c*ln(1-I*exp(I*(
b*x+a)))*x^2+12/b^4*d^3*a^2*c*ln(1-I*exp(I*(b*x+a)))-12/b^2*d^2*c^2*ln(1-I*
exp(I*(b*x+a)))*x-12/b^3*d^2*c^2*ln(1-I*exp(I*(b*x+a)))*a-12/b^4*d^3*a^2*c*
ln(1+I*exp(I*(b*x+a)))+12/b^2*d^2*c^2*ln(1+I*exp(I*(b*x+a)))*x+12/b^3*d^2*c
^2*ln(1+I*exp(I*(b*x+a)))*a-12*I/b^3*d^4*polylog(2,-I*exp(I*(b*x+a)))*x^2+1
2*I/b^3*d^4*polylog(2,I*exp(I*(b*x+a)))*x^2+12*I/b^3*d^2*c^2*polylog(2,I*ex
```

```
p(I*(b*x+a)))-8*I/b^5*d^4*a^3*arctan(exp(I*(b*x+a)))-12*I/b^3*d^2*c^2*polyl
og(2,-I*exp(I*(b*x+a)))+8*I/b^2*d*c^3*arctan(exp(I*(b*x+a)))-24*I/b^3*d^3*c
*polylog(2,-I*exp(I*(b*x+a)))*x+24*I/b^3*d^3*c*polylog(2,I*exp(I*(b*x+a)))*
x+24*I/b^4*d^3*c*a^2*arctan(exp(I*(b*x+a)))-24*I/b^3*d^2*c^2*a*arctan(exp(I
*(b*x+a)))+2*exp(I*(b*x+a))*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^
4)/b/(1+exp(2*I*(b*x+a)))+24/b^4*d^4*polylog(3,-I*exp(I*(b*x+a)))*x-24/b^4*
d^4*polylog(3,I*exp(I*(b*x+a)))*x+4/b^5*d^4*a^3*ln(1+I*exp(I*(b*x+a)))+24/b
^4*d^3*c*polylog(3,-I*exp(I*(b*x+a)))-24/b^4*d^3*c*polylog(3,I*exp(I*(b*x+a
)))+4/b^2*d^4*ln(1+I*exp(I*(b*x+a)))*x^3-4/b^2*d^4*ln(1-I*exp(I*(b*x+a)))*x
^3-4/b^5*d^4*a^3*ln(1-I*exp(I*(b*x+a)))
```

**maxima [B]** time = 0.79, size = 2944, normalized size = 12.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] (2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2*a
)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2
*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
 + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))*c^3
*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) -
 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2*a
)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2
*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
 + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))*a*c
^2*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*
b^2) + 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x
 + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)
^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos
(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1
))*a^2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
 + 1)*b^3) - 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin
(2*b*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x +
a) + 1))*a^3*d^4/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1)*b^4) + c^4/cos(b*x + a) - 4*a*c^3*d/(b*cos(b*x + a)) + 6*a^2*c^2*
d^2/(b^2*cos(b*x + a)) - 4*a^3*c*d^3/(b^3*cos(b*x + a)) + a^4*d^4/(b^4*cos(
b*x + a)) + ((4*(b*x + a)^3*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 4*((b*x + a)^3*d^4 + 3*(b*c*
d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a
))*cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*
(b*x + a)^2 + (-12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4)*(b*x + a)
)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (4*(b*x + a)^
3*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*(b*x + a) + 4*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 +
 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-4*
I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2 + (-12*I*b^2*c
^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan
2(cos(b*x + a), -sin(b*x + a) + 1) - (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 -
8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)
*(b*x + a)^2)*cos(b*x + a) + (12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^
2*d^4 + 12*a^2*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a) + 12*(b^2*c^2*d^2 - 2*a
*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2
```

```
*b*x + 2*a) - (-12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*(b*x + a)^2*d^4 -
12*I*a^2*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*di
log(I*e^(I*b*x + I*a)) - (12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^2*d^
4 + 12*a^2*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a) + 12*(b^2*c^2*d^2 - 2*a*b*c
*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x
 + 2*a) + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*(b*x + a)^2*d^4 + 12*I*
a^2*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I
*e^(I*b*x + I*a)) - (-2*I*(b*x + a)^3*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x
 + a)^2 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^4)*(b*x + a) + (-2
*I*(b*x + a)^3*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a)^2 + (-6*I*b^2*c^2
*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 2*((b*x
+ a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*sin(b*x + a) + 1) - (2*I*(b*x + a)^3*d^4 + (6*I*b*c*d^3 - 6*I*a*d^4)*
(b*x + a)^2 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a) +
(2*I*(b*x + a)^3*d^4 + (6*I*b*c*d^3 - 6*I*a*d^4)*(b*x + a)^2 + (6*I*b^2*c^2
*d^2 - 12*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 2*((b*x
+ a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 - 2*sin(b*x + a) + 1) - 24*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a)
 + d^4)*polylog(4, I*e^(I*b*x + I*a)) + 24*(d^4*cos(2*b*x + 2*a) + I*d^4*si
n(2*b*x + 2*a) + d^4)*polylog(4, -I*e^(I*b*x + I*a)) - (-24*I*b*c*d^3 - 24*
I*(b*x + a)*d^4 + 24*I*a*d^4 + (-24*I*b*c*d^3 - 24*I*(b*x + a)*d^4 + 24*I*a
*d^4)*cos(2*b*x + 2*a) + 24*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2
*a))*polylog(3, I*e^(I*b*x + I*a)) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 2
4*I*a*d^4 + (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 24*I*a*d^4)*cos(2*b*x + 2*
a) - 24*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(3, -I*e
^(I*b*x + I*a)) + 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*
(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2)*sin(b*x + a))/(-I*b^4*co
s(2*b*x + 2*a) + b^4*sin(2*b*x + 2*a) - I*b^4))/b
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(a+b\,x)\,(c+d\,x)^4}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x),x)
[Out] int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \tan(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)*tan(b*x+a),x)
[Out] Integral((c + d*x)**4*tan(a + b*x)*sec(a + b*x), x)
```

# 3.248 $\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=159

$$\frac{6d^3\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}+\frac{6id(c+dx)^2\tan^{-1}}{b^2}$$

```
[Out] 6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*polylog(2,-I*exp
(I*(b*x+a)))/b^3+6*I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3+6*d^3*poly
log(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+(d*x+c)^
3*sec(b*x+a)/b
```

**Rubi [A]** time = 0.13, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4409, 4181, 2531, 2282, 6589}

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{6d^3\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*Po
lyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(
I*(a + b*x))])/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*
PolyLog[3, I*E^(I*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\sec(a+bx)\,dx}{b}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\sec(a+bx)}{b} + \frac{\left(6d^2\right)\int (c+d$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c+$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c+$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c+$$

**Mathematica [A]** time = 0.81, size = 256, normalized size = 1.61

$$\frac{(c+dx)^3\sec(a+bx)}{b} - \frac{3d\left(-2ib^2c^2\tan^{-1}\left(e^{i(a+bx)}\right) + 2b^2cdx\log\left(1-ie^{i(a+bx)}\right) - 2b^2cdx\log\left(1+ie^{i(a+bx)}\right) + b^2d$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (-3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I*
(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 +
I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*(c
+ d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*
E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3
, I*E^(I*(a + b*x))]))/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

**fricas [C]** time = 0.53, size = 779, normalized size = 4.90

$$\frac{2\,b^3d^3x^3 + 6\,b^3cd^2x^2 + 6\,b^3c^2dx + 2\,b^3c^3 + 6\,d^3\cos(bx+a)\,\mathrm{polylog}(3, i\,\cos(bx+a) + \sin(bx+a)) - 6\,d^3\,\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*co
s(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*p
olylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, -I
*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x +
a) - sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(I*cos(b
*x + a) + sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(I*
cos(b*x + a) - sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*di
log(-I*cos(b*x + a) + sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x
+ a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I
) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*lo
g(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a
```

```
*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 3
*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*
cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c
*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 3*(b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*
x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b
*x + a) - I*sin(b*x + a) + I))/(b^4*cos(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sec(bx+a)\tan(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*tan(b*x + a), x)
```

**maple [B]** time = 0.13, size = 463, normalized size = 2.91

$$\frac{2\,e^{i(bx+a)}\left(d^3x^3+3c\,d^2x^2+3c^2dx+c^3\right)}{b\left(1+e^{2i(bx+a)}\right)}-\frac{6d^2c\ln\left(1-ie^{i(bx+a)}\right)a}{b^3}+\frac{3d^3\ln\left(1+ie^{i(bx+a)}\right)x^2}{b^2}+\frac{6id\,c^2\arctan\left(e^{i(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] 2*exp(I*(b*x+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(1+exp(2*I*(b*x+a)))
-6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a+3/b^2*d^3*ln(1+I*exp(I*(b*x+a)))*x^2+
6*I/b^2*d*c^2*arctan(exp(I*(b*x+a)))+6/b^3*d^2*c*ln(1+I*exp(I*(b*x+a)))*a-6
/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)))-3/
b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2-6*I*c*d^2*polylog(2,-I*exp(I*(b*x+a)))/b
^3+3/b^4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/
b^4+6*I/b^4*d^3*a^2*arctan(exp(I*(b*x+a)))-12*I/b^3*d^2*c*a*arctan(exp(I*(b
*x+a)))-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3+6*I*c*d^2*polylog(2,I*ex
p(I*(b*x+a)))/b^3-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+6/b^2*d^2*c*ln(1+I*
exp(I*(b*x+a)))*x+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3
```

**maxima [B]** time = 0.62, size = 1774, normalized size = 11.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(3*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x +
 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))
*c^2*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*
b) - 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x +
 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))
*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1
)*b^2) + 3*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b
*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x +
```

```
a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*c
os(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) +
 1))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
 + 1)*b^3) + 2*c^3/cos(b*x + a) - 6*a*c^2*d/(b*cos(b*x + a)) + 6*a^2*c*d^2/
(b^2*cos(b*x + a)) - 2*a^3*d^3/(b^3*cos(b*x + a)) + 2*((6*(b*x + a)^2*d^3 +
 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*
(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I
*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1
) + (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^
3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3
 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*
x + a), -sin(b*x + a) + 1) - (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2 - 12*I*a*
d^3)*(b*x + a)^2)*cos(b*x + a) + (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3
+ 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 -
12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a))
 - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3
- a*d^3)*cos(2*b*x + 2*a) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3
)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) - (-3*I*(b*x + a)^2*d^3 + (-6
*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 +
 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 -
 a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
2*sin(b*x + a) + 1) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x
 + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b
*x + 2*a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x +
 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (-12*I*d
^3*cos(2*b*x + 2*a) + 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*polylog(3, I*e^(I
*b*x + I*a)) - (12*I*d^3*cos(2*b*x + 2*a) - 12*d^3*sin(2*b*x + 2*a) + 12*I*
d^3)*polylog(3, -I*e^(I*b*x + I*a)) + 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d
^3)*(b*x + a)^2)*sin(b*x + a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x
 + 2*a) - 2*I*b^3))/b
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan (a+b\,x)\,\left(c+d\,x\right)^3}{\cos (a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x),x)
[Out] int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x), x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \tan (a+bx) \sec (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*tan(b*x+a),x)
[Out] Integral((c + d*x)**3*tan(a + b*x)*sec(a + b*x), x)
```

# 3.249 $\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=97

$$-\frac{2id^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}+\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

```
[Out] 4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*I*d^2*polylog(2,-I*exp(I*(b*x+a)
))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b
```

**Rubi** [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4409, 4181, 2279, 2391}

$$-\frac{2id^2\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, (-I
)*E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 + (
(c + d*x)^2*Sec[a + b*x])/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2\sec(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2d)\int (c+dx)\sec(a+bx)\,dx}{b}$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{\left(2d^2\right)\int \log\left(1\right.}{b^2}$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{\left(2id^2\right)\text{Subst}\left(\int\right.}{}$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}$$

**Mathematica [A]** time = 1.66, size = 174, normalized size = 1.79

$$b^2(c+dx)^2\sec(a+bx) - 4bcd\tanh^{-1}\left(\cos(a)\tan\left(\frac{bx}{2}\right)+\sin(a)\right) + \frac{2d^2\csc(a)\left(i\text{Li}_2\left(-e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)-i\text{Li}_2\left(e^{i\left(bx-\tan^{-1}(\cot\right.}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcT
anh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])*(
Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]])
)]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x
- ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + b^2*(c + d*x)^2*Sec[a + b*x])/b^3
```

**fricas [B]** time = 0.50, size = 446, normalized size = 4.60

$$b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + i\,d^2\cos(bx+a)\,\text{Li}_2\left(i\,\cos(bx+a)+\sin(bx+a)\right) + i\,d^2\cos(bx+a)\,\text{Li}_2\left(i\,\cos(bx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x +
 a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a
)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(b
*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b*c*d - a*d^2)*cos(b*x + a
)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log
(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*
cos(b*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos
(b*x + a) - sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b
*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x
 + a) - sin(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a)
+ I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*
sin(b*x + a) + I))/(b^3*cos(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\sec(bx+a)\tan(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*tan(b*x + a), x)
```

**maple [B]** time = 0.02, size = 239, normalized size = 2.46

$$\frac{d^2x^2}{b\cos(bx+a)}+\frac{2d^2\ln\left(1+ie^{i(bx+a)}\right)x}{b^2}+\frac{2d^2\ln\left(1+ie^{i(bx+a)}\right)a}{b^3}-\frac{2d^2\ln\left(1-ie^{i(bx+a)}\right)x}{b^2}-\frac{2d^2\ln\left(1-ie^{i(bx+a)}\right)a}{b^3}+\frac{2id}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] 1/b*d^2/cos(b*x+a)*x^2+2/b^2*d^2*ln(1+I*exp(I*(b*x+a)))*x+2/b^3*d^2*ln(1+I*
exp(I*(b*x+a)))*a-2/b^2*d^2*ln(1-I*exp(I*(b*x+a)))*x-2/b^3*d^2*ln(1-I*exp(I
*(b*x+a)))*a+2*I/b^3*d^2*dilog(1-I*exp(I*(b*x+a)))-2*I/b^3*d^2*dilog(1+I*ex
p(I*(b*x+a)))+2/b^3*a*d^2*ln(sec(b*x+a)+tan(b*x+a))+2/b*c*d/cos(b*x+a)*x-2/
b^2*c*d*ln(sec(b*x+a)+tan(b*x+a))+1/b/cos(b*x+a)*c^2
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(a+b\,x)\,\left(c+d\,x\right)^2}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x),x)
```

```
[Out] int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2\tan(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)*sec(a + b*x), x)
```

## 3.250 $\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=29

$$\frac{(c+dx)\sec(a+bx)}{b}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}$$

```
[Out] -d*arctanh(sin(b*x+a))/b^2+(d*x+c)*sec(b*x+a)/b
```

**Rubi [A]** time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4409, 3770}

$$\frac{(c+dx)\sec(a+bx)}{b}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] -((d*ArcTanh[Sin[a + b*x]])/b^2) + ((c + d*x)*Sec[a + b*x])/b
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx &= \frac{(c+dx)\sec(a+bx)}{b}-\frac{d\int \sec(a+bx)\,dx}{b}\\ &= -\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\sec(a+bx)}{b}\end{aligned}$$

**Mathematica [B]** time = 0.04, size = 93, normalized size = 3.21

$$\frac{d\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{d\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)+\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}+\frac{c\sec(a+bx)}{b}+\frac{dx\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (d*Log[Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2]])/b^2 - (d*Log[Cos[a/2 + (b*
x)/2] + Sin[a/2 + (b*x)/2]])/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[a + b*x])/
b
```

**fricas [B]** time = 0.47, size = 60, normalized size = 2.07

$$\frac{2\,bdx-d\cos\left(bx+a\right)\log\left(\sin\left(bx+a\right)+1\right)+d\cos\left(bx+a\right)\log\left(-\sin\left(bx+a\right)+1\right)+2\,bc}{2\,b^2\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x - d*cos(b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-s
in(b*x + a) + 1) + 2*b*c)/(b^2*cos(b*x + a))
```

**giac [B]** time = 1.31, size = 1537, normalized size = 53.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2*tan(1/2*a)^
2 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*
tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^
2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan
(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^2*tan(1/2*a)^2 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)
^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*
b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a
)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2 + 2*b*d*x*ta
n(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 +
2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^
4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1
/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) +
 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)
^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*
b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*
tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2
*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2 - 4*d*log(2*(tan(1/2*b*x)^4*tan
(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + t
an(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/
2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*
tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + 4*d*log(2*(ta
n(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*
tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b
*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*t
an(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)
 + 2*b*c*tan(1/2*a)^2 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*
x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/
2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*
tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2
*a)^2 + 1))*tan(1/2*a)^2 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2
*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan
(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 +
 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(
1/2*a)^2 + 1))*tan(1/2*a)^2 + 2*b*d*x + 2*b*c + d*log(2*(tan(1/2*b*x)^4*tan
(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + t
an(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/
2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*
tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1)) - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2
- 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x
)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan
(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a)
 + 1)/(tan(1/2*a)^2 + 1)))/(b^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*tan(1/2*b
*x)^2 - 4*b^2*tan(1/2*b*x)*tan(1/2*a) - b^2*tan(1/2*a)^2 + b^2)
```

**maple** [A] time = 0.02, size = 49, normalized size = 1.69

$$\frac{dx}{b\cos(bx+a)} - \frac{d\ln(\sec(bx+a)+\tan(bx+a))}{b^2} + \frac{c}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] 1/b*d/cos(b*x+a)*x-1/b^2*d*ln(sec(b*x+a)+tan(b*x+a))+1/b*c/cos(b*x+a)
```

**maxima** [B] time = 0.45, size = 259, normalized size = 8.93

$$\frac{\frac{\left(4\,(bx+a)\cos(2\,bx+2\,a)\cos(bx+a)+4\,(bx+a)\sin(2\,bx+2\,a)\sin(bx+a)+4\,(bx+a)\cos(bx+a)-\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)\log}{\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*((4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2
*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b
*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b
*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))*d
/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) + 2
*c/cos(b*x + a) - 2*a*d/(b*cos(b*x + a)))/b
```

**mupad** [B] time = 2.58, size = 78, normalized size = 2.69

$$\frac{d\,\ln\left(e^{a\,1i+b\,x\,1i}-i\right)}{b^2} - \frac{d\,\ln\left(e^{a\,1i+b\,x\,1i}+1i\right)}{b^2} + \frac{2\,e^{a\,1i+b\,x\,1i}\,(c+d\,x)}{b\,\left(e^{a\,2i+b\,x\,2i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x))/cos(a + b*x),x)
```

```
[Out] (d*log(exp(a*1i + b*x*1i) - 1i))/b^2 - (d*log(exp(a*1i + b*x*1i) + 1i))/b^2
 + (2*exp(a*1i + b*x*1i)*(c + d*x))/(b*(exp(a*2i + b*x*2i) + 1))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\tan(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)*sec(a + b*x), x)
```

## 3.251 $\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{\tan(a+bx)\sec(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

**Rubi** [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x),x]
[Out] Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$$

**Mathematica** [A] time = 11.05, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x),x]
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

**fricas** [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\tan(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(sec(b*x + a)*tan(b*x + a)/(d*x + c), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\tan(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)*tan(b*x + a)/(d*x + c), x)
```

**maple** [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\tan(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
[Out] int(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan\left(a+b\,x\right)}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)),x)
[Out] int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(a+bx\right)\sec\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
[Out] Integral(tan(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

## 3.252 $\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

**Rubi** [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
[Out] Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

**Mathematica** [A] time = 19.12, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

**fricas** [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\tan(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sec(b*x + a)*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\tan(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sec(b*x + a)*tan(b*x + a)/(d*x + c)^2, x)
```

**maple** [A]   time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)\tan\left(bx+a\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
[Out] int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

**maxima** [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A]   time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan\left(a+b\,x\right)}{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)^2),x)
[Out] int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)^2), x)
```

**sympy** [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan{\left(a+bx\right)}\sec{\left(a+bx\right)}}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)**2,x)
[Out] Integral(tan(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

# 3.253 $\int (c+dx)^m \tan^2(a+bx)\,dx$

**Optimal**. Leaf size=19

$$\mathrm{Int}\left(\tan^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable((d*x+c)^m*tan(b*x+a)^2,x)
```

**Rubi** [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int (c+dx)^m \tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Tan[a + b*x]^2,x]
[Out] Defer[Int][(c + d*x)^m*Tan[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \int (c+dx)^m \tan^2(a+bx)\,dx$$

**Mathematica** [A] time = 2.88, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Tan[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Tan[a + b*x]^2, x]
```

**fricas** [A] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((dx+c)^m \tan\left(bx+a\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*tan(b*x + a)^2, x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \tan\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*tan(b*x + a)^2, x)
```

**maple** [A] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\tan^2\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*tan(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*tan(b*x+a)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \tan(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*tan(b*x + a)^2, x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.05

$$\int \tan(a + b\,x)^2 \,(c + d\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(tan(a + b*x)^2*(c + d*x)^m, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \tan^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*tan(a + b*x)**2, x)
```

# 3.254 $\int (c+dx)^3 \tan^2(a+bx)\,dx$

**Optimal**. Leaf size=128

$$\frac{3d^3\mathrm{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^2(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{i(c+dx)^3}{b}$$

```
[Out] -I*(d*x+c)^3/b-1/4*(d*x+c)^4/d+3*d*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^2-3*I
*d^2*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^3+3/2*d^3*polylog(3,-exp(2*I*(b
*x+a)))/b^4+(d*x+c)^3*tan(b*x+a)/b
```

**Rubi [A]** time = 0.21, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3720, 3719, 2190, 2531, 2282, 6589, 32}

$$-\frac{3id^2(c+dx)\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^3\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^4}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^3\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tan[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) + (3*d*(c + d*x)^2*Log[1 + E^((2*I
)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])
/b^3 + (3*d^3*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^4) + ((c + d*x)^3*Tan[
a + b*x])/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
```

```
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3\tan^2(a+bx)\,dx &= \frac{(c+dx)^3\tan(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\tan(a+bx)\,dx}{b} - \int (c+dx)^3\,dx \\
&= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{(c+dx)^3\tan(a+bx)}{b} + \frac{(6id)\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx}{b} \\
&= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\tan(a+bx}{b} \\
&= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2\left(-e^{2}\right.}{b^3} \\
&= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2\left(-e^{2}\right.}{b^3} \\
&= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2\left(-e^{2}\right.}{b^3}
\end{aligned}$$

**Mathematica [B]** time = 6.55, size = 424, normalized size = 3.31

$$\frac{3c^2d\sec(a)(bx\sin(a)+\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx)))}{b^2\left(\sin^2(a)+\cos^2(a)\right)} + \frac{ie^{-ia}d^3\sec(a)\left(2b^2x^2\left(2bx-3i\left(1+e^{2ia}\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Tan[a + b*x]^2,x]
```

```
[Out] -1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)) + ((I/4)*d^3*(2*b^2*x^
2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1
+ E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a)
)*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^4*E^(I*a)) + (3*c^2*d*Sec[a
]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^2*(Cos[a
]^2 + Sin[a]^2)) + (3*c*d^2*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a
]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - A
rcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]]
 - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*
(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^3*Sqrt[Csc[a]^2*(
```

```
Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(c^3*Sin[b*x] + 3*c^2*d*x*Sin
[b*x] + 3*c*d^2*x^2*Sin[b*x] + d^3*x^3*Sin[b*x]))/b
```

**fricas [C]** time = 0.46, size = 373, normalized size = 2.91

$$-\frac{b^4d^3x^4 + 4\,b^4cd^2x^3 + 6\,b^4c^2dx^2 + 4\,b^4c^3x - 3\,d^3\text{polylog}\left(3, \frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) - 3\,d^3\text{polylog}\left(3, \frac{\tan(bx+a)^2-2}{\tan(bx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*x - 3*d^3
*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) -
 3*d^3*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 +
 1)) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x +
a)^2 + 1) + 1) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(2*(-I*tan(b*x + a) - 1)
/(tan(b*x + a)^2 + 1) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*lo
g(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^3*x^2 + 2*b^2*c*
d^2*x + b^2*c^2*d)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 4*(
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*tan(b*x + a))/b^4
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \tan(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*tan(b*x + a)^2, x)
```

**maple [B]** time = 0.13, size = 348, normalized size = 2.72

$$-\frac{d^3x^4}{4} - c\,d^2x^3 - \frac{3c^2d\,x^2}{2} - c^3x - \frac{6id^2c\,a^2}{b^3} + \frac{3d\,c^2\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)}{b^2} - \frac{6d\,c^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^2} - \frac{6d^3a^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^4} - \frac{6id^2c\,x^2}{b} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*tan(b*x+a)^2,x)
```

```
[Out] -1/4*d^3*x^4-c*d^2*x^3-3/2*c^2*d*x^2-c^3*x-6*I*d^2/b^3*c*a^2+3*d/b^2*c^2*ln
(1+exp(2*I*(b*x+a)))-6*d/b^2*c^2*ln(exp(I*(b*x+a)))-6*d^3/b^4*a^2*ln(exp(I*
(b*x+a)))-6*I*d^2/b*c*x^2+6*I*d^3/b^3*a^2*x+2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*
d*x+c^3)/b/(1+exp(2*I*(b*x+a)))+3*d^3/b^2*ln(1+exp(2*I*(b*x+a)))*x^2+4*I*d^
3/b^4*a^3+3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4+12*d^2/b^3*c*a*ln(exp(I*
(b*x+a)))-3*I*d^3/b^3*polylog(2,-exp(2*I*(b*x+a)))*x-2*I*d^3/b*x^3-3*I*d^2/
b^3*c*polylog(2,-exp(2*I*(b*x+a)))+6*d^2/b^2*c*ln(1+exp(2*I*(b*x+a)))*x-12*
I*d^2/b^2*c*a*x
```

**maxima [B]** time = 0.63, size = 1363, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a - tan(b*x + a))*c^3 - 6*(b*x + a - tan(b*x + a))*a*c^2*d/b
 + 6*(b*x + a - tan(b*x + a))*a^2*c*d^2/b^2 - 2*(b*x + a - tan(b*x + a))*a^
3*d^3/b^3 + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a
```

```
)^2 + 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 +
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*c^2
*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) -
 6*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*
x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)
^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^2/((cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b^2) + 3*((b*
x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*x + a)^
2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*
cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/((cos(2*b*x +
 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b^3) - 2*(I*(b*x + a
)^4*d^3 + (4*I*b*c*d^2 - 4*I*a*d^3)*(b*x + a)^3 + (12*(b*x + a)^2*d^3 + 24*
(b*c*d^2 - a*d^3)*(b*x + a) + 12*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*
x + a))*cos(2*b*x + 2*a) + (12*I*(b*x + a)^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d
^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a)
 + 1) + (I*(b*x + a)^4*d^3 + (4*I*b*c*d^2 - 4*(I*a + 2)*d^3)*(b*x + a)^3 -
24*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(2*b*x + 2*a) - (12*b*c*d^2 + 12*(b*x
+ a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a)
 - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*dilo
g(-e^(2*I*b*x + 2*I*a)) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d
^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x +
 a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*
x + 2*a) + 1) + (-6*I*d^3*cos(2*b*x + 2*a) + 6*d^3*sin(2*b*x + 2*a) - 6*I*d
^3)*polylog(3, -e^(2*I*b*x + 2*I*a)) - ((b*x + a)^4*d^3 + (4*b*c*d^2 - (4*a
 - 8*I)*d^3)*(b*x + a)^3 - (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a)^2)*sin(2*
b*x + 2*a))/(-4*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(2*b*x + 2*a) - 4*I*b^3))
/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(a + b\,x)^2\,(c + d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2*(c + d*x)^3,x)
[Out] int(tan(a + b*x)^2*(c + d*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \tan^2(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*tan(b*x+a)**2,x)
[Out] Integral((c + d*x)**3*tan(a + b*x)**2, x)
```

# 3.255 $\int (c+dx)^2 \tan^2(a+bx)\,dx$

**Optimal**. Leaf size=96

$$-\frac{id^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}$$

```
[Out] -I*(d*x+c)^2/b-1/3*(d*x+c)^3/d+2*d*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^2-I*d^2
*polylog(2,-exp(2*I*(b*x+a)))/b^3+(d*x+c)^2*tan(b*x+a)/b
```

**Rubi** [A] time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3720, 3719, 2190, 2279, 2391, 32}

$$-\frac{id^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Tan[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) + (2*d*(c + d*x)*Log[1 + E^((2*I)*
(a + b*x))])/b^2 - (I*d^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^3 + ((c + d*x
)^2*Tan[a + b*x])/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
```

```
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2 \tan^2(a+bx)\,dx = \frac{(c+dx)^2\tan(a+bx)}{b} - \frac{(2d)\int (c+dx)\tan(a+bx)\,dx}{b} - \int (c+dx)^2\,dx$$
$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{(c+dx)^2\tan(a+bx)}{b} + \frac{(4id)\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx}{b}$$
$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tan(a+bx)}{b}$$
$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tan(a+bx)}{b}$$
$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{id^2\operatorname{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3} + \cdots$$

**Mathematica [B]** time = 6.37, size = 276, normalized size = 2.88

$$\frac{2cd\sec(a)(bx\sin(a)+\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx)))}{b^2\left(\sin^2(a)+\cos^2(a)\right)} + \frac{d^2\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))} - \frac{\cot(a)\cdots}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Tan[a + b*x]^2,x]
```

```
[Out] -1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)) + (2*c*d*Sec[a]*(Cos[a]*Log[Cos[a]*Cos
[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^2*(Cos[a]^2 + Sin[a]^2)) + (d^2*
Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot
[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2
*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[
b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/S
qrt[1 + Cot[a]^2])*Sec[a])/(b^3*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Se
c[a]*Sec[a + b*x]*(c^2*Sin[b*x] + 2*c*d*x*Sin[b*x] + d^2*x^2*Sin[b*x]))/b
```

**fricas [B]** time = 0.47, size = 210, normalized size = 2.19

$$-\frac{2\,b^3d^2x^3 + 6\,b^3cdx^2 + 6\,b^3c^2x - 3i\,d^2\operatorname{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right) + 3i\,d^2\operatorname{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right) - 6\left(bd^2x+bcd\cdots\right.}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x - 3*I*d^2*dilog(2*(I*tan(
b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 3*I*d^2*dilog(2*(-I*tan(b*x + a)
- 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(b*d^2*x + b*c*d)*log(-2*(I*tan(b*x + a)
 - 1)/(tan(b*x + a)^2 + 1)) - 6*(b*d^2*x + b*c*d)*log(-2*(-I*tan(b*x + a) -
 1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*tan(b*x
 + a))/b^3
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \tan (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*tan(b*x + a)^2, x)
```

**maple [B]** time = 0.09, size = 191, normalized size = 1.99

$$-\frac{d^2x^3}{3} - cd\,x^2 - c^2x + \frac{2i\left(d^2x^2+2cdx+c^2\right)}{b\left(1+e^{2i(bx+a)}\right)} + \frac{2dc\ln\left(1+e^{2i(bx+a)}\right)}{b^2} - \frac{4dc\ln\left(e^{i(bx+a)}\right)}{b^2} - \frac{2id^2x^2}{b} - \frac{4id^2ax}{b^2} - \frac{2id^2a^2}{b^3} + \frac{2d^2 \mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*tan(b*x+a)^2,x)
```

```
[Out] -1/3*d^2*x^3-c*d*x^2-c^2*x+2*I*(d^2*x^2+2*c*d*x+c^2)/b/(1+exp(2*I*(b*x+a)))
+2*d/b^2*c*ln(1+exp(2*I*(b*x+a)))-4/b^2*d*c*ln(exp(I*(b*x+a)))-2*I/b*d^2*x^
2-4*I/b^2*d^2*a*x-2*I/b^3*d^2*a^2+2*d^2/b^2*ln(1+exp(2*I*(b*x+a)))*x-I*d^2*
polylog(2,-exp(2*I*(b*x+a)))/b^3+4/b^3*d^2*a*ln(exp(I*(b*x+a)))
```

**maxima [B]** time = 0.60, size = 417, normalized size = 4.34

$$\frac{i\,b^3d^2x^3 + 3i\,b^3cdx^2 + 3i\,b^3c^2x + 6\,b^2c^2 + \left(6\,bd^2x + 6\,bcd + 6\left(bd^2x + bcd\right)\cos\left(2\,bx + 2\,a\right) + \left(6i\,bd^2x + 6i\,bcd\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (I*b^3*d^2*x^3 + 3*I*b^3*c*d*x^2 + 3*I*b^3*c^2*x + 6*b^2*c^2 + (6*b*d^2*x +
 6*b*c*d + 6*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a) + (6*I*b*d^2*x + 6*I*b*c*d)
*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (I*b^3
*d^2*x^3 + (3*I*b^3*c*d - 6*b^2*d^2)*x^2 - 3*(-I*b^3*c^2 + 4*b^2*c*d)*x)*co
s(2*b*x + 2*a) - 3*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*di
log(-e^(2*I*b*x + 2*I*a)) + (-3*I*b*d^2*x - 3*I*b*c*d + (-3*I*b*d^2*x - 3*I
*b*c*d)*cos(2*b*x + 2*a) + 3*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(2*
b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (b^3*d^2*x^3
+ 3*(b^3*c*d + 2*I*b^2*d^2)*x^2 + (3*b^3*c^2 + 12*I*b^2*c*d)*x)*sin(2*b*x +
 2*a))/(-3*I*b^3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) - 3*I*b^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] int(tan(a + b*x)^2*(c + d*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \tan^2 (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)**2, x)
```

# 3.256 $\int (c+dx)\tan^2(a+bx)\,dx$

**Optimal**. Leaf size=40

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}-cx-\frac{dx^2}{2}$$

```
[Out] -c*x-1/2*d*x^2+d*ln(cos(b*x+a))/b^2+(d*x+c)*tan(b*x+a)/b
```

**Rubi [A]** time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3720, 3475}

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}-cx-\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tan[a + b*x]^2,x]
```

```
[Out] -(c*x) - (d*x^2)/2 + (d*Log[Cos[a + b*x]])/b^2 + ((c + d*x)*Tan[a + b*x])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\tan^2(a+bx)\,dx &= \frac{(c+dx)\tan(a+bx)}{b}-\frac{d\int \tan(a+bx)\,dx}{b}-\int (c+dx)\,dx\\ &= -cx-\frac{dx^2}{2}+\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}\end{aligned}$$

**Mathematica [A]** time = 0.28, size = 76, normalized size = 1.90

$$\frac{d\log(\cos(a+bx))}{b^2}-\frac{c\tan^{-1}(\tan(a+bx))}{b}+\frac{c\tan(a+bx)}{b}+\frac{dx\sec(a)\sin(bx)\sec(a+bx)}{b}-\frac{dx\sec(a)(bx\cos(a)-}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Tan[a + b*x]^2,x]
```

```
[Out] -((c*ArcTan[Tan[a + b*x]])/b) + (d*Log[Cos[a + b*x]])/b^2 - (d*x*Sec[a]*(b*
x*Cos[a] - 2*Sin[a]))/(2*b) + (d*x*Sec[a]*Sec[a + b*x]*Sin[b*x])/b + (c*Tan
[a + b*x])/b
```

**fricas [A]** time = 0.44, size = 53, normalized size = 1.32

$$-\frac{b^2dx^2+2\,b^2cx-d\log\left(\frac{1}{\tan(bx+a)^2+1}\right)-2\,(bdx+bc)\tan(bx+a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d*x^2 + 2*b^2*c*x - d*log(1/(tan(b*x + a)^2 + 1)) - 2*(b*d*x + b*
c)*tan(b*x + a))/b^2
```

**giac [B]** time = 0.56, size = 223, normalized size = 5.58

$$-\frac{b^2dx^2\tan(bx)\tan(a)+2b^2cx\tan(bx)\tan(a)-b^2dx^2-2b^2cx+2bdx\tan(bx)+2bdx\tan(a)-d\log\left(\frac{4\left(\tan(bx)^4\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d*x^2*tan(b*x)*tan(a) + 2*b^2*c*x*tan(b*x)*tan(a) - b^2*d*x^2 - 2
*b^2*c*x + 2*b*d*x*tan(b*x) + 2*b*d*x*tan(a) - d*log(4*(tan(b*x)^4*tan(a)^2
 - 2*tan(b*x)^3*tan(a) + tan(b*x)^2*tan(a)^2 + tan(b*x)^2 - 2*tan(b*x)*tan(
a) + 1)/(tan(a)^2 + 1))*tan(b*x)*tan(a) + 2*b*c*tan(b*x) + 2*b*c*tan(a) + d
*log(4*(tan(b*x)^4*tan(a)^2 - 2*tan(b*x)^3*tan(a) + tan(b*x)^2*tan(a)^2 + t
an(b*x)^2 - 2*tan(b*x)*tan(a) + 1)/(tan(a)^2 + 1)))/(b^2*tan(b*x)*tan(a) -
b^2)
```

**maple [A]** time = 0.05, size = 47, normalized size = 1.18

$$-\frac{d\,x^2}{2}-cx+\frac{d\tan(bx+a)\,x}{b}+\frac{d\ln(\cos(bx+a))}{b^2}+\frac{c\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*tan(b*x+a)^2,x)
```

```
[Out] -1/2*d*x^2-c*x+1/b*d*tan(b*x+a)*x+d*ln(cos(b*x+a))/b^2+1/b*c*tan(b*x+a)
```

**maxima [B]** time = 0.46, size = 237, normalized size = 5.92

$$-\frac{2(bx+a-\tan(bx+a))c-\frac{2(bx+a-\tan(bx+a))ad}{b}+\frac{\left((bx+a)^2\cos(2bx+2a)^2+(bx+a)^2\sin(2bx+2a)^2+2(bx+a)^2\cos(2bx+2a)+(bx+a)^2-\right.}{(c}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a - tan(b*x + a))*c - 2*(b*x + a - tan(b*x + a))*a*d/b + ((b
*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*x + a)
^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^
2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2
*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b))/b
```

**mupad [B]** time = 1.44, size = 52, normalized size = 1.30

$$-c\,x-\frac{d\,x^2}{2}-\frac{\frac{d\,\ln\left(\tan(a+b\,x)^2+1\right)}{2}-b\,\left(c\tan(a+b\,x)+d\,x\tan(a+b\,x)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2*(c + d*x),x)
```

```
[Out] - c*x - (d*x^2)/2 - ((d*log(tan(a + b*x)^2 + 1))/2 - b*(c*tan(a + b*x) + d*
x*tan(a + b*x)))/b^2
```

**sympy [A]** time = 0.25, size = 65, normalized size = 1.62

$$\begin{cases} -cx - \frac{dx^2}{2} + \frac{c\tan(a+bx)}{b} + \frac{dx\tan(a+bx)}{b} - \frac{d\log(\tan^2(a+bx)+1)}{2b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\tan^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)**2,x)
```

```
[Out] Piecewise((-c*x - d*x**2/2 + c*tan(a + b*x)/b + d*x*tan(a + b*x)/b - d*log(
tan(a + b*x)**2 + 1)/(2*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*tan(a)**2, True
))
```

# 3.257 $\int \frac{\tan^2(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\frac{\tan^2(a+bx)}{c+dx},x\right)$$

[Out] Unintegrable(tan(b*x+a)^2/(d*x+c),x)

**Rubi** [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + b*x]^2/(c + d*x),x]

[Out] Defer[Int][Tan[a + b*x]^2/(c + d*x), x]

Rubi steps

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

**Mathematica** [A] time = 3.83, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Tan[a + b*x]^2/(c + d*x),x]

[Out] Integrate[Tan[a + b*x]^2/(c + d*x), x]

**fricas** [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tan(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")

[Out] integral(tan(b*x + a)^2/(d*x + c), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="giac")

[Out] integrate(tan(b*x + a)^2/(d*x + c), x)

**maple** [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(tan(b*x+a)^2/(d*x+c),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\left(bdx + (bdx+bc)\cos(2\,bx+2\,a)^2 + (bdx+bc)\sin(2\,bx+2\,a)^2 + bc + 2\,(bdx+bc)\cos(2\,bx+2\,a)\right)\log(dx}{bd^2x + bcd + \left(bd^2x+bcd\right)\cos(2\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] (2*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x +
 b*c*d^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*inte
grate(sin(2*b*x + 2*a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*
d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x
 + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) - (b*d*
x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b
*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*log(d*x + c) + 2*d*sin(2*b*x + 2*a))
/(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d
)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\tan(a+b\,x)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2/(c + d*x),x)
```

```
[Out] int(tan(a + b*x)^2/(c + d*x), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)**2/(c + d*x), x)
```

# 3.258 $\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\frac{\tan^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^2/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]^2/(c + d*x)^2,x]
[Out] Defer[Int][Tan[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$$

**Mathematica [A]** time = 5.07, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^2/(c + d*x)^2,x]
[Out] Integrate[Tan[a + b*x]^2/(c + d*x)^2, x]
```

**fricas [A]** time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tan(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(tan(b*x + a)^2/(d*x + c)^2, x)
```

**maple** [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^2/(d*x+c)^2,x)
[Out] int(tan(b*x+a)^2/(d*x+c)^2,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\tan(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2/(c + d*x)^2,x)
[Out] int(tan(a + b*x)^2/(c + d*x)^2, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(tan(a + b*x)**2/(c + d*x)**2, x)
```

# 3.259 $\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$

**Optimal**. Leaf size=150

$$\text{Int}\left(\tan(a+bx)\sec(a+bx)(c+dx)^m,x\right)+\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\right.}{}$$

```
[Out] CannotIntegrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)+1/2*exp(I*(a-b*c/d))*(d*
x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+1/2*(d*x+c)^m*GAMMA
(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

**Rubi [A]** time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*(
((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(2
*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x)^m*Sec[
a + b*x]*Tan[a + b*x], x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx &= -\int (c+dx)^m \sin(a+bx)\,dx+\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx\\
&= -\left(\frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^m\,dx\right)+\frac{1}{2}i\int e^{i(a+bx)}(c+dx)^m\,dx+\int (c+dx\\
&= \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m}{}
\end{aligned}$$

**Mathematica [A]** time = 23.70, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2, x]
```

**fricas [A]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \sin(bx+a)\tan(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \sin(bx + a) \tan(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

**maple** [A] time = 0.09, size = 0, normalized size = 0.00

$$\int (dx + c)^m \sin(bx + a) \left(\tan^2(bx + a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \sin(bx + a) \tan(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a + b\,x)\,\tan(a + b\,x)^2\,(c + d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^m, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \sin(a + bx) \tan^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sin(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x)*tan(a + b*x)**2, x)
```

# 3.260 $\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\, dx$

**Optimal**. Leaf size=228

$$\frac{6d^3\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}+\frac{6d^3\sin(a+bx)}{b^4}-\frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\cdots$$

```
[Out] 6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*cos(b*x+a)/b^3+(d*
x+c)^3*cos(b*x+a)/b-6*I*d^2*(d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^3+6*I*d^
2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,-I*exp(I*(b*x+a))
)/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+(d*x+c)^3*sec(b*x+a)/b+6*d^3*si
n(b*x+a)/b^4-3*d*(d*x+c)^2*sin(b*x+a)/b^2
```

**Rubi [A]** time = 0.21, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4407, 3296, 2637, 4409, 4181, 2531, 2282, 6589}

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{6d^3\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{PolyLo}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - (6*d^2*(c + d*x)*Cos[a
 + b*x])/b^3 + ((c + d*x)^3*Cos[a + b*x])/b - ((6*I)*d^2*(c + d*x)*PolyLog[2
, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(a +
b*x))])/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog
[3, I*E^(I*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b + (6*d^3*Sin[a +
 b*x])/b^4 - (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \sin(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)^3 \sin(a+bx)\,dx + \int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,d$$
$$= \frac{(c+dx)^3\cos(a+bx)}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{(3d)\int (c+dx)^2 \cos}{b}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\cos(a+bx)}{b} + \frac{(c+dx)^3\sec}{b}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3 c}{b}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3 c}{b}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3 c}{b}$$

**Mathematica [B]** time = 1.54, size = 532, normalized size = 2.33

$$\frac{\sec(a+bx)\left(b^3c^3\cos(2(a+bx)) + 3b^3c^2dx\cos(2(a+bx)) + 3b^3cd^2x^2\cos(2(a+bx)) + b^3d^3x^3\cos(2(a+bx)) -\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (Sec[a + b*x]*(3*b^3*c^3 - 6*b*c*d^2 + 9*b^3*c^2*d*x - 6*b*d^3*x + 9*b^3*c*
d^2*x^2 + 3*b^3*d^3*x^3 + (12*I)*b^2*c^2*d*ArcTan[E^(I*(a + b*x))]*Cos[a +
b*x] + b^3*c^3*Cos[2*(a + b*x)] - 6*b*c*d^2*Cos[2*(a + b*x)] + 3*b^3*c^2*d*
x*Cos[2*(a + b*x)] - 6*b*d^3*x*Cos[2*(a + b*x)] + 3*b^3*c*d^2*x^2*Cos[2*(a
+ b*x)] + b^3*d^3*x^3*Cos[2*(a + b*x)] - 12*b^2*c*d^2*x*Cos[a + b*x]*Log[1
```

```
- I*E^(I*(a + b*x))] - 6*b^2*d^3*x^2*Cos[a + b*x]*Log[1 - I*E^(I*(a + b*x))
] + 12*b^2*c*d^2*x*Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] + 6*b^2*d^3*x^2*
Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] - (12*I)*b*d^2*(c + d*x)*Cos[a + b*
x]*PolyLog[2, (-I)*E^(I*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*Cos[a + b*x]*P
olyLog[2, I*E^(I*(a + b*x))] + 12*d^3*Cos[a + b*x]*PolyLog[3, (-I)*E^(I*(a
+ b*x))] - 12*d^3*Cos[a + b*x]*PolyLog[3, I*E^(I*(a + b*x))] - 3*b^2*c^2*d*
Sin[2*(a + b*x)] + 6*d^3*Sin[2*(a + b*x)] - 6*b^2*c*d^2*x*Sin[2*(a + b*x)]
- 3*b^2*d^3*x^2*Sin[2*(a + b*x)]))/(2*b^4)
```

**fricas [C]** time = 0.57, size = 892, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*co
s(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*p
olylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, -I
*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x +
a) - sin(b*x + a)) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2
 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + (6*I*b*d^3*x + 6*I*b*c*d^2)*
cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*
d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-6*I*b*d^3*x - 6*
I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-6*I*b*d^3
*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x
 + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x
 + a) - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2
- a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3
*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x +
a) - sin(b*x + a) + 1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2
*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2
 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a)
- sin(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*lo
g(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b^2*d^3*x^2 +
2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a))/(b^4*cos(b*x
+ a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \sin(bx + a) \tan(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sin(b*x + a)*tan(b*x + a)^2, x)
```

**maple [B]** time = 0.12, size = 677, normalized size = 2.97

$$\frac{\left(d^3x^3b^3 + 3b^3c\,d^2x^2 + 3ib^2d^3x^2 + 3b^3c^2dx + 6ib^2c\,d^2x + b^3c^3 + 3ib^2c^2d - 6b\,d^3x - 6c\,d^2b - 6id^3\right)e^{i(bx+a)}}{2b^4} + \frac{\left(d^3x^3b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] 1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*
d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))+1
```

```
/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d
^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))+2
*exp(I*(b*x+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(1+exp(2*I*(b*x+a)))+
3/b^4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))+3/b^2*d^3*ln(1+I*exp(I*(b*x+a)))*x^2-6
/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a-6/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x+6*
I*c*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3+6*I/b^4*d^3*a^2*arctan(exp(I*(b*x+a
)))+6/b^3*d^2*c*ln(1+I*exp(I*(b*x+a)))*a-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)
))+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3+6*I/b^2*d*c^2*arctan(exp(I*(b*
x+a)))-6*I*c*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3-12*I/b^3*d^2*c*a*arctan(e
xp(I*(b*x+a)))-3/b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2-6*d^3*polylog(3,I*exp(I
*(b*x+a)))/b^4-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3+6/b^2*d^2*c*ln(1+
I*exp(I*(b*x+a)))*x+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4
```

**maxima [B]** time = 2.38, size = 11054, normalized size = 48.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^3*(1/cos(b*x + a) + cos(b*x + a)) - 6*a*c^2*d*(1/cos(b*x + a) + co
s(b*x + a))/b + 6*a^2*c*d^2*(1/cos(b*x + a) + cos(b*x + a))/b^2 - 2*a^3*d^3
*(1/cos(b*x + a) + cos(b*x + a))/b^3 + 3*((b*x + (b*x + a)*cos(2*b*x + 2*a)
 + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^3 +
((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3 + 6*
(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos
(b*x + a) + 4*(b*x + a)*cos(b*x + a) + (3*(b*x + a)*sin(b*x + a) + cos(b*x
+ a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(
b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) +
 (b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)
 + 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + 6*(b*x +
a)*cos(b*x + a) - 2*sin(b*x + a))*sin(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x +
 a) - sin(b*x + a))*sin(2*b*x + 2*a)^2 + ((b*x + a)*cos(2*b*x + 2*a)^2 + 13
*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + (b*x + a)*sin(b*
x + a)^2 + b*x + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2 +
2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + c
os(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*cos(3*b*x + 3*a) + 2*
(3*(b*x + a)*cos(b*x + a)^3 + 3*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + (b*
x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + (b*x + a)*cos(b*x +
a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*co
s(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x
 + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2
*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
+ 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 +
sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*s
in(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x
+ a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + si
n(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x +
 a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x +
a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos
(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(
b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*
x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) +
 sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1)
+ (((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a)^2 +
 12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + 2*(((b*x + a)*sin(b*x + a) - cos(
```

```
b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)*cos(b*x + a) + sin(b*x + a))*sin(2*
b*x + 2*a) + (b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(3*b*x + 3*a) + (12*
(b*x + a)*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2 - 2)*
cos(2*b*x + 2*a) - cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 + ((b*x + a)*cos(b*x
 + a)^2 + 13*(b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^
2 - sin(b*x + a)^2 - 1)*sin(3*b*x + 3*a) + 6*((b*x + a)*cos(b*x + a)^2*sin(
b*x + a) + (b*x + a)*sin(b*x + a)^3)*sin(2*b*x + 2*a) - sin(b*x + a))*c^2*d
/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3
*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos
(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x +
3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*
x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2
*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin
(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(
b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a
) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*b) - 6*((b*x + (b*x +
a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^3 + 6*(b*x + a
)*cos(b*x + a)^3 + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(
3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + 2*(4*(b*x + a)*c
os(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*cos(b*x + a) + (3*(b*x + a)*sin(
b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + ((b*x + a)*
cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*sin(2*b*x +
2*a)*sin(b*x + a) + (b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a
))*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x
 + 2*a) + 6*(b*x + a)*cos(b*x + a) - 2*sin(b*x + a))*sin(3*b*x + 3*a)^2 + (
(b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 2*a)^2 + ((b*x + a)*cos(
2*b*x + 2*a)^2 + 13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2
 + (b*x + a)*sin(b*x + a)^2 + b*x + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a
)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x +
a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*co
s(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)^3 + 3*(b*x + a)*cos(b*x + a)*s
in(b*x + a)^2 + (b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + (
b*x + a)*cos(b*x + a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos
(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x
+ 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*
a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2
*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(c
os(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*
b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*
log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + ((cos(2*b*x + 2
*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (
cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 +
 sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x
 + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*
x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a)
+ cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(
2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*
b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))
*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2
*sin(b*x + a) + 1) + (((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*c
os(3*b*x + 3*a)^2 + 12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + 2*(((b*x + a)*
sin(b*x + a) - cos(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)*cos(b*x + a) + s
in(b*x + a))*sin(2*b*x + 2*a) + (b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(
3*b*x + 3*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - s
in(b*x + a)^2 - 2)*cos(2*b*x + 2*a) - cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 +
 ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a)
 - sin(2*b*x + 2*a)^2 - sin(b*x + a)^2 - 1)*sin(3*b*x + 3*a) + 6*((b*x + a)*
```

```
cos(b*x + a)^2*sin(b*x + a) + (b*x + a)*sin(b*x + a)^3)*sin(2*b*x + 2*a) -
sin(b*x + a))*a*c*d^2/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*
b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(
2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2
*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x +
 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a
)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*
(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(co
s(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b
*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*b
^2) + 3*((b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*
x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^3 + ((b*x + a)*sin(2*b*x + 2*a) - cos
(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)*sin(b*x +
a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*cos(b*x +
 a) + (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x
 + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a)^2 + (8
*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) + (b*x + (b*x + a)*cos(2*b*x + 2*a
) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a) -
sin(b*x + a))*cos(2*b*x + 2*a) + 6*(b*x + a)*cos(b*x + a) - 2*sin(b*x + a))
*sin(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 2
*a)^2 + ((b*x + a)*cos(2*b*x + 2*a)^2 + 13*(b*x + a)*cos(b*x + a)^2 + (b*x
+ a)*sin(2*b*x + 2*a)^2 + (b*x + a)*sin(b*x + a)^2 + b*x + (13*(b*x + a)*co
s(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (
12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2)*s
in(2*b*x + 2*a) + a)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)^3 + 3*(
b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + (b*x + a)*cos(b*x + a) - sin(b*x + a
))*cos(2*b*x + 2*a) + (b*x + a)*cos(b*x + a) - ((cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^
2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin
(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(
b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a
))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)
+ cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*
sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x +
3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a)
 + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)
*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2
+ (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*
b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(co
s(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b
*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2
 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^
2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b
*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (((b*x + a)*sin(2*b*x + 2*a) -
cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a)^2 + 12*(b*x + a)*cos(b*x + a)*sin(b*
x + a) + 2*(((b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(2*b*x + 2*a) + ((b*
x + a)*cos(b*x + a) + sin(b*x + a))*sin(2*b*x + 2*a) + (b*x + a)*sin(b*x +
a) - cos(b*x + a))*cos(3*b*x + 3*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x +
a) - cos(b*x + a)^2 - sin(b*x + a)^2 - 2)*cos(2*b*x + 2*a) - cos(2*b*x + 2*
a)^2 - cos(b*x + a)^2 + ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x +
a)^2)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 - sin(b*x + a)^2 - 1)*sin(3*b*x
 + 3*a) + 6*((b*x + a)*cos(b*x + a)^2*sin(b*x + a) + (b*x + a)*sin(b*x + a)
^3)*sin(2*b*x + 2*a) - sin(b*x + a))*a^2*d^3/(((cos(2*b*x + 2*a)^2 + sin(2*
b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2
 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(
b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b
```

```
*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a)
)*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) +
 cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*s
in(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3
*a) + sin(b*x + a)^2)*b^3) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + (6*a + 6*I)*d
^3 + (3*b*c*d^2 - (3*a + 3*I)*d^3)*(b*x + a)^2 + ((b*x + a)^3*d^3 - 6*b*c*d
^2 + (6*a - 6*I)*d^3 + (3*b*c*d^2 - (3*a - 3*I)*d^3)*(b*x + a)^2 + (6*I*b*c
*d^2 - 6*(I*a + 1)*d^3)*(b*x + a))*cos(3*b*x + 3*a)^2 + 6*((b*x + a)^3*d^3
 - 2*b*c*d^2 - 2*(b*x + a)*d^3 + 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*
cos(b*x + a)^2 - ((b*x + a)^3*d^3 - 6*b*c*d^2 + (6*a - 6*I)*d^3 + (3*b*c*d^
2 - (3*a - 3*I)*d^3)*(b*x + a)^2 - (-6*I*b*c*d^2 - 6*(-I*a - 1)*d^3)*(b*x +
 a))*sin(3*b*x + 3*a)^2 + (12*I*(b*x + a)^3*d^3 - 24*I*b*c*d^2 - 24*I*(b*x
+ a)*d^3 + 24*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2)*cos(b*x +
a)*sin(b*x + a) - 6*((b*x + a)^3*d^3 - 2*b*c*d^2 - 2*(b*x + a)*d^3 + 2*a*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*sin(b*x + a)^2 - 6*(I*b*c*d^2 + (-I*a
+ 1)*d^3)*(b*x + a) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(
b*x + a) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*co
s(2*b*x + 2*a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*
b*x + 2*a))*cos(3*b*x + 3*a) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I
*a*d^3)*(b*x + a))*cos(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*
(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^3 + (12*I*b*
c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(b*x + a) - (6*(b*x + a)^2*d^3 + 12*(b*c*
d^2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)
)*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(
b*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - (6*((b*x + a)^2*d^3 + 2*(b*c
*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*
d^2 + 12*I*a*d^3)*(b*x + a))*sin(b*x + a))*sin(2*b*x + 2*a) - 6*((b*x + a)^
2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*arctan2(cos(b*x + a),
sin(b*x + a) + 1) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*
x + a) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(
2*b*x + 2*a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*
x + 2*a))*cos(3*b*x + 3*a) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a
*d^3)*(b*x + a))*cos(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b
*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*
d^2 - 12*I*a*d^3)*(b*x + a))*cos(b*x + a) - (6*(b*x + a)^2*d^3 + 12*(b*c*d^
2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*
x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - (6*((b*x + a)^2*d^3 + 2*(b*c*d
^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^
2 + 12*I*a*d^3)*(b*x + a))*sin(b*x + a))*sin(2*b*x + 2*a) - 6*((b*x + a)^2*
d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*arctan2(cos(b*x + a), -s
in(b*x + a) + 1) + ((7*(b*x + a)^3*d^3 - 18*b*c*d^2 + (18*a - 6*I)*d^3 + (2
1*b*c*d^2 - (21*a - 3*I)*d^3)*(b*x + a)^2 + (6*I*b*c*d^2 - 6*(I*a + 3)*d^3)
*(b*x + a))*cos(b*x + a) + (7*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 - 6*(-3*I*a
- 1)*d^3 + (21*I*b*c*d^2 - 3*(7*I*a + 1)*d^3)*(b*x + a)^2 - (6*b*c*d^2 - (6
*a - 18*I)*d^3)*(b*x + a))*sin(b*x + a))*cos(3*b*x + 3*a) + ((b*x + a)^3*d^
3 - 6*b*c*d^2 + (6*a + 6*I)*d^3 + (3*b*c*d^2 - (3*a + 3*I)*d^3)*(b*x + a)^2
 - 6*(I*b*c*d^2 + (-I*a + 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + ((12*I*b*c*
d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3
- 12*I*a*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2
*b*x + 2*a))*cos(3*b*x + 3*a) + ((12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*
a*d^3)*cos(b*x + a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*co
s(2*b*x + 2*a) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(b*x +
 a) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d
^3 - a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a
*d^3)*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - (12*(b*c*d^2 + (b*x + a)*d^3 - a
*d^3)*cos(b*x + a) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(
b*x + a))*sin(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x +
 a))*dilog(I*e^(I*b*x + I*a)) + ((-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I
```

```
*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(2*b*x + 2*a)
 + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)
+ ((-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(b*x + a) + 12*(b*c
*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*cos(2*b*x + 2*a) + (-12*I*b*c*d
^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(b*x + a) + (12*b*c*d^2 + 12*(b*x
+ a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a)
 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(2*b*x + 2*a))*sin(3
*b*x + 3*a) + (12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(b*x + a) + (12*I*b*
c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(b*x + a))*sin(2*b*x + 2*a) + 1
2*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*dilog(-I*e^(I*b*x + I*a))
 - ((3*(b*x + a)^2*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a) + 3*((b*x + a)^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-3*I*(b*x + a)^2*d^3
+ (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)
+ (3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-3*I
*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*sin(b*x + a))*cos(
2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x
+ a) - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (-3*I
*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) +
 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*sin(
3*b*x + 3*a) - ((-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a
))*cos(b*x + a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b
*x + a))*sin(2*b*x + 2*a) - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d
^3)*(b*x + a))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*
x + a) + 1) + ((3*(b*x + a)^2*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a) + 3*((b*x
 + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (3*I*(b*x +
 a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*
x + 3*a) + (3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a
) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*sin(b*x + a
))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*c
os(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a) +
(3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a
) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*s
in(3*b*x + 3*a) + ((3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x +
a))*cos(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(
b*x + a))*sin(2*b*x + 2*a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^
3)*(b*x + a))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x
 + a) + 1) - (12*d^3*cos(b*x + a) + 12*I*d^3*sin(b*x + a) + 12*(d^3*cos(2*b
*x + 2*a) + I*d^3*sin(2*b*x + 2*a) + d^3)*cos(3*b*x + 3*a) + 12*(d^3*cos(b*
x + a) + I*d^3*sin(b*x + a))*cos(2*b*x + 2*a) - (-12*I*d^3*cos(2*b*x + 2*a)
 + 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*sin(3*b*x + 3*a) - (-12*I*d^3*cos(b*
x + a) + 12*d^3*sin(b*x + a))*sin(2*b*x + 2*a))*polylog(3, I*e^(I*b*x + I*a
)) + (12*d^3*cos(b*x + a) + 12*I*d^3*sin(b*x + a) + 12*(d^3*cos(2*b*x + 2*a
) + I*d^3*sin(2*b*x + 2*a) + d^3)*cos(3*b*x + 3*a) + 12*(d^3*cos(b*x + a) +
 I*d^3*sin(b*x + a))*cos(2*b*x + 2*a) + (12*I*d^3*cos(2*b*x + 2*a) - 12*d^3
*sin(2*b*x + 2*a) + 12*I*d^3)*sin(3*b*x + 3*a) + (12*I*d^3*cos(b*x + a) - 1
2*d^3*sin(b*x + a))*sin(2*b*x + 2*a))*polylog(3, -I*e^(I*b*x + I*a)) + ((2*
I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 - 12*(-I*a - 1)*d^3 + (6*I*b*c*d^2 - 6*(I*
a + 1)*d^3)*(b*x + a)^2 - (12*b*c*d^2 - (12*a - 12*I)*d^3)*(b*x + a))*cos(3
*b*x + 3*a) + (7*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 - 6*(-3*I*a - 1)*d^3 + (2
1*I*b*c*d^2 - 3*(7*I*a + 1)*d^3)*(b*x + a)^2 - (6*b*c*d^2 - (6*a - 18*I)*d^
3)*(b*x + a))*cos(b*x + a) - (7*(b*x + a)^3*d^3 - 18*b*c*d^2 + (18*a - 6*I)
*d^3 + (21*b*c*d^2 - (21*a - 3*I)*d^3)*(b*x + a)^2 - (-6*I*b*c*d^2 - 6*(-I*
a - 3)*d^3)*(b*x + a))*sin(b*x + a))*sin(3*b*x + 3*a) + (I*(b*x + a)^3*d^3
- 6*I*b*c*d^2 - 6*(-I*a + 1)*d^3 - 3*(-I*b*c*d^2 + (I*a - 1)*d^3)*(b*x + a)
^2 + (6*b*c*d^2 - (6*a + 6*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(2*b^3*cos(
b*x + a) + 2*I*b^3*sin(b*x + a) + (2*b^3*cos(2*b*x + 2*a) + 2*I*b^3*sin(2*b
*x + 2*a) + 2*b^3)*cos(3*b*x + 3*a) + 2*(b^3*cos(b*x + a) + I*b^3*sin(b*x +
 a))*cos(2*b*x + 2*a) - (-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a)
 - 2*I*b^3)*sin(3*b*x + 3*a) - (-2*I*b^3*cos(b*x + a) + 2*b^3*sin(b*x + a))
```

```
*sin(2*b*x + 2*a)))/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \sin \left(a+b\,x\right)\,\tan \left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \sin (a+bx) \tan ^2(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sin(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*sin(a + b*x)*tan(a + b*x)**2, x)
```

# 3.261 $\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx)\, dx$

**Optimal**. Leaf size=145

$$-\frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{2d^2\cos(a+bx)}{b^3}-\frac{2d(c+dx)\sin(a+bx)}{b^2}+\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}$$

[Out] 4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*d^2*cos(b*x+a)/b^3+(d*x+c)^2*cos(b*x+a)/b-2*I*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b-2*d*(d*x+c)*sin(b*x+a)/b^2

**Rubi** [A] time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4407, 3296, 2638, 4409, 4181, 2279, 2391}

$$-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)\sin(a+bx)}{b^2}+\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x]^2,x]

[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - (2*d^2*Cos[a + b*x])/b^3 + ((c + d*x)^2*Cos[a + b*x])/b - ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 + ((c + d*x)^2*Sec[a + b*x])/b - (2*d*(c + d*x)*Sin[a + b*x])/b^2

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
```

```
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2\sin(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)^2\sin(a+bx)\,dx + \int (c+dx)^2\sec(a+bx)\tan(a+bx)\,dx$$

$$= \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2d)\int (c+dx)\cos(a+\ldots}{b}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{(c+dx)^2\sec(a+\ldots}{b}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a+bx)}{b} - \ldots$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a+bx)}{b} - \ldots$$

**Mathematica [B]** time = 3.12, size = 362, normalized size = 2.50

$$\frac{\cos(bx)\left(\cos(a)\left(b^2(c+dx)^2-2d^2\right)-2bd\sin(a)(c+dx)\right)-\sin(bx)\left(\sin(a)\left(b^2(c+dx)^2-2d^2\right)+2bd\cos(a)(c+\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcT
anh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])*(
Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]])
)]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x
- ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + b^2*(c + d*x)^2*Sec[a] + Cos[b*x]*((
-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - (2*b*d*(c + d*
x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] + (b^2*(c + d*x)^2*
Sin[(b*x)/2])/((Cos[a/2] - Sin[a/2])*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2]))
 - (b^2*(c + d*x)^2*Sin[(b*x)/2])/((Cos[a/2] + Sin[a/2])*(Cos[(a + b*x)/2]
+ Sin[(a + b*x)/2])))/b^3
```

**fricas [B]** time = 0.53, size = 511, normalized size = 3.52

$$\frac{b^2d^2x^2+2\,b^2cdx+b^2c^2+i\,d^2\cos\left(bx+a\right){\rm Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)+i\,d^2\cos\left(bx+a\right){\rm Li}_2\left(i\,\cos\left(bx+a\right)\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x +
 a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a
```

```
)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(b
*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*
x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a
) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a)
 + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) -
sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + si
n(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(
b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x
+ a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a)
 + I) - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \sin(bx+a)\tan(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sin(b*x + a)*tan(b*x + a)^2, x)
```

**maple [B]** time = 0.13, size = 345, normalized size = 2.38

$$\frac{\left(d^2x^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)\mathrm{e}^{i(bx+a)}}{2b^3}+\frac{\left(d^2x^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2\right)\mathrm{e}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] 1/2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I
*(b*x+a))+1/2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)
/b^3*exp(-I*(b*x+a))+2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(1+exp(2*I*(b
*x+a)))+4*I*d/b^2*c*arctan(exp(I*(b*x+a)))+2/b^2*d^2*ln(1+I*exp(I*(b*x+a)))
*x+2/b^3*d^2*ln(1+I*exp(I*(b*x+a)))*a-2/b^2*d^2*ln(1-I*exp(I*(b*x+a)))*x-2/
b^3*d^2*ln(1-I*exp(I*(b*x+a)))*a-2*I/b^3*d^2*dilog(1+I*exp(I*(b*x+a)))+2*I/
b^3*d^2*dilog(1-I*exp(I*(b*x+a)))-4*I*d^2/b^3*a*arctan(exp(I*(b*x+a)))
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)\,\tan(a+b\,x)^2\,(c+d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \sin(a+bx)\tan^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sin(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*sin(a + b*x)*tan(a + b*x)**2, x)
```

# 3.262 $\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$

**Optimal**. Leaf size=56

$$-\frac{d\sin(a+bx)}{b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{(c+dx)\sec(a+bx)}{b}$$

```
[Out] -d*arctanh(sin(b*x+a))/b^2+(d*x+c)*cos(b*x+a)/b+(d*x+c)*sec(b*x+a)/b-d*sin(
b*x+a)/b^2
```

**Rubi** [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4407, 3296, 2637, 4409, 3770}

$$-\frac{d\sin(a+bx)}{b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{(c+dx)\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -((d*ArcTanh[Sin[a + b*x]])/b^2) + ((c + d*x)*Cos[a + b*x])/b + ((c + d*x)*
Sec[a + b*x])/b - (d*Sin[a + b*x])/b^2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)\sin(a+bx)\,dx + \int (c+dx)\sec(a+bx)\tan(a+bx)\,dx$$
$$= \frac{(c+dx)\cos(a+bx)}{b} + \frac{(c+dx)\sec(a+bx)}{b} - \frac{d\int \cos(a+bx)\,dx}{b} - \frac{d\int}{}$$
$$= -\frac{d\tanh^{-1}(\sin(a+bx))}{b^2} + \frac{(c+dx)\cos(a+bx)}{b} + \frac{(c+dx)\sec(a+bx)}{b} -$$

**Mathematica [A]** time = 0.30, size = 107, normalized size = 1.91

$$\frac{\sec(a+bx)\left(b(c+dx)\cos(2(a+bx)) - d\sin(2(a+bx)) + 2d\cos(a+bx)\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right) - \sin\left(\frac{1}{2}(a+bx)\right)\right.\right.\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (Sec[a + b*x]*(3*b*c + 3*b*d*x + b*(c + d*x)*Cos[2*(a + b*x)] + 2*d*Cos[a +
 b*x]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2] + Si
n[(a + b*x)/2]]) - d*Sin[2*(a + b*x)]))/(2*b^2)
```

**fricas [A]** time = 0.48, size = 93, normalized size = 1.66

$$\frac{2\,bdx + 2\,(bdx + bc)\cos(bx+a)^2 - d\cos(bx+a)\log(\sin(bx+a)+1) + d\cos(bx+a)\log(-\sin(bx+a)+1) -}{2\,b^2\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x + 2*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*log(sin(b*x
+ a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) - 2*d*cos(b*x + a)*sin(b*
x + a) + 2*b*c)/(b^2*cos(b*x + a))
```

**giac [B]** time = 7.90, size = 2762, normalized size = 49.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b*c*tan(1/2*b*x)^4*tan(1/2*a)^
4 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*
tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^
2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan
(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^4*tan(1/2*a)^4 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)
^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*
b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a
)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 - 16*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^
3 - 16*b*c*tan(1/2*b*x)^3*tan(1/2*a)^3 - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*
a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/
2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x
)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1
/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^3 + 4*d*log(2*(tan
(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*t
an(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*
```

x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*ta
n(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a
)^3 + 4*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + 4
*b*d*x*tan(1/2*b*x)^4 + 16*b*d*x*tan(1/2*b*x)^3*tan(1/2*a) + 48*b*d*x*tan(1
/2*b*x)^2*tan(1/2*a)^2 + 16*b*d*x*tan(1/2*b*x)*tan(1/2*a)^3 + 4*b*d*x*tan(1
/2*a)^4 + 4*b*c*tan(1/2*b*x)^4 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*t
an(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 +
 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*
a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)
/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2
- 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x
)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan
(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a)
 + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4 + 16*b*c*tan(1/2*b*x)^3*tan(1/2*a)
 - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2
*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)
^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + ta
n(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2
*b*x)^3*tan(1/2*a) + 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x
)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2
*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*t
an(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*
a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a) - 4*d*tan(1/2*b*x)^4*tan(1/2*a) + 48*b
*c*tan(1/2*b*x)^2*tan(1/2*a)^2 - 24*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 16*b*c*
tan(1/2*b*x)*tan(1/2*a)^3 - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(
1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*
tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^
2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(t
an(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^3 + 4*d*log(2*(tan(1/2*b*x)^4*tan
(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + t
an(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/
2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*
tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^3 - 24*d*tan(1/
2*b*x)^2*tan(1/2*a)^3 + 4*b*c*tan(1/2*a)^4 - d*log(2*(tan(1/2*b*x)^4*tan(1/
2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(
1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b
*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan
(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a)^4 + d*log(2*(tan(1/2*b*x)^4*tan
(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + t
an(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/
2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*
tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a)^4 - 4*d*tan(1/2*b*x)*tan(1/2
*a)^4 - 16*b*d*x*tan(1/2*b*x)*tan(1/2*a) + 4*d*tan(1/2*b*x)^3 - 16*b*c*tan(
1/2*b*x)*tan(1/2*a) - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*
x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/
2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*
tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2
*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^
2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b
*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*t
an(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*
a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + 24*d*tan(1/2*b*x)^2*t
an(1/2*a) + 24*d*tan(1/2*b*x)*tan(1/2*a)^2 + 4*d*tan(1/2*a)^3 + 4*b*d*x + 4
*b*c + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) +
 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*
a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 +
tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1)) - d*l
og(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2
*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*t

```
an(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)
^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1)) - 4*d*tan(1/2*b
*x) - 4*d*tan(1/2*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^4 - 4*b^2*tan(1/2*b*x)
^3*tan(1/2*a)^3 - b^2*tan(1/2*b*x)^4 - 4*b^2*tan(1/2*b*x)^3*tan(1/2*a) - 4*
b^2*tan(1/2*b*x)*tan(1/2*a)^3 - b^2*tan(1/2*a)^4 - 4*b^2*tan(1/2*b*x)*tan(1
/2*a) + b^2)
```

**maple [C]** time = 0.16, size = 123, normalized size = 2.20

$$\frac{(bdx+cb+id)\,e^{i(bx+a)}}{2b^2}+\frac{(bdx+cb-id)\,e^{-i(bx+a)}}{2b^2}+\frac{2\,e^{i(bx+a)}\,(dx+c)}{b\left(1+e^{2i(bx+a)}\right)}-\frac{d\ln\left(e^{i(bx+a)}+i\right)}{b^2}+\frac{d\ln\left(e^{i(bx+a)}-i\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] 1/2*(b*d*x+c*b+I*d)/b^2*exp(I*(b*x+a))+1/2*(b*d*x+c*b-I*d)/b^2*exp(-I*(b*x+
a))+2*exp(I*(b*x+a))*(d*x+c)/b/(1+exp(2*I*(b*x+a)))-d/b^2*ln(exp(I*(b*x+a))
+I)+d/b^2*ln(exp(I*(b*x+a))-I)
```

**maxima [B]** time = 0.48, size = 2123, normalized size = 37.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*(1/cos(b*x + a) + cos(b*x + a)) - 2*a*d*(1/cos(b*x + a) + cos(b*x
+ a))/b + ((b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*
b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^3 + ((b*x + a)*sin(2*b*x + 2*a) - c
os(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)*sin(b*x
+ a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*cos(b*x
 + a) + (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a))*cos(3*b
*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a)^2 +
(8*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) + (b*x + (b*x + a)*cos(2*b*x + 2
*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)
- sin(b*x + a))*cos(2*b*x + 2*a) + 6*(b*x + a)*cos(b*x + a) - 2*sin(b*x + a
))*sin(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x +
 2*a)^2 + ((b*x + a)*cos(2*b*x + 2*a)^2 + 13*(b*x + a)*cos(b*x + a)^2 + (b*
x + a)*sin(2*b*x + 2*a)^2 + (b*x + a)*sin(b*x + a)^2 + b*x + (13*(b*x + a)*
cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) +
 (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2)
*sin(2*b*x + 2*a) + a)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)^3 + 3
*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + (b*x + a)*cos(b*x + a) - sin(b*x +
 a))*cos(2*b*x + 2*a) + (b*x + a)*cos(b*x + a) - ((cos(2*b*x + 2*a)^2 + sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a
)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + s
in(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + co
s(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x +
 a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a
) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^
2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x
+ 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x +
a) + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) +
1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^
2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(
3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(
cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)
```

```
^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a
)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin
(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x +
a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (((b*x + a)*sin(2*b*x + 2*a)
- cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a)^2 + 12*(b*x + a)*cos(b*x + a)*sin(
b*x + a) + 2*(((b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(2*b*x + 2*a) + ((
b*x + a)*cos(b*x + a) + sin(b*x + a))*sin(2*b*x + 2*a) + (b*x + a)*sin(b*x
+ a) - cos(b*x + a))*cos(3*b*x + 3*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x
+ a) - cos(b*x + a)^2 - sin(b*x + a)^2 - 2)*cos(2*b*x + 2*a) - cos(2*b*x +
2*a)^2 - cos(b*x + a)^2 + ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x
+ a)^2)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 - sin(b*x + a)^2 - 1)*sin(3*b
*x + 3*a) + 6*((b*x + a)*cos(b*x + a)^2*sin(b*x + a) + (b*x + a)*sin(b*x +
a)^3)*sin(2*b*x + 2*a) - sin(b*x + a))*d/(((cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + s
in(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^
2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x
+ a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x +
 a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*co
s(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos
(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b
*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a)
+ sin(b*x + a)^2)*b))/b
```

**mupad [B]** time = 1.16, size = 151, normalized size = 2.70

$$e^{a\,1i+b\,x\,1i}\,\left(\frac{b\,c+d\,1i}{2\,b^2}+\frac{d\,x}{2\,b}\right)-e^{-a\,1i-b\,x\,1i}\,\left(\frac{-b\,c+d\,1i}{2\,b^2}-\frac{d\,x}{2\,b}\right)+\frac{d\,\ln\left(e^{a\,1i+b\,x\,1i}-i\right)}{b^2}-\frac{d\,\ln\left(e^{a\,1i+b\,x\,1i}+1i\right)}{b^2}+\frac{e^{a\,1i+b}}{b\,\left(e^{a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x),x)
```

```
[Out] exp(a*1i + b*x*1i)*((d*1i + b*c)/(2*b^2) + (d*x)/(2*b)) - exp(- a*1i - b*x*
1i)*((d*1i - b*c)/(2*b^2) - (d*x)/(2*b)) + (d*log(exp(a*1i + b*x*1i) - 1i))
/b^2 - (d*log(exp(a*1i + b*x*1i) + 1i))/b^2 + (exp(a*1i + b*x*1i)*(c + d*x)
*2i)/(b*(exp(a*2i + b*x*2i)*1i + 1i))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*sin(a + b*x)*tan(a + b*x)**2, x)
```

# 3.263 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=76

$$\mathrm{Int}\left(\frac{\tan(a+bx)\sec(a+bx)}{c+dx},x\right)-\frac{\sin\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)-cos(a-b*c/d)*Si(b*c/d+b*x)
/d-Ci(b*c/d+b*x)*sin(a-b*c/d)/d
```

**Rubi** [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -((CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d) - (Cos[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x
), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx &= -\int \frac{\sin(a+bx)}{c+dx}\,dx+\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx\\ &= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)-\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\int \frac{\mathrm{se}}{}\\ &= -\frac{\mathrm{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\sec(a+bx)\tan(a+}{c+dx}\end{aligned}$$

**Mathematica** [A] time = 3.78, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]
```

**fricas** [A] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sin(bx+a)\tan(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sin(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)\tan(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(sin(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

**maple** [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)\left(\tan^2(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
[Out] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)\,\tan(a+b\,x)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x),x)
[Out] int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)**2/(d*x+c),x)
[Out] Integral(sin(a + b*x)*tan(a + b*x)**2/(c + d*x), x)
```

# 3.264 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=94

$$\operatorname{Int}\left(\frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)-\frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}$$

```
[Out] CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)-b*Ci(b*c/d+b*x)*cos(a-b*
c/d)/d^2+b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2+sin(b*x+a)/d/(d*x+c)
```

**Rubi [A]** time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2) + Sin[a + b*x]/(d*(c
 + d*x)) + (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int]
[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx+\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}-\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}+\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}-\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}\\
&= -\frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}+\frac{b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int \frac{\text{se}}{}
\end{aligned}$$

**Mathematica [A]** time = 4.07, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]
```

**fricas [A]** time = 0.49, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sin(bx+a)\tan(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sin(b*x + a)*tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)\tan(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sin(b*x + a)*tan(b*x + a)^2/(d*x + c)^2, x)
```

**maple** [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)\left(\tan^2(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
[Out] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)\,\tan(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x)^2,x)
[Out] int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x)^2, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(sin(a + b*x)*tan(a + b*x)**2/(c + d*x)**2, x)
```

# 3.265 $\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\csc(a+bx)\sec^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

**Rubi** [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx$$

**Mathematica** [A] time = 24.57, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]
```

**fricas** [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \csc(bx+a)\sec(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

**maple** [A] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\left(\sec^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2\,\sin(a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)),x)
```

```
[Out] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.266 $\int (c+dx)^4 \csc(a+bx)\sec^2(a+bx)\,dx$

**Optimal**. Leaf size=469

$$\frac{24id^4\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^5}-\frac{24id^4\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^5}+\frac{24d^4\text{Li}_5\left(-e^{i(a+bx)}\right)}{b^5}-\frac{24d^4\text{Li}_5\left(e^{i(a+bx)}\right)}{b^5}+\frac{24d^3(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-$$

```
[Out] 8*I*d*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b^2-2*(d*x+c)^4*arctanh(exp(I*(b*x+a
)))/b-12*I*d^2*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^3-24*I*d^4*polylog(
4,I*exp(I*(b*x+a)))/b^5+12*I*d^2*(d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^3-
24*I*d^3*(d*x+c)*polylog(4,-exp(I*(b*x+a)))/b^4-12*d^2*(d*x+c)^2*polylog(3,
-exp(I*(b*x+a)))/b^3+24*d^3*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^4-24*d^3
*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^4+12*d^2*(d*x+c)^2*polylog(3,exp(I*(
b*x+a)))/b^3+24*I*d^3*(d*x+c)*polylog(4,exp(I*(b*x+a)))/b^4+4*I*d*(d*x+c)^3
*polylog(2,-exp(I*(b*x+a)))/b^2-4*I*d*(d*x+c)^3*polylog(2,exp(I*(b*x+a)))/b
^2+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5+24*d^4*polylog(5,-exp(I*(b*x+a
)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5+(d*x+c)^4*sec(b*x+a)/b
```

**Rubi [A]** time = 0.79, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {2622, 321, 207, 4420, 6741, 12, 6742, 6273, 4183, 2531, 6609, 2282, 6589, 4181}

$$\frac{24d^3(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4}-\frac{24id^3(c+dx)\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] ((8*I)*d*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^4*ArcTanh[
E^(I*(a + b*x))])/b + ((4*I)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^
2 - ((12*I)*d^2*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((12*I)
*d^2*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((4*I)*d*(c + d*x)^3*
PolyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a
+ b*x))])/b^3 + (24*d^3*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (
24*d^3*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + (12*d^2*(c + d*x)^2*P
olyLog[3, E^(I*(a + b*x))])/b^3 - ((24*I)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a
 + b*x))])/b^4 + ((24*I)*d^4*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^5 - ((24*I
)*d^4*PolyLog[4, I*E^(I*(a + b*x))])/b^5 + ((24*I)*d^3*(c + d*x)*PolyLog[4,
 E^(I*(a + b*x))])/b^4 + (24*d^4*PolyLog[5, -E^(I*(a + b*x))])/b^5 - (24*d^
4*PolyLog[5, E^(I*(a + b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^4\csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}-(4d)\int(c+\ldots$$

$$= -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}-(4d)\int\frac{(c+\ldots}{\ldots}$$

$$= -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}-\frac{(4d)\int(c+d\ldots}{\ldots}$$

$$= -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}-\frac{(4d)\int\left(-(c\ldots\right.}{\ldots}$$

$$= -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}+\frac{(4d)\int(c+d\ldots}{\ldots}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^4\sec(a+bx)}{b}+\frac{\int b(c+dx)^4\csc\ldots}{b}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{12id^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{12id^2(c\ldots}{\ldots}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{12id^2(c+d\ldots}{\ldots}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{4id(c+dx\ldots}{\ldots}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{4id(c+dx\ldots}{\ldots}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{4id(c+dx\ldots}{\ldots}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{4id(c+dx\ldots}{\ldots}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{4id(c+dx\ldots}{\ldots}$$

**Mathematica [A]** time = 3.37, size = 694, normalized size = 1.48

$$\frac{b^4(c+dx)^4\log\left(1-e^{i(a+bx)}\right)-b^4(c+dx)^4\log\left(1+e^{i(a+bx)}\right)+b^4(c+dx)^4\sec(a+bx)-4d\left(-2ib^3c^3\tan^{-1}\left(e^{i(a+}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (b^4*(c + d*x)^4*Log[1 - E^(I*(a + b*x))] - b^4*(c + d*x)^4*Log[1 + E^(I*(a
 + b*x))] - 4*d*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log
[1 - I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*
d^3*x^3*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x)
)] - 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(
I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] -
(3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3
, (-I)*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*
c*d^2*PolyLog[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x)
)] - (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^
(I*(a + b*x))]) + (4*I)*d*(b^3*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))] + (
3*I)*b^2*d*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))] - 6*d^2*(b*(c + d*x)*Po
lyLog[4, -E^(I*(a + b*x))] + I*d*PolyLog[5, -E^(I*(a + b*x))])) - (4*I)*d*(
b^3*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyL
og[3, E^(I*(a + b*x))] - 6*d^2*(b*(c + d*x)*PolyLog[4, E^(I*(a + b*x))] + I
*d*PolyLog[5, E^(I*(a + b*x))])) + b^4*(c + d*x)^4*Sec[a + b*x])/b^5
```

**fricas [C]** time = 0.81, size = 2507, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x +
 2*b^4*c^4 - 24*d^4*cos(b*x + a)*polylog(5, cos(b*x + a) + I*sin(b*x + a))
- 24*d^4*cos(b*x + a)*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 24*d^4*co
s(b*x + a)*polylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*cos(b*x + a)
*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) - 24*I*d^4*cos(b*x + a)*polylog
(4, I*cos(b*x + a) + sin(b*x + a)) - 24*I*d^4*cos(b*x + a)*polylog(4, I*cos
(b*x + a) - sin(b*x + a)) + 24*I*d^4*cos(b*x + a)*polylog(4, -I*cos(b*x + a
) + sin(b*x + a)) + 24*I*d^4*cos(b*x + a)*polylog(4, -I*cos(b*x + a) - sin(
b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4
*I*b^3*c^3*d)*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (4*I*b^3*
d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*cos(b*x
+ a)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (12*I*b^2*d^4*x^2 + 24*I*b^2*c*
d^3*x + 12*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a))
 + (12*I*b^2*d^4*x^2 + 24*I*b^2*c*d^3*x + 12*I*b^2*c^2*d^2)*cos(b*x + a)*di
log(I*cos(b*x + a) - sin(b*x + a)) + (-12*I*b^2*d^4*x^2 - 24*I*b^2*c*d^3*x
- 12*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-
12*I*b^2*d^4*x^2 - 24*I*b^2*c*d^3*x - 12*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(
-I*cos(b*x + a) - sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 -
12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*cos(b*x + a)*dilog(-cos(b*x + a) + I*si
n(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x +
4*I*b^3*c^3*d)*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^4*d^
4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*cos(
b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*c^3*d - 3*a*b^2*c^
2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x
+ a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*
d*x + b^4*c^4)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 4*(b^3
*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(cos(b*
```

```
x + a) - I*sin(b*x + a) + I) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2
*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2
*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(
b*x + a) - sin(b*x + a) + 1) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2
*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*cos
(b*x + a) + sin(b*x + a) + 1) + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^
2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*co
s(b*x + a) - sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d
^2 - 4*a^3*b*c*d^3 + a^4*d^4)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*si
n(b*x + a) + 1/2) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) +
1/2) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x +
 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*cos(b*x + a)*
log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) +
I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4
*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*cos(b*x + a)*lo
g(-cos(b*x + a) - I*sin(b*x + a) + 1) + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*
a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I)
 + (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)*polylog(4, cos(b*x + a) + I*s
in(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x + a)*polylog(4, cos(b
*x + a) - I*sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)*poly
log(4, -cos(b*x + a) + I*sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos
(b*x + a)*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x
 + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*polylo
g(3, cos(b*x + a) - I*sin(b*x + a)) + 24*(b*d^4*x + b*c*d^3)*cos(b*x + a)*p
olylog(3, I*cos(b*x + a) + sin(b*x + a)) - 24*(b*d^4*x + b*c*d^3)*cos(b*x +
 a)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 24*(b*d^4*x + b*c*d^3)*cos(
b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 24*(b*d^4*x + b*c*d^3
)*cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2
 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*polylog(3, -cos(b*x + a) + I*s
in(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*
polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^5*cos(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^4 \csc(bx + a) \sec(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a)^2, x)
```

**maple [B]** time = 0.60, size = 1866, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x)
```

```
[Out] 1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)-12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a))
)+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))-1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)
))+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2-12/b^3*d^4*polylog(3,-exp(I*(b*
x+a)))*x^2+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5+24*d^4*polylog(5,-exp(
I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5-24*I*d^4*polylog(4,I*e
xp(I*(b*x+a)))/b^5+12*I/b^3*c^2*d^2*dilog(1-I*exp(I*(b*x+a)))-12*I/b^5*a^2*
d^4*polylog(2,I*exp(I*(b*x+a)))-12*I/b^5*a^2*d^4*dilog(1+I*exp(I*(b*x+a)))+
```

```
12*I/b^5*a^2*d^4*dilog(1-I*exp(I*(b*x+a)))+12*I/b^5*a^2*d^4*polylog(2,-I*ex
p(I*(b*x+a)))-12*I/b^3*c^2*d^2*dilog(1+I*exp(I*(b*x+a)))+12/b^2*d^3*c*ln(1+
I*exp(I*(b*x+a)))*x^2-12/b^2*d^3*c*ln(1-I*exp(I*(b*x+a)))*x^2+12/b^4*d^3*a^
2*c*ln(1-I*exp(I*(b*x+a)))-12/b^2*d^2*c^2*ln(1-I*exp(I*(b*x+a)))*x-12/b^3*d
^2*c^2*ln(1-I*exp(I*(b*x+a)))*a-12/b^4*d^3*a^2*c*ln(1+I*exp(I*(b*x+a)))+12/
b^2*d^2*c^2*ln(1+I*exp(I*(b*x+a)))*x+12/b^3*d^2*c^2*ln(1+I*exp(I*(b*x+a)))*
a-12*I/b^3*d^4*polylog(2,-I*exp(I*(b*x+a)))*x^2+12*I/b^3*d^4*polylog(2,I*ex
p(I*(b*x+a)))*x^2-8*I/b^5*d^4*a^3*arctan(exp(I*(b*x+a)))+8*I/b^2*d*c^3*arct
an(exp(I*(b*x+a)))-24*I/b^3*d^3*c*polylog(2,-I*exp(I*(b*x+a)))*x+24*I/b^3*d
^3*c*polylog(2,I*exp(I*(b*x+a)))*x+24*I/b^4*d^3*c*a^2*arctan(exp(I*(b*x+a))
)-24*I/b^3*d^2*c^2*a*arctan(exp(I*(b*x+a)))-1/b*c^4*ln(exp(I*(b*x+a))+1)+1/
b*c^4*ln(exp(I*(b*x+a))-1)+2*exp(I*(b*x+a))*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*
x^2+4*c^3*d*x+c^4)/b/(1+exp(2*I*(b*x+a)))-24*I/b^4*d^4*polylog(4,-exp(I*(b*
x+a)))*x-24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))+4*I/b^2*c^3*d*polylog(2,
-exp(I*(b*x+a)))+4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+24/b^4*d^4*poly
log(3,-I*exp(I*(b*x+a)))*x-24/b^4*d^4*polylog(3,I*exp(I*(b*x+a)))*x+4/b^5*d
^4*a^3*ln(1+I*exp(I*(b*x+a)))+24/b^4*d^3*c*polylog(3,-I*exp(I*(b*x+a)))-24/
b^4*d^3*c*polylog(3,I*exp(I*(b*x+a)))+4/b^2*d^4*ln(1+I*exp(I*(b*x+a)))*x^3-
4/b^2*d^4*ln(1-I*exp(I*(b*x+a)))*x^3-4/b^5*d^4*a^3*ln(1-I*exp(I*(b*x+a)))-4
/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*
d*ln(1-exp(I*(b*x+a)))*a-6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2-24/b^3*c*d^3*
polylog(3,-exp(I*(b*x+a)))*x-6/b^3*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))+6/b*c^2
*d^2*ln(1-exp(I*(b*x+a)))*x^2+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x+24*I
/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*
x^3+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b^2*c^3*d*polylog(2,exp(I*
(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a))-1)+6/b^3*c^2*d^2*a^2*ln(exp(I*(
b*x+a))-1)-4/b^2*c^3*d*a*ln(exp(I*(b*x+a))-1)+1/b*d^4*ln(1-exp(I*(b*x+a)))*
x^4-1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+
a)))*x-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-4/b*c*d^3*ln(exp(I*(b*x
+a))+1)*x^3+4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+
a)))*a^3+12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+12*I/b^2*c^2*d^2*pol
ylog(2,-exp(I*(b*x+a)))*x-24*I/b^4*c*d^3*polylog(2,-I*exp(I*(b*x+a)))*a+24*
I/b^4*a*c*d^3*dilog(1+I*exp(I*(b*x+a)))-24*I/b^4*a*c*d^3*dilog(1-I*exp(I*(b
*x+a)))+24*I/b^4*c*d^3*polylog(2,I*exp(I*(b*x+a)))*a
```

**maxima [B]** time = 2.57, size = 5695, normalized size = 12.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^4*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 4*a*c^3*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 6*a^2*c^2*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x +
a) - 1))/b^2 - 4*a^3*c*d^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(co
s(b*x + a) - 1))/b^3 + a^4*d^4*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + lo
g(cos(b*x + a) - 1))/b^4 + 2*((8*b^3*c^3*d - 24*a*b^2*c^2*d^2 + 24*a^2*b*c*
d^3 + 8*(b*x + a)^3*d^4 - 8*a^3*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 24
*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 8*(b^3*c^3*d - 3*a*b^2*c
^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b
*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x +
2*a) - (-8*I*b^3*c^3*d + 24*I*a*b^2*c^2*d^2 - 24*I*a^2*b*c*d^3 - 8*I*(b*x +
 a)^3*d^4 + 8*I*a^3*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a)^2 + (-24*I
*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*
arctan2(cos(b*x + a), sin(b*x + a) + 1) + (8*b^3*c^3*d - 24*a*b^2*c^2*d^2 +
 24*a^2*b*c*d^3 + 8*(b*x + a)^3*d^4 - 8*a^3*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x
 + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 8*(b^3*c^3*d
 - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3
 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*
```

```
cos(2*b*x + 2*a) - (-8*I*b^3*c^3*d + 24*I*a*b^2*c^2*d^2 - 24*I*a^2*b*c*d^3
- 8*I*(b*x + a)^3*d^4 + 8*I*a^3*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a
)^2 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a^2*d^4)*(b*x + a))*sin(2*
b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - (2*(b*x + a)^4*d^4 +
 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a)^2 + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b
*x + a) + 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2
*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2
+ 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^4*d
^4 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c
*d^3 + 12*I*a^2*d^4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + 24
*I*a^2*b*c*d^3 - 8*I*a^3*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x
+ a), cos(b*x + a) + 1) - (2*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a
)^3 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 8*(b^3*c^3*d -
 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a) + 2*((b*x + a)^4*d^4
+ 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b
*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - 8*I*a*d^4
)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a
)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + 24*I*a^2*b*c*d^3 - 8*I*a^3*d^4)
*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (4
*I*(b*x + a)^4*d^4 + (16*I*b*c*d^3 - 16*I*a*d^4)*(b*x + a)^3 + (24*I*b^2*c^
2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^2*d^4)*(b*x + a)^2 + (16*I*b^3*c^3*d - 48*I
*a*b^2*c^2*d^2 + 48*I*a^2*b*c*d^3 - 16*I*a^3*d^4)*(b*x + a))*cos(b*x + a) +
 (24*b^2*c^2*d^2 - 48*a*b*c*d^3 + 24*(b*x + a)^2*d^4 + 24*a^2*d^4 + 48*(b*c
*d^3 - a*d^4)*(b*x + a) + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 +
 a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-24*I*b^2*c^2
*d^2 + 48*I*a*b*c*d^3 - 24*I*(b*x + a)^2*d^4 - 24*I*a^2*d^4 + (-48*I*b*c*d^
3 + 48*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (24
*b^2*c^2*d^2 - 48*a*b*c*d^3 + 24*(b*x + a)^2*d^4 + 24*a^2*d^4 + 48*(b*c*d^3
 - a*d^4)*(b*x + a) + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2
*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (24*I*b^2*c^2*d^2
- 48*I*a*b*c*d^3 + 24*I*(b*x + a)^2*d^4 + 24*I*a^2*d^4 + (48*I*b*c*d^3 - 48
*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + (8*b^3*c
^3*d - 24*a*b^2*c^2*d^2 + 24*a^2*b*c*d^3 + 8*(b*x + a)^3*d^4 - 8*a^3*d^4 +
24*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d
^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d
^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-8*I*b^3*c^3*d + 24*I*a*b^2*c^
2*d^2 - 24*I*a^2*b*c*d^3 - 8*I*(b*x + a)^3*d^4 + 8*I*a^3*d^4 + (-24*I*b*c*d
^3 + 24*I*a*d^4)*(b*x + a)^2 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a
^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - (8*b^3*c^3*d
 - 24*a*b^2*c^2*d^2 + 24*a^2*b*c*d^3 + 8*(b*x + a)^3*d^4 - 8*a^3*d^4 + 24*(
b*c*d^3 - a*d^4)*(b*x + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*
x + a) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 -
 a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
 a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2
 + 24*I*a^2*b*c*d^3 + 8*I*(b*x + a)^3*d^4 - 8*I*a^3*d^4 + (24*I*b*c*d^3 - 2
4*I*a*d^4)*(b*x + a)^2 + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^2*d^4)
*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-I*(b*x + a)^4*d^4
+ (-4*I*b*c*d^3 + 4*I*a*d^4)*(b*x + a)^3 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d
^3 - 6*I*a^2*d^4)*(b*x + a)^2 + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 - 12*I
*a^2*b*c*d^3 + 4*I*a^3*d^4)*(b*x + a) + (-I*(b*x + a)^4*d^4 + (-4*I*b*c*d^3
 + 4*I*a*d^4)*(b*x + a)^3 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^
4)*(b*x + a)^2 + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 - 12*I*a^2*b*c*d^3 +
4*I*a^3*d^4)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^4*d^4 + 4*(b*c*d^3 -
a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 +
4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*sin(2*
b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*
```

(b*x + a)^4*d^4 + (4*I*b*c*d^3 - 4*I*a*d^4)*(b*x + a)^3 + (6*I*b^2*c^2*d^2
- 12*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a)^2 + (4*I*b^3*c^3*d - 12*I*a*b^2*c
^2*d^2 + 12*I*a^2*b*c*d^3 - 4*I*a^3*d^4)*(b*x + a) + (I*(b*x + a)^4*d^4 + (
4*I*b*c*d^3 - 4*I*a*d^4)*(b*x + a)^3 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 +
6*I*a^2*d^4)*(b*x + a)^2 + (4*I*b^3*c^3*d - 12*I*a*b^2*c^2*d^2 + 12*I*a^2*b
*c*d^3 - 4*I*a^3*d^4)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^4*d^4 + 4*(b
*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x
+ a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a
))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) +
1) - (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 - 12*I*a^2*b*c*d^3 - 4*I*(b*x +
a)^3*d^4 + 4*I*a^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2 + (-12*I*
b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4)*(b*x + a) + (-4*I*b^3*c^3*d +
12*I*a*b^2*c^2*d^2 - 12*I*a^2*b*c*d^3 - 4*I*(b*x + a)^3*d^4 + 4*I*a^3*d^4 +
(-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2 + (-12*I*b^2*c^2*d^2 + 24*I*a*b*c
*d^3 - 12*I*a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 4*(b^3*c^3*d - 3*a*b^2*c
^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b
*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*x +
2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (4*I*b^3*
c^3*d - 12*I*a*b^2*c^2*d^2 + 12*I*a^2*b*c*d^3 + 4*I*(b*x + a)^3*d^4 - 4*I*a
^3*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^2 + (12*I*b^2*c^2*d^2 - 24*I
*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a) + (4*I*b^3*c^3*d - 12*I*a*b^2*c^2*d^2
+ 12*I*a^2*b*c*d^3 + 4*I*(b*x + a)^3*d^4 - 4*I*a^3*d^4 + (12*I*b*c*d^3 - 12
*I*a*d^4)*(b*x + a)^2 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*
(b*x + a))*cos(2*b*x + 2*a) - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^
3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^
2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a
)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (48*I*d^4*cos(2*b*x + 2*a) - 4
8*d^4*sin(2*b*x + 2*a) + 48*I*d^4)*polylog(5, -e^(I*b*x + I*a)) - (-48*I*d^
4*cos(2*b*x + 2*a) + 48*d^4*sin(2*b*x + 2*a) - 48*I*d^4)*polylog(5, e^(I*b*
x + I*a)) - 48*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) + d^4)*polylo
g(4, I*e^(I*b*x + I*a)) + 48*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a)
+ d^4)*polylog(4, -I*e^(I*b*x + I*a)) - (48*b*c*d^3 + 48*(b*x + a)*d^4 - 4
8*a*d^4 + 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) + (48*I*b*c
*d^3 + 48*I*(b*x + a)*d^4 - 48*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, -e^(I*
b*x + I*a)) + (48*b*c*d^3 + 48*(b*x + a)*d^4 - 48*a*d^4 + 48*(b*c*d^3 + (b*
x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) - (-48*I*b*c*d^3 - 48*I*(b*x + a)*d^4
+ 48*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, e^(I*b*x + I*a)) - (-48*I*b*c*d^
3 - 48*I*(b*x + a)*d^4 + 48*I*a*d^4 + (-48*I*b*c*d^3 - 48*I*(b*x + a)*d^4 +
48*I*a*d^4)*cos(2*b*x + 2*a) + 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*
b*x + 2*a))*polylog(3, I*e^(I*b*x + I*a)) - (48*I*b*c*d^3 + 48*I*(b*x + a)*
d^4 - 48*I*a*d^4 + (48*I*b*c*d^3 + 48*I*(b*x + a)*d^4 - 48*I*a*d^4)*cos(2*b
*x + 2*a) - 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(
3, -I*e^(I*b*x + I*a)) - (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*(b*x +
a)^2*d^4 - 24*I*a^2*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a) + (-24*I*b
^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*(b*x + a)^2*d^4 - 24*I*a^2*d^4 + (-48*I*
b*c*d^3 + 48*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 24*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b
*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) - (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^
3 + 24*I*(b*x + a)^2*d^4 + 24*I*a^2*d^4 + (48*I*b*c*d^3 - 48*I*a*d^4)*(b*x
+ a) + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*(b*x + a)^2*d^4 + 24*I*a^2
*d^4 + (48*I*b*c*d^3 - 48*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 24*(b^2*c^
2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x
+ a))*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) + 4*((b*x + a)^4*d^4 +
4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(
b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x
+ a))*sin(b*x + a))/(-2*I*b^4*cos(2*b*x + 2*a) + 2*b^4*sin(2*b*x + 2*a) -
2*I*b^4))/b

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(cos(a + b*x)^2*sin(a + b*x)),x)
[Out] \text{Hanged}
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a)**2,x)
[Out] Timed out
```

# 3.267 $\int (c + dx)^3 \csc(a + bx) \sec^2(a + bx) \, dx$

**Optimal**. Leaf size=343

$$\frac{6d^3\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4} - \frac{6d^3\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4} - \frac{6id^3\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4} + \frac{6id^3\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4} - \frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2}{}$$

```
[Out] 6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-2*(d*x+c)^3*arctanh(exp(I*(b*x+a
)))/b+3*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*polylo
g(2,-I*exp(I*(b*x+a)))/b^3+6*I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3-
3*I*d*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-exp(
I*(b*x+a)))/b^3+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*ex
p(I*(b*x+a)))/b^4+6*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-6*I*d^3*polyl
og(4,-exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+(d*x+c)^3*s
ec(b*x+a)/b
```

**Rubi [A]** time = 0.57, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2622, 321, 207, 4420, 6741, 12, 6742, 6273, 4183, 2531, 6609, 2282, 6589, 4181}

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} - \frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^3*ArcTanh[
E^(I*(a + b*x))])/b + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^
2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2
*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d*(c + d*x)^2*PolyLo
g[2, E^(I*(a + b*x))])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])
/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog[3, I*E
^(I*(a + b*x))])/b^4 + (6*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 -
((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, E^(I*(
a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - (3d)\int (c$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - (3d)\int \frac{(c}{}$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{(3d)\int (c}{}$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{(3d)\int \left(-\right.}{}$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} + \frac{(3d)\int (c}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{\int b(-c-dx)^3}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6id^2(c}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+}{}$$

**Mathematica [A]** time = 1.34, size = 473, normalized size = 1.38

$$\frac{b^3(c+dx)^3\sec(a+bx) - 2b^3(c+dx)^3\tanh^{-1}(\cos(a+bx)+i\sin(a+bx)) - 3d\left(-2ib^2c^2\tan^{-1}\left(e^{i(a+bx)}\right) + 2b^2c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] - 3*d*((-2*I)*b^
2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] + b^
2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x)
)] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2
, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))]
 - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3, I*E^(I*(a + b*
x))]) + (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]]
 + (2*I)*b*d*(c + d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*P
olyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyL
og[2, Cos[a + b*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a
 + b*x] + I*Sin[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]
) + b^3*(c + d*x)^3*Sec[a + b*x])/b^4
```

**fricas [C]** time = 0.65, size = 1697, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*I*d^3*
cos(b*x + a)*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x +
a)*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*cos(b*x + a)*polylog
(4, -cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x + a)*polylog(4, -cos(
b*x + a) - I*sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) +
 sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a
)) + 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*
cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*d^3*x^2
 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*cos(b*x + a)*dilog(cos(b*x + a) + I*sin
(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*cos(b*x +
a)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b
*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*
cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c
*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-6*I*b*d^3*x -
6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2
*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*cos(b*x + a)*dilog(-cos(b*x + a
) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*c
os(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*
x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*
x + a) + I*sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d
*x + b^3*c^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 3*(b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x +
a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x +
 a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
 + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) +
1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*l
og(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2
*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)
+ (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)*log(-1/2
*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a
) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) +
 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a)
+ I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*s
in(b*x + a) + 1) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-
cos(b*x + a) - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)*pol
ylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)
```

```
*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x
+ a)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos
(b*x + a)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^4*cos(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \csc(bx+a) \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a)^2, x)
```

**maple [B]** time = 0.47, size = 1152, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x)
```

```
[Out] 6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^
4-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I/b^3*d^2*c*a*arctan(exp(I*(
b*x+a)))+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+6*I*d^3*x*polylog(2,I*exp(I*
(b*x+a)))/b^3-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3-1/b^4*d^3*a^3*ln(e
xp(I*(b*x+a))-1)-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*c*d^2*polylog
(3,exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-6/b^3*d^3*polylog(
3,-exp(I*(b*x+a)))*x-6*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4-6*I/b^3*d^2*c*d
ilog(1+I*exp(I*(b*x+a)))+6*I/b^3*d^2*c*dilog(1-I*exp(I*(b*x+a)))+6*I/b^4*a*
d^3*dilog(1+I*exp(I*(b*x+a)))-6*I/b^4*a*d^3*dilog(1-I*exp(I*(b*x+a)))+6*I/b
^4*d^3*polylog(2,I*exp(I*(b*x+a)))*a-6*I/b^4*d^3*polylog(2,-I*exp(I*(b*x+a)
))*a+1/b*c^3*ln(exp(I*(b*x+a))-1)-1/b*c^3*ln(exp(I*(b*x+a))+1)+2*exp(I*(b*x
+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(1+exp(2*I*(b*x+a)))+6*I/b^2*c*d
^2*polylog(2,-exp(I*(b*x+a)))*x+6/b^2*d^2*c*ln(1+I*exp(I*(b*x+a)))*x+6*I/b^
4*d^3*a^2*arctan(exp(I*(b*x+a)))+6*I/b^2*d*c^2*arctan(exp(I*(b*x+a)))+3*I/b
^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a
)))-6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a+6/b^3*d^2*c*ln(1+I*exp(I*(b*x+a)))
*a-6/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1
)-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,exp(I*(b*x+
a)))*x^2-3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+
3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+3/b
*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/b^2*c^
2*d*a*ln(exp(I*(b*x+a))-1)+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-
exp(I*(b*x+a)))*a^3-1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+3/b^2*d^3*ln(1+I*exp(I
*(b*x+a)))*x^2-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)))-3/b^2*d^3*ln(1-I*exp(I*
(b*x+a)))*x^2+3/b^4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))
```

**maxima [B]** time = 1.14, size = 3205, normalized size = 9.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 3*a*c^2*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 3*a^2*c*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a)
 - 1))/b^2 - a^3*d^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x
+ a) - 1))/b^3 + 2*((6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2
*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x +
```

```
a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6
*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*
b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), si
n(b*x + a) + 1) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d
^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a
)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I
*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*
c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin
(b*x + a) + 1) - (2*(b*x + a)^3*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(
b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) + 2*((b*x + a)^3*d^3 + 3*(b*c*
d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))
*cos(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x +
 a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*a^2*d^3)*(b*x + a))*sin(2*b*x
 + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*(b*x + a)^3*d^3 + 6*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x +
 a) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3
+ (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 +
 6*I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x +
 a) + 1) - (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^2 +
 (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3)*(b*x + a))*cos(b*x + a) +
 (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 -
a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)
*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (12*b*c*d^2 + 12*(b*x + a)*d^
3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (12*
I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-I*e^(
I*b*x + I*a)) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d^3
 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^
2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*b
^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*
d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - (6
*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d^3 + 12*(b*c*d^2 - a
*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 +
2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*b^2*c^2*d - 12*I*a*b
*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b
*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-I*(b*x + a)^3*d^3 + (
-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3
*I*a^2*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b
*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*a^2*d^3)*(b*x + a))*cos(2
*b*x + 2*a) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*(b*x + a)^3*d^3 + (3*I*b*c*d
^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*a^2*d^3)
*(b*x + a) + (I*(b*x + a)^3*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (
3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) -
((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 - 2*cos(b*x + a) + 1) - (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*(b*x +
a)^2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (-3*I*b^2*c
^2*d + 6*I*a*b*c*d^2 - 3*I*(b*x + a)^2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 +
6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x
+ a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log
(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (3*I*b^2*c^2*d - 6
*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3
)*(b*x + a) + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^
2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*(b^2*c^2*
d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)
)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) +
1) - 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) + d^3)*polylog(4, -e
```

```
^(I*b*x + I*a)) + 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) + d^3)*
polylog(4, e^(I*b*x + I*a)) - (-12*I*d^3*cos(2*b*x + 2*a) + 12*d^3*sin(2*b*
x + 2*a) - 12*I*d^3)*polylog(3, I*e^(I*b*x + I*a)) - (12*I*d^3*cos(2*b*x +
2*a) - 12*d^3*sin(2*b*x + 2*a) + 12*I*d^3)*polylog(3, -I*e^(I*b*x + I*a)) -
 (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(
b*x + a)*d^3 + 12*I*a*d^3)*cos(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 -
 a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) - (12*I*b*c*d^2 + 12
*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a
*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2
*a))*polylog(3, e^(I*b*x + I*a)) + 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(b*x + a
))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a) - 2*I*b^3))/b
```

**mupad** [F(-1)]  time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

**sympy** [F]  time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*csc(a + b*x)*sec(a + b*x)**2, x)
```

# 3.268 $\int (c+dx)^2 \csc(a+bx) \sec^2(a+bx)\,dx$

**Optimal**. Leaf size=219

$$-\frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{2d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+\cdots}{}$$

```
[Out] 4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*(d*x+c)^2*arctanh(exp(I*(b*x+a))
)/b+2*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*d^2*polylog(2,-I*exp(I
*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3-2*I*d*(d*x+c)*polylo
g(2,exp(I*(b*x+a)))/b^2-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(
3,exp(I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b
```

**Rubi [A]** time = 0.38, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {2622, 321, 207, 4420, 6741, 12, 6742, 6273, 4183, 2531, 2282, 6589, 4181, 2279, 2391}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}\left(\cdots\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^2*ArcTanh[E^
(I*(a + b*x))])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 -
((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*
E^(I*(a + b*x))])/b^3 - ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2
 - (2*d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, E^(I*(a + b
*x))])/b^3 + ((c + d*x)^2*Sec[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - (2d)\int (c+d$$

$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - (2d)\int \frac{(c+}{}$$

$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2d)\int (c+d}{}$$

$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2d)\int \left(-(c\right.}{}$$

$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{(2d)\int (c+d}{}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{\int b(c+dx)^2\csc(}{b}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{\left(2id^2\right)\text{Subst}\left(\int \frac{1}{}\right.}{}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2id^2\text{Li}_2\left(-i\right.}{b^3}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)}{}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)}{}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)}{}$$

**Mathematica [A]** time = 2.47, size = 317, normalized size = 1.45

$$\frac{b^2(c+dx)^2\log\left(1-e^{i(a+bx)}\right) - b^2(c+dx)^2\log\left(1+e^{i(a+bx)}\right) + b^2(c+dx)^2\sec(a+bx) + 2id\left(b(c+dx)\text{Li}_2\left(-e^{i(a+bx}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcT
anh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))
] - b^2*(c + d*x)^2*Log[1 + E^(I*(a + b*x))] + (2*d^2*Csc[a]*((b*x - ArcTan
```

```
[Cot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcT
an[Cot[a]]))]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2,
 E^(I*(b*x - ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + (2*I)*d*(b*(c + d*x)*Poly
Log[2, -E^(I*(a + b*x))] + I*d*PolyLog[3, -E^(I*(a + b*x))]) + 2*d*((-I)*b*
(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + d*PolyLog[3, E^(I*(a + b*x))]) + b^
2*(c + d*x)^2*Sec[a + b*x])/b^3
```

**fricas [C]** time = 0.57, size = 1031, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*I*d^2*cos(b*x + a)*dilog(I
*cos(b*x + a) + sin(b*x + a)) + 2*I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) -
 sin(b*x + a)) - 2*I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a))
 - 2*I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b
*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)*poly
log(3, cos(b*x + a) - I*sin(b*x + a)) - 2*d^2*cos(b*x + a)*polylog(3, -cos(
b*x + a) + I*sin(b*x + a)) - 2*d^2*cos(b*x + a)*polylog(3, -cos(b*x + a) -
I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)*dilog(cos(b*x + a
) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)*dilog(cos(b*x
+ a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)*dilog(-cos
(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)*dilog(
-cos(b*x + a) - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*c*d - a*d^2)*cos(b*
x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 2*(b*c*d -
 a*d^2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I) - 2*(b*d^2*x +
a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*d^2*x + a
*d^2)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 2*(b*d^2*x + a*
d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*d^2*x + a*
d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) +
1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)*log(-1/2*cos(b*x + a) -
 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d
^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*c*d - a*d^2
)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b
^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x
+ a) + 1) + 2*(b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x +
a) + I))/(b^3*cos(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \csc(bx + a) \sec(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a)^2, x)
```

**maple [B]** time = 0.27, size = 568, normalized size = 2.59

$$\frac{d^2 a^2 \ln\left(e^{i(bx+a)} - 1\right)}{b^3} + \frac{d^2 \ln\left(1 - e^{i(bx+a)}\right) x^2}{b} - \frac{d^2 \ln\left(1 - e^{i(bx+a)}\right) a^2}{b^3} - \frac{d^2 \ln\left(e^{i(bx+a)} + 1\right) x^2}{b} + \frac{2d^2 \ln\left(1 + ie^{i(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x)
```

```
[Out] 1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d
^2*ln(1-exp(I*(b*x+a)))*a^2-1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2-2*d^2*polylog(
3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3-2/b^3*d^2*ln(1-I
*exp(I*(b*x+a)))*a+2*I/b^3*d^2*dilog(1-I*exp(I*(b*x+a)))-2*I/b^3*d^2*dilog(
1+I*exp(I*(b*x+a)))+2/b^2*d^2*ln(1+I*exp(I*(b*x+a)))*x+2/b^3*d^2*ln(1+I*exp
(I*(b*x+a)))*a-2/b^2*d^2*ln(1-I*exp(I*(b*x+a)))*x+1/b*c^2*ln(exp(I*(b*x+a))
-1)-1/b*c^2*ln(exp(I*(b*x+a))+1)+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*l
n(1-exp(I*(b*x+a)))*a-2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*a*ln(exp(I*(
b*x+a))-1)-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,ex
p(I*(b*x+a)))+4*I*d/b^2*c*arctan(exp(I*(b*x+a)))-4*I*d^2/b^3*a*arctan(exp(I
*(b*x+a)))+2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(1+exp(2*I*(b*x+a)))+2*
I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)
))
```

**maxima** [B] time = 0.69, size = 1598, normalized size = 7.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 2*a*c*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
 + a^2*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b^2 + 2*((4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 -
 a*d^2)*cos(2*b*x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin
(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (4*b*c*d + 4*(b*x
+ a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (
-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b
*x + a), -sin(b*x + a) + 1) - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x +
 a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) +
(2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))
*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*(b*x + a)^2*d^2 + 4*(b*c*d -
a*d^2)*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*
b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(
2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (4*I*(b*x + a)^2*d
^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a))*cos(b*x + a) + 4*(d^2*cos(2*b*x + 2
*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*dilog(I*e^(I*b*x + I*a)) - 4*(d^2*cos(2
*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*dilog(-I*e^(I*b*x + I*a)) + (4*
b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2
*b*x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a)
)*dilog(-e^(I*b*x + I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d
 + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2
 - 4*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-I*(b*x + a)^2*d^
2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d
+ 2*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*
d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*c
os(b*x + a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) +
 (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) -
 ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(
b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-2*I*b*c*d - 2*I*(b*x
+ a)*d^2 + 2*I*a*d^2 + (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*cos(2*b
*x + 2*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x
 + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (2*I*b*c*d + 2*I*(b*x + a)
*d^2 - 2*I*a*d^2 + (2*I*b*c*d + 2*I*(b*x + a)*d^2 - 2*I*a*d^2)*cos(2*b*x +
2*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (-4*I*d^2*cos(2*b*x + 2*a) + 4*
d^2*sin(2*b*x + 2*a) - 4*I*d^2)*polylog(3, -e^(I*b*x + I*a)) - (4*I*d^2*cos
```

```
(2*b*x + 2*a) - 4*d^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, e^(I*b*x + I*a
)) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(b*x + a))/(-2*I*
b^2*cos(2*b*x + 2*a) + 2*b^2*sin(2*b*x + 2*a) - 2*I*b^2))/b
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \csc(a + bx) \sec^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x)**2, x)
```

# 3.269 $\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx$

**Optimal**. Leaf size=113

$$\frac{id\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{c\sec(a+bx)}{b}-\frac{c\tanh^{-1}(\cos(a+bx))}{b}+\frac{dx\sec(a+bx)}{b}-$$

```
[Out] -2*d*x*arctanh(exp(I*(b*x+a)))/b-c*arctanh(cos(b*x+a))/b-d*arctanh(sin(b*x+
a))/b^2+I*d*polylog(2,-exp(I*(b*x+a)))/b^2-I*d*polylog(2,exp(I*(b*x+a)))/b^
2+c*sec(b*x+a)/b+d*x*sec(b*x+a)/b
```

**Rubi [A]** time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2622, 321, 207, 4420, 6271, 12, 4183, 2279, 2391, 3770}

$$\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\sec(a+bx)}{b}-\frac{(c+dx)\tanh^{-1}(\text{c}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (-2*d*x*ArcTanh[E^(I*(a + b*x))])/b + (d*x*ArcTanh[Cos[a + b*x]])/b - ((c +
 d*x)*ArcTanh[Cos[a + b*x]])/b - (d*ArcTanh[Sin[a + b*x]])/b^2 + (I*d*PolyL
og[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2 + ((c
+ d*x)*Sec[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
```

```
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)\sec(a+bx)}{b}-d\int\left(-\frac{\tanh\right.$$
$$= -\frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)\sec(a+bx)}{b}+\frac{d\int\tanh^{-1}(}{}$$
$$= \frac{dx\tanh^{-1}(\cos(a+bx))}{b}-\frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b}-\frac{d\tanh^{-1}(s}{b}$$
$$= \frac{dx\tanh^{-1}(\cos(a+bx))}{b}-\frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b}-\frac{d\tanh^{-1}(s}{b}$$
$$= -\frac{2dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{dx\tanh^{-1}(\cos(a+bx))}{b}-\frac{(c+dx)\tanh^{-1}(c}{b}$$
$$= -\frac{2dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{dx\tanh^{-1}(\cos(a+bx))}{b}-\frac{(c+dx)\tanh^{-1}(c}{b}$$
$$= -\frac{2dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{dx\tanh^{-1}(\cos(a+bx))}{b}-\frac{(c+dx)\tanh^{-1}(c}{b}$$

**Mathematica [A]** time = 0.49, size = 212, normalized size = 1.88

$$\frac{d\left(i\left(\text{Li}_2\left(-e^{i(a+bx)}\right)-\text{Li}_2\left(e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)\right)}{b^2}-\frac{ad\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] -((c*Log[Cos[(a + b*x)/2]])/b) + (d*Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]
])/b^2 + (c*Log[Sin[(a + b*x)/2]])/b - (d*Log[Cos[(a + b*x)/2] + Sin[(a + b
*x)/2]])/b^2 - (a*d*Log[Tan[(a + b*x)/2]])/b^2 + (d*((a + b*x)*(Log[1 - E^(
I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x))]
 - PolyLog[2, E^(I*(a + b*x))])))/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[a + b
*x])/b
```

**fricas [B]** time = 0.47, size = 366, normalized size = 3.24

$$2\,bdx - i\,d\cos\left(bx + a\right)\mathrm{Li}_2\left(\cos\left(bx + a\right) + i\,\sin\left(bx + a\right)\right) + i\,d\cos\left(bx + a\right)\mathrm{Li}_2\left(\cos\left(bx + a\right) - i\,\sin\left(bx + a\right)\right) - i\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x - I*d*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*
cos(b*x + a)*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*cos(b*x + a)*dilog(
-cos(b*x + a) + I*sin(b*x + a)) + I*d*cos(b*x + a)*dilog(-cos(b*x + a) - I*
sin(b*x + a)) - (b*d*x + b*c)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a
) + 1) - (b*d*x + b*c)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1)
+ (b*c - a*d)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2
) + (b*c - a*d)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1
/2) + (b*d*x + a*d)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) +
(b*d*x + a*d)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - d*cos(
b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) + 2*
b*c)/(b^2*cos(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\csc\left(bx + a\right)\sec\left(bx + a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a)^2, x)
```

**maple [A]** time = 0.11, size = 160, normalized size = 1.42

$$\frac{2\,\mathrm{e}^{i(bx+a)}\,(dx + c)}{b\left(1 + \mathrm{e}^{2i(bx+a)}\right)} + \frac{c\ln\left(\mathrm{e}^{i(bx+a)} - 1\right)}{b} - \frac{c\ln\left(\mathrm{e}^{i(bx+a)} + 1\right)}{b} + \frac{2id\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^2} + \frac{id\,\mathrm{dilog}\left(\mathrm{e}^{i(bx+a)}\right)}{b^2} + \frac{id\,\mathrm{dilog}\left(\mathrm{e}^{i(bx+}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x)
```

```
[Out] 2*exp(I*(b*x+a))*(d*x+c)/b/(1+exp(2*I*(b*x+a)))+1/b*c*ln(exp(I*(b*x+a))-1)-
1/b*c*ln(exp(I*(b*x+a))+1)+2*I/b^2*d*arctan(exp(I*(b*x+a)))+I/b^2*d*dilog(e
xp(I*(b*x+a)))+I/b^2*d*dilog(exp(I*(b*x+a))+1)-1/b*d*ln(exp(I*(b*x+a))+1)*x
-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

**maxima [B]** time = 0.62, size = 806, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d)*arctan2(2*(cos(b*x + 2*
a)*cos(a) + sin(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*
```

```
sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2), (c
os(b*x + 2*a)^2 - cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos(b*x + 2*a)^2
 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)
*sin(a) + sin(a)^2)) + (2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a)
- (-2*I*b*d*x - 2*I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x +
a) + 1) - (2*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin(2*b*x + 2*a) + 2*b*c)*arcta
n2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*
sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (-4*
I*b*d*x - 4*I*b*c)*cos(b*x + a) - 2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2
*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x +
2*a) + d)*dilog(e^(I*b*x + I*a)) - (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos
(2*b*x + 2*a) - (b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos
(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*cos(b*x + a) + 1) - (-I*d*cos(2*b*x + 2*a) + d*sin(2*b*x + 2*a
) - I*d)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b
*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a
)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a)
+ sin(a)^2)) - 4*(b*d*x + b*c)*sin(b*x + a))/(-2*I*b^2*cos(2*b*x + 2*a) + 2
*b^2*sin(2*b*x + 2*a) - 2*I*b^2)
```

**mupad** [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)*sec(a + b*x)**2, x)
```

## 3.270 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx} dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\frac{\csc(a+bx)\sec^2(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

**Rubi [A]** time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x),x]
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx} dx = \int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx} dx$$

**Mathematica [A]** time = 9.65, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]
```

**fricas [A]** time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)\sec(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)*sec(b*x + a)^2/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\sec(bx+a)^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)*sec(b*x + a)^2/(d*x + c), x)
```

**maple** [**A**] time = 2.33, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)\left(\sec^{2}\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
[Out] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

**maxima** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

**mupad** [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\cos\left(a+b\,x\right)}^2\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)), x)
```

**sympy** [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(a+bx\right)\sec^{2}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**2/(d*x+c),x)
[Out] Integral(csc(a + b*x)*sec(a + b*x)**2/(c + d*x), x)
```

# 3.271 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

**Rubi** [A] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2,x]
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx = \int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$$

**Mathematica** [A] time = 9.76, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

**fricas** [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)\sec(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] sage0*x
```

**maple** [A] time = 2.56, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\left(\sec^2(bx+a)\right)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(a+bx\right)\sec^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)**2/(c + d*x)**2, x)
```

# 3.272 $\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$

**Optimal**. Leaf size=27

$$\text{Int}\left(\csc^2(a+bx)\sec^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

**Rubi** [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$$

**Mathematica** [A] time = 2.63, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]
```

**fricas** [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \csc\left(bx+a\right)^2 \sec\left(bx+a\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc\left(bx+a\right)^2 \sec\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

**maple** [A] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^2\left(bx+a\right)\right)\left(\sec^2\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2 \sin(a+b\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

```
[Out] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.273 $\int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx)\,dx$

**Optimal**. Leaf size=118

$$\frac{3d^3\text{Li}_3\left(e^{4i(a+bx)}\right)}{8b^4}-\frac{3id^2(c+dx)\text{Li}_2\left(e^{4i(a+bx)}\right)}{2b^3}+\frac{3d(c+dx)^2\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^3\cot(2a+2bx)}{b}-\frac{2i(c+dx}{b}$$

```
[Out] -2*I*(d*x+c)^3/b-2*(d*x+c)^3*cot(2*b*x+2*a)/b+3*d*(d*x+c)^2*ln(1-exp(4*I*(b
*x+a)))/b^2-3/2*I*d^2*(d*x+c)*polylog(2,exp(4*I*(b*x+a)))/b^3+3/8*d^3*polyl
og(3,exp(4*I*(b*x+a)))/b^4
```

**Rubi [A]** time = 0.28, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4419, 4184, 3717, 2190, 2531, 2282, 6589}

$$-\frac{3id^2(c+dx)\text{PolyLog}\left(2,e^{4i(a+bx)}\right)}{2b^3}+\frac{3d^3\text{PolyLog}\left(3,e^{4i(a+bx)}\right)}{8b^4}+\frac{3d(c+dx)^2\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^3\cot(2a}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-2*I)*(c + d*x)^3)/b - (2*(c + d*x)^3*Cot[2*a + 2*b*x])/b + (3*d*(c + d*x
)^2*Log[1 - E^((4*I)*(a + b*x))])/b^2 - (((3*I)/2)*d^2*(c + d*x)*PolyLog[2,
 E^((4*I)*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((4*I)*(a + b*x))])/(8*b^4
)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\csc^2(a+bx)\sec^2(a+bx)\,dx = 4\int (c+dx)^3\csc^2(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{(6d)\int (c+dx)^2\cot(2a+2bx)\,dx}{b}$$
$$= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} - \frac{(12id)\int \frac{e^{2i(2a+2bx)}(c+dx)^2}{1-e^{2i(2a+2bx)}}\,d}{b}$$
$$= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{4i(}\right.}{b^2}$$
$$= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{4i(}\right.}{b^2}$$
$$= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{4i(}\right.}{b^2}$$
$$= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{4i(}\right.}{b^2}$$

**Mathematica [B]** time = 2.13, size = 285, normalized size = 2.42

$$\frac{-\frac{8ib^3(c+dx)^3}{-1+e^{4ia}} + 4b^3\csc(2a)\sin(2bx)(c+dx)^3\csc(2(a+bx)) + 6b^2d(c+dx)^2\log\left(1-e^{-i(a+bx)}\right) + 6b^2d(c+dx)^2\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (((-8*I)*b^3*(c + d*x)^3)/(-1 + E^((4*I)*a)) + 6*b^2*d*(c + d*x)^2*Log[1 -
E^((-I)*(a + b*x))] + 6*b^2*d*(c + d*x)^2*Log[1 + E^((-I)*(a + b*x))] + 6*b
^2*d*(c + d*x)^2*Log[1 + E^((-2*I)*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*Pol
yLog[2, -E^((-I)*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*PolyLog[2, E^((-I)*(a
 + b*x))] + (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((-2*I)*(a + b*x))] + 12*d^
3*PolyLog[3, -E^((-I)*(a + b*x))] + 12*d^3*PolyLog[3, E^((-I)*(a + b*x))] +
 3*d^3*PolyLog[3, -E^((-2*I)*(a + b*x))] + 4*b^3*(c + d*x)^3*Csc[2*a]*Csc[2
*(a + b*x)]*Sin[2*b*x])/(2*b^4)
```

**fricas [C]** time = 0.62, size = 1627, normalized size = 13.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")

[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*cos(b*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3*cos(b*x + a)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 6*d^3*cos(b*x + a)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3*cos(b*x + a)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2)/(b^4*cos(b*x + a)*sin(b*x + a))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \csc(bx+a)^2 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a)^2, x)

**maple [B]** time = 0.13, size = 687, normalized size = 5.82

$$-\frac{24id^2cax}{b^2}+\frac{3d\,c^2\ln\left(1+e^{2i(bx+a)}\right)}{b^2}-\frac{12d\,c^2\ln\left(e^{i(bx+a)}\right)}{b^2}-\frac{12d^3a^2\ln\left(e^{i(bx+a)}\right)}{b^4}+\frac{3d^3\ln\left(1+e^{2i(bx+a)}\right)x^2}{b^2}+\frac{8id^3a^3}{b^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

```
[Out] 3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4+6*d^3*polylog(3,-exp(I*(b*x+a)))/b
^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+3*d/b^2*c^2*ln(1+exp(2*I*(b*x+a)))-1
2*d/b^2*c^2*ln(exp(I*(b*x+a)))-12*d^3/b^4*a^2*ln(exp(I*(b*x+a)))+3*d^3/b^2*
ln(1+exp(2*I*(b*x+a)))*x^2+3/b^2*c^2*d*ln(exp(I*(b*x+a))-1)+3/b^2*c^2*d*ln(
exp(I*(b*x+a))+1)+3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-1)+3/b^2*d^3*ln(exp(I*(b*
x+a))+1)*x^2+3/b^2*d^3*ln(1-exp(I*(b*x+a)))*x^2-3/b^4*d^3*ln(1-exp(I*(b*x+a
)))*a^2-6/b^3*c*d^2*a*ln(exp(I*(b*x+a))-1)+8*I*d^3/b^4*a^3-4*I*d^3/b*x^3-6*
I*d^2/b^3*c*polylog(2,-exp(I*(b*x+a)))+12*I*d^3/b^3*a^2*x-12*I*d^2/b*c*x^2-
12*I*d^2/b^3*c*a^2-6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x-6*I/b^3*d^3*pol
ylog(2,exp(I*(b*x+a)))*x+6/b^2*d^2*c*ln(exp(I*(b*x+a))+1)*x-6*I/b^3*d^2*c*p
olylog(2,exp(I*(b*x+a)))-24*I*d^2/b^2*c*a*x-4*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*
d*x+c^3)/b/(1+exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))-1)+6/b^2*d^2*c*ln(1-exp(I
*(b*x+a)))*x+6/b^3*d^2*c*ln(1-exp(I*(b*x+a)))*a+24*d^2/b^3*c*a*ln(exp(I*(b*
x+a)))+6*d^2/b^2*c*ln(1+exp(2*I*(b*x+a)))*x-3*I*d^3/b^3*polylog(2,-exp(2*I*
(b*x+a)))*x-3*I*d^2/b^3*c*polylog(2,-exp(2*I*(b*x+a)))
```

**maxima [B]** time = 0.74, size = 2355, normalized size = 19.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^3*(1/tan(b*x + a) - tan(b*x + a)) - 6*a*c^2*d*(1/tan(b*x + a) - t
an(b*x + a))/b + 6*a^2*c*d^2*(1/tan(b*x + a) - tan(b*x + a))/b^2 - 2*a^3*d^
3*(1/tan(b*x + a) - tan(b*x + a))/b^3 - 3*((cos(4*b*x + 4*a)^2 + sin(4*b*x
+ 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2 + 2*cos(2*b*x + 2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 -
2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a
) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)
*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*si
n(4*b*x + 4*a))*c^2*d/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b
*x + 4*a) + 1)*b) + 6*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b
*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) +
 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*a*c*
d^2/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b^2
) - 3*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*l
og(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (cos
(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4
*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*a^2*d^3/((cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b^3) + 2*((6*(b*x +
 a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^
2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d
^2 - 12*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos
(2*b*x + 2*a) + 1) + (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) -
```

```
6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (6*I
*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))
*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (6*(b*x + a)^2*d^3 + 12*(b*c*d^2
 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*c
os(4*b*x + 4*a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x
 + a))*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 8*((b*x
 + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(4*b*x + 4*a) - (6*b*c*d^
2 + 6*(b*x + a)*d^3 - 6*a*d^3 - 6*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(4*b
*x + 4*a) + (-6*I*b*c*d^2 - 6*I*(b*x + a)*d^3 + 6*I*a*d^3)*sin(4*b*x + 4*a)
)*dilog(-e^(2*I*b*x + 2*I*a)) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 -
 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(4*b*x + 4*a) + (-12*I*b*c*d^2 - 1
2*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(4*b*x + 4*a))*dilog(-e^(I*b*x + I*a)) -
 (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 -
a*d^3)*cos(4*b*x + 4*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)
*sin(4*b*x + 4*a))*dilog(e^(I*b*x + I*a)) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c
*d^2 - 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a
*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3
)*(b*x + a))*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
+ 2*cos(2*b*x + 2*a) + 1) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3
)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))
*cos(4*b*x + 4*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin
(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) -
(3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a
)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 3*((b*x
+ a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-3*I*d^3*cos(4*b*x + 4*a) +
 3*d^3*sin(4*b*x + 4*a) + 3*I*d^3)*polylog(3, -e^(2*I*b*x + 2*I*a)) - (-12*
I*d^3*cos(4*b*x + 4*a) + 12*d^3*sin(4*b*x + 4*a) + 12*I*d^3)*polylog(3, -e^
(I*b*x + I*a)) - (-12*I*d^3*cos(4*b*x + 4*a) + 12*d^3*sin(4*b*x + 4*a) + 12
*I*d^3)*polylog(3, e^(I*b*x + I*a)) - (-8*I*(b*x + a)^3*d^3 + (-24*I*b*c*d^
2 + 24*I*a*d^3)*(b*x + a)^2)*sin(4*b*x + 4*a))/(-2*I*b^3*cos(4*b*x + 4*a) +
 2*b^3*sin(4*b*x + 4*a) + 2*I*b^3))/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{\cos(a+b\,x)^2\,\sin(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

```
[Out] int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.274 $\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx$

**Optimal**. Leaf size=88

$$-\frac{id^2\text{Li}_2\left(e^{4i(a+bx)}\right)}{2b^3}+\frac{2d(c+dx)\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}-\frac{2i(c+dx)^2}{b}$$

```
[Out] -2*I*(d*x+c)^2/b-2*(d*x+c)^2*cot(2*b*x+2*a)/b+2*d*(d*x+c)*ln(1-exp(4*I*(b*x
+a)))/b^2-1/2*I*d^2*polylog(2,exp(4*I*(b*x+a)))/b^3
```

**Rubi [A]** time = 0.19, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4419, 4184, 3717, 2190, 2279, 2391}

$$-\frac{id^2\text{PolyLog}\left(2,e^{4i(a+bx)}\right)}{2b^3}+\frac{2d(c+dx)\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}-\frac{2i(c+dx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-2*I)*(c + d*x)^2)/b - (2*(c + d*x)^2*Cot[2*a + 2*b*x])/b + (2*d*(c + d*x
)*Log[1 - E^((4*I)*(a + b*x))])/b^2 - ((I/2)*d^2*PolyLog[2, E^((4*I)*(a + b
*x))])/b^3
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
```

```
x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx = 4\int (c+dx)^2 \csc^2(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)^2\cot(2a+2bx)}{b} + \frac{(4d)\int (c+dx)\cot(2a+2bx)\,dx}{b}$$
$$= -\frac{2i(c+dx)^2}{b} - \frac{2(c+dx)^2\cot(2a+2bx)}{b} - \frac{(8id)\int \frac{e^{2i(2a+2bx)}(c+dx)}{1-e^{2i(2a+2bx)}}\,dx}{b}$$
$$= -\frac{2i(c+dx)^2}{b} - \frac{2(c+dx)^2\cot(2a+2bx)}{b} + \frac{2d(c+dx)\log\left(1-e^{4i(a+bx}\right.}{b^2}$$
$$= -\frac{2i(c+dx)^2}{b} - \frac{2(c+dx)^2\cot(2a+2bx)}{b} + \frac{2d(c+dx)\log\left(1-e^{4i(a+bx}\right.}{b^2}$$
$$= -\frac{2i(c+dx)^2}{b} - \frac{2(c+dx)^2\cot(2a+2bx)}{b} + \frac{2d(c+dx)\log\left(1-e^{4i(a+bx}\right.}{b^2}$$

**Mathematica [B]** time = 1.72, size = 277, normalized size = 3.15

$$\frac{2b^2\csc(2a)\sin(2bx)(c+dx)^2\csc(2(a+bx)) - \frac{ie^{4ia}\left(4e^{-4ia}b^2(c+dx)^2+2i\left(1-e^{-4ia}\right)bd(c+dx)\log\left(1-e^{-i(a+bx)}\right)+2i\left(1-e^{-4ia}\right)bd(c+dx)\log(1\cdots\right.}{\cdots}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (((-I)*E^((4*I)*a)*((4*b^2*(c + d*x)^2)/E^((4*I)*a) + (2*I)*b*d*(1 - E^((-4
*I)*a))*(c + d*x)*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b*d*(1 - E^((-4*I)*a)
)*(c + d*x)*Log[1 + E^((-I)*(a + b*x))] + (2*I)*b*d*(1 - E^((-4*I)*a))*(c +
 d*x)*Log[1 + E^((-2*I)*(a + b*x))] - 2*d^2*(1 - E^((-4*I)*a))*PolyLog[2, -
E^((-I)*(a + b*x))] - 2*d^2*(1 - E^((-4*I)*a))*PolyLog[2, E^((-I)*(a + b*x)
)] - d^2*(1 - E^((-4*I)*a))*PolyLog[2, -E^((-2*I)*(a + b*x))]))/(-1 + E^((4
*I)*a)) + 2*b^2*(c + d*x)^2*Csc[2*a]*Csc[2*(a + b*x)]*Sin[2*b*x])/b^3
```

**fricas [B]** time = 0.56, size = 950, normalized size = 10.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + 2*b^2*c*d*x - I*d^2*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(
b*x + a))*sin(b*x + a) + I*d^2*cos(b*x + a)*dilog(cos(b*x + a) - I*sin(b*x
+ a))*sin(b*x + a) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)
)*sin(b*x + a) - I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a))*si
n(b*x + a) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b
*x + a) + I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x
+ a) + I*d^2*cos(b*x + a)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a
) - I*d^2*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) +
 b^2*c^2 + (b*d^2*x + b*c*d)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a)
 + 1)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a) + I*sin(
b*x + a) + I)*sin(b*x + a) + (b*d^2*x + b*c*d)*cos(b*x + a)*log(cos(b*x + a
) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos
(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + (b*d^2*x + a*d^2)*cos(b*x +
```

```
a)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x + a*d^2)*
cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x
 + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a)
 + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*s
in(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*si
n(b*x + a) + 1/2)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-1/2*cos(
b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + (b*d^2*x + a*d^2)*cos(b
*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d
^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + (b*
d^2*x + a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x
 + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I
)*sin(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a)^2)/(b
^3*cos(b*x + a)*sin(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc(bx+a)^2 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

**maple [B]** time = 0.12, size = 351, normalized size = 3.99

$$-\frac{4i\left(d^2x^2+2cdx+c^2\right)}{b\left(1+e^{2i(bx+a)}\right)\left(e^{2i(bx+a)}-1\right)}+\frac{2dc\ln\left(e^{i(bx+a)}-1\right)}{b^2}+\frac{2dc\ln\left(1+e^{2i(bx+a)}\right)}{b^2}+\frac{2dc\ln\left(e^{i(bx+a)}+1\right)}{b^2}-\frac{8dc\ln\left(e^{i(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

```
[Out] -2*I/b^3*d^2*polylog(2,-exp(I*(b*x+a)))+2/b^2*d*c*ln(exp(I*(b*x+a))-1)+2*d/
b^2*c*ln(1+exp(2*I*(b*x+a)))+2/b^2*d*c*ln(exp(I*(b*x+a))+1)-8/b^2*d*c*ln(ex
p(I*(b*x+a)))-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3+2/b^2*d^2*ln(exp(I*(b*x
+a))+1)*x-4*I*(d^2*x^2+2*c*d*x+c^2)/b/(1+exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a)
)-1)-4*I*d^2/b^3*a^2+2*d^2/b^2*ln(1+exp(2*I*(b*x+a)))*x+2/b^2*d^2*ln(1-exp(
I*(b*x+a)))*x+2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a-I*d^2*polylog(2,-exp(2*I*(b*
x+a)))/b^3-4*I*d^2/b*x^2-8*I*d^2/b^2*a*x-2/b^3*d^2*a*ln(exp(I*(b*x+a))-1)+8
/b^3*d^2*a*ln(exp(I*(b*x+a)))
```

**maxima [B]** time = 0.69, size = 777, normalized size = 8.83

$$-\frac{4\,b^2c^2+\left(2\,bd^2x+2\,bcd-2\left(bd^2x+bcd\right)\cos\left(4\,bx+4\,a\right)-\left(2i\,bd^2x+2i\,bcd\right)\sin\left(4\,bx+4\,a\right)\right)\arctan\left(\sin\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(4*b^2*c^2 + (2*b*d^2*x + 2*b*c*d - 2*(b*d^2*x + b*c*d)*cos(4*b*x + 4*a) -
 (2*I*b*d^2*x + 2*I*b*c*d)*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(
2*b*x + 2*a) + 1) + (2*b*d^2*x + 2*b*c*d - 2*(b*d^2*x + b*c*d)*cos(4*b*x +
4*a) - (2*I*b*d^2*x + 2*I*b*c*d)*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), co
s(b*x + a) + 1) - (2*b*c*d*cos(4*b*x + 4*a) + 2*I*b*c*d*sin(4*b*x + 4*a) -
2*b*c*d)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d^2*x*cos(4*b*x + 4
*a) + 2*I*b*d^2*x*sin(4*b*x + 4*a) - 2*b*d^2*x)*arctan2(sin(b*x + a), -cos(
b*x + a) + 1) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x)*cos(4*b*x + 4*a) + (d^2*cos(4
*b*x + 4*a) + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(-e^(2*I*b*x + 2*I*a)) + 2
```

```
*(d^2*cos(4*b*x + 4*a) + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(-e^(I*b*x + I*
a)) + 2*(d^2*cos(4*b*x + 4*a) + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(e^(I*b*
x + I*a)) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(4*b*x + 4*a)
+ (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b
*c*d)*cos(4*b*x + 4*a) + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log(cos(b*x +
a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*
d^2*x - I*b*c*d)*cos(4*b*x + 4*a) + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log
(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-4*I*b^2*d^2*x^2
- 8*I*b^2*c*d*x)*sin(4*b*x + 4*a))/(-I*b^3*cos(4*b*x + 4*a) + b^3*sin(4*b*x
 + 4*a) + I*b^3)
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{\cos(a+b\,x)^2\sin(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^2),x)
[Out] int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2\csc^2(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a)**2,x)
[Out] Integral((c + d*x)**2*csc(a + b*x)**2*sec(a + b*x)**2, x)
```

## 3.275 $\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx$

**Optimal**. Leaf size=35

$$\frac{d\log(\sin(2a+2bx))}{b^2}-\frac{2(c+dx)\cot(2a+2bx)}{b}$$

```
[Out] -2*(d*x+c)*cot(2*b*x+2*a)/b+d*ln(sin(2*b*x+2*a))/b^2
```

**Rubi [A]** time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4419, 4184, 3475}

$$\frac{d\log(\sin(2a+2bx))}{b^2}-\frac{2(c+dx)\cot(2a+2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
[Out] (-2*(c + d*x)*Cot[2*a + 2*b*x])/b + (d*Log[Sin[2*a + 2*b*x]])/b^2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx &= 4\int (c+dx)\csc^2(2a+2bx)\,dx\\ &= -\frac{2(c+dx)\cot(2a+2bx)}{b}+\frac{(2d)\int \cot(2a+2bx)\,dx}{b}\\ &= -\frac{2(c+dx)\cot(2a+2bx)}{b}+\frac{d\log(\sin(2a+2bx))}{b^2}\end{aligned}$$

**Mathematica [A]** time = 0.20, size = 32, normalized size = 0.91

$$\frac{d\log(\sin(2(a+bx)))-2b(c+dx)\cot(2(a+bx))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
[Out] (-2*b*(c + d*x)*Cot[2*(a + b*x)] + d*Log[Sin[2*(a + b*x)]])/b^2
```

**fricas [B]** time = 0.45, size = 75, normalized size = 2.14

$$\frac{d\cos\left(bx+a\right)\log\left(-\frac{1}{2}\cos\left(bx+a\right)\sin\left(bx+a\right)\right)\sin\left(bx+a\right)+bdx-2\left(bdx+bc\right)\cos\left(bx+a\right)^{2}+bc}{b^{2}\cos\left(bx+a\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (d*cos(b*x + a)*log(-1/2*cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + b*d*x -
2*(b*d*x + b*c)*cos(b*x + a)^2 + b*c)/(b^2*cos(b*x + a)*sin(b*x + a))
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.09, size = 182, normalized size = 5.20

$$\frac{\frac{c}{2b}+\frac{dx}{2b}-\frac{3c\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{c\left(\tan^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}-\frac{3dx\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{dx\left(\tan^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}+\frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^{2}}+\frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

```
[Out] (1/2*c/b+1/2*d*x/b-3*c/b*tan(1/2*b*x+1/2*a)^2+1/2*c/b*tan(1/2*b*x+1/2*a)^4-
3/b*d*x*tan(1/2*b*x+1/2*a)^2+1/2/b*d*x*tan(1/2*b*x+1/2*a)^4)/tan(1/2*b*x+1/
2*a)/(tan(1/2*b*x+1/2*a)^2-1)+d/b^2*ln(tan(1/2*b*x+1/2*a))+d/b^2*ln(tan(1/2
*b*x+1/2*a)-1)+d/b^2*ln(tan(1/2*b*x+1/2*a)+1)-2*d/b^2*ln(1+tan(1/2*b*x+1/2*
a)^2)
```

**maxima [B]** time = 0.48, size = 308, normalized size = 8.80

$$-\frac{2\,c\left(\frac{1}{\tan\left(bx+a\right)}-\tan\left(bx+a\right)\right)-\frac{2\,ad\left(\frac{1}{\tan\left(bx+a\right)}-\tan\left(bx+a\right)\right)}{b}-\frac{\left(\left(\cos\left(4\,bx+4\,a\right)^{2}+\sin\left(4\,bx+4\,a\right)^{2}-2\,\cos\left(4\,bx+4\,a\right)+1\right)\log\left(\cos\left(2\,bx+2\,a\right)^{2}+\sin\ldots\right.\right.}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*(1/tan(b*x + a) - tan(b*x + a)) - 2*a*d*(1/tan(b*x + a) - tan(b*x
 + a))/b - ((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) +
 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) +
 (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos
(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*d/((cos(4*b*x +
 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b))/b
```

**mupad [B]** time = 1.66, size = 55, normalized size = 1.57

$$\frac{d\,\ln\left({\mathrm{e}}^{a\,4{}\mathrm{i}}\,{\mathrm{e}}^{b\,x\,4{}\mathrm{i}}-1\right)}{b^2}-\frac{\left(c+d\,x\right)\,4{}\mathrm{i}}{b\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}-1\right)}-\frac{d\,x\,4{}\mathrm{i}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

```
[Out] (d*log(exp(a*4i)*exp(b*x*4i) - 1))/b^2 - ((c + d*x)*4i)/(b*(exp(a*4i + b*x*
4i) - 1)) - (d*x*4i)/b
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x)**2, x)
```

## 3.276 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=24

$$4\text{Int}\left(\frac{\csc^2(2a+2bx)}{c+dx},x\right)$$

```
[Out] 4*Unintegrable(csc(2*b*x+2*a)^2/(d*x+c),x)
```

**Rubi** [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x),x]
[Out] 4*Defer[Int][Csc[2*a + 2*b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = 4\int \frac{\csc^2(2a+2bx)}{c+dx}\,dx$$

**Mathematica** [A] time = 7.06, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x), x]
```

**fricas** [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^2\sec(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^2\sec(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c), x)
```

**maple** [**A**] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2(bx+a)\right)\left(\sec^2(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x)
[Out] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x)
```

**maxima** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

**mupad** [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)^2\,\sin(a+b\,x)^2\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)), x)
```

**sympy** [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**2/(d*x+c),x)
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**2/(c + d*x), x)
```

# 3.277 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=24

$$4\text{Int}\left(\frac{\csc^2(2a+2bx)}{(c+dx)^2},x\right)$$

```
[Out] 4*Unintegrable(csc(2*b*x+2*a)^2/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] 4*Defer[Int][Csc[2*a + 2*b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = 4\int \frac{\csc^2(2a+2bx)}{(c+dx)^2}\,dx$$

**Mathematica [A]** time = 7.33, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c)^2, x)
```

**maple** [A] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{2}\left(bx+a\right)\right)\left(\sec^{2}\left(bx+a\right)\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x)
[Out] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^{2}\,\sin\left(a+b\,x\right)^{2}\,\left(c+d\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{2}\left(a+bx\right)\sec^{2}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**2/(c + d*x)**2, x)
```

# 3.278 $\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx$

**Optimal**. Leaf size=27

$$\text{Int}\left(\csc^3(a+bx)\sec^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

**Rubi [A]** time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx=\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

**Mathematica [A]** time = 25.51, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

**fricas [A]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

**maple [A]** time = 0.14, size = 0, normalized size = 0.00

$$\int (dx+c)^m\left(\csc^3\left(bx+a\right)\right)\left(\sec^2\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2 \sin(a+b\,x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.279 $\int (c+dx)^3 \csc^3(a+bx)\sec^2(a+bx)\,dx$

**Optimal**. Leaf size=601

$$\frac{3id^3\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_2\left(e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{9id^3\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{9id^3\text{Li}_4\left(e^{i(a+bx)}\right.}{b^4}$$

```
[Out] -3*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^
4+9*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-3*c*d^2*x*csc(b*x+a)/b^2-6*I*c*d^2*
polylog(2,-I*exp(I*(b*x+a)))/b^3-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3
-9/2*I*d*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2+6*I*d^3*x^2*arctan(exp(I*(
b*x+a)))/b^2+6*I*c*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3+6*I*d^3*x*polylog(2,
I*exp(I*(b*x+a)))/b^3-9*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^3+9*d^2*(d
*x+c)*polylog(3,exp(I*(b*x+a)))/b^3+12*I*c*d^2*x*arctan(exp(I*(b*x+a)))/b^2
+9/2*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2+6*d^3*polylog(3,-I*exp(I*
(b*x+a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(2,-exp(
I*(b*x+a)))/b^4+3/2*(d*x+c)^3*sec(b*x+a)/b-6*d^3*x*arctanh(exp(I*(b*x+a)))/
b^3-3*c*d^2*arctanh(cos(b*x+a))/b^3-3*c^2*d*arctanh(sin(b*x+a))/b^2-3/2*c^2
*d*csc(b*x+a)/b^2-3/2*d^3*x^2*csc(b*x+a)/b^2-1/2*(d*x+c)^3*csc(b*x+a)^2*sec
(b*x+a)/b-9*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4
```

**Rubi** [A] time = 2.31, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 64, number of rules used = 24, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2622, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4183, 2531, 6609, 2282, 6589, 4133, 453, 206, 4181, 2279, 2391, 2621, 6271, 3770, 14}

$$-\frac{6icd^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6icd^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{9d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{9d^2(c+dx)\text{PolyL}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((12*I)*c*d^2*x*ArcTan[E^(I*(a + b*x))])/b^2 + ((6*I)*d^3*x^2*ArcTan[E^(I*(
a + b*x))])/b^2 - (6*d^3*x*ArcTanh[E^(I*(a + b*x))])/b^3 - (3*(c + d*x)^3*A
rcTanh[E^(I*(a + b*x))])/b - (3*c*d^2*ArcTanh[Cos[a + b*x]])/b^3 - (3*c^2*d
*ArcTanh[Sin[a + b*x]])/b^2 - (3*c^2*d*Csc[a + b*x])/(2*b^2) - (3*c*d^2*x*C
sc[a + b*x])/b^2 - (3*d^3*x^2*Csc[a + b*x])/(2*b^2) + ((3*I)*d^3*PolyLog[2,
 -E^(I*(a + b*x))])/b^4 + (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*
x))])/b^2 - ((6*I)*c*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3
*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*c*d^2*PolyLog[2, I*E^(I*(
a + b*x))])/b^3 + ((6*I)*d^3*x*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*
d^3*PolyLog[2, E^(I*(a + b*x))])/b^4 - (((9*I)/2)*d*(c + d*x)^2*PolyLog[2,
E^(I*(a + b*x))])/b^2 - (9*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3
+ (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog[3, I*E^(I*(
a + b*x))])/b^4 + (9*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((9*I
)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((9*I)*d^3*PolyLog[4, E^(I*(a + b
*x))])/b^4 + (3*(c + d*x)^3*Sec[a + b*x])/(2*b) - ((c + d*x)^3*Csc[a + b*x]
^2*Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3\csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3}{}$$

$$= \frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b}-\frac{3\int b(-c}{}$$

$$= \frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b}-\frac{3}{2}\int(-c$$

$$= -\frac{3(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3\csc^2}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{3c^2d\csc(a+bx)}{2b^2}+\frac{9id(c+dx)^2\text{Li}_2\left(-e\right.}{2b^2}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3(c+dx)^3\tanh^{-1}\left(\right.}{b}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3(c+dx)^3\tanh^{-1}\left(\right.}{b}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3(c+dx)^3\tanh^{-1}\left(\right.}{b}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6d^3x\tanh^{-1}\left(e^{i(a+b}\right.}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6d^3x\tanh^{-1}\left(e^{i(a+b}\right.}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6d^3x\tanh^{-1}\left(e^{i(a+b}\right.}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6d^3x\tanh^{-1}\left(e^{i(a+b}\right.}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6d^3x\tanh^{-1}\left(e^{i(a+b}\right.}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6d^3x\tanh^{-1}\left(e^{i(a+b}\right.}{b^3}$$

**Mathematica** [A] time = 8.35, size = 907, normalized size = 1.51

$$-\frac{\sec(a+bx)\left(-bc^3+3b\cos(2a+2bx)c^3-3bdxc^2+9bdx\cos(2a+2bx)c^2+3d\sin(2a+2bx)c^2-3bd^2x^2c+9bd\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I*
(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 +
I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*(c
+ d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*
E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3
, I*E^(I*(a + b*x))]))/b^4 + (3*(b^3*c^3*Log[1 - E^(I*(a + b*x))] + 2*b*c*d
^2*Log[1 - E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - E^(I*(a + b*x))] + 2*b*
d^3*x*Log[1 - E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - E^(I*(a + b*x))] +
 b^3*d^3*x^3*Log[1 - E^(I*(a + b*x))] - b^3*c^3*Log[1 + E^(I*(a + b*x))] -
2*b*c*d^2*Log[1 + E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + E^(I*(a + b*x))]
 - 2*b*d^3*x*Log[1 + E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 + E^(I*(a + b
*x))] - b^3*d^3*x^3*Log[1 + E^(I*(a + b*x))] + I*d*(2*d^2 + 3*b^2*(c + d*x)
^2)*PolyLog[2, -E^(I*(a + b*x))] - I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*PolyLog[
2, E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, -E^(I*(a + b*x))] - 6*b*d^3*x*Po
lyLog[3, -E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, E^(I*(a + b*x))] + 6*b*d^
3*x*PolyLog[3, E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))] +
(6*I)*d^3*PolyLog[4, E^(I*(a + b*x))]))/(2*b^4) - (Csc[a + b*x]^2*Sec[a + b
*x]*(-(b*c^3) - 3*b*c^2*d*x - 3*b*c*d^2*x^2 - b*d^3*x^3 + 3*b*c^3*Cos[2*a +
 2*b*x] + 9*b*c^2*d*x*Cos[2*a + 2*b*x] + 9*b*c*d^2*x^2*Cos[2*a + 2*b*x] + 3
*b*d^3*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b*x] + 6*c*d^2*x*Sin[2*a
+ 2*b*x] + 3*d^3*x^2*Sin[2*a + 2*b*x]))/(4*b^2)
```

**fricas [C]** time = 0.81, size = 3173, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 4*b^3*c^3 - 6*(b^
3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2 - 6*(
b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) - ((-9*I
*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^3 +
 (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a
))*dilog(cos(b*x + a) + I*sin(b*x + a)) - ((9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^
2*x + 9*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a)^3 + (-9*I*b^2*d^3*x^2 - 18*I*b^
2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a))*dilog(cos(b*x + a) - I*s
in(b*x + a)) - ((12*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a)^3 + (-12*I*b*d^3
*x - 12*I*b*c*d^2)*cos(b*x + a))*dilog(I*cos(b*x + a) + sin(b*x + a)) - ((1
2*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a)^3 + (-12*I*b*d^3*x - 12*I*b*c*d^2)
*cos(b*x + a))*dilog(I*cos(b*x + a) - sin(b*x + a)) - ((-12*I*b*d^3*x - 12*
I*b*c*d^2)*cos(b*x + a)^3 + (12*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a))*dil
og(-I*cos(b*x + a) + sin(b*x + a)) - ((-12*I*b*d^3*x - 12*I*b*c*d^2)*cos(b*
x + a)^3 + (12*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a))*dilog(-I*cos(b*x + a
) - sin(b*x + a)) - ((-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d -
 6*I*d^3)*cos(b*x + a)^3 + (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^
2*d + 6*I*d^3)*cos(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) - ((9*I*
b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a)^3 +
(-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a
))*dilog(-cos(b*x + a) - I*sin(b*x + a)) + 3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^
2 + b^3*c^3 + 2*b*c*d^2 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^3 - (b^3*
d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 2*b*c*d^2 + (3*b^3*c^2*d + 2*b*d^3)*x
)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1) + 6*((b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*cos(b*x + a)^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*c
os(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + I) + 3*((b^3*d^3*x^3 + 3*b
^3*c*d^2*x^2 + b^3*c^3 + 2*b*c*d^2 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a
)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 2*b*c*d^2 + (3*b^3*c^2*d +
 2*b*d^3)*x)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 6*((b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)^3 - (b^2*c^2*d - 2*a*b*c*d^2 +
 a^2*d^3)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + I) + 6*((b^2*d^
```

```
3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)^3 - (b^2*d^3*x^
2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a))*log(I*cos(b*x + a)
 + sin(b*x + a) + 1) - 6*((b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*
d^3)*cos(b*x + a)^3 - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)
*cos(b*x + a))*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 6*((b^2*d^3*x^2 + 2
*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)^3 - (b^2*d^3*x^2 + 2*b^2
*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a))*log(-I*cos(b*x + a) + sin(b
*x + a) + 1) - 6*((b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos
(b*x + a)^3 - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x
 + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 3*((b^3*c^3 - 3*a*b^2*c^2*
d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^3 - (b^3*c^3 - 3*a*
b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a))*log(-1/2*c
os(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*((b^3*c^3 - 3*a*b^2*c^2*d + (3*
a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^3 - (b^3*c^3 - 3*a*b^2*c^2
*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a))*log(-1/2*cos(b*x
+ a) - 1/2*I*sin(b*x + a) + 1/2) - 3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*co
s(b*x + a)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a))*log(-cos(b*
x + a) + I*sin(b*x + a) + 1) + 6*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b
*x + a)^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a))*log(-cos(b*x
+ a) + I*sin(b*x + a) + I) - 3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x
+ a)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (
a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a))*log(-cos(b*x + a)
 - I*sin(b*x + a) + 1) - 6*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a
)^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a))*log(-cos(b*x + a) -
 I*sin(b*x + a) + I) - (18*I*d^3*cos(b*x + a)^3 - 18*I*d^3*cos(b*x + a))*po
lylog(4, cos(b*x + a) + I*sin(b*x + a)) - (-18*I*d^3*cos(b*x + a)^3 + 18*I*
d^3*cos(b*x + a))*polylog(4, cos(b*x + a) - I*sin(b*x + a)) - (18*I*d^3*cos
(b*x + a)^3 - 18*I*d^3*cos(b*x + a))*polylog(4, -cos(b*x + a) + I*sin(b*x +
 a)) - (-18*I*d^3*cos(b*x + a)^3 + 18*I*d^3*cos(b*x + a))*polylog(4, -cos(b
*x + a) - I*sin(b*x + a)) - 18*((b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - (b*d^3
*x + b*c*d^2)*cos(b*x + a))*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 18*
((b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cos(b*x + a))*pol
ylog(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(d^3*cos(b*x + a)^3 - d^3*cos(b
*x + a))*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 12*(d^3*cos(b*x + a)^3
 - d^3*cos(b*x + a))*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*(d^3*co
s(b*x + a)^3 - d^3*cos(b*x + a))*polylog(3, -I*cos(b*x + a) + sin(b*x + a))
 + 12*(d^3*cos(b*x + a)^3 - d^3*cos(b*x + a))*polylog(3, -I*cos(b*x + a) -
sin(b*x + a)) + 18*((b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - (b*d^3*x + b*c*d^2
)*cos(b*x + a))*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 18*((b*d^3*x +
 b*c*d^2)*cos(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cos(b*x + a))*polylog(3, -co
s(b*x + a) - I*sin(b*x + a)))/(b^4*cos(b*x + a)^3 - b^4*cos(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

**maple [B]** time = 0.54, size = 1613, normalized size = 2.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] 6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^
4-12*I/b^3*d^2*c*a*arctan(exp(I*(b*x+a)))+3*I*d^3*polylog(2,-exp(I*(b*x+a))
)/b^4+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3+9*I*d^3*polylog(4,exp(I*(b*
x+a)))/b^4-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3-3/2/b^4*d^3*a^3*ln(ex
p(I*(b*x+a))-1)-9/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+9/b^3*c*d^2*polylog(
3,exp(I*(b*x+a)))+9/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-9/b^3*d^3*polylog(3
,-exp(I*(b*x+a)))*x-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-9*I*d^3*polylog(4
,-exp(I*(b*x+a)))/b^4+1/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*
d^3*x^3*b*exp(5*I*(b*x+a))+9*c*d^2*x^2*b*exp(5*I*(b*x+a))+9*c^2*d*x*b*exp(5
*I*(b*x+a))-2*d^3*x^3*b*exp(3*I*(b*x+a))+3*c^3*b*exp(5*I*(b*x+a))-6*c*d^2*x
^2*b*exp(3*I*(b*x+a))-3*I*c^2*d*exp(5*I*(b*x+a))-6*c^2*d*x*b*exp(3*I*(b*x+a
))+3*d^3*x^3*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(5*I*(b*x+a))-2*c^3*b*exp(3*I*
(b*x+a))+9*c*d^2*x^2*b*exp(I*(b*x+a))+6*I*c*d^2*x*exp(I*(b*x+a))+9*c^2*d*x*
b*exp(I*(b*x+a))+3*c^3*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(5*I*(b*x+a))+3*I*c^
2*d*exp(I*(b*x+a))+3*I*d^3*x^2*exp(I*(b*x+a)))-6*I/b^3*d^2*c*dilog(1+I*exp(
I*(b*x+a)))+6*I/b^3*d^2*c*dilog(1-I*exp(I*(b*x+a)))+6*I/b^4*a*d^3*dilog(1+I
*exp(I*(b*x+a)))-6*I/b^4*a*d^3*dilog(1-I*exp(I*(b*x+a)))+6*I/b^4*d^3*polylo
g(2,I*exp(I*(b*x+a)))*a-6*I/b^4*d^3*polylog(2,-I*exp(I*(b*x+a)))*a+3/2/b*c^
3*ln(exp(I*(b*x+a))-1)-3/2/b*c^3*ln(exp(I*(b*x+a))+1)+3/b^3*d^2*c*ln(exp(I*
(b*x+a))-1)-3/b^3*d^2*c*ln(exp(I*(b*x+a))+1)-3/b^3*d^3*ln(exp(I*(b*x+a))+1)
*x+3/b^3*d^3*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a-3/b^4*
d^3*a*ln(exp(I*(b*x+a))-1)+9/2*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-9/2*I
/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+9/2*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)
))*x^2-9/2*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+6/b^2*d^2*c*ln(1+I*exp(I
*(b*x+a)))*x+6*I/b^4*d^3*a^2*arctan(exp(I*(b*x+a)))+6*I/b^2*d*c^2*arctan(ex
p(I*(b*x+a)))-6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a+6/b^3*d^2*c*ln(1+I*exp(I
*(b*x+a)))*a-6/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x+9/2/b^3*c*d^2*a^2*ln(exp(
I*(b*x+a))-1)-9/2/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+9/2/b*c^2*d*ln(1-exp(I*(b*
x+a)))*x+9/2/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-9/2/b^3*c*d^2*a^2*ln(1-exp(I*
(b*x+a)))+9/2/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-9/2/b*c*d^2*ln(exp(I*(b*x+a)
)+1)*x^2-9/2/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+3/2/b*d^3*ln(1-exp(I*(b*x+a))
)*x^3+3/2/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-3/2/b*d^3*ln(exp(I*(b*x+a))+1)*x
^3+3/b^2*d^3*ln(1+I*exp(I*(b*x+a)))*x^2-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a))
)-3/b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2+3/b^4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))
-9*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+9*I/b^2*c*d^2*polylog(2,-exp(I*(
b*x+a)))*x
```

**maxima [B]** time = 6.03, size = 8043, normalized size = 13.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*(3*cos(b*x + a)
^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(c
os(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3
 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^2 -
 a^3*d^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^3 + 4*((12*b^2*c^2*d - 24*a*
b*c*d^2 + 12*(b*x + a)^2*d^3 + 12*a^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a)
+ 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*
d^3)*(b*x + a))*cos(6*b*x + 6*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^
2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 12*(b^2
*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
 + a))*cos(2*b*x + 2*a) - (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a
)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(6*b*x
```

```
+ 6*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + 12*I*a^2
*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (12*I*b^2*
c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^
2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x
+ a) + 1) + (12*b^2*c^2*d - 24*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 12*a^2*d^3
+ 24*(b*c*d^2 - a*d^3)*(b*x + a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^
2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 12*(b^2
*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
 + a))*cos(4*b*x + 4*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a
^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-12*I*b^2*c^2*d
 + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b*c*d^2 +
24*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2
+ 12*I*(b*x + a)^2*d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x +
a))*sin(4*b*x + 4*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*
d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a
))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - (6*(b*x + a)^3*d^3 + 12*b*c*d
^2 - 12*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d - 6*a*b*c
*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 +
 2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a
*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^
2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 -
2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3
*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^3 + 12*I*b*
c*d^2 - 12*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^
2*d - 36*I*a*b*c*d^2 + (18*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(6*b*x + 6*a) +
 (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*
a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)
*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 +
12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d +
36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arcta
n2(sin(b*x + a), cos(b*x + a) + 1) + (12*b*c*d^2 - 12*a*d^3 + 12*(b*c*d^2 -
 a*d^3)*cos(6*b*x + 6*a) - 12*(b*c*d^2 - a*d^3)*cos(4*b*x + 4*a) - 12*(b*c*
d^2 - a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 + 12*I*a*d^3)*sin(6*b*x + 6*
a) - (12*I*b*c*d^2 - 12*I*a*d^3)*sin(4*b*x + 4*a) - (12*I*b*c*d^2 - 12*I*a*
d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - (6*(b*x +
a)^3*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d - 6*a*b*c*d^2
+ (3*a^2 + 2)*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*
x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(6*b
*x + 6*a) - 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c
^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 6*((b*x
 + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2
+ (3*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^3 + (18
*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (
18*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^3*d^3 +
 (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d
^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^
3*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*
a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(si
n(b*x + a), -cos(b*x + a) + 1) - (12*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*
a*b*c*d^2 + 12*a^2*d^3 - 12*(-3*I*b*c*d^2 + (3*I*a - 1)*d^3)*(b*x + a)^2 +
(36*I*b^2*c^2*d - 24*(3*I*a - 1)*b*c*d^2 + (36*I*a^2 - 24*a)*d^3)*(b*x + a)
)*cos(5*b*x + 5*a) - (-8*I*(b*x + a)^3*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(
b*x + a)^2 + (-24*I*b^2*c^2*d + 48*I*a*b*c*d^2 - 24*I*a^2*d^3)*(b*x + a))*c
os(3*b*x + 3*a) - (12*I*(b*x + a)^3*d^3 - 12*b^2*c^2*d + 24*a*b*c*d^2 - 12*
a^2*d^3 + (36*I*b*c*d^2 - 12*(3*I*a + 1)*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d
 - 24*(3*I*a + 1)*b*c*d^2 + (36*I*a^2 + 24*a)*d^3)*(b*x + a))*cos(b*x + a)
+ (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 + 24*(b*c*d^2 + (b*x + a)*d^3 -
 a*d^3)*cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(4*b*x +
```

```
4*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (-24*I*b*c*
d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(6*b*x + 6*a) - (24*I*b*c*d^2 + 2
4*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(4*b*x + 4*a) - (24*I*b*c*d^2 + 24*I*(b*
x + a)*d^3 - 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (24*b
*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)
*cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(4*b*x + 4*a) -
 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (24*I*b*c*d^2 + 24
*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(6*b*x + 6*a) + (-24*I*b*c*d^2 - 24*I*(b*
x + a)*d^3 + 24*I*a*d^3)*sin(4*b*x + 4*a) + (-24*I*b*c*d^2 - 24*I*(b*x + a)
*d^3 + 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + (18*b^2*c^
2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^3 + 36*(b*c*d^2 -
 a*d^3)*(b*x + a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a
^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*(3*b^2*c^
2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^
3)*(b*x + a))*cos(4*b*x + 4*a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)
^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a)
- (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 1
2*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) - (18*I
*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 12*I)*d^3
+ (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (18*I*b^2*c^2*d
 - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 12*I)*d^3 + (36*I*b*
c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) -
(18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^3 + 36*
(b*c*d^2 - a*d^3)*(b*x + a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*
d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6
*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*
d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3
*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b
*x + 2*a) + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I
*a^2 + 12*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a)
+ (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 1
2*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-18*
I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 12*I)*d^
3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*
x + I*a)) - (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2
 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 -
6*I)*d^3)*(b*x + a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9
*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-
9*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (3*I*(b*x + a)^3*d^3 + 6*
I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^
2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (
3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(
b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x +
a))*cos(2*b*x + 2*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x
 + a))*sin(6*b*x + 6*a) - 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c
*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(
b*x + a))*sin(4*b*x + 4*a) - 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3
)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(
b*x + a) + 1) - (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d
^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 +
6*I)*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I
*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*
a^2 + 6*I)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b
*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2
*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (
-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)
*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*
```

x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*
c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*
(b*x + a))*sin(6*b*x + 6*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*
(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^
3)*(b*x + a))*sin(4*b*x + 4*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 +
 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)
*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*
cos(b*x + a) + 1) - (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3
 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a) + (-6*I*b^2*c^2*d +
12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*
a*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*
(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(
4*b*x + 4*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*
a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*(b^2*
c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
 + a))*sin(6*b*x + 6*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2
*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 6*(b^2*c^2*d - 2*a
*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (6
*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b
*c*d^2 - 12*I*a*d^3)*(b*x + a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x
 + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(6*b*
x + 6*a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2
*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-6*I*b^2
*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^
2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 +
(b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(6*b*x + 6*a)
 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*
d^3)*(b*x + a))*sin(4*b*x + 4*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2
*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b
*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (36*d^3*cos(6*b*x + 6*a)
 - 36*d^3*cos(4*b*x + 4*a) - 36*d^3*cos(2*b*x + 2*a) + 36*I*d^3*sin(6*b*x +
 6*a) - 36*I*d^3*sin(4*b*x + 4*a) - 36*I*d^3*sin(2*b*x + 2*a) + 36*d^3)*pol
ylog(4, -e^(I*b*x + I*a)) + (36*d^3*cos(6*b*x + 6*a) - 36*d^3*cos(4*b*x + 4
*a) - 36*d^3*cos(2*b*x + 2*a) + 36*I*d^3*sin(6*b*x + 6*a) - 36*I*d^3*sin(4*
b*x + 4*a) - 36*I*d^3*sin(2*b*x + 2*a) + 36*d^3)*polylog(4, e^(I*b*x + I*a)
) - (-24*I*d^3*cos(6*b*x + 6*a) + 24*I*d^3*cos(4*b*x + 4*a) + 24*I*d^3*cos(
2*b*x + 2*a) + 24*d^3*sin(6*b*x + 6*a) - 24*d^3*sin(4*b*x + 4*a) - 24*d^3*s
in(2*b*x + 2*a) - 24*I*d^3)*polylog(3, I*e^(I*b*x + I*a)) - (24*I*d^3*cos(6
*b*x + 6*a) - 24*I*d^3*cos(4*b*x + 4*a) - 24*I*d^3*cos(2*b*x + 2*a) - 24*d^
3*sin(6*b*x + 6*a) + 24*d^3*sin(4*b*x + 4*a) + 24*d^3*sin(2*b*x + 2*a) + 24
*I*d^3)*polylog(3, -I*e^(I*b*x + I*a)) - (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^
3 + 36*I*a*d^3 + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3)*cos(6*b*
x + 6*a) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a
) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(2*b*x + 2*a) + 36*
(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(6*b*x + 6*a) - 36*(b*c*d^2 + (b*x + a
)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(
2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) - (36*I*b*c*d^2 + 36*I*(b*x + a)
*d^3 - 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(6*
b*x + 6*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3)*cos(4*b*x +
4*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3)*cos(2*b*x + 2*a) -
 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(6*b*x + 6*a) + 36*(b*c*d^2 + (b*x
 + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*
sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) + (12*(b*x + a)^3*d^3 - 12*I*
b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (36*b*c*d^2 - (36*a + 12*I)*d^3
)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a + 24*I)*b*c*d^2 + 12*(3*a^2 + 2*I*a)*
d^3)*(b*x + a))*sin(5*b*x + 5*a) - 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(3*b*x +
 3*a) + (12*(b*x + a)^3*d^3 + 12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^

```
3 + (36*b*c*d^2 - (36*a - 12*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a -
24*I)*b*c*d^2 + 12*(3*a^2 - 2*I*a)*d^3)*(b*x + a))*sin(b*x + a))/(-4*I*b^3*
cos(6*b*x + 6*a) + 4*I*b^3*cos(4*b*x + 4*a) + 4*I*b^3*cos(2*b*x + 2*a) + 4*
b^3*sin(6*b*x + 6*a) - 4*b^3*sin(4*b*x + 4*a) - 4*b^3*sin(2*b*x + 2*a) - 4*
I*b^3))/b
```

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] \text{Hanged}
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a)**2,x)
[Out] Timed out
```

# 3.280 $\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx)\,dx$

**Optimal**. Leaf size=305

$$-\frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{3d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{3d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}+\frac{3id(c+\ldots}{}$$

```
[Out] 4*I*d^2*x*arctan(exp(I*(b*x+a)))/b^2-3*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-
d^2*arctanh(cos(b*x+a))/b^3-2*c*d*arctanh(sin(b*x+a))/b^2-c*d*csc(b*x+a)/b^
2-d^2*x*csc(b*x+a)/b^2+3*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*d^2
*polylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3-3
*I*d*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^2-3*d^2*polylog(3,-exp(I*(b*x+a)))
/b^3+3*d^2*polylog(3,exp(I*(b*x+a)))/b^3+3/2*(d*x+c)^2*sec(b*x+a)/b-1/2*(d*
x+c)^2*csc(b*x+a)^2*sec(b*x+a)/b
```

**Rubi [A]** time = 0.87, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 22, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {2622, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4183, 2531, 2282, 6589, 4133, 453, 206, 4181, 2279, 2391, 2621, 6271, 3770}

$$\frac{3id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}\left(\ldots\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((4*I)*d^2*x*ArcTan[E^(I*(a + b*x))])/b^2 - (3*(c + d*x)^2*ArcTanh[E^(I*(a
+ b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]])/b^3 - (2*c*d*ArcTanh[Sin[a + b*x]
])/b^2 - (c*d*Csc[a + b*x])/b^2 - (d^2*x*Csc[a + b*x])/b^2 + ((3*I)*d*(c +
d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d*(
c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (3*d^2*PolyLog[3, -E^(I*(a + b*
x))])/b^3 + (3*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 + (3*(c + d*x)^2*Sec[a
+ b*x])/(2*b) - ((c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2\csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+d}{}$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+d}{}$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+d}{}$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+d}{}$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+d}{}$$

$$= \frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}+\frac{3\int b}{}$$

$$= \frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}+\frac{3}{2}\int ($$

$$= -\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2 c}{}$$

$$= -\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{cd\csc(a+bx)}{b^2}+\frac{3id(c+dx)\mathrm{Li}_2\left(-e\right.}{b^2}$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{2cd\tanh^{-1}(\mathrm{s}}{b^2}$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{2cd\tanh^{-1}(\mathrm{s}}{b^2}$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d^2\tanh^{-1}(\mathrm{co}}{b^3}$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d^2\tanh^{-1}(\mathrm{co}}{b^3}$$

$$= \frac{4id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d^2\tanh^{-1}(\mathrm{co}}{b^3}$$

$$= \frac{4id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d^2\tanh^{-1}(\mathrm{co}}{b^3}$$

$$= \frac{4id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{d^2\tanh^{-1}(\mathrm{co}}{b^3}$$

**Mathematica [B]** time = 7.97, size = 889, normalized size = 2.91

$$-\frac{2\left(\frac{2\tan^{-1}(\cot(a))\tanh^{-1}\left(\frac{\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}}-\frac{\csc(a)\left(\left(bx-\tan^{-1}(\cot(a))\right)\left(\log\left(1-e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)-\log\left(1+e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)\right)+i\left(\mathrm{Li}_2\left(-e^{i}\right.\right.\right.}{\sqrt{\cot^2(a)+1}}\right.}{b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/2]^2)/(8*b) + (3*b^2*c^2*Log[1
- E^(I*(a + b*x))] + 2*d^2*Log[1 - E^(I*(a + b*x))] + 6*b^2*c*d*x*Log[1 - E
^(I*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] - 3*b^2*c^2*Log[1
+ E^(I*(a + b*x))] - 2*d^2*Log[1 + E^(I*(a + b*x))] - 6*b^2*c*d*x*Log[1 + E
^(I*(a + b*x))] - 3*b^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] + (6*I)*b*d*(c + d
*x)*PolyLog[2, -E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, E^(I*(a +
 b*x))] - 6*d^2*PolyLog[3, -E^(I*(a + b*x))] + 6*d^2*PolyLog[3, E^(I*(a + b
*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a/2 + (b*x)/2]^2)/(8*b) + (
(c + d*x)*Csc[a]*Sec[a]*(-(d*Cos[a]) + b*c*Sin[a] + b*d*x*Sin[a]))/b^2 - ((
4*I)*c*d*ArcTan[((-I)*Sin[a] - I*Cos[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a
]^2]])/(b^2*Sqrt[Cos[a]^2 + Sin[a]^2]) - (2*d^2*(-((Csc[a]*((b*x - ArcTan[C
ot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan
[Cot[a]]))]) + I*(PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - PolyLog[2, E^
(I*(b*x - ArcTan[Cot[a]]))])))/Sqrt[1 + Cot[a]^2]) + (2*ArcTan[Cot[a]]*ArcT
anh[(Sin[a] + Cos[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^
2 + Sin[a]^2]))/b^3 + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*Sin[(b*x)/2]) - d
^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d*Sin[(b*x)/2
] + d^2*x*Sin[(b*x)/2]))/(2*b^2) + (c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2]
 + d^2*x^2*Sin[(b*x)/2])/(b*(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin
[a/2 + (b*x)/2])) + (-(c^2*Sin[(b*x)/2]) - 2*c*d*x*Sin[(b*x)/2] - d^2*x^2*S
in[(b*x)/2])/(b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)
/2]))
```

**fricas [C]** time = 0.66, size = 1801, normalized size = 5.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - 6*(b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a)
 - ((-6*I*b*d^2*x - 6*I*b*c*d)*cos(b*x + a)^3 + (6*I*b*d^2*x + 6*I*b*c*d)*c
os(b*x + a))*dilog(cos(b*x + a) + I*sin(b*x + a)) - ((6*I*b*d^2*x + 6*I*b*c
*d)*cos(b*x + a)^3 + (-6*I*b*d^2*x - 6*I*b*c*d)*cos(b*x + a))*dilog(cos(b*x
 + a) - I*sin(b*x + a)) - (4*I*d^2*cos(b*x + a)^3 - 4*I*d^2*cos(b*x + a))*d
ilog(I*cos(b*x + a) + sin(b*x + a)) - (4*I*d^2*cos(b*x + a)^3 - 4*I*d^2*cos
(b*x + a))*dilog(I*cos(b*x + a) - sin(b*x + a)) - (-4*I*d^2*cos(b*x + a)^3
+ 4*I*d^2*cos(b*x + a))*dilog(-I*cos(b*x + a) + sin(b*x + a)) - (-4*I*d^2*c
os(b*x + a)^3 + 4*I*d^2*cos(b*x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a))
 - ((-6*I*b*d^2*x - 6*I*b*c*d)*cos(b*x + a)^3 + (6*I*b*d^2*x + 6*I*b*c*d)*c
os(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) - ((6*I*b*d^2*x + 6*I*b*
c*d)*cos(b*x + a)^3 + (-6*I*b*d^2*x - 6*I*b*c*d)*cos(b*x + a))*dilog(-cos(b
*x + a) - I*sin(b*x + a)) + ((3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d
^2)*cos(b*x + a)^3 - (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(
b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1) + 4*((b*c*d - a*d^2)*cos(b
*x + a)^3 - (b*c*d - a*d^2)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a)
 + I) + ((3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a)^3 -
 (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a))*log(cos(b*
x + a) - I*sin(b*x + a) + 1) - 4*((b*c*d - a*d^2)*cos(b*x + a)^3 - (b*c*d -
 a*d^2)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + I) + 4*((b*d^2*x
+ a*d^2)*cos(b*x + a)^3 - (b*d^2*x + a*d^2)*cos(b*x + a))*log(I*cos(b*x + a
) + sin(b*x + a) + 1) - 4*((b*d^2*x + a*d^2)*cos(b*x + a)^3 - (b*d^2*x + a*
d^2)*cos(b*x + a))*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 4*((b*d^2*x + a
*d^2)*cos(b*x + a)^3 - (b*d^2*x + a*d^2)*cos(b*x + a))*log(-I*cos(b*x + a)
+ sin(b*x + a) + 1) - 4*((b*d^2*x + a*d^2)*cos(b*x + a)^3 - (b*d^2*x + a*d^
```

```
2)*cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - ((3*b^2*c^2 - 6*
a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^3 - (3*b^2*c^2 - 6*a*b*c*d + (3*a^2
 + 2)*d^2)*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)
- ((3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^3 - (3*b^2*c^2 -
6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a))*log(-1/2*cos(b*x + a) - 1/2*I*si
n(b*x + a) + 1/2) - 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*co
s(b*x + a)^3 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 4*((b*c*d - a*d^2)*cos(b*x +
a)^3 - (b*c*d - a*d^2)*cos(b*x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + I
) - 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^3 - (
b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a))*log(-cos(b*x
 + a) - I*sin(b*x + a) + 1) - 4*((b*c*d - a*d^2)*cos(b*x + a)^3 - (b*c*d -
a*d^2)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(d^2*cos(b
*x + a)^3 - d^2*cos(b*x + a))*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 6
*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*polylog(3, cos(b*x + a) - I*sin(b*
x + a)) + 6*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*polylog(3, -cos(b*x + a
) + I*sin(b*x + a)) + 6*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*polylog(3,
-cos(b*x + a) - I*sin(b*x + a)))/(b^3*cos(b*x + a)^3 - b^3*cos(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

**maple [B]** time = 0.31, size = 802, normalized size = 2.63

$$\frac{3d^2a^2\ln\left(e^{i(bx+a)}-1\right)}{2b^3}+\frac{3d^2\ln\left(1-e^{i(bx+a)}\right)x^2}{2b}-\frac{3d^2\ln\left(1-e^{i(bx+a)}\right)a^2}{2b^3}-\frac{3d^2\ln\left(e^{i(bx+a)}+1\right)x^2}{2b}+\frac{2d^2\ln\left(1+ie^{i(}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] 3/2/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+3/2/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/2
/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2-3*d^2*
polylog(3,-exp(I*(b*x+a)))/b^3+3*d^2*polylog(3,exp(I*(b*x+a)))/b^3-2/b^3*d^
2*ln(1-I*exp(I*(b*x+a)))*a+2*I/b^3*d^2*dilog(1-I*exp(I*(b*x+a)))-2*I/b^3*d^
2*dilog(1+I*exp(I*(b*x+a)))+2/b^2*d^2*ln(1+I*exp(I*(b*x+a)))*x+2/b^3*d^2*ln
(1+I*exp(I*(b*x+a)))*a-2/b^2*d^2*ln(1-I*exp(I*(b*x+a)))*x+3/2/b*c^2*ln(exp(
I*(b*x+a))-1)-3/2/b*c^2*ln(exp(I*(b*x+a))+1)-3*I/b^2*c*d*polylog(2,exp(I*(b
*x+a)))+3*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))+3/b*c*d*ln(1-exp(I*(b*x+a)))
*x+3/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-3/b*c*d*ln(exp(I*(b*x+a))+1)*x-3/b^2*c*
d*a*ln(exp(I*(b*x+a))-1)-1/b^3*d^2*ln(exp(I*(b*x+a))+1)+1/b^3*d^2*ln(exp(I*
(b*x+a))-1)+4*I*d/b^2*c*arctan(exp(I*(b*x+a)))-4*I*d^2/b^3*a*arctan(exp(I*(
b*x+a)))+3*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-3*I/b^2*polylog(2,exp(I*(
b*x+a)))*d^2*x+1/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*d^2*x^2
*b*exp(5*I*(b*x+a))+6*c*d*x*b*exp(5*I*(b*x+a))+3*c^2*b*exp(5*I*(b*x+a))-2*d
^2*x^2*b*exp(3*I*(b*x+a))-4*c*d*x*b*exp(3*I*(b*x+a))-2*I*d^2*x*exp(5*I*(b*x
+a))-2*c^2*b*exp(3*I*(b*x+a))+3*d^2*x^2*b*exp(I*(b*x+a))-2*I*c*d*exp(5*I*(b
*x+a))+6*c*d*x*b*exp(I*(b*x+a))+3*c^2*b*exp(I*(b*x+a))+2*I*d^2*x*exp(I*(b*x
+a))+2*I*d*c*exp(I*(b*x+a)))
```

**maxima [B]** time = 1.62, size = 3820, normalized size = 12.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 2*a*c*d*(2*(3*cos(b*x + a)^2
 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos
(b*x + a) - 1))/b + a^2*d^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos
(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^2 + 4*((8
*b*c*d + 8*(b*x + a)*d^2 - 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(
6*b*x + 6*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 8*(b*c*
d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-8*I*b*c*d - 8*I*(b*x + a)*d
^2 + 8*I*a*d^2)*sin(6*b*x + 6*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d
^2)*sin(4*b*x + 4*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*
x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (8*b*c*d + 8*(b*x + a)*
d^2 - 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 8*(b*c
*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 8*(b*c*d + (b*x + a)*d^2 - a
*d^2)*cos(2*b*x + 2*a) - (-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(6
*b*x + 6*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(4*b*x + 4*a)
- (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(cos
(b*x + a), -sin(b*x + a) + 1) - (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*
x + a) + 4*d^2 + 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2
)*cos(6*b*x + 6*a) - 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2
*d^2)*cos(4*b*x + 4*a) - 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a)
 + 2*d^2)*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^
2)*(b*x + a) + 4*I*d^2)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b
*c*d + 12*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^
2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(2*b*x + 2*a))*a
rctan2(sin(b*x + a), cos(b*x + a) + 1) + (4*d^2*cos(6*b*x + 6*a) - 4*d^2*co
s(4*b*x + 4*a) - 4*d^2*cos(2*b*x + 2*a) + 4*I*d^2*sin(6*b*x + 6*a) - 4*I*d^
2*sin(4*b*x + 4*a) - 4*I*d^2*sin(2*b*x + 2*a) + 4*d^2)*arctan2(sin(b*x + a)
, cos(b*x + a) - 1) - (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) + 6
*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(6*b*x + 6*a) - 6*((b*x
 + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a) - 6*((b*x + a)^
2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^
2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a))*sin(6*b*x + 6*a) + (-6*I*(b*x + a)
^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) + (-6*I*(b*
x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arct
an2(sin(b*x + a), -cos(b*x + a) + 1) + 4*(-3*I*(b*x + a)^2*d^2 - 2*b*c*d +
2*a*d^2 + 2*(-3*I*b*c*d + (3*I*a - 1)*d^2)*(b*x + a))*cos(5*b*x + 5*a) - (-
8*I*(b*x + a)^2*d^2 + (-16*I*b*c*d + 16*I*a*d^2)*(b*x + a))*cos(3*b*x + 3*a
) - (12*I*(b*x + a)^2*d^2 - 8*b*c*d + 8*a*d^2 + (24*I*b*c*d - 8*(3*I*a + 1)
*d^2)*(b*x + a))*cos(b*x + a) + (8*d^2*cos(6*b*x + 6*a) - 8*d^2*cos(4*b*x +
 4*a) - 8*d^2*cos(2*b*x + 2*a) + 8*I*d^2*sin(6*b*x + 6*a) - 8*I*d^2*sin(4*b
*x + 4*a) - 8*I*d^2*sin(2*b*x + 2*a) + 8*d^2)*dilog(I*e^(I*b*x + I*a)) - (8
*d^2*cos(6*b*x + 6*a) - 8*d^2*cos(4*b*x + 4*a) - 8*d^2*cos(2*b*x + 2*a) + 8
*I*d^2*sin(6*b*x + 6*a) - 8*I*d^2*sin(4*b*x + 4*a) - 8*I*d^2*sin(2*b*x + 2*
a) + 8*d^2)*dilog(-I*e^(I*b*x + I*a)) + (12*b*c*d + 12*(b*x + a)*d^2 - 12*a
*d^2 + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 12*(b*c*d + (b
*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*
cos(2*b*x + 2*a) - (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*sin(6*b*
x + 6*a) - (12*I*b*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(4*b*x + 4*a)
- (12*I*b*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e
^(I*b*x + I*a)) - (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d^2 + 12*(b*c*d + (b*
x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*c
os(4*b*x + 4*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (12
*I*b*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(6*b*x + 6*a) + (-12*I*b*c*d
 - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*sin(4*b*x + 4*a) + (-12*I*b*c*d - 12*I*
(b*x + a)*d^2 + 12*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-3*
I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2 + (-3*I*(b
```

```
*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(6*b*x + 6
*a) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*I*d^2)*c
os(4*b*x + 4*a) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a)
+ 2*I*d^2)*cos(2*b*x + 2*a) + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x +
 a) + 2*d^2)*sin(6*b*x + 6*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x
 + a) + 2*d^2)*sin(4*b*x + 4*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b
*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*
cos(b*x + a) + 1) - (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a
) + 2*I*d^2 + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*
I*d^2)*cos(6*b*x + 6*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*
(b*x + a) - 2*I*d^2)*cos(4*b*x + 4*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d
 + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) - (3*(b*x + a)^2*d^2 +
6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(6*b*x + 6*a) + (3*(b*x + a)^2*d^2
+ 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(4*b*x + 4*a) + (3*(b*x + a)^2*d^
2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2
 + 4*I*a*d^2 + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*cos(6*b*x + 6*a
) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos(4*b*x + 4*a) + (4*I*b*c
*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos(2*b*x + 2*a) + 4*(b*c*d + (b*x + a)
*d^2 - a*d^2)*sin(6*b*x + 6*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(4*b*
x + 4*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (4*I*b*c*d + 4*I*(b*x + a)*
d^2 - 4*I*a*d^2 + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos(6*b*x + 6
*a) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*cos(4*b*x + 4*a) + (-4*I
*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*cos(2*b*x + 2*a) - 4*(b*c*d + (b*x
+ a)*d^2 - a*d^2)*sin(6*b*x + 6*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(
4*b*x + 4*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(
b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (-12*I*d^2*cos(6*b*x +
6*a) + 12*I*d^2*cos(4*b*x + 4*a) + 12*I*d^2*cos(2*b*x + 2*a) + 12*d^2*sin(6
*b*x + 6*a) - 12*d^2*sin(4*b*x + 4*a) - 12*d^2*sin(2*b*x + 2*a) - 12*I*d^2)
*polylog(3, -e^(I*b*x + I*a)) - (12*I*d^2*cos(6*b*x + 6*a) - 12*I*d^2*cos(4
*b*x + 4*a) - 12*I*d^2*cos(2*b*x + 2*a) - 12*d^2*sin(6*b*x + 6*a) + 12*d^2*
sin(4*b*x + 4*a) + 12*d^2*sin(2*b*x + 2*a) + 12*I*d^2)*polylog(3, e^(I*b*x
+ I*a)) + (12*(b*x + a)^2*d^2 - 8*I*b*c*d + 8*I*a*d^2 + (24*b*c*d - (24*a +
 8*I)*d^2)*(b*x + a))*sin(5*b*x + 5*a) - 8*((b*x + a)^2*d^2 + 2*(b*c*d - a*
d^2)*(b*x + a))*sin(3*b*x + 3*a) + (12*(b*x + a)^2*d^2 + 8*I*b*c*d - 8*I*a*
d^2 + (24*b*c*d - (24*a - 8*I)*d^2)*(b*x + a))*sin(b*x + a))/(-4*I*b^2*cos(
6*b*x + 6*a) + 4*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 4*b^2*
sin(6*b*x + 6*a) - 4*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 4*I*b^
2))/b
```

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

## 3.281 $\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx$

**Optimal**. Leaf size=154

$$\frac{3id\text{Li}_2\left(-e^{i(a+bx)}\right)}{2b^2}-\frac{3id\text{Li}_2\left(e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+dx)\csc^2(a$$

```
[Out] -3*d*x*arctanh(exp(I*(b*x+a)))/b-3/2*c*arctanh(cos(b*x+a))/b-d*arctanh(sin(
b*x+a))/b^2-1/2*d*csc(b*x+a)/b^2+3/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-3/2
*I*d*polylog(2,exp(I*(b*x+a)))/b^2+3/2*(d*x+c)*sec(b*x+a)/b-1/2*(d*x+c)*csc
(b*x+a)^2*sec(b*x+a)/b
```

**Rubi [A]** time = 0.19, antiderivative size = 174, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2622, 288, 321, 207, 4420, 6271, 12, 4183, 2279, 2391, 3770, 2621}

$$\frac{3id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{3id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{3(c+dx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*d*x*ArcTanh[E^(I*(a + b*x))])/b + (3*d*x*ArcTanh[Cos[a + b*x]])/(2*b) -
 (3*(c + d*x)*ArcTanh[Cos[a + b*x]])/(2*b) - (d*ArcTanh[Sin[a + b*x]])/b^2
- (d*Csc[a + b*x])/(2*b^2) + (((3*I)/2)*d*PolyLog[2, -E^(I*(a + b*x))])/b^2
 - (((3*I)/2)*d*PolyLog[2, E^(I*(a + b*x))])/b^2 + (3*(c + d*x)*Sec[a + b*x
])/(2*b) - ((c + d*x)*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+dx)\csc}{}$$

$$= -\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+dx)\csc}{}$$

$$= \frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3d\tanh^{-1}}{}$$

$$= \frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3d\tanh^{-1}}{}$$

$$= -\frac{3dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}(}{2b}$$

$$= -\frac{3dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}(}{2b}$$

$$= -\frac{3dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}(}{2b}$$

**Mathematica [B]** time = 4.92, size = 520, normalized size = 3.38

$$\frac{3d\left(i\left(\mathrm{Li}_2\left(-e^{i(a+bx)}\right)-\mathrm{Li}_2\left(e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)\right)}{2b^2}-\frac{d\tan\left(\frac{1}{2}(a+bx)\right)}{4b^2}-\frac{d\cot\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (d*x)/b - (d*Cot[(a + b*x)/2])/(4*b^2) - (c*Csc[(a + b*x)/2]^2)/(8*b) - (d*
x*Csc[(a + b*x)/2]^2)/(8*b) - (3*c*Log[Cos[(a + b*x)/2]])/(2*b) + (d*Log[Co
s[(a + b*x)/2] - Sin[(a + b*x)/2]])/b^2 + (3*c*Log[Sin[(a + b*x)/2]])/(2*b)
 - (d*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]])/b^2 - (3*a*d*Log[Tan[(a + b
*x)/2]])/(2*b^2) + (3*d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I
*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x)
)])))/(2*b^2) + (c*Sec[(a + b*x)/2]^2)/(8*b) + (d*x*Sec[(a + b*x)/2]^2)/(8*
b) + (c*Sin[(a + b*x)/2])/(b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (c*Si
n[(a + b*x)/2])/(b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (d*(a*Sin[(a +
b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/(b^2*(Cos[(a + b*x)/2] + Sin[(a + b*
x)/2])) + (d*(-(a*Sin[(a + b*x)/2]) + (a + b*x)*Sin[(a + b*x)/2]))/(b^2*(Co
s[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*Tan[(a + b*x)/2])/(4*b^2)
```

**fricas [B]** time = 0.52, size = 621, normalized size = 4.03

$$-\frac{4\,bdx-6\,(bdx+bc)\cos\left(bx+a\right)^2-2\,d\cos\left(bx+a\right)\sin\left(bx+a\right)+4\,bc-\left(-3i\,d\cos\left(bx+a\right)^3+3i\,d\cos\left(bx+a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*d*x - 6*(b*d*x + b*c)*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x +
 a) + 4*b*c - (-3*I*d*cos(b*x + a)^3 + 3*I*d*cos(b*x + a))*dilog(cos(b*x +
a) + I*sin(b*x + a)) - (3*I*d*cos(b*x + a)^3 - 3*I*d*cos(b*x + a))*dilog(co
s(b*x + a) - I*sin(b*x + a)) - (-3*I*d*cos(b*x + a)^3 + 3*I*d*cos(b*x + a))
*dilog(-cos(b*x + a) + I*sin(b*x + a)) - (3*I*d*cos(b*x + a)^3 - 3*I*d*cos(
```

```
b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) + 3*((b*d*x + b*c)*cos(b*x
+ a)^3 - (b*d*x + b*c)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1)
 + 3*((b*d*x + b*c)*cos(b*x + a)^3 - (b*d*x + b*c)*cos(b*x + a))*log(cos(b*
x + a) - I*sin(b*x + a) + 1) - 3*((b*c - a*d)*cos(b*x + a)^3 - (b*c - a*d)*
cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*((b*c -
 a*d)*cos(b*x + a)^3 - (b*c - a*d)*cos(b*x + a))*log(-1/2*cos(b*x + a) - 1/
2*I*sin(b*x + a) + 1/2) - 3*((b*d*x + a*d)*cos(b*x + a)^3 - (b*d*x + a*d)*c
os(b*x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 3*((b*d*x + a*d)*cos
(b*x + a)^3 - (b*d*x + a*d)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a
) + 1) + 2*(d*cos(b*x + a)^3 - d*cos(b*x + a))*log(sin(b*x + a) + 1) - 2*(d
*cos(b*x + a)^3 - d*cos(b*x + a))*log(-sin(b*x + a) + 1))/(b^2*cos(b*x + a)
^3 - b^2*cos(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\csc (bx + a)^3 \sec (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

**maple [A]** time = 0.16, size = 267, normalized size = 1.73

$$\frac{3bdx\,e^{5i(bx+a)} + 3cb\,e^{5i(bx+a)} - 2bdx\,e^{3i(bx+a)} - 2cb\,e^{3i(bx+a)} - id\,e^{5i(bx+a)} + 3bdx\,e^{i(bx+a)} + 3cb\,e^{i(bx+a)} + id\,e^{i(bx+a}}{b^2\left(e^{2i(bx+a)} - 1\right)^2\left(1 + e^{2i(bx+a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*b*d*x*exp(5*I*(b*x+a))
+3*c*b*exp(5*I*(b*x+a))-2*b*d*x*exp(3*I*(b*x+a))-2*c*b*exp(3*I*(b*x+a))-I*d
*exp(5*I*(b*x+a))+3*b*d*x*exp(I*(b*x+a))+3*c*b*exp(I*(b*x+a))+I*d*exp(I*(b*
x+a)))+3/2/b*c*ln(exp(I*(b*x+a))-1)-3/2/b*c*ln(exp(I*(b*x+a))+1)-3/2/b^2*d*
a*ln(exp(I*(b*x+a))-1)+2*I/b^2*d*arctan(exp(I*(b*x+a)))+3/2*I/b^2*d*dilog(e
xp(I*(b*x+a)))+3/2*I/b^2*d*dilog(exp(I*(b*x+a))+1)-3/2/b*d*ln(exp(I*(b*x+a)
)+1)*x
```

**maxima [B]** time = 0.90, size = 1503, normalized size = 9.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -((4*d*cos(6*b*x + 6*a) - 4*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 4*I
*d*sin(6*b*x + 6*a) - 4*I*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 4*d
)*arctan2(2*(cos(b*x + 2*a)*cos(a) + sin(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)
^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*
a)*sin(a) + sin(a)^2), (cos(b*x + 2*a)^2 - cos(a)^2 + sin(b*x + 2*a)^2 - si
n(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) + (6*b*d*x + 6*b*c + 6*(b*d*x
 + b*c)*cos(6*b*x + 6*a) - 6*(b*d*x + b*c)*cos(4*b*x + 4*a) - 6*(b*d*x + b*
c)*cos(2*b*x + 2*a) - (-6*I*b*d*x - 6*I*b*c)*sin(6*b*x + 6*a) - (6*I*b*d*x
+ 6*I*b*c)*sin(4*b*x + 4*a) - (6*I*b*d*x + 6*I*b*c)*sin(2*b*x + 2*a))*arcta
n2(sin(b*x + a), cos(b*x + a) + 1) - (6*b*c*cos(6*b*x + 6*a) - 6*b*c*cos(4*
b*x + 4*a) - 6*b*c*cos(2*b*x + 2*a) + 6*I*b*c*sin(6*b*x + 6*a) - 6*I*b*c*si
n(4*b*x + 4*a) - 6*I*b*c*sin(2*b*x + 2*a) + 6*b*c)*arctan2(sin(b*x + a), co
```

```
s(b*x + a) - 1) + (6*b*d*x*cos(6*b*x + 6*a) - 6*b*d*x*cos(4*b*x + 4*a) - 6*
b*d*x*cos(2*b*x + 2*a) + 6*I*b*d*x*sin(6*b*x + 6*a) - 6*I*b*d*x*sin(4*b*x +
 4*a) - 6*I*b*d*x*sin(2*b*x + 2*a) + 6*b*d*x)*arctan2(sin(b*x + a), -cos(b*
x + a) + 1) - (-12*I*b*d*x - 12*I*b*c - 4*d)*cos(5*b*x + 5*a) - (8*I*b*d*x
+ 8*I*b*c)*cos(3*b*x + 3*a) - (-12*I*b*d*x - 12*I*b*c + 4*d)*cos(b*x + a) -
 (6*d*cos(6*b*x + 6*a) - 6*d*cos(4*b*x + 4*a) - 6*d*cos(2*b*x + 2*a) + 6*I*
d*sin(6*b*x + 6*a) - 6*I*d*sin(4*b*x + 4*a) - 6*I*d*sin(2*b*x + 2*a) + 6*d)
*dilog(-e^(I*b*x + I*a)) + (6*d*cos(6*b*x + 6*a) - 6*d*cos(4*b*x + 4*a) - 6
*d*cos(2*b*x + 2*a) + 6*I*d*sin(6*b*x + 6*a) - 6*I*d*sin(4*b*x + 4*a) - 6*I
*d*sin(2*b*x + 2*a) + 6*d)*dilog(e^(I*b*x + I*a)) - (3*I*b*d*x + 3*I*b*c +
(3*I*b*d*x + 3*I*b*c)*cos(6*b*x + 6*a) + (-3*I*b*d*x - 3*I*b*c)*cos(4*b*x +
 4*a) + (-3*I*b*d*x - 3*I*b*c)*cos(2*b*x + 2*a) - 3*(b*d*x + b*c)*sin(6*b*x
 + 6*a) + 3*(b*d*x + b*c)*sin(4*b*x + 4*a) + 3*(b*d*x + b*c)*sin(2*b*x + 2*
a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-3*I*b*d*x
 - 3*I*b*c + (-3*I*b*d*x - 3*I*b*c)*cos(6*b*x + 6*a) + (3*I*b*d*x + 3*I*b*c
)*cos(4*b*x + 4*a) + (3*I*b*d*x + 3*I*b*c)*cos(2*b*x + 2*a) + 3*(b*d*x + b*
c)*sin(6*b*x + 6*a) - 3*(b*d*x + b*c)*sin(4*b*x + 4*a) - 3*(b*d*x + b*c)*si
n(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) -
 (-2*I*d*cos(6*b*x + 6*a) + 2*I*d*cos(4*b*x + 4*a) + 2*I*d*cos(2*b*x + 2*a)
 + 2*d*sin(6*b*x + 6*a) - 2*d*sin(4*b*x + 4*a) - 2*d*sin(2*b*x + 2*a) - 2*I
*d)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 +
 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin
(a)^2)) - (12*b*d*x + 12*b*c - 4*I*d)*sin(5*b*x + 5*a) + 8*(b*d*x + b*c)*si
n(3*b*x + 3*a) - (12*b*d*x + 12*b*c + 4*I*d)*sin(b*x + a))/(-4*I*b^2*cos(6*
b*x + 6*a) + 4*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 4*b^2*si
n(6*b*x + 6*a) - 4*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 4*I*b^2)
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \csc^3 (a + bx) \sec^2 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a)**2,x)
[Out] Integral((c + d*x)*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

# 3.282 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=27

$$\text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx},x\right)$$

[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)

**Rubi [A]** time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x),x]

[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx=\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

**Mathematica [A]** time = 22.22, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x),x]

[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]

**fricas [A]** time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^3\sec(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")

[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/(d*x + c), x)

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")

[Out] Timed out

**maple [A]** time = 2.67, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^3(bx+a)\right)\left(\sec^2(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**2/(c + d*x), x)
```

## 3.283 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=27

$$\operatorname{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2},x\right)$$

[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)

**Rubi** [A] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2,x]

[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

**Mathematica** [A] time = 30.37, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2,x]

[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2, x]

**fricas** [A] time = 0.58, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\csc(bx+a)^3\sec(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")

[Out] Timed out

**maple** [A] time = 5.20, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^3(bx+a)\right)\left(\sec^2(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)^2\,\sin(a+b\,x)^3\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**2/(c + d*x)**2, x)
```

# 3.284 $\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(x^m\csc^3(a+bx)\sec^2(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

**Rubi** [A] time = 0.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
[Out] Defer[Int][x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int x^m\csc^3(a+bx)\sec^2(a+bx)\,dx=\int x^m\csc^3(a+bx)\sec^2(a+bx)\,dx$$

**Mathematica** [A] time = 39.65, size = 0, normalized size = 0.00

$$\int x^m\csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
[Out] Integrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

**fricas** [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

**maple** [A] time = 0.11, size = 0, normalized size = 0.00

$$\int x^m\left(\csc^3\left(bx+a\right)\right)\left(\sec^2\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
[Out] int(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\cos(a+b\,x)^2 \sin(a+b\,x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] int(x^m/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csc(b*x+a)**3*sec(b*x+a)**2,x)
[Out] Timed out
```

# 3.285 $\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx$

**Optimal**. Leaf size=387

$$\frac{3i\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^4}-\frac{3i\text{Li}_2\left(e^{i(a+bx)}\right)}{b^4}+\frac{6\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{6\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{9i\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{9i\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}-\frac{6ix\text{Li}_2\left(\right.}{}$$

```
[Out] 9*I*polylog(4,exp(I*(b*x+a)))/b^4-6*x*arctanh(exp(I*(b*x+a)))/b^3-3*x^3*arc
tanh(exp(I*(b*x+a)))/b-3/2*x^2*csc(b*x+a)/b^2-9*I*polylog(4,-exp(I*(b*x+a))
)/b^4-6*I*x*polylog(2,-I*exp(I*(b*x+a)))/b^3+9/2*I*x^2*polylog(2,-exp(I*(b*
x+a)))/b^2+6*I*x^2*arctan(exp(I*(b*x+a)))/b^2+3*I*polylog(2,-exp(I*(b*x+a))
)/b^4+6*I*x*polylog(2,I*exp(I*(b*x+a)))/b^3-9*x*polylog(3,-exp(I*(b*x+a)))/
b^3+6*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*polylog(3,I*exp(I*(b*x+a)))/b^4+9*
x*polylog(3,exp(I*(b*x+a)))/b^3-9/2*I*x^2*polylog(2,exp(I*(b*x+a)))/b^2-3*I
*polylog(2,exp(I*(b*x+a)))/b^4+3/2*x^3*sec(b*x+a)/b-1/2*x^3*csc(b*x+a)^2*se
c(b*x+a)/b
```

**Rubi** [A] time = 0.96, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2622, 288, 321, 207, 4420, 14, 6273, 12, 4183, 2531, 6609, 2282, 6589, 6742, 4181, 2621, 2279, 2391}

$$\frac{9ix^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{9ix^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{6ix\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6ix\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((6*I)*x^2*ArcTan[E^(I*(a + b*x))])/b^2 - (6*x*ArcTanh[E^(I*(a + b*x))])/b^
3 - (3*x^3*ArcTanh[E^(I*(a + b*x))])/b - (3*x^2*Csc[a + b*x])/(2*b^2) + ((3
*I)*PolyLog[2, -E^(I*(a + b*x))])/b^4 + (((9*I)/2)*x^2*PolyLog[2, -E^(I*(a
+ b*x))])/b^2 - ((6*I)*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*x*P
olyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*PolyLog[2, E^(I*(a + b*x))])/b^4
 - (((9*I)/2)*x^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (9*x*PolyLog[3, -E^(I*
(a + b*x))])/b^3 + (6*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*PolyLog[3,
 I*E^(I*(a + b*x))])/b^4 + (9*x*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((9*I)*P
olyLog[4, -E^(I*(a + b*x))])/b^4 + ((9*I)*PolyLog[4, E^(I*(a + b*x))])/b^4
+ (3*x^3*Sec[a + b*x])/(2*b) - (x^3*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
```

```
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3x^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}$$
$$= -\frac{3x^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}$$
$$= -\frac{3x^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}$$
$$= \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3\int bx^3\csc(a+bx)\,dx}{2b} + \frac{3}{}$$
$$= \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3}{2}\int x^3\csc(a+bx)\,dx + \frac{3\int}{}$$
$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3x^2\tanh^{-1}(\sin(a+bx))}{2b^2} - \frac{3x^2}{}$$
$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3x^2\tanh^{-1}(\sin(a+bx))}{2b^2} - \frac{3x^2}{}$$
$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3x^2\tanh^{-1}(\sin(a+bx))}{2b^2} - \frac{3x^2}{}$$
$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(}{2b}$$
$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(}{2b}$$
$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(}{2b}$$
$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(}{2b}$$
$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(}{2b}$$
$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(}{2b}$$

**Mathematica [A]** time = 7.21, size = 672, normalized size = 1.74

$$\frac{3x^2\csc\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)}{4b^2} - \frac{3x^2\sec\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)}{4b^2} + \frac{x^2\csc(a)\sec(a)(2bx\sin(a)-3\cos(a))}{2b^2} + \frac{6\left(ib\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] -1/8*(x^3*Csc[a/2 + (b*x)/2]^2)/b + (6*(I*b^2*x^2*ArcTan[Cos[a + b*x] + I*S
in[a + b*x]] + I*b*x*PolyLog[2, I*Cos[a + b*x] - Sin[a + b*x]] - I*b*x*Poly
Log[2, (-I)*Cos[a + b*x] + Sin[a + b*x]] - PolyLog[3, I*Cos[a + b*x] - Sin[
a + b*x]] + PolyLog[3, (-I)*Cos[a + b*x] + Sin[a + b*x]]))/b^4 + (3*(2*b*x*
Log[1 - Cos[a + b*x] - I*Sin[a + b*x]] + b^3*x^3*Log[1 - Cos[a + b*x] - I*S
in[a + b*x]] - 2*b*x*Log[1 + Cos[a + b*x] + I*Sin[a + b*x]] - b^3*x^3*Log[1
 + Cos[a + b*x] + I*Sin[a + b*x]] + I*(2 + 3*b^2*x^2)*PolyLog[2, -Cos[a + b
*x] - I*Sin[a + b*x]] - I*(2 + 3*b^2*x^2)*PolyLog[2, Cos[a + b*x] + I*Sin[a
```

```
 + b*x]] - 6*b*x*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] + 6*b*x*PolyLog
[3, Cos[a + b*x] + I*Sin[a + b*x]] - (6*I)*PolyLog[4, -Cos[a + b*x] - I*Sin
[a + b*x]] + (6*I)*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]))/(2*b^4) + (x
^3*Sec[a/2 + (b*x)/2]^2)/(8*b) + (x^2*Csc[a]*Sec[a]*(-3*Cos[a] + 2*b*x*Sin[
a]))/(2*b^2) + (3*x^2*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x)/2])/(4*b^2) - (
3*x^2*Sec[a/2]*Sec[a/2 + (b*x)/2]*Sin[(b*x)/2])/(4*b^2) + (x^3*Sin[(b*x)/2]
)/(b*(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) - (x^
3*Sin[(b*x)/2])/(b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b
*x)/2]))
```

**fricas [C]** time = 0.62, size = 1735, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b^3*x^3*cos(b*x + a)^2 - 4*b^3*x^3 + 6*b^2*x^2*cos(b*x + a)*sin(b*x
+ a) + ((-9*I*b^2*x^2 - 6*I)*cos(b*x + a)^3 + (9*I*b^2*x^2 + 6*I)*cos(b*x +
 a))*dilog(cos(b*x + a) + I*sin(b*x + a)) + ((9*I*b^2*x^2 + 6*I)*cos(b*x +
a)^3 + (-9*I*b^2*x^2 - 6*I)*cos(b*x + a))*dilog(cos(b*x + a) - I*sin(b*x +
a)) + (12*I*b*x*cos(b*x + a)^3 - 12*I*b*x*cos(b*x + a))*dilog(I*cos(b*x + a
) + sin(b*x + a)) + (12*I*b*x*cos(b*x + a)^3 - 12*I*b*x*cos(b*x + a))*dilog
(I*cos(b*x + a) - sin(b*x + a)) + (-12*I*b*x*cos(b*x + a)^3 + 12*I*b*x*cos(
b*x + a))*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-12*I*b*x*cos(b*x + a)^3
 + 12*I*b*x*cos(b*x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a)) + ((-9*I*b^
2*x^2 - 6*I)*cos(b*x + a)^3 + (9*I*b^2*x^2 + 6*I)*cos(b*x + a))*dilog(-cos(
b*x + a) + I*sin(b*x + a)) + ((9*I*b^2*x^2 + 6*I)*cos(b*x + a)^3 + (-9*I*b^
2*x^2 - 6*I)*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*((b^3*
x^3 + 2*b*x)*cos(b*x + a)^3 - (b^3*x^3 + 2*b*x)*cos(b*x + a))*log(cos(b*x +
 a) + I*sin(b*x + a) + 1) - 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*x + a))*log(c
os(b*x + a) + I*sin(b*x + a) + I) - 3*((b^3*x^3 + 2*b*x)*cos(b*x + a)^3 - (
b^3*x^3 + 2*b*x)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(
a^2*cos(b*x + a)^3 - a^2*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) +
I) - 6*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(
I*cos(b*x + a) + sin(b*x + a) + 1) + 6*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b
^2*x^2 - a^2)*cos(b*x + a))*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 6*((b^
2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + 6*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a
^2)*cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 3*((a^3 + 2*a)*
cos(b*x + a)^3 - (a^3 + 2*a)*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*si
n(b*x + a) + 1/2) - 3*((a^3 + 2*a)*cos(b*x + a)^3 - (a^3 + 2*a)*cos(b*x + a
))*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*((b^3*x^3 + a^3 +
2*b*x + 2*a)*cos(b*x + a)^3 - (b^3*x^3 + a^3 + 2*b*x + 2*a)*cos(b*x + a))*l
og(-cos(b*x + a) + I*sin(b*x + a) + 1) - 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*
x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*((b^3*x^3 + a^3 + 2*b*x
 + 2*a)*cos(b*x + a)^3 - (b^3*x^3 + a^3 + 2*b*x + 2*a)*cos(b*x + a))*log(-c
os(b*x + a) - I*sin(b*x + a) + 1) + 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*x + a
))*log(-cos(b*x + a) - I*sin(b*x + a) + I) + (18*I*cos(b*x + a)^3 - 18*I*co
s(b*x + a))*polylog(4, cos(b*x + a) + I*sin(b*x + a)) + (-18*I*cos(b*x + a)
^3 + 18*I*cos(b*x + a))*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + (18*I*c
os(b*x + a)^3 - 18*I*cos(b*x + a))*polylog(4, -cos(b*x + a) + I*sin(b*x + a
)) + (-18*I*cos(b*x + a)^3 + 18*I*cos(b*x + a))*polylog(4, -cos(b*x + a) -
I*sin(b*x + a)) + 18*(b*x*cos(b*x + a)^3 - b*x*cos(b*x + a))*polylog(3, cos
(b*x + a) + I*sin(b*x + a)) + 18*(b*x*cos(b*x + a)^3 - b*x*cos(b*x + a))*po
lylog(3, cos(b*x + a) - I*sin(b*x + a)) + 12*(cos(b*x + a)^3 - cos(b*x + a)
)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 12*(cos(b*x + a)^3 - cos(b*x
+ a))*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 12*(cos(b*x + a)^3 - cos(
b*x + a))*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*(cos(b*x + a)^3 -
 cos(b*x + a))*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 18*(b*x*cos(b*x
```

```
 + a)^3 - b*x*cos(b*x + a))*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 18
*(b*x*cos(b*x + a)^3 - b*x*cos(b*x + a))*polylog(3, -cos(b*x + a) - I*sin(b
*x + a)))/(b^4*cos(b*x + a)^3 - b^4*cos(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

**maple [F]** time = 1.62, size = 0, normalized size = 0.00

$$\int x^3 \left(\csc^3(bx+a)\right) \left(\sec^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] int(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

**maxima [B]** time = 1.24, size = 3989, normalized size = 10.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log
(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 4*((12*(b*x + a)^2 - 24*(b*
x + a)*a + 12*a^2 + 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(6*b*x + 6*a)
 - 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(4*b*x + 4*a) - 12*((b*x + a)^
2 - 2*(b*x + a)*a + a^2)*cos(2*b*x + 2*a) - (-12*I*(b*x + a)^2 + 24*I*(b*x
+ a)*a - 12*I*a^2)*sin(6*b*x + 6*a) - (12*I*(b*x + a)^2 - 24*I*(b*x + a)*a
+ 12*I*a^2)*sin(4*b*x + 4*a) - (12*I*(b*x + a)^2 - 24*I*(b*x + a)*a + 12*I*
a^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (12*(b*x +
 a)^2 - 24*(b*x + a)*a + 12*a^2 + 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*co
s(6*b*x + 6*a) - 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(4*b*x + 4*a) -
12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(2*b*x + 2*a) - (-12*I*(b*x + a)^
2 + 24*I*(b*x + a)*a - 12*I*a^2)*sin(6*b*x + 6*a) - (12*I*(b*x + a)^2 - 24*
I*(b*x + a)*a + 12*I*a^2)*sin(4*b*x + 4*a) - (12*I*(b*x + a)^2 - 24*I*(b*x
+ a)*a + 12*I*a^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) +
1) - (6*(b*x + a)^3 - 18*(b*x + a)^2*a + 6*(3*a^2 + 2)*(b*x + a) + 6*((b*x
+ a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*cos(6*b*x + 6*a) -
6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*cos(4*b*x +
 4*a) - 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*cos
(2*b*x + 2*a) + (6*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + (18*I*a^2 + 12*I)*(
b*x + a) - 12*I*a)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^3 + 18*I*(b*x + a)^2*
a + (-18*I*a^2 - 12*I)*(b*x + a) + 12*I*a)*sin(4*b*x + 4*a) + (-6*I*(b*x +
a)^3 + 18*I*(b*x + a)^2*a + (-18*I*a^2 - 12*I)*(b*x + a) + 12*I*a)*sin(2*b*
x + 2*a) - 12*a)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (12*a*cos(6*b*x
+ 6*a) - 12*a*cos(4*b*x + 4*a) - 12*a*cos(2*b*x + 2*a) + 12*I*a*sin(6*b*x +
 6*a) - 12*I*a*sin(4*b*x + 4*a) - 12*I*a*sin(2*b*x + 2*a) + 12*a)*arctan2(s
in(b*x + a), cos(b*x + a) - 1) - (6*(b*x + a)^3 - 18*(b*x + a)^2*a + 6*(3*a
^2 + 2)*(b*x + a) + 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a
))*cos(6*b*x + 6*a) - 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x +
 a))*cos(4*b*x + 4*a) - 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x
```

```
 + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + (18*I*a^2
 + 12*I)*(b*x + a))*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^3 + 18*I*(b*x + a)^2
*a + (-18*I*a^2 - 12*I)*(b*x + a))*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^3 + 1
8*I*(b*x + a)^2*a + (-18*I*a^2 - 12*I)*(b*x + a))*sin(2*b*x + 2*a))*arctan2
(sin(b*x + a), -cos(b*x + a) + 1) - (12*I*(b*x + a)^3 - 12*(b*x + a)^2*(3*I
*a - 1) + (36*I*a^2 - 24*a)*(b*x + a) + 12*a^2)*cos(5*b*x + 5*a) - (-8*I*(b
*x + a)^3 + 24*I*(b*x + a)^2*a - 24*I*(b*x + a)*a^2)*cos(3*b*x + 3*a) - (12
*I*(b*x + a)^3 - 12*(b*x + a)^2*(3*I*a + 1) + (36*I*a^2 + 24*a)*(b*x + a) -
 12*a^2)*cos(b*x + a) + (24*b*x*cos(6*b*x + 6*a) - 24*b*x*cos(4*b*x + 4*a)
- 24*b*x*cos(2*b*x + 2*a) + 24*I*b*x*sin(6*b*x + 6*a) - 24*I*b*x*sin(4*b*x
+ 4*a) - 24*I*b*x*sin(2*b*x + 2*a) + 24*b*x)*dilog(I*e^(I*b*x + I*a)) - (24
*b*x*cos(6*b*x + 6*a) - 24*b*x*cos(4*b*x + 4*a) - 24*b*x*cos(2*b*x + 2*a) +
 24*I*b*x*sin(6*b*x + 6*a) - 24*I*b*x*sin(4*b*x + 4*a) - 24*I*b*x*sin(2*b*x
 + 2*a) + 24*b*x)*dilog(-I*e^(I*b*x + I*a)) + (18*(b*x + a)^2 - 36*(b*x + a
)*a + 18*a^2 + 6*(3*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(6*b*x + 6*
a) - 6*(3*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(4*b*x + 4*a) - 6*(3*
(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(2*b*x + 2*a) - (-18*I*(b*x + a
)^2 + 36*I*(b*x + a)*a - 18*I*a^2 - 12*I)*sin(6*b*x + 6*a) - (18*I*(b*x + a
)^2 - 36*I*(b*x + a)*a + 18*I*a^2 + 12*I)*sin(4*b*x + 4*a) - (18*I*(b*x + a
)^2 - 36*I*(b*x + a)*a + 18*I*a^2 + 12*I)*sin(2*b*x + 2*a) + 12)*dilog(-e^(
I*b*x + I*a)) - (18*(b*x + a)^2 - 36*(b*x + a)*a + 18*a^2 + 6*(3*(b*x + a)^
2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(6*b*x + 6*a) - 6*(3*(b*x + a)^2 - 6*(b*x
 + a)*a + 3*a^2 + 2)*cos(4*b*x + 4*a) - 6*(3*(b*x + a)^2 - 6*(b*x + a)*a +
3*a^2 + 2)*cos(2*b*x + 2*a) + (18*I*(b*x + a)^2 - 36*I*(b*x + a)*a + 18*I*a
^2 + 12*I)*sin(6*b*x + 6*a) + (-18*I*(b*x + a)^2 + 36*I*(b*x + a)*a - 18*I*
a^2 - 12*I)*sin(4*b*x + 4*a) + (-18*I*(b*x + a)^2 + 36*I*(b*x + a)*a - 18*I
*a^2 - 12*I)*sin(2*b*x + 2*a) + 12)*dilog(e^(I*b*x + I*a)) - (-3*I*(b*x + a
)^3 + 9*I*(b*x + a)^2*a + (-9*I*a^2 - 6*I)*(b*x + a) + (-3*I*(b*x + a)^3 +
9*I*(b*x + a)^2*a + (-9*I*a^2 - 6*I)*(b*x + a) + 6*I*a)*cos(6*b*x + 6*a) +
(3*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + (9*I*a^2 + 6*I)*(b*x + a) - 6*I*a)*c
os(4*b*x + 4*a) + (3*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + (9*I*a^2 + 6*I)*(b
*x + a) - 6*I*a)*cos(2*b*x + 2*a) + 3*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a
^2 + 2)*(b*x + a) - 2*a)*sin(6*b*x + 6*a) - 3*((b*x + a)^3 - 3*(b*x + a)^2*
a + (3*a^2 + 2)*(b*x + a) - 2*a)*sin(4*b*x + 4*a) - 3*((b*x + a)^3 - 3*(b*x
 + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*sin(2*b*x + 2*a) + 6*I*a)*log(cos(
b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (3*I*(b*x + a)^3 - 9*I*
(b*x + a)^2*a + (9*I*a^2 + 6*I)*(b*x + a) + (3*I*(b*x + a)^3 - 9*I*(b*x + a
)^2*a + (9*I*a^2 + 6*I)*(b*x + a) - 6*I*a)*cos(6*b*x + 6*a) + (-3*I*(b*x +
a)^3 + 9*I*(b*x + a)^2*a + (-9*I*a^2 - 6*I)*(b*x + a) + 6*I*a)*cos(4*b*x +
4*a) + (-3*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a + (-9*I*a^2 - 6*I)*(b*x + a) +
 6*I*a)*cos(2*b*x + 2*a) - 3*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(
b*x + a) - 2*a)*sin(6*b*x + 6*a) + 3*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^
2 + 2)*(b*x + a) - 2*a)*sin(4*b*x + 4*a) + 3*((b*x + a)^3 - 3*(b*x + a)^2*a
 + (3*a^2 + 2)*(b*x + a) - 2*a)*sin(2*b*x + 2*a) - 6*I*a)*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-6*I*(b*x + a)^2 + 12*I*(b*x +
a)*a - 6*I*a^2 + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a - 6*I*a^2)*cos(6*b*x
+ 6*a) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a + 6*I*a^2)*cos(4*b*x + 4*a) +
(6*I*(b*x + a)^2 - 12*I*(b*x + a)*a + 6*I*a^2)*cos(2*b*x + 2*a) + 6*((b*x +
 a)^2 - 2*(b*x + a)*a + a^2)*sin(6*b*x + 6*a) - 6*((b*x + a)^2 - 2*(b*x + a
)*a + a^2)*sin(4*b*x + 4*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (6*I
*(b*x + a)^2 - 12*I*(b*x + a)*a + 6*I*a^2 + (6*I*(b*x + a)^2 - 12*I*(b*x +
a)*a + 6*I*a^2)*cos(6*b*x + 6*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a - 6
*I*a^2)*cos(4*b*x + 4*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a - 6*I*a^2)*
cos(2*b*x + 2*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*sin(6*b*x + 6*a) +
 6*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*sin(4*b*x + 4*a) + 6*((b*x + a)^2 -
2*(b*x + a)*a + a^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
- 2*sin(b*x + a) + 1) - (36*cos(6*b*x + 6*a) - 36*cos(4*b*x + 4*a) - 36*cos
(2*b*x + 2*a) + 36*I*sin(6*b*x + 6*a) - 36*I*sin(4*b*x + 4*a) - 36*I*sin(2*
```

```
b*x + 2*a) + 36)*polylog(4, -e^(I*b*x + I*a)) + (36*cos(6*b*x + 6*a) - 36*c
os(4*b*x + 4*a) - 36*cos(2*b*x + 2*a) + 36*I*sin(6*b*x + 6*a) - 36*I*sin(4*
b*x + 4*a) - 36*I*sin(2*b*x + 2*a) + 36)*polylog(4, e^(I*b*x + I*a)) - (-24
*I*cos(6*b*x + 6*a) + 24*I*cos(4*b*x + 4*a) + 24*I*cos(2*b*x + 2*a) + 24*si
n(6*b*x + 6*a) - 24*sin(4*b*x + 4*a) - 24*sin(2*b*x + 2*a) - 24*I)*polylog(
3, I*e^(I*b*x + I*a)) - (24*I*cos(6*b*x + 6*a) - 24*I*cos(4*b*x + 4*a) - 24
*I*cos(2*b*x + 2*a) - 24*sin(6*b*x + 6*a) + 24*sin(4*b*x + 4*a) + 24*sin(2*
b*x + 2*a) + 24*I)*polylog(3, -I*e^(I*b*x + I*a)) - (-36*I*b*x*cos(6*b*x +
6*a) + 36*I*b*x*cos(4*b*x + 4*a) + 36*I*b*x*cos(2*b*x + 2*a) + 36*b*x*sin(6
*b*x + 6*a) - 36*b*x*sin(4*b*x + 4*a) - 36*b*x*sin(2*b*x + 2*a) - 36*I*b*x)
*polylog(3, -e^(I*b*x + I*a)) - (36*I*b*x*cos(6*b*x + 6*a) - 36*I*b*x*cos(4
*b*x + 4*a) - 36*I*b*x*cos(2*b*x + 2*a) - 36*b*x*sin(6*b*x + 6*a) + 36*b*x*
sin(4*b*x + 4*a) + 36*b*x*sin(2*b*x + 2*a) + 36*I*b*x)*polylog(3, e^(I*b*x
+ I*a)) + (12*(b*x + a)^3 - (b*x + a)^2*(36*a + 12*I) + 12*(3*a^2 + 2*I*a)*
(b*x + a) - 12*I*a^2)*sin(5*b*x + 5*a) - 8*((b*x + a)^3 - 3*(b*x + a)^2*a +
 3*(b*x + a)*a^2)*sin(3*b*x + 3*a) + (12*(b*x + a)^3 - (b*x + a)^2*(36*a -
12*I) + 12*(3*a^2 - 2*I*a)*(b*x + a) + 12*I*a^2)*sin(b*x + a))/(-4*I*cos(6*
b*x + 6*a) + 4*I*cos(4*b*x + 4*a) + 4*I*cos(2*b*x + 2*a) + 4*sin(6*b*x + 6*
a) - 4*sin(4*b*x + 4*a) - 4*sin(2*b*x + 2*a) - 4*I))/b^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\cos(a+b\,x)^2\,\sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] int(x^3/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.286 $\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$

**Optimal**. Leaf size=235

$$-\frac{2i\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2i\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{\tanh^{-1}(\cos(a+bx))}{b^3}+\frac{3ix\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}$$

```
[Out] 4*I*x*arctan(exp(I*(b*x+a)))/b^2-3*x^2*arctanh(exp(I*(b*x+a)))/b-arctanh(co
s(b*x+a))/b^3-x*csc(b*x+a)/b^2+3*I*x*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*pol
ylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*polylog(2,I*exp(I*(b*x+a)))/b^3-3*I*x*pol
ylog(2,exp(I*(b*x+a)))/b^2-3*polylog(3,-exp(I*(b*x+a)))/b^3+3*polylog(3,exp
(I*(b*x+a)))/b^3+3/2*x^2*sec(b*x+a)/b-1/2*x^2*csc(b*x+a)^2*sec(b*x+a)/b
```

**Rubi [A]** time = 0.54, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.950, Rules used = {2622, 288, 321, 207, 4420, 14, 6273, 12, 4183, 2531, 2282, 6589, 6742, 4181, 2279, 2391, 2621, 6271, 3770}

$$\frac{3ix\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3ix\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2i\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2i\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-3\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((4*I)*x*ArcTan[E^(I*(a + b*x))])/b^2 - (3*x^2*ArcTanh[E^(I*(a + b*x))])/b
- ArcTanh[Cos[a + b*x]]/b^3 - (x*Csc[a + b*x])/b^2 + ((3*I)*x*PolyLog[2, -E
^(I*(a + b*x))])/b^2 - ((2*I)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((2*I
)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*x*PolyLog[2, E^(I*(a + b*x))]
)/b^2 - (3*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (3*PolyLog[3, E^(I*(a + b*x)
)])/b^3 + (3*x^2*Sec[a + b*x])/(2*b) - (x^2*Csc[a + b*x]^2*Sec[a + b*x])/(2
*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
```

```
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3x^2\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b} -$$

$$= -\frac{3x^2\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b} -$$

$$= -\frac{3x^2\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b} +$$

$$= \frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}+\frac{\int\left(-3x\sec(a+bx)+x\csc^2\right.}{b}$$

$$= \frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}+\frac{3}{2}\int x^2\csc(a+bx)\,dx+\frac{\int}{}$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{x\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{x\csc(}{b}$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{x\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{x\csc(}{b}$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{x\csc(a}{b^2}$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{x\csc(a}{b^2}$$

$$= \frac{4ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{x\csc(a}{b^2}$$

$$= \frac{4ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{x\csc(a}{b^2}$$

$$= \frac{4ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{x\csc(a}{b^2}$$

**Mathematica** [**B**] time = 6.62, size = 613, normalized size = 2.61

$$-\frac{2\left(i\left(\text{Li}_2\left(-e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)-\text{Li}_2\left(e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)\right)+\left(-a-bx+\frac{\pi}{2}\right)\left(\log\left(1-e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)-\log\left(1+e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)\right)-\left(\frac{\pi}{2}\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] -1/8*(x^2*Csc[a/2 + (b*x)/2]^2)/b - (2*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a
 + Pi/2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[
(-a + Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[2
, E^(I*(-a + Pi/2 - b*x))])))/b^3 + (2*Log[1 - Cos[a + b*x] - I*Sin[a + b*x
]] + 3*b^2*x^2*Log[1 - Cos[a + b*x] - I*Sin[a + b*x]] - 2*Log[1 + Cos[a + b
*x] + I*Sin[a + b*x]] - 3*b^2*x^2*Log[1 + Cos[a + b*x] + I*Sin[a + b*x]] +
(6*I)*b*x*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] - (6*I)*b*x*PolyLog[2,
 Cos[a + b*x] + I*Sin[a + b*x]] - 6*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*
x]] + 6*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]])/(2*b^3) + (x^2*Sec[a/2 +
 (b*x)/2]^2)/(8*b) + (x*Csc[a]*Sec[a]*(-Cos[a] + b*x*Sin[a]))/b^2 + (x*Csc[
a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x)/2])/(2*b^2) - (x*Sec[a/2]*Sec[a/2 + (b*x)
/2]*Sin[(b*x)/2])/(2*b^2) + (x^2*Sin[(b*x)/2])/(b*(Cos[a/2] - Sin[a/2])*(Co
s[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) - (x^2*Sin[(b*x)/2])/(b*(Cos[a/2] +
 Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2]))
```

**fricas [C]** time = 0.56, size = 1229, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b^2*x^2*cos(b*x + a)^2 - 4*b^2*x^2 + 4*b*x*cos(b*x + a)*sin(b*x + a)
 + (-6*I*b*x*cos(b*x + a)^3 + 6*I*b*x*cos(b*x + a))*dilog(cos(b*x + a) + I*
sin(b*x + a)) + (6*I*b*x*cos(b*x + a)^3 - 6*I*b*x*cos(b*x + a))*dilog(cos(b
*x + a) - I*sin(b*x + a)) + (4*I*cos(b*x + a)^3 - 4*I*cos(b*x + a))*dilog(I
*cos(b*x + a) + sin(b*x + a)) + (4*I*cos(b*x + a)^3 - 4*I*cos(b*x + a))*dil
og(I*cos(b*x + a) - sin(b*x + a)) + (-4*I*cos(b*x + a)^3 + 4*I*cos(b*x + a)
)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-4*I*cos(b*x + a)^3 + 4*I*cos(b*
x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (-6*I*b*x*cos(b*x + a)^3 +
6*I*b*x*cos(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (6*I*b*x*cos(
b*x + a)^3 - 6*I*b*x*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) -
((3*b^2*x^2 + 2)*cos(b*x + a)^3 - (3*b^2*x^2 + 2)*cos(b*x + a))*log(cos(b*x
 + a) + I*sin(b*x + a) + 1) + 4*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(cos
(b*x + a) + I*sin(b*x + a) + I) - ((3*b^2*x^2 + 2)*cos(b*x + a)^3 - (3*b^2*
x^2 + 2)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(a*cos(b*
x + a)^3 - a*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + I) - 4*((b*x
 + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*log(I*cos(b*x + a) + sin(b*x
 + a) + 1) + 4*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*log(I*co
s(b*x + a) - sin(b*x + a) + 1) - 4*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*co
s(b*x + a))*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 4*((b*x + a)*cos(b*x
+ a)^3 - (b*x + a)*cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) +
((3*a^2 + 2)*cos(b*x + a)^3 - (3*a^2 + 2)*cos(b*x + a))*log(-1/2*cos(b*x +
a) + 1/2*I*sin(b*x + a) + 1/2) + ((3*a^2 + 2)*cos(b*x + a)^3 - (3*a^2 + 2)*
cos(b*x + a))*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*((b^2*x
^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(-cos(b*x + a)
+ I*sin(b*x + a) + 1) + 4*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(-cos(b*x
+ a) + I*sin(b*x + a) + I) + 3*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 -
 a^2)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(a*cos(b*x
+ a)^3 - a*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 6*(cos(b
*x + a)^3 - cos(b*x + a))*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(co
s(b*x + a)^3 - cos(b*x + a))*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*
(cos(b*x + a)^3 - cos(b*x + a))*polylog(3, -cos(b*x + a) + I*sin(b*x + a))
- 6*(cos(b*x + a)^3 - cos(b*x + a))*polylog(3, -cos(b*x + a) - I*sin(b*x +
a)))/(b^3*cos(b*x + a)^3 - b^3*cos(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

**maple [B]** time = 0.22, size = 429, normalized size = 1.83

$$\frac{x\left(3bx\,e^{5i(bx+a)} - 2bx\,e^{3i(bx+a)} - 2ie^{5i(bx+a)} + 3bx\,e^{i(bx+a)} + 2ie^{i(bx+a)}\right)}{b^2\left(e^{2i(bx+a)}-1\right)^2\left(1+e^{2i(bx+a)}\right)} - \frac{3a^2\ln\left(1-e^{i(bx+a)}\right)}{2b^3} - \frac{3ix\,\mathrm{polylog}\left(2, e^{i(b}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] x/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*b*x*exp(5*I*(b*x+a))-2
*b*x*exp(3*I*(b*x+a))-2*I*exp(5*I*(b*x+a))+3*b*x*exp(I*(b*x+a))+2*I*exp(I*(
b*x+a)))-3/2/b^3*a^2*ln(1-exp(I*(b*x+a)))-2*I/b^3*dilog(1+I*exp(I*(b*x+a)))
+3*polylog(3,exp(I*(b*x+a)))/b^3-3*polylog(3,-exp(I*(b*x+a)))/b^3+2/b^2*ln(
1+I*exp(I*(b*x+a)))*x+3*I*x*polylog(2,-exp(I*(b*x+a)))/b^2-3/2/b*ln(exp(I*(
b*x+a))+1)*x^2-3*I*x*polylog(2,exp(I*(b*x+a)))/b^2+3/2/b*ln(1-exp(I*(b*x+a)
))*x^2-2/b^2*ln(1-I*exp(I*(b*x+a)))*x+2/b^3*ln(1+I*exp(I*(b*x+a)))*a-2/b^3*
ln(1-I*exp(I*(b*x+a)))*a+1/b^3*ln(exp(I*(b*x+a))-1)-1/b^3*ln(exp(I*(b*x+a))
+1)+3/2/b^3*a^2*ln(exp(I*(b*x+a))-1)-4*I/b^3*a*arctan(exp(I*(b*x+a)))+2*I/b
^3*dilog(1-I*exp(I*(b*x+a)))
```

**maxima [B]** time = 0.74, size = 2219, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) + 4*((8*b*x*cos(6*b*x + 6*a) -
 8*b*x*cos(4*b*x + 4*a) - 8*b*x*cos(2*b*x + 2*a) + 8*I*b*x*sin(6*b*x + 6*a)
 - 8*I*b*x*sin(4*b*x + 4*a) - 8*I*b*x*sin(2*b*x + 2*a) + 8*b*x)*arctan2(cos
(b*x + a), sin(b*x + a) + 1) + (8*b*x*cos(6*b*x + 6*a) - 8*b*x*cos(4*b*x +
4*a) - 8*b*x*cos(2*b*x + 2*a) + 8*I*b*x*sin(6*b*x + 6*a) - 8*I*b*x*sin(4*b*
x + 4*a) - 8*I*b*x*sin(2*b*x + 2*a) + 8*b*x)*arctan2(cos(b*x + a), -sin(b*x
 + a) + 1) - (6*(b*x + a)^2 - 12*(b*x + a)*a + 2*(3*(b*x + a)^2 - 6*(b*x +
a)*a + 2)*cos(6*b*x + 6*a) - 2*(3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*cos(4*b*
x + 4*a) - 2*(3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*cos(2*b*x + 2*a) + (6*I*(b
*x + a)^2 - 12*I*(b*x + a)*a + 4*I)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2 +
12*I*(b*x + a)*a - 4*I)*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x +
a)*a - 4*I)*sin(2*b*x + 2*a) + 4)*arctan2(sin(b*x + a), cos(b*x + a) + 1) +
 (4*cos(6*b*x + 6*a) - 4*cos(4*b*x + 4*a) - 4*cos(2*b*x + 2*a) + 4*I*sin(6*
b*x + 6*a) - 4*I*sin(4*b*x + 4*a) - 4*I*sin(2*b*x + 2*a) + 4)*arctan2(sin(b
*x + a), cos(b*x + a) - 1) - (6*(b*x + a)^2 - 12*(b*x + a)*a + 6*((b*x + a)
^2 - 2*(b*x + a)*a)*cos(6*b*x + 6*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a)*cos(
4*b*x + 4*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a)*cos(2*b*x + 2*a) + (6*I*(b*x
 + a)^2 - 12*I*(b*x + a)*a)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2 + 12*I*(b*
x + a)*a)*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*sin(2*b*
x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (12*I*(b*x + a)^2 - 8*
(b*x + a)*(3*I*a - 1) - 8*a)*cos(5*b*x + 5*a) - (-8*I*(b*x + a)^2 + 16*I*(b
*x + a)*a)*cos(3*b*x + 3*a) - (12*I*(b*x + a)^2 - 8*(b*x + a)*(3*I*a + 1) +
 8*a)*cos(b*x + a) + (8*cos(6*b*x + 6*a) - 8*cos(4*b*x + 4*a) - 8*cos(2*b*x
 + 2*a) + 8*I*sin(6*b*x + 6*a) - 8*I*sin(4*b*x + 4*a) - 8*I*sin(2*b*x + 2*a
) + 8)*dilog(I*e^(I*b*x + I*a)) - (8*cos(6*b*x + 6*a) - 8*cos(4*b*x + 4*a)
- 8*cos(2*b*x + 2*a) + 8*I*sin(6*b*x + 6*a) - 8*I*sin(4*b*x + 4*a) - 8*I*si
n(2*b*x + 2*a) + 8)*dilog(-I*e^(I*b*x + I*a)) + (12*b*x*cos(6*b*x + 6*a) -
12*b*x*cos(4*b*x + 4*a) - 12*b*x*cos(2*b*x + 2*a) + 12*I*b*x*sin(6*b*x + 6*
a) - 12*I*b*x*sin(4*b*x + 4*a) - 12*I*b*x*sin(2*b*x + 2*a) + 12*b*x)*dilog(
-e^(I*b*x + I*a)) - (12*b*x*cos(6*b*x + 6*a) - 12*b*x*cos(4*b*x + 4*a) - 12
*b*x*cos(2*b*x + 2*a) + 12*I*b*x*sin(6*b*x + 6*a) - 12*I*b*x*sin(4*b*x + 4*
a) - 12*I*b*x*sin(2*b*x + 2*a) + 12*b*x)*dilog(e^(I*b*x + I*a)) - (-3*I*(b*
x + a)^2 + 6*I*(b*x + a)*a + (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a - 2*I)*cos
(6*b*x + 6*a) + (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2*I)*cos(4*b*x + 4*a)
+ (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2*I)*cos(2*b*x + 2*a) + (3*(b*x + a)
^2 - 6*(b*x + a)*a + 2)*sin(6*b*x + 6*a) - (3*(b*x + a)^2 - 6*(b*x + a)*a +
 2)*sin(4*b*x + 4*a) - (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*sin(2*b*x + 2*a)
 - 2*I)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (3*I*(b
*x + a)^2 - 6*I*(b*x + a)*a + (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2*I)*cos
(6*b*x + 6*a) + (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a - 2*I)*cos(4*b*x + 4*a)
 + (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a - 2*I)*cos(2*b*x + 2*a) - (3*(b*x +
```

```
a)^2 - 6*(b*x + a)*a + 2)*sin(6*b*x + 6*a) + (3*(b*x + a)^2 - 6*(b*x + a)*a
 + 2)*sin(4*b*x + 4*a) + (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*sin(2*b*x + 2*
a) + 2*I)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-4*I
*b*x*cos(6*b*x + 6*a) + 4*I*b*x*cos(4*b*x + 4*a) + 4*I*b*x*cos(2*b*x + 2*a)
 + 4*b*x*sin(6*b*x + 6*a) - 4*b*x*sin(4*b*x + 4*a) - 4*b*x*sin(2*b*x + 2*a)
 - 4*I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (4*
I*b*x*cos(6*b*x + 6*a) - 4*I*b*x*cos(4*b*x + 4*a) - 4*I*b*x*cos(2*b*x + 2*a
) - 4*b*x*sin(6*b*x + 6*a) + 4*b*x*sin(4*b*x + 4*a) + 4*b*x*sin(2*b*x + 2*a
) + 4*I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (-
12*I*cos(6*b*x + 6*a) + 12*I*cos(4*b*x + 4*a) + 12*I*cos(2*b*x + 2*a) + 12*
sin(6*b*x + 6*a) - 12*sin(4*b*x + 4*a) - 12*sin(2*b*x + 2*a) - 12*I)*polylo
g(3, -e^(I*b*x + I*a)) - (12*I*cos(6*b*x + 6*a) - 12*I*cos(4*b*x + 4*a) - 1
2*I*cos(2*b*x + 2*a) - 12*sin(6*b*x + 6*a) + 12*sin(4*b*x + 4*a) + 12*sin(2
*b*x + 2*a) + 12*I)*polylog(3, e^(I*b*x + I*a)) + (12*(b*x + a)^2 - (b*x +
a)*(24*a + 8*I) + 8*I*a)*sin(5*b*x + 5*a) - 8*((b*x + a)^2 - 2*(b*x + a)*a)
*sin(3*b*x + 3*a) + (12*(b*x + a)^2 - (b*x + a)*(24*a - 8*I) - 8*I*a)*sin(b
*x + a))/(-4*I*cos(6*b*x + 6*a) + 4*I*cos(4*b*x + 4*a) + 4*I*cos(2*b*x + 2*
a) + 4*sin(6*b*x + 6*a) - 4*sin(4*b*x + 4*a) - 4*sin(2*b*x + 2*a) - 4*I))/b
^3
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] int(x^2/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \csc^{3}{\left (a + b x \right )} \sec^{2}{\left (a + b x \right )}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Integral(x**2*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

# 3.287 $\int x \csc^3(a+bx) \sec^2(a+bx)\, dx$

**Optimal**. Leaf size=126

$$\frac{3i\text{Li}_2\left(-e^{i(a+bx)}\right)}{2b^2}-\frac{3i\text{Li}_2\left(e^{i(a+bx)}\right)}{2b^2}-\frac{\csc(a+bx)}{2b^2}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{x\csc^2}{}$$

```
[Out] -3*x*arctanh(exp(I*(b*x+a)))/b-arctanh(sin(b*x+a))/b^2-1/2*csc(b*x+a)/b^2+3
/2*I*polylog(2,-exp(I*(b*x+a)))/b^2-3/2*I*polylog(2,exp(I*(b*x+a)))/b^2+3/2
*x*sec(b*x+a)/b-1/2*x*csc(b*x+a)^2*sec(b*x+a)/b
```

**Rubi [A]** time = 0.17, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2622, 288, 321, 207, 4420, 6271, 12, 4183, 2279, 2391, 3770, 2621}

$$\frac{3i\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{3i\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{\csc(a+bx)}{2b^2}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{3x\tanh^{-1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*x*ArcTanh[E^(I*(a + b*x))])/b - ArcTanh[Sin[a + b*x]]/b^2 - Csc[a + b*x
]/(2*b^2) + (((3*I)/2)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (((3*I)/2)*PolyL
og[2, E^(I*(a + b*x))])/b^2 + (3*x*Sec[a + b*x])/(2*b) - (x*Csc[a + b*x]^2*
Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int x\csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3x\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}-\int$$

$$= -\frac{3x\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}+\frac{\int}{}$$

$$= -\frac{3\tanh^{-1}(\sin(a+bx))}{2b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}-\frac{\text{Sub}}{}$$

$$= -\frac{3\tanh^{-1}(\sin(a+bx))}{2b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(}{2b}$$

$$= -\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3x\sec(a+bx)}{2b}$$

$$= -\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3x\sec(a+bx)}{2b}$$

$$= -\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3i\text{Li}_2\left(-e^{i(a+bx)}\right)}{2b^2}$$

**Mathematica [B]** time = 2.41, size = 282, normalized size = 2.24

$$\frac{12i\left(\text{Li}_2\left(-e^{i(a+bx)}\right)-\text{Li}_2\left(e^{i(a+bx)}\right)\right)+12(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)-2\tan\left(\frac{1}{2}(a+bx)\right)-2\cot}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (8*b*x - 2*Cot[(a + b*x)/2] - b*x*Csc[(a + b*x)/2]^2 + 12*(a + b*x)*(Log[1
- E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + 8*Log[Cos[(a + b*x)/2] - S
in[(a + b*x)/2]] - 8*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]] - 12*a*Log[Ta
n[(a + b*x)/2]] + (12*I)*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a
 + b*x))]) + b*x*Sec[(a + b*x)/2]^2 + (8*b*x*Sin[(a + b*x)/2])/(Cos[(a + b*
x)/2] - Sin[(a + b*x)/2]) - (8*b*x*Sin[(a + b*x)/2])/(Cos[(a + b*x)/2] + Si
n[(a + b*x)/2]) - 2*Tan[(a + b*x)/2])/(8*b^2)
```

**fricas [B]** time = 0.51, size = 527, normalized size = 4.18

$$\frac{6\,bx\cos\left(bx+a\right)^2-4\,bx+\left(-3i\,\cos\left(bx+a\right)^3+3i\,\cos\left(bx+a\right)\right)\text{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+\left(3i\,\cos\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b*x*cos(b*x + a)^2 - 4*b*x + (-3*I*cos(b*x + a)^3 + 3*I*cos(b*x + a)
)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (3*I*cos(b*x + a)^3 - 3*I*cos(b*x
+ a))*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-3*I*cos(b*x + a)^3 + 3*I*cos
(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (3*I*cos(b*x + a)^3 - 3*
I*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*x*cos(b*x + a)
^3 - b*x*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*x*cos(
b*x + a)^3 - b*x*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 3*(
a*cos(b*x + a)^3 - a*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x +
a) + 1/2) - 3*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(-1/2*cos(b*x + a) - 1
/2*I*sin(b*x + a) + 1/2) + 3*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x
```

```
+ a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*((b*x + a)*cos(b*x + a)^3
 - (b*x + a)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 2*(cos
(b*x + a)^3 - cos(b*x + a))*log(sin(b*x + a) + 1) + 2*(cos(b*x + a)^3 - cos
(b*x + a))*log(-sin(b*x + a) + 1) + 2*cos(b*x + a)*sin(b*x + a))/(b^2*cos(b
*x + a)^3 - b^2*cos(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

**maple [A]** time = 0.14, size = 182, normalized size = 1.44

$$\frac{3bx\,e^{5i(bx+a)} - 2bx\,e^{3i(bx+a)} - ie^{5i(bx+a)} + 3bx\,e^{i(bx+a)} + ie^{i(bx+a)}}{b^2\left(e^{2i(bx+a)}-1\right)^2\left(1+e^{2i(bx+a)}\right)} - \frac{3a\ln\left(e^{i(bx+a)}-1\right)}{2b^2} + \frac{2i\arctan\left(e^{i(bx+a)}\right)}{b^2} + \frac{3i\,\mathrm{dil}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*b*x*exp(5*I*(b*x+a))-2
*b*x*exp(3*I*(b*x+a))-I*exp(5*I*(b*x+a))+3*b*x*exp(I*(b*x+a))+I*exp(I*(b*x+
a)))-3/2/b^2*a*ln(exp(I*(b*x+a))-1)+2*I/b^2*arctan(exp(I*(b*x+a)))+3/2*I/b^
2*dilog(exp(I*(b*x+a)))+3/2*I/b^2*dilog(exp(I*(b*x+a))+1)-3/2/b*ln(exp(I*(b
*x+a))+1)*x
```

**maxima [B]** time = 0.69, size = 1184, normalized size = 9.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (8*I*b*x*cos(3*b*x + 3*a) - 8*b*x*sin(3*b*x + 3*a) - (4*cos(6*b*x + 6*a) -
4*cos(4*b*x + 4*a) - 4*cos(2*b*x + 2*a) + 4*I*sin(6*b*x + 6*a) - 4*I*sin(4*
b*x + 4*a) - 4*I*sin(2*b*x + 2*a) + 4)*arctan2(2*(cos(b*x + 2*a)*cos(a) + s
in(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a
) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2), (cos(b*x + 2*a)
^2 - cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 +
 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin
(a)^2)) - (6*b*x*cos(6*b*x + 6*a) - 6*b*x*cos(4*b*x + 4*a) - 6*b*x*cos(2*b*
x + 2*a) + 6*I*b*x*sin(6*b*x + 6*a) - 6*I*b*x*sin(4*b*x + 4*a) - 6*I*b*x*si
n(2*b*x + 2*a) + 6*b*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (6*b*x*co
s(6*b*x + 6*a) - 6*b*x*cos(4*b*x + 4*a) - 6*b*x*cos(2*b*x + 2*a) + 6*I*b*x*
sin(6*b*x + 6*a) - 6*I*b*x*sin(4*b*x + 4*a) - 6*I*b*x*sin(2*b*x + 2*a) + 6*
b*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(3*I*b*x + 1)*cos(5*b*x +
 5*a) - 4*(3*I*b*x - 1)*cos(b*x + a) + (6*cos(6*b*x + 6*a) - 6*cos(4*b*x +
4*a) - 6*cos(2*b*x + 2*a) + 6*I*sin(6*b*x + 6*a) - 6*I*sin(4*b*x + 4*a) - 6
*I*sin(2*b*x + 2*a) + 6)*dilog(-e^(I*b*x + I*a)) - (6*cos(6*b*x + 6*a) - 6*
cos(4*b*x + 4*a) - 6*cos(2*b*x + 2*a) + 6*I*sin(6*b*x + 6*a) - 6*I*sin(4*b*
x + 4*a) - 6*I*sin(2*b*x + 2*a) + 6)*dilog(e^(I*b*x + I*a)) + (3*I*b*x*cos(
6*b*x + 6*a) - 3*I*b*x*cos(4*b*x + 4*a) - 3*I*b*x*cos(2*b*x + 2*a) - 3*b*x*
sin(6*b*x + 6*a) + 3*b*x*sin(4*b*x + 4*a) + 3*b*x*sin(2*b*x + 2*a) + 3*I*b*
x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (-3*I*b*x*co
s(6*b*x + 6*a) + 3*I*b*x*cos(4*b*x + 4*a) + 3*I*b*x*cos(2*b*x + 2*a) + 3*b*
```

```
x*sin(6*b*x + 6*a) - 3*b*x*sin(4*b*x + 4*a) - 3*b*x*sin(2*b*x + 2*a) - 3*I*
b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (-2*I*cos(
6*b*x + 6*a) + 2*I*cos(4*b*x + 4*a) + 2*I*cos(2*b*x + 2*a) + 2*sin(6*b*x +
6*a) - 2*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a) - 2*I)*log((cos(b*x + 2*a)^2
 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)
*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a)
+ sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) + (12*b*x - 4*I)*
sin(5*b*x + 5*a) + (12*b*x + 4*I)*sin(b*x + a))/(-4*I*b^2*cos(6*b*x + 6*a)
+ 4*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(6*b*x + 6
*a) - 4*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 4*I*b^2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] int(x/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\csc^3\left(a+bx\right)\sec^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)**3*sec(b*x+a)**2,x)
[Out] Integral(x*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

## 3.288 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

**Rubi [A]** time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x,x]
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$$

**Mathematica [A]** time = 47.01, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x,x]
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]
```

**fricas [A]** time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^3\sec(bx+a)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/x, x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^3\sec(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^2/x, x)
```

**maple** [A] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{3}\left(bx+a\right)\right)\left(\sec^{2}\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] int(1/(x*cos(a + b*x)^2*sin(a + b*x)^3), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}\left(a+bx\right)\sec^{2}\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/x,x)
```

```
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**2/x, x)
```

## 3.289 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

**Rubi [A]** time = 0.51, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2,x]
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

**Mathematica [A]** time = 19.78, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2,x]
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2, x]
```

**fricas [A]** time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^3\sec(bx+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/x^2, x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^3\sec(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^2/x^2, x)
```

**maple** [A] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{3}(bx+a)\right)\left(\sec^{2}(bx+a)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^{2}\cos\left(a+b\,x\right)^{2}\sin\left(a+b\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] int(1/(x^2*cos(a + b*x)^2*sin(a + b*x)^3), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}(a+bx)\sec^{2}(a+bx)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/x**2,x)
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**2/x**2, x)
```

# 3.290 $\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\tan(a+bx)\sec^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

**Rubi** [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$$

**Mathematica** [A] time = 5.41, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]
```

**fricas** [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \sec(bx+a)^2 \tan(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)^2 \tan(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

**maple** [A] time = 0.10, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\sec^2(bx+a)\right)\tan(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
[Out] int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)^2 \tan(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a+b\,x)\,(c+d\,x)^m}{\cos(a+b\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x)^2,x)
[Out] int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x)^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan(a+bx)\sec^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)**2*tan(b*x+a),x)
[Out] Integral((c + d*x)**m*tan(a + b*x)*sec(a + b*x)**2, x)
```

## 3.291 $\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=139

$$-\frac{3d^4\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^5}+\frac{6id^3(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^4}-\frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)^3\tan(a+bx)}{b^2}+$$

```
[Out] 2*I*d*(d*x+c)^3/b^2-6*d^2*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^3+6*I*d^3*(d*x
+c)*polylog(2,-exp(2*I*(b*x+a)))/b^4-3*d^4*polylog(3,-exp(2*I*(b*x+a)))/b^5
+1/2*(d*x+c)^4*sec(b*x+a)^2/b-2*d*(d*x+c)^3*tan(b*x+a)/b^2
```

**Rubi [A]** time = 0.26, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4409, 4184, 3719, 2190, 2531, 2282, 6589}

$$\frac{6id^3(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^4}-\frac{3d^4\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^5}-\frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)^3}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((2*I)*d*(c + d*x)^3)/b^2 - (6*d^2*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))]
)/b^3 + ((6*I)*d^3*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 - (3*d^4
*PolyLog[3, -E^((2*I)*(a + b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b)
 - (2*d*(c + d*x)^3*Tan[a + b*x])/b^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4\sec^2(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{(2d)\int (c+dx)^3\sec^2(a+bx)\,dx}{b}$$

$$= \frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{2d(c+dx)^3\tan(a+bx)}{b^2} + \frac{\left(6d^2\right)\int (c+dx)^2\,\mathrm{ta}}{b^2}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{2d(c+dx)^3\tan(a+bx)}{b^2} - \frac{(12}{}$$

$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\sec^2(a+bx)}{2b}$$

$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{6id^3(c+dx)\mathrm{Li}_2\left(-e^{2i(}\right.}{b^4}$$

$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{6id^3(c+dx)\mathrm{Li}_2\left(-e^{2i(}\right.}{b^4}$$

$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{6id^3(c+dx)\mathrm{Li}_2\left(-e^{2i(}\right.}{b^4}$$

**Mathematica [B]** time = 6.56, size = 418, normalized size = 3.01

$$-\frac{6c^2d^2\sec(a)(bx\sin(a)+\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx)))}{b^3\left(\sin^2(a)+\cos^2(a)\right)} - \frac{2\sec(a)\sec(a+bx)\left(c^3d\sin(bx)+3c^2d^2\right.}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((-1/2*I)*d^4*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)
*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] -
 (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^5*E^
(I*a)) + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b) - (6*c^2*d^2*Sec[a]*(Cos[a]*Log
[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^
2)) - (6*c*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi
 - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]]
```

```
)*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[C
ot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan
[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Si
n[a]^2)]) - (2*Sec[a]*Sec[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] +
 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2
```

**fricas [C]** time = 0.57, size = 888, normalized size = 6.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4 - 12*d^4*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 1
2*d^4*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*d^4*cos
(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*d^4*cos(b*x + a
)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + (-12*I*b*d^4*x - 12*I*b*c*
d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (12*I*b*d^4*x +
12*I*b*c*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (12*I*b
*d^4*x + 12*I*b*c*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a))
 + (-12*I*b*d^4*x - 12*I*b*c*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - si
n(b*x + a)) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(co
s(b*x + a) + I*sin(b*x + a) + I) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2*log(I*cos(b*x + a) + si
n(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*c
os(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b
^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2*log(-I*cos(b*x + a) + si
n(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*c
os(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 6*(b^2*c^2*d^2 - 2*
a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I)
 - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(-cos(b*x + a)
 - I*sin(b*x + a) + I) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x
 + b^3*c^3*d)*cos(b*x + a)*sin(b*x + a))/(b^5*cos(b*x + a)^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \sec(bx+a)^2 \tan(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)^2*tan(b*x + a), x)
```

**maple [B]** time = 0.11, size = 489, normalized size = 3.52

$$\frac{2b\,d^4x^4e^{2i(bx+a)} + 8bc\,d^3x^3e^{2i(bx+a)} + 12b\,c^2d^2x^2e^{2i(bx+a)} + 8b\,c^3dx\,e^{2i(bx+a)} - 4id^4x^3e^{2i(bx+a)} + 2b\,c^4e^{2i(bx+a)} - 1}{b^2\left(1+e^{2i(bx+a)}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x)
```

```
[Out] 2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^
2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a)
)+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2
*I*(b*x+a))-2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))-6*I*c*d^3*x^2-6*I*c^2*d^
2*x-2*I*c^3*d)/b^2/(1+exp(2*I*(b*x+a)))^2-6/b^3*d^2*c^2*ln(1+exp(2*I*(b*x+a
```

```
)))+12/b^3*d^2*c^2*ln(exp(I*(b*x+a)))+12/b^5*d^4*a^2*ln(exp(I*(b*x+a)))+24*
I/b^3*d^3*c*a*x+6*I/b^4*d^3*c*polylog(2,-exp(2*I*(b*x+a)))+12*I/b^4*d^3*c*a
^2-12/b^3*d^3*c*ln(1+exp(2*I*(b*x+a)))*x-6/b^3*d^4*ln(1+exp(2*I*(b*x+a)))*x
^2+6*I/b^4*d^4*polylog(2,-exp(2*I*(b*x+a)))*x-3*d^4*polylog(3,-exp(2*I*(b*x
+a)))/b^5-24/b^4*d^3*c*a*ln(exp(I*(b*x+a)))-12*I/b^4*d^4*a^2*x+12*I/b^2*d^3
*c*x^2-8*I/b^5*d^4*a^3+4*I/b^2*d^4*x^3
```

**maxima** [B] time = 0.68, size = 3438, normalized size = 24.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^4*tan(b*x + a)^2 - 4*a*c^3*d*tan(b*x + a)^2/b + 6*a^2*c^2*d^2*tan(b*
x + a)^2/b^2 - 4*a^3*c*d^3*tan(b*x + a)^2/b^3 + a^4*d^4*tan(b*x + a)^2/b^4
+ 8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(
b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a
)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*
sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*c^3*d/((2*(2*cos(2*b*x + 2*a) + 1)*cos
(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a
)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*
b*x + 2*a) + 1)*b) - 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2
*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x
 + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - co
s(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a*c^2*d^2/((2*(2*c
os(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x +
2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2
*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) + 24*(4*(b*x + a)*cos(2*b*x +
2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + s
in(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x
+ a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x
+ 2*a))*a^2*c*d^3/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x
 + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^3) - 8*
(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x
+ a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*co
s(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(
4*b*x + 4*a) - sin(2*b*x + 2*a))*a^3*d^4/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4
*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^
2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*
x + 2*a) + 1)*b^4) + 6*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*si
n(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2*b*x + 2*a) + 4*((b*x + a)^2*cos(2*b*
x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b*x +
 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + s
in(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a
)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
 2*cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*sin(2*b*x + 2*a) - b*x - (b*x + a
)*cos(2*b*x + 2*a) - a)*sin(4*b*x + 4*a) - 4*(b*x + a)*sin(2*b*x + 2*a))*c^
2*d^2/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 +
4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) - 12*(8*(b*x + a
)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*c
os(2*b*x + 2*a) + 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2
*a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(
4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x +
4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*((b*x
 + a)^2*sin(2*b*x + 2*a) - b*x - (b*x + a)*cos(2*b*x + 2*a) - a)*sin(4*b*x
+ 4*a) - 4*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/((2*(2*cos(2*b*x + 2*a) + 1)
```

```
*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x +
 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*co
s(2*b*x + 2*a) + 1)*b^3) + 6*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a
)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2*b*x + 2*a) + 4*((b*x + a)^2*co
s(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (2*(2*cos(2
*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)
^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x
 + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*
a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*sin(2*b*x + 2*a) - b*x - (b
*x + a)*cos(2*b*x + 2*a) - a)*sin(4*b*x + 4*a) - 4*(b*x + a)*sin(2*b*x + 2*
a))*a^2*d^4/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a
)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*
b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^4) - 2*((6*(b
*x + a)^2*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a) + 6*((b*x + a)^2*d^4 + 2*(b*
c*d^3 - a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 12*((b*x + a)^2*d^4 + 2*(b*c*d
^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^4 + (12*I*b*c*
d^3 - 12*I*a*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (12*I*(b*x + a)^2*d^4 + (24
*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2
*a), cos(2*b*x + 2*a) + 1) - 4*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x
+ a)^2)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - 4*(2*I*a +
 1)*d^4)*(b*x + a)^3 - 12*(b*c*d^3 - a*d^4)*(b*x + a)^2)*cos(2*b*x + 2*a) -
 (6*b*c*d^3 + 6*(b*x + a)*d^4 - 6*a*d^4 + 6*(b*c*d^3 + (b*x + a)*d^4 - a*d^
4)*cos(4*b*x + 4*a) + 12*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a)
 - (-6*I*b*c*d^3 - 6*I*(b*x + a)*d^4 + 6*I*a*d^4)*sin(4*b*x + 4*a) - (-12*I
*b*c*d^3 - 12*I*(b*x + a)*d^4 + 12*I*a*d^4)*sin(2*b*x + 2*a))*dilog(-e^(2*I
*b*x + 2*I*a)) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x +
a) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a))*cos(4*b*
x + 4*a) + (-6*I*(b*x + a)^2*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a))*
cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(
4*b*x + 4*a) + 6*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*
x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
+ 1) + (-3*I*d^4*cos(4*b*x + 4*a) - 6*I*d^4*cos(2*b*x + 2*a) + 3*d^4*sin(4*
b*x + 4*a) + 6*d^4*sin(2*b*x + 2*a) - 3*I*d^4)*polylog(3, -e^(2*I*b*x + 2*I
*a)) + (-4*I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2)*si
n(4*b*x + 4*a) - (2*(b*x + a)^4*d^4 + (8*b*c*d^3 - (8*a - 4*I)*d^4)*(b*x +
a)^3 - (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-I*b^4*
cos(4*b*x + 4*a) - 2*I*b^4*cos(2*b*x + 2*a) + b^4*sin(4*b*x + 4*a) + 2*b^4*
sin(2*b*x + 2*a) - I*b^4))/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(a+b\,x)\,\left(c+d\,x\right)^4}{\cos(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x)^2,x)
```

```
[Out] int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \tan(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*tan(a + b*x)*sec(a + b*x)**2, x)
```

# 3.292 $\int (c+dx)^3 \sec^2(a+bx) \tan(a+bx) \, dx$

**Optimal**. Leaf size=115

$$\frac{3id^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\tan(a+bx)}{2b^2}+\frac{(c+dx)^3\sec^2(a+bx)}{2b}+\frac{3id(c+dx)}{2b^2}$$

```
[Out] 3/2*I*d*(d*x+c)^2/b^2-3*d^2*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^3+3/2*I*d^3*po
lylog(2,-exp(2*I*(b*x+a)))/b^4+1/2*(d*x+c)^3*sec(b*x+a)^2/b-3/2*d*(d*x+c)^2
*tan(b*x+a)/b^2
```

**Rubi** [A] time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4409, 4184, 3719, 2190, 2279, 2391}

$$\frac{3id^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\tan(a+bx)}{2b^2}+\frac{(c+dx)^3\sec^2(a+bx)}{2b}+\frac{3id}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 - (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b^3 + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + ((c + d*x)
^3*Sec[a + b*x]^2)/(2*b) - (3*d*(c + d*x)^2*Tan[a + b*x])/(2*b^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\sec^2(a+bx)\tan(a+bx)\,dx=\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{(3d)\int (c+dx)^2\sec^2(a+bx)\,dx}{2b}$$
$$=\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3d(c+dx)^2\tan(a+bx)}{2b^2}+\frac{\left(3d^2\right)\int (c+dx)}{b^2}$$
$$=\frac{3id(c+dx)^2}{2b^2}+\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3d(c+dx)^2\tan(a+bx)}{2b^2}-$$
$$=\frac{3id(c+dx)^2}{2b^2}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}+\frac{(c+dx)^3\sec^2(a+b}{2b}$$
$$=\frac{3id(c+dx)^2}{2b^2}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}+\frac{(c+dx)^3\sec^2(a+b}{2b}$$
$$=\frac{3id(c+dx)^2}{2b^2}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4}$$

**Mathematica [B]** time = 6.38, size = 286, normalized size = 2.49

$$-\frac{3cd^2\sec(a)(bx\sin(a)+\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx)))}{b^3\left(\sin^2(a)+\cos^2(a)\right)}-\frac{3\sec(a)\sec(a+bx)\left(c^2d\sin(bx)+2cd^2\right.}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((c + d*x)^3*Sec[a + b*x]^2)/(2*b) - (3*c*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos
[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (3*d^
3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[C
ot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^(
(2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Si
n[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))
/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) -
 (3*Sec[a]*Sec[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[
b*x]))/(2*b^2)
```

**fricas [B]** time = 0.53, size = 540, normalized size = 4.70

$$\frac{b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3-3i\,d^3\cos\left(bx+a\right)^2\text{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)+3i\,d^3\cos\left(bx+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 3*I*d^3*cos(
b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + 3*I*d^3*cos(b*x + a)^2*di
log(I*cos(b*x + a) - sin(b*x + a)) + 3*I*d^3*cos(b*x + a)^2*dilog(-I*cos(b*
```

```
x + a) + sin(b*x + a)) - 3*I*d^3*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin
(b*x + a)) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*
x + a) + I) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b
*x + a) + I) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(
b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin
(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(-I*cos(b*x + a) + s
in(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(-I*cos(b*x + a) -
 sin(b*x + a) + 1) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) +
 I*sin(b*x + a) + I) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(-cos(b*x + a)
 - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*
x + a)*sin(b*x + a))/(b^4*cos(b*x + a)^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sec(bx+a)^2 \tan(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)^2*tan(b*x + a), x)
```

**maple [B]** time = 0.10, size = 301, normalized size = 2.62

$$\frac{2b\,d^3x^3\mathrm{e}^{2i(bx+a)} - 3id^3x^2\mathrm{e}^{2i(bx+a)} + 6bc\,d^2x^2\mathrm{e}^{2i(bx+a)} - 6ic\,d^2x\,\mathrm{e}^{2i(bx+a)} + 6b\,c^2dx\,\mathrm{e}^{2i(bx+a)} - 3ic^2d\,\mathrm{e}^{2i(bx+a)} - 3id^3x^2}{b^2\left(1+\mathrm{e}^{2i(bx+a)}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x)
```

```
[Out] (2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*ex
p(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*
I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))-6*I*c*d^2*x-3
*I*c^2*d)/b^2/(1+exp(2*I*(b*x+a)))^2-3/b^3*d^2*c*ln(1+exp(2*I*(b*x+a)))+6/b
^3*d^2*c*ln(exp(I*(b*x+a)))+3*I/b^2*d^3*x^2+6*I/b^3*d^3*a*x+3*I/b^4*d^3*a^2
-3/b^3*d^3*ln(1+exp(2*I*(b*x+a)))*x+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/
b^4-6/b^4*d^3*a*ln(exp(I*(b*x+a)))
```

**maxima [B]** time = 0.63, size = 667, normalized size = 5.80

$$-\frac{6\,b^2c^2d + \left(6\,bd^3x + 6\,bcd^2 + 6\left(bd^3x + bcd^2\right)\cos\left(4\,bx + 4\,a\right) + 12\left(bd^3x + bcd^2\right)\cos\left(2\,bx + 2\,a\right) + \left(6i\,bd^3x + 6i\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] -(6*b^2*c^2*d + (6*b*d^3*x + 6*b*c*d^2 + 6*(b*d^3*x + b*c*d^2)*cos(4*b*x +
4*a) + 12*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a) + (6*I*b*d^3*x + 6*I*b*c*d^2
)*sin(4*b*x + 4*a) + (12*I*b*d^3*x + 12*I*b*c*d^2)*sin(2*b*x + 2*a))*arctan
2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x)
*cos(4*b*x + 4*a) + (4*I*b^3*d^3*x^3 + 4*I*b^3*c^3 + 6*b^2*c^2*d + (12*I*b^
3*c*d^2 - 6*b^2*d^3)*x^2 + (12*I*b^3*c^2*d - 12*b^2*c*d^2)*x)*cos(2*b*x + 2
*a) - (3*d^3*cos(4*b*x + 4*a) + 6*d^3*cos(2*b*x + 2*a) + 3*I*d^3*sin(4*b*x
+ 4*a) + 6*I*d^3*sin(2*b*x + 2*a) + 3*d^3)*dilog(-e^(2*I*b*x + 2*I*a)) + (-
3*I*b*d^3*x - 3*I*b*c*d^2 + (-3*I*b*d^3*x - 3*I*b*c*d^2)*cos(4*b*x + 4*a) +
 (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(2*b*x + 2*a) + 3*(b*d^3*x + b*c*d^2)*sin(
4*b*x + 4*a) + 6*(b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (-6*I*b^2*d^3*x^2 - 12*
```

```
I*b^2*c*d^2*x)*sin(4*b*x + 4*a) - (4*b^3*d^3*x^3 + 4*b^3*c^3 - 6*I*b^2*c^2*
d + 6*(2*b^3*c*d^2 + I*b^2*d^3)*x^2 + 12*(b^3*c^2*d + I*b^2*c*d^2)*x)*sin(2
*b*x + 2*a))/(-2*I*b^4*cos(4*b*x + 4*a) - 4*I*b^4*cos(2*b*x + 2*a) + 2*b^4*
sin(4*b*x + 4*a) + 4*b^4*sin(2*b*x + 2*a) - 2*I*b^4)
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(a+b\,x\right)\,{\left(c+d\,x\right)}^3}{{\cos\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x)^2,x)
[Out] int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x)^2, x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \tan(a+bx) \sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)**2*tan(b*x+a),x)
[Out] Integral((c + d*x)**3*tan(a + b*x)*sec(a + b*x)**2, x)
```

# 3.293 $\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=55

$$-\frac{d^2\log(\cos(a+bx))}{b^3}-\frac{d(c+dx)\tan(a+bx)}{b^2}+\frac{(c+dx)^2\sec^2(a+bx)}{2b}$$

```
[Out] -d^2*ln(cos(b*x+a))/b^3+1/2*(d*x+c)^2*sec(b*x+a)^2/b-d*(d*x+c)*tan(b*x+a)/b
^2
```

**Rubi [A]** time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4409, 4184, 3475}

$$-\frac{d(c+dx)\tan(a+bx)}{b^2}-\frac{d^2\log(\cos(a+bx))}{b^3}+\frac{(c+dx)^2\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] -((d^2*Log[Cos[a + b*x]])/b^3) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) - (d*(c
 + d*x)*Tan[a + b*x])/b^2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\sec^2(a+bx)\tan(a+bx)\,dx &= \frac{(c+dx)^2\sec^2(a+bx)}{2b}-\frac{d\int (c+dx)\sec^2(a+bx)\,dx}{b}\\ &= \frac{(c+dx)^2\sec^2(a+bx)}{2b}-\frac{d(c+dx)\tan(a+bx)}{b^2}+\frac{d^2\int \tan(a+bx)\,dx}{b^2}\\ &= -\frac{d^2\log(\cos(a+bx))}{b^3}+\frac{(c+dx)^2\sec^2(a+bx)}{2b}-\frac{d(c+dx)\tan(a+bx)}{b^2}\end{aligned}$$

**Mathematica [A]** time = 0.51, size = 66, normalized size = 1.20

$$\frac{b^2(c+dx)^2\sec^2(a+bx)-2bd\sec(a)\sin(bx)(c+dx)\sec(a+bx)-2d^2(bx\tan(a)+\log(\cos(a+bx)))}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (b^2*(c + d*x)^2*Sec[a + b*x]^2 - 2*b*d*(c + d*x)*Sec[a]*Sec[a + b*x]*Sin[b
*x] - 2*d^2*(Log[Cos[a + b*x]] + b*x*Tan[a]))/(2*b^3)
```

**fricas [A]** time = 0.45, size = 86, normalized size = 1.56

$$\frac{b^2d^2x^2 + 2\,b^2cdx - 2\,d^2\cos\left(bx + a\right)^2\log\left(-\cos\left(bx + a\right)\right) + b^2c^2 - 2\left(bd^2x + bcd\right)\cos\left(bx + a\right)\sin\left(bx + a\right)}{2\,b^3\cos\left(bx + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x - 2*d^2*cos(b*x + a)^2*log(-cos(b*x + a)) +
b^2*c^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)^
2)
```

**giac [B]** time = 1.81, size = 4474, normalized size = 81.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4*t
an(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d^2*x^2*tan
(1/2*b*x)^2*tan(1/2*a)^4 + b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^2*c*d*
x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*b
^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^
4 - d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2
- 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2
*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*ta
n(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4
*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/
2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1
/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(t
an(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 + b^2*d^2*x^
2*tan(1/2*b*x)^4 + 4*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b^2*c^2*ta
n(1/2*b*x)^4*tan(1/2*a)^2 + 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*d^2*x
^2*tan(1/2*a)^4 + 2*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b*c*d*tan(1/2*b
*x)^3*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4 - 4*b*d^2*x*tan(1/2*b*x)^4*
tan(1/2*a) + 8*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 24*b*d^2*x*tan(1/2*b
*x)^3*tan(1/2*a)^2 + 2*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b
*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan
(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*
tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a)
+ 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/
2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8
*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*t
an(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/
2*a)^2 - 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^3 + 8*d^2*log(4*(tan(1/2*b*x)
^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*
a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan
(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 +
 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b
*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*ta
n(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*
tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^
2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^3 + 2*b^2*c*d*x*tan(1/2*a)^4 - 4*b*d^2*x*
tan(1/2*b*x)*tan(1/2*a)^4 + 2*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*ta
```

n(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8
+ 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*
b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(
1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2
*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a
)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a
) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2
*tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^2 + b^2*c^2*tan(1/2*b*x)^4 - 4*b
*c*d*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^2*d^2*x^2*tan(1/2*a)^2 + 4*b^2*c^2*tan
(1/2*b*x)^2*tan(1/2*a)^2 - 24*b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 24*b*c*d*
tan(1/2*b*x)^2*tan(1/2*a)^3 + b^2*c^2*tan(1/2*a)^4 - 4*b*c*d*tan(1/2*b*x)*t
an(1/2*a)^4 + 4*b^2*c*d*x*tan(1/2*b*x)^2 + 4*b*d^2*x*tan(1/2*b*x)^3 - d^2*l
og(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1
/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*
tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x
)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*
a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*
tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 +
tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)
^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4 + 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/
2*a) - 8*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*
a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*ta
n(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 -
2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b
*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*t
an(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*
tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 +
1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a) + 4*b^2*c
*d*x*tan(1/2*a)^2 + 24*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^2 - 20*d^2*log(4*(ta
n(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^
7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*
b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(
1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8
*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*
a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2
*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*t
an(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b*d^2*x*tan(1/2*a)^3 - 8*
d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*
tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a)
+ 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/
2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan
(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*
x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a
)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1
/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^3 - d^2*log(4*(tan(1
/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*t
an(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x
)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2
*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*ta
n(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a)
+ 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)
^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(
1/2*a)^2 + 1))*tan(1/2*a)^4 + b^2*d^2*x^2 + 2*b^2*c^2*tan(1/2*b*x)^2 + 4*b*
c*d*tan(1/2*b*x)^3 + 24*b*c*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c^2*tan(1/2
*a)^2 + 24*b*c*d*tan(1/2*b*x)*tan(1/2*a)^2 + 4*b*c*d*tan(1/2*a)^3 + 2*b^2*c
*d*x - 4*b*d^2*x*tan(1/2*b*x) + 2*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 -
2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x
)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(
1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*

```
tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3
 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1
/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1
/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*
x)^2 - 4*b*d^2*x*tan(1/2*a) + 8*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*
tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^
8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/
2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*ta
n(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 -
 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2
*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2
*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)
*tan(1/2*a) + 2*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*t
an(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*
x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2
*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*ta
n(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^
4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/
2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*
a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*a)^2 + b^2*c^2 - 4*b
*c*d*tan(1/2*b*x) - 4*b*c*d*tan(1/2*a) - d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*
a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(
1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 -
 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*
b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1
/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^
2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x
)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)))/(b
^3*tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*b^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 8*b^3*
tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*b^3*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^3*tan(1
/2*b*x)^4 + 8*b^3*tan(1/2*b*x)^3*tan(1/2*a) + 20*b^3*tan(1/2*b*x)^2*tan(1/2
*a)^2 + 8*b^3*tan(1/2*b*x)*tan(1/2*a)^3 + b^3*tan(1/2*a)^4 - 2*b^3*tan(1/2*
b*x)^2 - 8*b^3*tan(1/2*b*x)*tan(1/2*a) - 2*b^3*tan(1/2*a)^2 + b^3)
```

**maple [A]** time = 0.03, size = 95, normalized size = 1.73

$$\frac{d^{2} x^{2}}{2 b \cos \left(b x +a \right)^{2}}-\frac{d^{2} \tan \left(b x +a \right) x}{b^{2}}-\frac{d^{2} \ln \left(\cos \left(b x +a \right)\right)}{b^{3}}+\frac{c d x}{b \cos \left(b x +a \right)^{2}}-\frac{c d \tan \left(b x +a \right)}{b^{2}}+\frac{c^{2}}{2 b \cos \left(b x +a \right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x)
```

```
[Out] 1/2/b*d^2/cos(b*x+a)^2*x^2-1/b^2*d^2*tan(b*x+a)*x-d^2*ln(cos(b*x+a))/b^3+1/
b*c*d/cos(b*x+a)^2*x-1/b^2*c*d*tan(b*x+a)+1/2/b*c^2/cos(b*x+a)^2
```

**maxima [B]** time = 0.52, size = 988, normalized size = 17.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*tan(b*x + a)^2 - 2*a*c*d*tan(b*x + a)^2/b + a^2*d^2*tan(b*x + a)^2
/b^2 + 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 +
 (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*
x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
- 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*c*d/((2*(2*cos(2*b*x + 2*a) + 1)*
cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x +
4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos
```

```
(2*b*x + 2*a) + 1)*b) - 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin
(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b
*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) -
cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a*d^2/((2*(2*cos
(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*
a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b
*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) + (8*(b*x + a)^2*cos(2*b*x + 2*a
)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2*b*x + 2*a) + 4
*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*
a) - (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*
cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*
a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2
+ sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*sin(2*b*x +
 2*a) - b*x - (b*x + a)*cos(2*b*x + 2*a) - a)*sin(4*b*x + 4*a) - 4*(b*x + a
)*sin(2*b*x + 2*a))*d^2/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos
(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x +
 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2
))/b
```

**mupad [B]** time = 3.06, size = 150, normalized size = 2.73

$$-\frac{\frac{(c+d\,x)^2}{b}-\frac{e^{a\,2i+b\,x\,2i}\,(c+d\,x)^2}{b}}{2\,e^{a\,2i+b\,x\,2i}+e^{a\,4i+b\,x\,4i}+1}+\frac{d^2\,x\,2i}{b^2}+\frac{b\,c^2+2\,b\,c\,d\,x-c\,d\,2i+b\,d^2\,x^2-d^2\,x\,2i}{b^2\,\left(e^{a\,2i+b\,x\,2i}+1\right)}-\frac{d^2\,\ln\left(e^{a\,2i}\,e^{b\,x\,2i}+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x)^2,x)
```

```
[Out] (d^2*x*2i)/b^2 - ((c + d*x)^2/b - (exp(a*2i + b*x*2i)*(c + d*x)^2)/b)/(2*ex
p(a*2i + b*x*2i) + exp(a*4i + b*x*4i) + 1) + (b*c^2 - c*d*2i - d^2*x*2i + b
*d^2*x^2 + 2*b*c*d*x)/(b^2*(exp(a*2i + b*x*2i) + 1)) - (d^2*log(exp(a*2i)*e
xp(b*x*2i) + 1))/b^3
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \tan(a+bx) \sec^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)*sec(a + b*x)**2, x)
```

# 3.294 $\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx$

**Optimal**. Leaf size=35

$$\frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}$$

```
[Out] 1/2*(d*x+c)*sec(b*x+a)^2/b-1/2*d*tan(b*x+a)/b^2
```

**Rubi [A]** time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4409, 3767, 8}

$$\frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((c + d*x)*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx &= \frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\int \sec^2(a+bx)\,dx}{2b}\\ &= \frac{(c+dx)\sec^2(a+bx)}{2b}+\frac{d\,\text{Subst}\left(\int 1\,dx,x,-\tan(a+bx)\right)}{2b^2}\\ &= \frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 48, normalized size = 1.37

$$-\frac{d\tan(a+bx)}{2b^2}+\frac{c\sec^2(a+bx)}{2b}+\frac{dx\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (c*Sec[a + b*x]^2)/(2*b) + (d*x*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(2
*b^2)
```

**fricas [A]** time = 0.41, size = 36, normalized size = 1.03

$$\frac{bdx - d\cos(bx+a)\sin(bx+a) + bc}{2\,b^2\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x - d*cos(b*x + a)*sin(b*x + a) + b*c)/(b^2*cos(b*x + a)^2)
```

**giac [B]** time = 1.84, size = 571, normalized size = 16.31

$$\frac{bdx\tan\left(\frac{1}{2}\,bx\right)^4\tan\left(\frac{1}{2}\,a\right)^4 + bc\tan\left(\frac{1}{2}\,bx\right)^4\tan\left(\frac{1}{2}\,a\right)^4 + 2\,bdx\tan\left(\frac{1}{2}\,bx\right)^4\tan\left(\frac{1}{2}\,a\right)^2 + 2\,bdx\tan\left(\frac{1}{2}\,bx\right)^2\tan\left(\frac{1}{2}\,a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
2*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 +
 2*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 2*b*
c*tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x*tan
(1/2*b*x)^4 + 4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*tan(1/2*a)^4 + b*
c*tan(1/2*b*x)^4 - 2*d*tan(1/2*b*x)^4*tan(1/2*a) + 4*b*c*tan(1/2*b*x)^2*tan
(1/2*a)^2 - 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 12*d*tan(1/2*b*x)^2*tan(1/2*
a)^3 + b*c*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b*d*x*tan(1/2*b
*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 + 2*d*tan(1/2*b*x)^3 +
12*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)*tan
(1/2*a)^2 + 2*d*tan(1/2*a)^3 + b*d*x + b*c - 2*d*tan(1/2*b*x) - 2*d*tan(1/2
*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*b^2*tan(1/2*b*x)^4*tan(1/2*a)^2 -
 8*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^
2*tan(1/2*b*x)^4 + 8*b^2*tan(1/2*b*x)^3*tan(1/2*a) + 20*b^2*tan(1/2*b*x)^2*
tan(1/2*a)^2 + 8*b^2*tan(1/2*b*x)*tan(1/2*a)^3 + b^2*tan(1/2*a)^4 - 2*b^2*t
an(1/2*b*x)^2 - 8*b^2*tan(1/2*b*x)*tan(1/2*a) - 2*b^2*tan(1/2*a)^2 + b^2)
```

**maple [A]** time = 0.03, size = 61, normalized size = 1.74

$$\frac{\frac{d\left(\frac{bx+a}{2\cos(bx+a)^2}-\frac{\tan(bx+a)}{2}\right)}{b} - \frac{da}{2b\cos(bx+a)^2} + \frac{c}{2\cos(bx+a)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x)
```

```
[Out] 1/b*(1/b*d*(1/2*(b*x+a)/cos(b*x+a)^2-1/2*tan(b*x+a))-1/2/b*d*a/cos(b*x+a)^2
+1/2*c/cos(b*x+a)^2)
```

**maxima [B]** time = 0.35, size = 283, normalized size = 8.09

$$\frac{c\tan(bx+a)^2 - \frac{ad\tan(bx+a)^2}{b} + \frac{2\left(4\,(bx+a)\cos(2\,bx+2\,a)^2+4\,(bx+a)\sin(2\,bx+2\,a)^2+(2\,(bx+a)\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a))\cos(4\,bx+4\,a)+2\,(}{\left(2\,(2\,\cos(2\,bx+2\,a)+1)\cos(4\,bx+4\,a)+\cos(4\,bx+4\,a)^2+4\,\cos(2\,bx+2\,a)^2+\sin(4\,bx+4\,a)^2\right.}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c*tan(b*x + a)^2 - a*d*tan(b*x + a)^2/b + 2*(4*(b*x + a)*cos(2*b*x + 2
*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + si
n(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x +
 a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x +
 2*a))*d/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2
 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x
 + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b))/b
```

**mupad** [B] time = 2.19, size = 53, normalized size = 1.51

$$-\frac{d\,1\mathrm{i}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,\left(-b\,\left(2\,c+2\,d\,x\right)+d\,1\mathrm{i}\right)}{b^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x))/cos(a + b*x)^2,x)
```

```
[Out] -(d*1i + exp(a*2i + b*x*2i)*(d*1i - b*(2*c + 2*d*x)))/(b^2*(exp(a*2i + b*x*
2i) + 1)^2)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \tan (a + bx) \sec^2 (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)*sec(a + b*x)**2, x)
```

## 3.295 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\frac{\tan(a+bx)\sec^2(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
[Out] Defer[Int][(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

**Mathematica [A]** time = 7.24, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
[Out] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

**fricas [A]** time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(bx+a\right)^2\tan\left(bx+a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(sec(b*x + a)^2*tan(b*x + a)/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)^2\tan\left(bx+a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)^2*tan(b*x + a)/(d*x + c), x)
```

**maple** [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^{2}\left(bx+a\right)\right)\tan\left(bx+a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan\left(a+b\,x\right)}{{\cos\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(a+bx\right)\sec^{2}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*tan(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)*sec(a + b*x)**2/(c + d*x), x)
```

## 3.296 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2} \, dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\frac{\tan(a+bx)\sec^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
[Out] Defer[Int][(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2} \, dx = \int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2} \, dx$$

**Mathematica [A]** time = 10.39, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

**fricas [A]** time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(bx+a\right)^2\tan\left(bx+a\right)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sec(b*x + a)^2*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)^2\tan\left(bx+a\right)}{\left(dx+c\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sec(b*x + a)^2*tan(b*x + a)/(d*x + c)^2, x)
```

**maple** [A] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^2(bx+a)\right)\tan(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
[Out] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a+b\,x)}{\cos(a+b\,x)^2\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)^2),x)
[Out] int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)^2), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*tan(b*x+a)/(d*x+c)**2,x)
[Out] Integral(tan(a + b*x)*sec(a + b*x)**2/(c + d*x)**2, x)
```

# 3.297 $\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$

**Optimal**. Leaf size=39

$$\mathrm{Int}\left(\sec^3(a+bx)(c+dx)^m,x\right)-\mathrm{Int}\left(\sec(a+bx)(c+dx)^m,x\right)$$

```
[Out] -Unintegrable((d*x+c)^m*sec(b*x+a),x)+Unintegrable((d*x+c)^m*sec(b*x+a)^3,x
)
```

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -Defer[Int][(c + d*x)^m*Sec[a + b*x], x] + Defer[Int][(c + d*x)^m*Sec[a + b
*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)^m \sec(a+bx)\,dx + \int (c+dx)^m \sec^3(a+bx)\,dx$$

**Mathematica [F]** time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2,x]
[Out] $Aborted
```

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((dx+c)^m \sec(bx+a)\tan(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\tan(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

**maple [A]** time = 0.09, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\left(\tan^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec (bx+a)\tan (bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan (a+b\,x)^2\,(c+d\,x)^m}{\cos (a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)^2*(c + d*x)^m)/cos(a + b*x),x)
```

```
[Out] int((tan(a + b*x)^2*(c + d*x)^m)/cos(a + b*x), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan^2 (a+bx) \sec (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*tan(a + b*x)**2*sec(a + b*x), x)
```

# 3.298 $\int (c+dx)^3 \sec(a+bx) \tan^2(a+bx) \, dx$

**Optimal**. Leaf size=337

$$\frac{3id^3\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^4}+\frac{3id^3\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}-\frac{3d^2(}{}$$

[Out] -6*I*d^2*(d*x+c)*arctan(exp(I*(b*x+a)))/b^3+I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b+3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^2-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2+3*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^3-3*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3+3*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-3/2*d*(d*x+c)^2*sec(b*x+a)/b^2+1/2*(d*x+c)^3*sec(b*x+a)*tan(b*x+a)/b

**Rubi [A]** time = 0.41, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4413, 4181, 2531, 6609, 2282, 6589, 4186, 2279, 2391}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}-\frac{3d^2(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}+\frac{3id(c}{}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x]^2,x]

[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 + (I*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*PolyLog[2, I*E^(I*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 - (3*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^4 - ((3*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Sec[a + b*x]*Tan[a + b*x])/(2*b)

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \sec(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)^3 \sec(a+bx)\,dx + \int (c+dx)^3 \sec^3(a+bx)\,dx$$
$$= \frac{2i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2 \sec(a+bx)}{2b^2} + \frac{(c+dx)^3 \sec(a$$
$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3id(c+dx$$
$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3id(c+dx$$
$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id^3\mathrm{Li}_2\left(-\right.}{b^4}$$
$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id^3\mathrm{Li}_2\left(-\right.}{b^4}$$
$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id^3\mathrm{Li}_2\left(-\right.}{b^4}$$

**Mathematica [A]** time = 3.40, size = 530, normalized size = 1.57

$$\frac{2ib^3c^3\tan^{-1}\left(e^{i(a+bx)}\right) - 3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right) + 3b^3c^2dx\log\left(1+ie^{i(a+bx)}\right) - 3b^3cd^2x^2\log\left(1-ie^{i(a+bx)}\right) + 3b^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] ((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] - (12*I)*b*c*d^2*ArcTan[E^(I*(a + b*
x))] - 3*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 - I*E^(I*
(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[
1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 6*b*d^3
*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))]
+ b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x))] - (3*I)*d*(-2*d^2 + b^2*(c + d*x)^
2)*PolyLog[2, (-I)*E^(I*(a + b*x))] + (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*Po
lyLog[2, I*E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] +
6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a
 + b*x))] - 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4,
(-I)*E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))] + b^2*(c +
d*x)^2*Sec[a + b*x]*(-3*d + b*(c + d*x)*Tan[a + b*x]))/(2*b^4)
```

**fricas [C]** time = 0.64, size = 1311, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(-6*I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 6*
I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 6*I*d^3*co
s(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*cos(b*x +
 a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b
^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) +
 sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d
^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*d^3*x^2
 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b
*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2
```

```
*d + 6*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^3*c
^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2*
log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2
- 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x
 + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*
x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a
)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3
)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^
3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1
) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b
*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*
d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2*log(-cos(b*x + a)
 - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*
cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog
(3, I*cos(b*x + a) - sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*p
olylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x +
 a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b^2*d^3*x^2 + 2*b^2*c
*d^2*x + b^2*c^2*d)*cos(b*x + a) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3
*c^2*d*x + b^3*c^3)*sin(b*x + a))/(b^4*cos(b*x + a)^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sec(bx+a)\tan(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*tan(b*x + a)^2, x)
```

**maple [B]** time = 0.21, size = 1127, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] 3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(4,-I*exp(I*(b*x+a)
))/b^4-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(4,I*exp(I*(b
*x+a)))/b^4-3/2*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2+3/2*I/b^2*d^3*po
lylog(2,I*exp(I*(b*x+a)))*x^2-I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))+6*I/b^4*
d^3*a*arctan(exp(I*(b*x+a)))-3/2*I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))-6
*I/b^3*c*d^2*arctan(exp(I*(b*x+a)))+3/2*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+
a)))+1/2/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)))-3/b^3*d^3*polylog(3,I*exp(I*(b*
x+a)))*x-1/2/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3+1/2/b*d^3*ln(1+I*exp(I*(b*x+a
)))*x^3+3/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x-3/b^3*d^2*c*polylog(3,I*ex
p(I*(b*x+a)))+3/b^3*d^2*c*polylog(3,-I*exp(I*(b*x+a)))-1/2/b^4*a^3*d^3*ln(1
-I*exp(I*(b*x+a)))-3/2/b*d^2*c*ln(1-I*exp(I*(b*x+a)))*x^2+3/2/b*d^2*c*ln(1+
I*exp(I*(b*x+a)))*x^2-3/2/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x-3/2/b^2*c^2*d*ln
(1-I*exp(I*(b*x+a)))*a-3/2/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x+a)))+3/2/b*c^2*d
*ln(1+I*exp(I*(b*x+a)))*x+3/2/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a+3/2/b^3*a^
2*c*d^2*ln(1-I*exp(I*(b*x+a)))+I/b*c^3*arctan(exp(I*(b*x+a)))+3/b^3*d^3*ln(
1-I*exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-I*exp(I*(b*x+a)))*a-3/b^3*d^3*ln(1+I*e
xp(I*(b*x+a)))*x-3/b^4*d^3*ln(1+I*exp(I*(b*x+a)))*a-3*I/b^2*c*d^2*polylog(2
,-I*exp(I*(b*x+a)))*x+3*I/b^2*c*d^2*polylog(2,I*exp(I*(b*x+a)))*x+3*I/b^3*c
*d^2*a^2*arctan(exp(I*(b*x+a)))-I/b^2/(1+exp(2*I*(b*x+a)))^2*(d^3*x^3*b*exp
```

```
(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))-d
^3*x^3*b*exp(I*(b*x+a))+c^3*b*exp(3*I*(b*x+a))-3*c*d^2*x^2*b*exp(I*(b*x+a))
-3*I*d^3*x^2*exp(3*I*(b*x+a))-3*c^2*d*x*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(3*
I*(b*x+a))-c^3*b*exp(I*(b*x+a))-3*I*c^2*d*exp(3*I*(b*x+a))-3*I*d^3*x^2*exp(
I*(b*x+a))-6*I*c*d^2*x*exp(I*(b*x+a))-3*I*c^2*d*exp(I*(b*x+a)))-3*I/b^2*c^2
*d*a*arctan(exp(I*(b*x+a)))
```

**maxima** [B] time = 2.01, size = 3828, normalized size = 11.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - lo
g(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log
(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*sin(b*x + a)
/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b^2
- a^3*d^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - lo
g(sin(b*x + a) - 1))/b^3 - 4*((2*(b*x + a)^3*d^3 - 12*b*c*d^2 + 12*a*d^3 +
6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^
3)*(b*x + a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos
(4*b*x + 4*a) + 4*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d
^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*co
s(2*b*x + 2*a) - (-2*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 - 12*I*a*d^3 + (-6*I*
b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I
*a^2 + 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-4*I*(b*x + a)^3*d^3 + 24*
I*b*c*d^2 - 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*
b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*x + a))*sin(2*b*x +
 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (2*(b*x + a)^3*d^3 - 12*b*
c*d^2 + 12*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c
*d^2 + (a^2 - 2)*d^3)*(b*x + a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3
+ 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*
d^3)*(b*x + a))*cos(4*b*x + 4*a) + 4*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3
 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)
*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-2*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 -
12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*
I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-4*I*(b
*x + a)^3*d^3 + 24*I*b*c*d^2 - 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b
*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*
x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - (4*(b*
x + a)^3*d^3 - 12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (12*b*c*d^2
 - (12*a + 12*I)*d^3)*(b*x + a)^2 + (12*b^2*c^2*d - (24*a + 24*I)*b*c*d^2 +
 12*(a^2 + 2*I*a)*d^3)*(b*x + a))*cos(3*b*x + 3*a) + (4*(b*x + a)^3*d^3 + 1
2*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3 + (12*b*c*d^2 - (12*a - 12*I)
*d^3)*(b*x + a)^2 + (12*b^2*c^2*d - (24*a - 24*I)*b*c*d^2 + 12*(a^2 - 2*I*a
)*d^3)*(b*x + a))*cos(b*x + a) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^
2*d^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2
*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a
))*cos(4*b*x + 4*a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2
- 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*b^2*c^2*
d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 + (-6*I*a^2 + 12*I)*d^3 + (-12*I*b
*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-12*I*b^2*c^2*d + 24*I*
a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 + (-12*I*a^2 + 24*I)*d^3 + (-24*I*b*c*d^2
+ 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (6*b^
2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^2
- a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 -
2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*(b^2*c^2*d -
2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
```

```
a))*cos(2*b*x + 2*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^
3 + (6*I*a^2 - 12*I)*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(4*b*x
 + 4*a) + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + (12*I*a
^2 - 24*I)*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*d
ilog(-I*e^(I*b*x + I*a)) - (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 +
(-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 +
(-3*I*a^2 + 6*I)*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a
*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c
*d^2 + (-3*I*a^2 + 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^
3*d^3 + 12*I*b*c*d^2 - 12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2
+ (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a))*cos(
2*b*x + 2*a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(4
*b*x + 4*a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(
2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (
I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*
x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 - 6*I)*d^3)*(b*x + a)
+ (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*
(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 - 6*I)*d^3)*(b*x +
a))*cos(4*b*x + 4*a) + (2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (
6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6
*I*a^2 - 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^3*d^3 - 6*b*c*
d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + (a^2 - 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 - 6*b*c
*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d
^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b
*x + a)^2 - 2*sin(b*x + a) + 1) - (12*d^3*cos(4*b*x + 4*a) + 24*d^3*cos(2*b
*x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a) + 24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)
*polylog(4, I*e^(I*b*x + I*a)) + (12*d^3*cos(4*b*x + 4*a) + 24*d^3*cos(2*b*
x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a) + 24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)*
polylog(4, -I*e^(I*b*x + I*a)) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I
*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(4*b*x + 4*a)
 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*cos(2*b*x + 2*a) + 12*
(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a
)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, I*e^(I*b*x + I*a)) - (12*I*b*c*
d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3
- 12*I*a*d^3)*cos(4*b*x + 4*a) + (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*
a*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x +
4*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -I
*e^(I*b*x + I*a)) - (4*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12
*a^2*d^3 - 12*(-I*b*c*d^2 + (I*a - 1)*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d -
24*(I*a - 1)*b*c*d^2 + (12*I*a^2 - 24*a)*d^3)*(b*x + a))*sin(3*b*x + 3*a) -
 (-4*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 + (-12*I*
b*c*d^2 - 12*(-I*a - 1)*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d - 24*(-I*a - 1)
*b*c*d^2 + (-12*I*a^2 - 24*a)*d^3)*(b*x + a))*sin(b*x + a))/(-4*I*b^3*cos(4
*b*x + 4*a) - 8*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(4*b*x + 4*a) + 8*b^3*sin
(2*b*x + 2*a) - 4*I*b^3))/b
```

**mupad** [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)^2*(c + d*x)^3)/cos(a + b*x),x)
```

```
[Out] \text{Hanged}
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \tan^2(a + bx) \sec(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*tan(a + b*x)**2*sec(a + b*x), x)
```

# 3.299 $\int (c+dx)^2 \sec(a+bx) \tan^2(a+bx)\,dx$

**Optimal**. Leaf size=193

$$\frac{d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}-\frac{d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}$$

```
[Out] I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+d^2*arctanh(sin(b*x+a))/b^3-I*d*(d*x+c
)*polylog(2,-I*exp(I*(b*x+a)))/b^2+I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/
b^2+d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3-d^2*polylog(3,I*exp(I*(b*x+a)))/b^
3-d*(d*x+c)*sec(b*x+a)/b^2+1/2*(d*x+c)^2*sec(b*x+a)*tan(b*x+a)/b
```

**Rubi [A]** time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4413, 4181, 2531, 2282, 6589, 4186, 3770}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}-\frac{d^2\text{PolyLog}\left(3\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (I*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (d^2*ArcTanh[Sin[a + b*x]])/b^3
 - (I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + (I*d*(c + d*x)*Po
lyLog[2, I*E^(I*(a + b*x))])/b^2 + (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 - (d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Sec[a + b*x])/b
^2 + ((c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)^2\sec(a+bx)\,dx + \int (c+dx)^2\sec^3(a+bx)\,dx$$

$$= \frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2\sec(a+}{2b}$$

$$= \frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3} - \frac{2id(c+dx)\text{Li}_2\left(-\right.}{b^2}$$

$$= \frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3} - \frac{id(c+dx)\text{Li}_2\left(-i\right.}{b^2}$$

$$= \frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3} - \frac{id(c+dx)\text{Li}_2\left(-i\right.}{b^2}$$

$$= \frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3} - \frac{id(c+dx)\text{Li}_2\left(-i\right.}{b^2}$$

**Mathematica [B]** time = 7.18, size = 526, normalized size = 2.73

$$\frac{ibc^2\tan^{-1}\left(e^{i(a+bx)}\right) - id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right) + id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right) - bcdx\log\left(1-ie^{i(a+bx)}\right) + bcdx\log\left(1+\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (I*b*c^2*ArcTan[E^(I*(a + b*x))] - ((2*I)*d^2*ArcTan[E^(I*(a + b*x))])/b -
b*c*d*x*Log[1 - I*E^(I*(a + b*x))] - (b*d^2*x^2*Log[1 - I*E^(I*(a + b*x))])
/2 + b*c*d*x*Log[1 + I*E^(I*(a + b*x))] + (b*d^2*x^2*Log[1 + I*E^(I*(a + b*
x))])/2 - I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] + I*d*(c + d*x)*Po
lyLog[2, I*E^(I*(a + b*x))] + (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b - (d
^2*PolyLog[3, I*E^(I*(a + b*x))])/b)/b^2 - (d*(c + d*x)*Sec[a])/b^2 + (c^2
+ 2*c*d*x + d^2*x^2)/(4*b*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])^2) + (-
```

```
(c*d*Sin[(b*x)/2]) - d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] - Sin[a/2])*(Cos[a/
2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-c^2 - 2*c*d*x - d^2*x^2)/(4*b*(Cos[
a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])^2) + (c*d*Sin[(b*x)/2] + d^2*x*Sin[(b*
x)/2])/(b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])
)
```

**fricas [C]** time = 0.55, size = 791, normalized size = 4.10

$$\frac{2\,d^2\cos(bx+a)^2\operatorname{polylog}(3,i\cos(bx+a)+\sin(bx+a)) - 2\,d^2\cos(bx+a)^2\operatorname{polylog}(3,i\cos(bx+a)-\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2
*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x +
 a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x + a)^2*pol
ylog(3, -I*cos(b*x + a) - sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x
 + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*co
s(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c
*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x -
2*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2
- 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x +
a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(cos(b*x
+ a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d
^2)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 +
2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(
b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
 a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) +
1) - (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a)
 + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)
^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 4*(b*d^2*x + b*c*d)*cos(b*x +
a) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(b*x + a))/(b^3*cos(b*x + a
)^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\sec(bx+a)\tan(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*tan(b*x + a)^2, x)
```

**maple [B]** time = 0.17, size = 584, normalized size = 3.03

$$-\frac{icd\operatorname{polylog}\left(2,-ie^{i(bx+a)}\right)}{b^2}-\frac{cd\ln\left(1-ie^{i(bx+a)}\right)a}{b^2}+\frac{d^2\operatorname{polylog}\left(3,-ie^{i(bx+a)}\right)}{b^3}+\frac{cd\ln\left(1+ie^{i(bx+a)}\right)a}{b^2}+\frac{d^2\ln\left(1+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] -I/b^2/(1+exp(2*I*(b*x+a)))^2*(d^2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I
*(b*x+a))+c^2*b*exp(3*I*(b*x+a))-d^2*x^2*b*exp(I*(b*x+a))-2*c*d*x*b*exp(I*(
b*x+a))-2*I*d^2*x*exp(3*I*(b*x+a))-c^2*b*exp(I*(b*x+a))-2*I*d*c*exp(3*I*(b*
x+a))-2*I*d^2*x*exp(I*(b*x+a))-2*I*c*d*exp(I*(b*x+a)))-1/b^2*c*d*ln(1-I*exp
```

```
(I*(b*x+a)))*a+d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+1/b^2*c*d*ln(1+I*exp(I*
(b*x+a)))*a+1/2/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2-1/2/b^3*a^2*d^2*ln(1+I*exp
(I*(b*x+a)))-1/2/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-2*I/b^3*d^2*arctan(exp(I*
(b*x+a)))+I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x+I/b^2*c*d*polylog(2,I*exp
(I*(b*x+a)))-2*I/b^2*a*c*d*arctan(exp(I*(b*x+a)))-I/b^2*c*d*polylog(2,-I*ex
p(I*(b*x+a)))+I/b^3*a^2*d^2*arctan(exp(I*(b*x+a)))+1/b*c*d*ln(1+I*exp(I*(b*
x+a)))*x-d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-I/b^2*d^2*polylog(2,-I*exp(I*(
b*x+a)))*x+I/b*c^2*arctan(exp(I*(b*x+a)))+1/2/b^3*a^2*d^2*ln(1-I*exp(I*(b*x
+a)))-1/b*c*d*ln(1-I*exp(I*(b*x+a)))*x
```

**maxima [B]** time = 0.85, size = 1893, normalized size = 9.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - lo
g(sin(b*x + a) - 1)) - 2*a*c*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(s
in(b*x + a) + 1) - log(sin(b*x + a) - 1))/b + a^2*d^2*(2*sin(b*x + a)/(sin(
b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b^2 - 4*((
2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - 4*d^2 + 2*((b*x + a)^2*d^
2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^2*
d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) - (-2*I*(b*x +
a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(4*b*x + 4*a) -
 (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) + 8*I*d^2)*sin(
2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (2*(b*x + a)^2*d^2
+ 4*(b*c*d - a*d^2)*(b*x + a) - 4*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d
^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a
*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) - (-2*I*(b*x + a)^2*d^2 + (-4*I*b
*c*d + 4*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(4*b*x + 4*a) - (-4*I*(b*x + a)^2
*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) + 8*I*d^2)*sin(2*b*x + 2*a))*arct
an2(cos(b*x + a), -sin(b*x + a) + 1) - (4*(b*x + a)^2*d^2 - 8*I*b*c*d + 8*I
*a*d^2 + (8*b*c*d - (8*a + 8*I)*d^2)*(b*x + a))*cos(3*b*x + 3*a) + (4*(b*x
+ a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (8*b*c*d - (8*a - 8*I)*d^2)*(b*x + a))
*cos(b*x + a) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)
*d^2 - a*d^2)*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*
x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(4*b*x + 4*a) -
(-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(I*e^(I
*b*x + I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*
d^2 - a*d^2)*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x
 + 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (8
*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b
*x + I*a)) - (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + 2*I
*d^2 + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + 2*I*d^2)*
cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x +
a) + 4*I*d^2)*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a) - 2*d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*
x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*s
in(b*x + a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) -
 2*I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) - 2*I*d^2
)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x +
a) - 4*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a) - 2*d^2)*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*
x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*s
in(b*x + a) + 1) - (-4*I*d^2*cos(4*b*x + 4*a) - 8*I*d^2*cos(2*b*x + 2*a) +
4*d^2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a) - 4*I*d^2)*polylog(3, I*e^(
I*b*x + I*a)) - (4*I*d^2*cos(4*b*x + 4*a) + 8*I*d^2*cos(2*b*x + 2*a) - 4*d^
2*sin(4*b*x + 4*a) - 8*d^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, -I*e^(I*b
*x + I*a)) + 4*(-I*(b*x + a)^2*d^2 - 2*b*c*d + 2*a*d^2 + 2*(-I*b*c*d + (I*a
```

```
 - 1)*d^2)*(b*x + a))*sin(3*b*x + 3*a) - (-4*I*(b*x + a)^2*d^2 + 8*b*c*d -
8*a*d^2 + (-8*I*b*c*d - 8*(-I*a - 1)*d^2)*(b*x + a))*sin(b*x + a))/(-4*I*b^
2*cos(4*b*x + 4*a) - 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) + 8*
b^2*sin(2*b*x + 2*a) - 4*I*b^2))/b
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)^2*(c + d*x)^2)/cos(a + b*x),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \tan^2(a + bx) \sec(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)**2*sec(a + b*x), x)
```

# 3.300 $\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx$

**Optimal**. Leaf size=117

$$-\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}+\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

```
[Out] I*(d*x+c)*arctan(exp(I*(b*x+a)))/b-1/2*I*d*polylog(2,-I*exp(I*(b*x+a)))/b^2
+1/2*I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-1/2*d*sec(b*x+a)/b^2+1/2*(d*x+c)*s
ec(b*x+a)*tan(b*x+a)/b
```

**Rubi [A]** time = 0.13, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4413, 4181, 2279, 2391, 4185}

$$-\frac{id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}+\frac{id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)\tan(a+b}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (I*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - ((I/2)*d*PolyLog[2, (-I)*E^(I*(a
+ b*x))])/b^2 + ((I/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[a + b*
x])/(2*b^2) + ((c + d*x)*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rubi steps

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)\sec(a+bx)\,dx + \int (c+dx)\sec^3(a+bx)\,dx$$
$$= \frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)\tan}{2b}$$
$$= \frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)\tan(}{2b}$$
$$= \frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{id\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} + \frac{id\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{b^2} - \frac{d\,\mathrm{se}}{}$$
$$= \frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{id\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2} + \frac{id\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2} - \frac{d\,\mathrm{se}}{}$$

**Mathematica [B]** time = 6.50, size = 555, normalized size = 4.74

$$-\frac{d\sin\left(\frac{1}{2}(a+bx)\right)}{2b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)}+\frac{d\sin\left(\frac{1}{2}(a+bx)\right)}{2b^2\left(\sin\left(\frac{1}{2}(a+bx)\right)+\cos\left(\frac{1}{2}(a+bx)\right)\right)}-\frac{c\tanh^{-1}(\sin(a+bx))}{2b}+\frac{c}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -1/2*(c*ArcTanh[Sin[a + b*x]])/b + (d*x*(a*Log[1 - Tan[(a + b*x)/2]] + I*Lo
g[1 + I*Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] - I*Log
[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)/2])] - I*Log[
1 + I*Tan[(a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + I*Log[1 -
 I*Tan[(a + b*x)/2]]*Log[(1/2 + I/2)*(1 + Tan[(a + b*x)/2])] - a*Log[1 + Ta
n[(a + b*x)/2]] - I*PolyLog[2, ((1 + I) - (1 - I)*Tan[(a + b*x)/2])/2] + I*
PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/2])] - I*PolyLog[2, ((1 + I) + (
1 - I)*Tan[(a + b*x)/2])/2] + I*PolyLog[2, ((1 - I) + (1 + I)*Tan[(a + b*x)
/2])/2]))/(2*b*(a - I*Log[1 - I*Tan[(a + b*x)/2]] + I*Log[1 + I*Tan[(a + b*
x)/2]])) + (d*x)/(4*b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])^2) - (d*Sin[(a
+ b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*x)/(4*b*(Cos[
(a + b*x)/2] + Sin[(a + b*x)/2])^2) + (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a +
 b*x)/2] + Sin[(a + b*x)/2])) + (c*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

**fricas [B]** time = 0.53, size = 435, normalized size = 3.72

$$\frac{i\,d\cos(bx+a)^2\,\mathrm{Li}_2\left(i\cos(bx+a)+\sin(bx+a)\right)+i\,d\cos(bx+a)^2\,\mathrm{Li}_2\left(i\cos(bx+a)-\sin(bx+a)\right)-i\,d\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*cos(b*x
+ a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-I*c
os(b*x + a) + sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - si
n(b*x + a)) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a)
+ I) + (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) -
(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*
x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a
*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*
cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c - a*d)*cos(b*
```

```
x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*cos(b*x + a)
^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*d*cos(b*x + a) + 2*(b*d*x +
b*c)*sin(b*x + a))/(b^2*cos(b*x + a)^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\sec(bx+a)\tan(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*tan(b*x + a)^2, x)
```

**maple [B]** time = 0.12, size = 267, normalized size = 2.28

$$-\frac{i\left(bdx\,\mathrm{e}^{3i(bx+a)}-id\,\mathrm{e}^{3i(bx+a)}+cb\,\mathrm{e}^{3i(bx+a)}-bdx\,\mathrm{e}^{i(bx+a)}-id\,\mathrm{e}^{i(bx+a)}-cb\,\mathrm{e}^{i(bx+a)}\right)}{b^2\left(1+\mathrm{e}^{2i(bx+a)}\right)^2}+\frac{ic\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b}+\frac{d\ln\left(1+i\mathrm{e}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] -I/b^2/(1+exp(2*I*(b*x+a)))^2*(b*d*x*exp(3*I*(b*x+a))-I*d*exp(3*I*(b*x+a))+
c*b*exp(3*I*(b*x+a))-b*d*x*exp(I*(b*x+a))-I*d*exp(I*(b*x+a))-c*b*exp(I*(b*x
+a)))+I/b*c*arctan(exp(I*(b*x+a)))+1/2/b*d*ln(1+I*exp(I*(b*x+a)))*x+1/2/b^2
*d*ln(1+I*exp(I*(b*x+a)))*a-1/2/b*d*ln(1-I*exp(I*(b*x+a)))*x-1/2/b^2*d*ln(1
-I*exp(I*(b*x+a)))*a-1/2*I/b^2*d*dilog(1+I*exp(I*(b*x+a)))+1/2*I/b^2*d*dilo
g(1-I*exp(I*(b*x+a)))-I/b^2*d*a*arctan(exp(I*(b*x+a)))
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)^2*(c + d*x))/cos(a + b*x),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\tan^2(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)**2*sec(a + b*x), x)
```

## 3.301 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=39

$$\text{Int}\left(\frac{\sec^3(a+bx)}{c+dx},x\right)-\text{Int}\left(\frac{\sec(a+bx)}{c+dx},x\right)$$

```
[Out] -Unintegrable(sec(b*x+a)/(d*x+c),x)+Unintegrable(sec(b*x+a)^3/(d*x+c),x)
```

**Rubi** [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
[Out] -Defer[Int][Sec[a + b*x]/(c + d*x), x] + Defer[Int][Sec[a + b*x]^3/(c + d*x
), x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx=-\int \frac{\sec(a+bx)}{c+dx}\,dx+\int \frac{\sec^3(a+bx)}{c+dx}\,dx$$

**Mathematica** [A] time = 26.39, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]
```

**fricas** [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(bx+a\right)\tan\left(bx+a\right)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(sec(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)\tan\left(bx+a\right)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

**maple** [A] time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{\sec (bx+a) \left(\tan^2 (bx+a)\right)}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
[Out] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan (a+b\,x)^2}{\cos (a+b\,x)\,\left(c+d\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)),x)
[Out] int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2 (a+bx) \sec (a+bx)}{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)**2/(d*x+c),x)
[Out] Integral(tan(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

## 3.302 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=39

$$\operatorname{Int}\left(\frac{\sec^3(a+bx)}{(c+dx)^2},x\right)-\operatorname{Int}\left(\frac{\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] -Unintegrable(sec(b*x+a)/(d*x+c)^2,x)+Unintegrable(sec(b*x+a)^3/(d*x+c)^2,x
)
```

**Rubi** [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -Defer[Int][Sec[a + b*x]/(c + d*x)^2, x] + Defer[Int][Sec[a + b*x]^3/(c + d
*x)^2, x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx=-\int \frac{\sec(a+bx)}{(c+dx)^2}\,dx+\int \frac{\sec^3(a+bx)}{(c+dx)^2}\,dx$$

**Mathematica** [A] time = 29.12, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]
```

**fricas** [A] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sec(bx+a)\tan(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\tan(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*tan(b*x + a)^2/(d*x + c)^2, x)
```

**maple [A]** time = 2.60, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\left(\tan^2(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
[Out] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(a+b\,x)^2}{\cos(a+b\,x)\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2),x)
[Out] int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(tan(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

## 3.303 $\int (c+dx)^m \tan^3(a+bx)\,dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\tan^3(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable((d*x+c)^m*tan(b*x+a)^3,x)
```

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \tan^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Tan[a + b*x]^3,x]
[Out] Defer[Int][(c + d*x)^m*Tan[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \int (c+dx)^m \tan^3(a+bx)\,dx$$

**Mathematica [A]** time = 9.70, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Tan[a + b*x]^3,x]
[Out] Integrate[(c + d*x)^m*Tan[a + b*x]^3, x]
```

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \tan(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="fricas")
[Out] integral((d*x + c)^m*tan(b*x + a)^3, x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \tan(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*tan(b*x + a)^3, x)
```

**maple [A]** time = 0.12, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\tan^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*tan(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*tan(b*x+a)^3,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \tan(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*tan(b*x + a)^3, x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.05

$$\int \tan(a+b\,x)^3\,(c+d\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3*(c + d*x)^m,x)
```

```
[Out] int(tan(a + b*x)^3*(c + d*x)^m, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*tan(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**m*tan(a + b*x)**3, x)
```

# 3.304 $\int (c+dx)^3 \tan^3(a+bx)\,dx$

**Optimal**. Leaf size=259

$$\frac{3id^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4}+\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{3id(c+}{}$$

```
[Out] 3/2*I*d*(d*x+c)^2/b^2+1/2*(d*x+c)^3/b-1/4*I*(d*x+c)^4/d-3*d^2*(d*x+c)*ln(1+
exp(2*I*(b*x+a)))/b^3+(d*x+c)^3*ln(1+exp(2*I*(b*x+a)))/b+3/2*I*d^3*polylog(
2,-exp(2*I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2
+3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,-exp(
2*I*(b*x+a)))/b^4-3/2*d*(d*x+c)^2*tan(b*x+a)/b^2+1/2*(d*x+c)^3*tan(b*x+a)^2
/b
```

**Rubi** [A] time = 0.36, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3720, 3719, 2190, 2279, 2391, 32, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4}+\frac{3id^3\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tan[a + b*x]^3,x]
```

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 + (c + d*x)^3/(2*b) - ((I/4)*(c + d*x)^4)/d -
 (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^3 + ((c + d*x)^3*Log[1 +
E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/
b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (3*d
^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*Pol
yLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Tan[a + b*x])/(2*b^2)
 + ((c + d*x)^3*Tan[a + b*x]^2)/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\tan^3(a+bx)\,dx = \frac{(c+dx)^3\tan^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)^2\tan^2(a+bx)\,dx}{2b} - \int (c+dx)^3\tan(a+$$

$$= -\frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\tan(a+bx)}{2b^2} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}}{1+e}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - 3d(c+dx$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + (c+dx$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + (c+dx$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + (c+dx$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + (c+dx$$

**Mathematica [B]** time = 6.89, size = 803, normalized size = 3.10

$$\frac{\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))c^3}{b\left(\cos^2(a)+\sin^2(a)\right)} + \frac{3d\csc(a)\left(b^2e^{-i\tan^{-1}(\cot(a))}x^2 - \frac{\cot(a)\left(ibx\left(-2\tan\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Tan[a + b*x]^3,x]
```

```
[Out] ((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*
(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] -
(3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(
I*a)) + (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))*
x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*Poly
Log[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))]
 - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] + ((c + d*
x)^3*Sec[a + b*x]^2)/(2*b) + (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[
a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c*d^2*Sec[a]*(Co
s[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 +
 Sin[a]^2)) + (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*
b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan
[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*
ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x
- ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos
[a]^2 + Sin[a]^2)]) - (3*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[
a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x -
ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]
] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)
*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^4*Sqrt[Csc[a]^
2*(Cos[a]^2 + Sin[a]^2)]) - (3*Sec[a]*Sec[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^
2*x*Sin[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) - (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2
*x^2 + d^3*x^3)*Tan[a])/4
```

**fricas [C]** time = 0.45, size = 590, normalized size = 2.28

$$\frac{4\,b^3d^3x^3 + 12\,b^3cd^2x^2 + 12\,b^3c^2dx - 3i\,d^3\mathrm{polylog}\left(4, \frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + 3i\,d^3\mathrm{polylog}\left(4, \frac{\tan(bx+a)^2-2i\,\tan}{\tan(bx+a)^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 3*I*d^3*polylog(4,
 (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 3*I*d^3*po
lylog(4, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 4*
(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*tan(b*x + a)^2 +
(6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d - 6*I*d^3)*dilog(2*(I*t
an(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c
*d^2*x - 6*I*b^2*c^2*d + 6*I*d^3)*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x +
a)^2 + 1) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 3*b*c*d^2 + 3
*(b^3*c^2*d - b*d^3)*x)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) +
 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 3*b*c*d^2 + 3*(b^3*c^2*d - b*
d^3)*x)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 6*(b*d^3*x + b
*c*d^2)*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2
+ 1)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a)
 - 1)/(tan(b*x + a)^2 + 1)) - 12*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*
tan(b*x + a))/b^4
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \tan (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*tan(b*x + a)^3, x)
```

**maple [B]** time = 0.13, size = 720, normalized size = 2.78

$$\frac{3d^3 \operatorname{polylog}\left(3,-e^{2i(bx+a)}\right) x}{2b^3}+\frac{d^3 \ln\left(1+e^{2i(bx+a)}\right) x^3}{b}+\frac{6ic\,d^2a^2x}{b^2}-\frac{6ic^2dax}{b}+\frac{2d^3a^3\ln\left(e^{i(bx+a)}\right)}{b^4}-\frac{3ia^4d^3}{2b^4}-ic\,d^2x^3-\frac{3i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*tan(b*x+a)^3,x)
```

```
[Out] 6*I/b^2*c*d^2*a^2*x-6*I/b*c^2*d*a*x+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/
b^4+3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+I*c^3*x-3/b^3*d^2*c*ln(1+exp
(2*I*(b*x+a)))-3/b^3*d^3*ln(1+exp(2*I*(b*x+a)))*x+1/b*c^3*ln(1+exp(2*I*(b*x
+a)))+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))-3/2*I/b^4*a^4*d^3-I*c*d^2*x^3-3/2*I*
c^2*d*x^2+3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))+3/2/b^3*d^3*polylog(3,
-exp(2*I*(b*x+a)))*x+(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x
+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*
x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*c^3*exp(2*I*(
b*x+a))-6*I*c*d^2*x-3*I*c^2*d)/b^2/(1+exp(2*I*(b*x+a)))^2-1/4*I*d^3*x^4-2/b
*c^3*ln(exp(I*(b*x+a)))+3*I/b^2*d^3*x^2+3*I/b^4*d^3*a^2+6/b^3*d^2*c*ln(exp(
I*(b*x+a)))-6/b^4*d^3*a*ln(exp(I*(b*x+a)))-3/2*I/b^2*d^3*polylog(2,-exp(2*I
*(b*x+a)))*x^2-3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-6/b^3*c*d^2*a^2
*ln(exp(I*(b*x+a)))-3*I/b^2*c^2*d*a^2-2*I/b^3*a^3*d^3*x+4*I/b^3*c*d^2*a^3+6
/b^2*c^2*d*a*ln(exp(I*(b*x+a)))+1/b*d^3*ln(1+exp(2*I*(b*x+a)))*x^3-3*I/b^2*
polylog(2,-exp(2*I*(b*x+a)))*c*d^2*x+6*I/b^3*d^3*a*x+3/b*c^2*d*ln(1+exp(2*I
*(b*x+a)))*x+3/b*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2
```

**maxima [B]** time = 1.19, size = 2405, normalized size = 9.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) - 3*a*c^2*d*(1
/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b + 3*a^2*c*d^2*(1/(sin(b*
x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b^2 - a^3*d^3*(1/(sin(b*x + a)^2 -
 1) - log(sin(b*x + a)^2 - 1))/b^3 + 2*(3*(b*x + a)^4*d^3 + 36*b^2*c^2*d -
72*a*b*c*d^2 + 36*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 18*(b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)^2 - (16*(b*x + a)^3*d^3 - 36*b*c*d^2 +
 36*a*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 36*(b^2*c^2*d - 2*a*b*c*d^2
+ (a^2 - 1)*d^3)*(b*x + a) + 4*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9
*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3
)*(b*x + a))*cos(4*b*x + 4*a) + 8*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3
+ 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*
d^3)*(b*x + a))*cos(2*b*x + 2*a) + (16*I*(b*x + a)^3*d^3 - 36*I*b*c*d^2 + 3
6*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*
I*a*b*c*d^2 + (36*I*a^2 - 36*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (32*I*(b
*x + a)^3*d^3 - 72*I*b*c*d^2 + 72*I*a*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*
x + a)^2 + (72*I*b^2*c^2*d - 144*I*a*b*c*d^2 + (72*I*a^2 - 72*I)*d^3)*(b*x
+ a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + 3
*((b*x + a)^4*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*
c*d^2 + (a^2 - 2)*d^3)*(b*x + a)^2 - 24*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*
b*x + 4*a) + (6*(b*x + a)^4*d^3 + 36*b^2*c^2*d - 72*a*b*c*d^2 + 36*a^2*d^3
+ (24*b*c*d^2 - (24*a - 24*I)*d^3)*(b*x + a)^3 + (36*b^2*c^2*d - (72*a - 72
*I)*b*c*d^2 + 36*(a^2 - 2*I*a - 1)*d^3)*(b*x + a)^2 - (-72*I*b^2*c^2*d - 72
*(-2*I*a - 1)*b*c*d^2 + (-72*I*a^2 - 72*a)*d^3)*(b*x + a))*cos(2*b*x + 2*a)
 + (18*b^2*c^2*d - 36*a*b*c*d^2 + 24*(b*x + a)^2*d^3 + 18*(a^2 - 1)*d^3 + 3
6*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^
2*d^3 + 3*(a^2 - 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) +
 12*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 - 1)*d^3 + 6*(b
*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-18*I*b^2*c^2*d + 36*I*a*b*c
*d^2 - 24*I*(b*x + a)^2*d^3 + (-18*I*a^2 + 18*I)*d^3 + (-36*I*b*c*d^2 + 36*
I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 -
48*I*(b*x + a)^2*d^3 + (-36*I*a^2 + 36*I)*d^3 + (-72*I*b*c*d^2 + 72*I*a*d^3
)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) - (-8*I*(b*x + a
)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a
)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 + 18*I)*d^3)*(b*x + a)
 + (-8*I*(b*x + a)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*
I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 + 18*
I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-16*I*(b*x + a)^3*d^3 + 36*I*b*c*d^2
 - 36*I*a*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d
 + 72*I*a*b*c*d^2 + (-36*I*a^2 + 36*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 2
*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2
 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a)
+ 4*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a
)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(2*b*x + 2*
a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) -
 (12*d^3*cos(4*b*x + 4*a) + 24*d^3*cos(2*b*x + 2*a) + 12*I*d^3*sin(4*b*x +
4*a) + 24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)*polylog(4, -e^(2*I*b*x + 2*I*a))
 - (-18*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 18*I*a*d^3 + (-18*I*b*c*d^2 - 24*I
*(b*x + a)*d^3 + 18*I*a*d^3)*cos(4*b*x + 4*a) + (-36*I*b*c*d^2 - 48*I*(b*x
+ a)*d^3 + 36*I*a*d^3)*cos(2*b*x + 2*a) + 6*(3*b*c*d^2 + 4*(b*x + a)*d^3 -
3*a*d^3)*sin(4*b*x + 4*a) + 12*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*sin(
2*b*x + 2*a))*polylog(3, -e^(2*I*b*x + 2*I*a)) - (-3*I*(b*x + a)^4*d^3 + (-
12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^3 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2
+ (-18*I*a^2 + 36*I)*d^3)*(b*x + a)^2 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x +
a))*sin(4*b*x + 4*a) - (-6*I*(b*x + a)^4*d^3 - 36*I*b^2*c^2*d + 72*I*a*b*c*
d^2 - 36*I*a^2*d^3 + (-24*I*b*c*d^2 - 24*(-I*a - 1)*d^3)*(b*x + a)^3 + (-36
*I*b^2*c^2*d - 72*(-I*a - 1)*b*c*d^2 + (-36*I*a^2 - 72*a + 36*I)*d^3)*(b*x
+ a)^2 + (72*b^2*c^2*d - (144*a - 72*I)*b*c*d^2 + 72*(a^2 - I*a)*d^3)*(b*x
+ a))*sin(2*b*x + 2*a))/(-12*I*b^3*cos(4*b*x + 4*a) - 24*I*b^3*cos(2*b*x +
```

```
2*a) + 12*b^3*sin(4*b*x + 4*a) + 24*b^3*sin(2*b*x + 2*a) - 12*I*b^3))/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \tan\left(a + b\,x\right)^3\,{\left(c + d\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3*(c + d*x)^3,x)
[Out] int(tan(a + b*x)^3*(c + d*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx\right)^{3} \tan^{3}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*tan(b*x+a)**3,x)
[Out] Integral((c + d*x)**3*tan(a + b*x)**3, x)
```

# 3.305 $\int (c+dx)^2 \tan^3(a+bx)\,dx$

**Optimal**. Leaf size=169

$$\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{d^2\log(\cos(a+bx))}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\tan(a+bx)}{b^2}+\frac{(c+dx)^2\log\left(1+e\right.}{b}$$

```
[Out] c*d*x/b+1/2*d^2*x^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b-
d^2*ln(cos(b*x+a))/b^3-I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/2*d^2
*polylog(3,-exp(2*I*(b*x+a)))/b^3-d*(d*x+c)*tan(b*x+a)/b^2+1/2*(d*x+c)^2*ta
n(b*x+a)^2/b
```

**Rubi** [A] time = 0.22, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3720, 3475, 3719, 2190, 2531, 2282, 6589}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\tan(a+bx)}{b^2}-\frac{d^2\log(\cos(a+bx))}{b^3}+\frac{(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Tan[a + b*x]^3,x]
```

```
[Out] (c*d*x)/b + (d^2*x^2)/(2*b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 +
E^((2*I)*(a + b*x))])/b - (d^2*Log[Cos[a + b*x]])/b^3 - (I*d*(c + d*x)*Poly
Log[2, -E^((2*I)*(a + b*x))])/b^2 + (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/
(2*b^3) - (d*(c + d*x)*Tan[a + b*x])/b^2 + ((c + d*x)^2*Tan[a + b*x]^2)/(2*
b)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\tan^3(a+bx)\,dx &= \frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{d\int (c+dx)\tan^2(a+bx)\,dx}{b}-\int (c+dx)^2\tan(a+bx)\,dx \\ &= -\frac{i(c+dx)^3}{3d}-\frac{d(c+dx)\tan(a+bx)}{b^2}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}+2i\int \frac{e^{2i(a+bx)}(c+d}{1+e^{2i(a+bx}} \\ &= \frac{cdx}{b}+\frac{d^2x^2}{2b}-\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{d^2\log(\cos(a+bx))}{b^3}-\frac{d(}{} \\ &= \frac{cdx}{b}+\frac{d^2x^2}{2b}-\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{d^2\log(\cos(a+bx))}{b^3}-\frac{id(}{} \\ &= \frac{cdx}{b}+\frac{d^2x^2}{2b}-\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{d^2\log(\cos(a+bx))}{b^3}-\frac{id(}{} \\ &= \frac{cdx}{b}+\frac{d^2x^2}{2b}-\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{d^2\log(\cos(a+bx))}{b^3}-\frac{id(}{}\end{aligned}$$

**Mathematica [B]** time = 6.66, size = 454, normalized size = 2.69

$$-\frac{d^2\sec(a)(bx\sin(a)+\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx)))}{b^3\left(\sin^2(a)+\cos^2(a)\right)}+\frac{\sec(a)\sec(a+bx)\left(d^2(-x)\sin(bx)-cd\sin(b\right.}{b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Tan[a + b*x]^3,x]
```

```
[Out] ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(
a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (
3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I
*a)) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) + (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*
Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (d^2
*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*
(Cos[a]^2 + Sin[a]^2)) + (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot
[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x -
```

```
ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x
]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I
)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2
*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(-(c*d*Sin[b*x]) - d^2*x*Si
n[b*x]))/b^2 - (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3
```

**fricas [C]** time = 0.45, size = 352, normalized size = 2.08

$$\frac{2\,b^2d^2x^2 + 4\,b^2cdx + d^2\mathrm{polylog}\left(3, \frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + d^2\mathrm{polylog}\left(3, \frac{\tan(bx+a)^2-2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + 2\left(b^2d^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + d^2*polylog(3, (tan(b*x + a)^2 + 2*I*tan
(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + d^2*polylog(3, (tan(b*x + a)^2 - 2*I
*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*tan(b*x + a)^2 + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(2*(I*tan(b*x + a)
- 1)/(tan(b*x + a)^2 + 1) + 1) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(2*(-I*tan
(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2 - d^2)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2 - d^2)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x
+ a)^2 + 1)) - 4*(b*d^2*x + b*c*d)*tan(b*x + a))/b^3
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \tan(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*tan(b*x + a)^3, x)
```

**maple [B]** time = 0.10, size = 400, normalized size = 2.37

$$-\frac{id^2x^3}{3} - \frac{4icdax}{b} + \frac{4id^2a^3}{3b^3} + \frac{2b\,d^2x^2\mathrm{e}^{2i(bx+a)} - 2id^2x\,\mathrm{e}^{2i(bx+a)} + 4bcdx\,\mathrm{e}^{2i(bx+a)} - 2icd\,\mathrm{e}^{2i(bx+a)} + 2b\,c^2\mathrm{e}^{2i(bx+a)} - 2i}{b^2\left(1+\mathrm{e}^{2i(bx+a)}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*tan(b*x+a)^3,x)
```

```
[Out] -1/3*I*d^2*x^3-4*I/b*c*d*a*x+4/3*I/b^3*d^2*a^3+2*(b*d^2*x^2*exp(2*I*(b*x+a)
)-I*d^2*x*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+a)
)+b*c^2*exp(2*I*(b*x+a))-I*d^2*x-I*c*d)/b^2/(1+exp(2*I*(b*x+a)))^2-I/b^2*c*
d*polylog(2,-exp(2*I*(b*x+a)))+I*c^2*x+2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x+1/2
*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-1/b^3*d^2*ln(1+exp(2*I*(b*x+a)))+2/b^
3*d^2*ln(exp(I*(b*x+a)))-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+1/b*c^2*ln(1+exp(
2*I*(b*x+a)))-2/b*c^2*ln(exp(I*(b*x+a)))+2*I/b^2*d^2*a^2*x-I*c*d*x^2+1/b*d^
2*ln(1+exp(2*I*(b*x+a)))*x^2-2*I/b^2*c*d*a^2+4/b^2*c*d*a*ln(exp(I*(b*x+a)))
-I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x
```

**maxima [B]** time = 0.67, size = 1226, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) - 2*a*c*d*(1/(
sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b + a^2*d^2*(1/(sin(b*x + a)
^2 - 1) - log(sin(b*x + a)^2 - 1))/b^2 + 2*(2*(b*x + a)^3*d^2 + 6*(b*c*d -
a*d^2)*(b*x + a)^2 + 12*b*c*d - 12*a*d^2 - (6*(b*x + a)^2*d^2 + 12*(b*c*d -
 a*d^2)*(b*x + a) - 6*d^2 + 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a
) - d^2)*cos(4*b*x + 4*a) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x +
a) - d^2)*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^
2)*(b*x + a) - 6*I*d^2)*sin(4*b*x + 4*a) + (12*I*(b*x + a)^2*d^2 + (24*I*b*
c*d - 24*I*a*d^2)*(b*x + a) - 12*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x
 + 2*a), cos(2*b*x + 2*a) + 1) + 2*((b*x + a)^3*d^2 + 3*(b*c*d - a*d^2)*(b*
x + a)^2 - 6*(b*x + a)*d^2)*cos(4*b*x + 4*a) + (4*(b*x + a)^3*d^2 + (12*b*c
*d - (12*a - 12*I)*d^2)*(b*x + a)^2 + 12*b*c*d - 12*a*d^2 - (-24*I*b*c*d -
12*(-2*I*a - 1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (6*b*c*d + 6*(b*x + a)*d
^2 - 6*a*d^2 + 6*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 12*(b*c
*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-6*I*b*c*d - 6*I*(b*x + a)*
d^2 + 6*I*a*d^2)*sin(4*b*x + 4*a) - (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 12*
I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) - (-3*I*(b*x + a)^2*
d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2 + (-3*I*(b*x + a)^2*d^2
+ (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2)*cos(4*b*x + 4*a) + (-6*I*(b
*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) + 6*I*d^2)*cos(2*b*x +
 2*a) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(4*b*x +
 4*a) + 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(2*b*x +
 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1
) - (-3*I*d^2*cos(4*b*x + 4*a) - 6*I*d^2*cos(2*b*x + 2*a) + 3*d^2*sin(4*b*x
 + 4*a) + 6*d^2*sin(2*b*x + 2*a) - 3*I*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)
) - (-2*I*(b*x + a)^3*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a)^2 + 12*I*(b*
x + a)*d^2)*sin(4*b*x + 4*a) - (-4*I*(b*x + a)^3*d^2 + (-12*I*b*c*d - 12*(-
I*a - 1)*d^2)*(b*x + a)^2 - 12*I*b*c*d + 12*I*a*d^2 + (24*b*c*d - (24*a - 1
2*I)*d^2)*(b*x + a))*sin(2*b*x + 2*a))/(-6*I*b^2*cos(4*b*x + 4*a) - 12*I*b^
2*cos(2*b*x + 2*a) + 6*b^2*sin(4*b*x + 4*a) + 12*b^2*sin(2*b*x + 2*a) - 6*I
*b^2))/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(a+b\,x)^3\,(c+d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] int(tan(a + b*x)^3*(c + d*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2\tan^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*tan(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)**3, x)
```

# 3.306 $\int (c+dx)\tan^3(a+bx)\,dx$

**Optimal**. Leaf size=108

$$-\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{(c+dx)\tan^2(a+bx)}{2b}+\frac{dx}{2b}-\frac{i(c+dx)^2}{2d}$$

```
[Out] 1/2*d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b-1/2*I*d*polylo
g(2,-exp(2*I*(b*x+a)))/b^2-1/2*d*tan(b*x+a)/b^2+1/2*(d*x+c)*tan(b*x+a)^2/b
```

**Rubi** [A] time = 0.12, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3720, 3473, 8, 3719, 2190, 2279, 2391}

$$-\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{(c+dx)\tan^2(a+bx)}{2b}+\frac{dx}{2b}-\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tan[a + b*x]^3,x]
```

```
[Out] (d*x)/(2*b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b - ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d*Tan[a + b*x])/(
2*b^2) + ((c + d*x)*Tan[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\tan^3(a+bx)\,dx = \frac{(c+dx)\tan^2(a+bx)}{2b} - \frac{d\int \tan^2(a+bx)\,dx}{2b} - \int (c+dx)\tan(a+bx)\,dx$$
$$= -\frac{i(c+dx)^2}{2d} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c+dx)\tan^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx + \frac{d}{}$$
$$= \frac{dx}{2b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c+dx)\tan^2(a+b}{2b}$$
$$= \frac{dx}{2b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c+dx)\tan^2(a+b}{2b}$$
$$= \frac{dx}{2b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{d\tan(a+bx)}{2b^2} +$$

**Mathematica [B]** time = 6.15, size = 240, normalized size = 2.22

$$\frac{d\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))} - \frac{\cot(a)\left(i\mathrm{Li}_2\left(e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan^{-1}(\cot\right.}\right.}{\sqrt{\cot^2(a)+1}}\right.}{2b^2\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Tan[a + b*x]^3,x]
```

```
[Out] (d*x*Sec[a + b*x]^2)/(2*b) + (d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (C
ot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x
 - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b
*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2
*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[
a]^2*(Cos[a]^2 + Sin[a]^2)]) - (d*Sec[a]*Sec[a + b*x]*Sin[b*x])/(2*b^2) - (
d*x^2*Tan[a])/2 + (c*(2*Log[Cos[a + b*x]] + Tan[a + b*x]^2))/(2*b)
```

**fricas [A]** time = 0.43, size = 168, normalized size = 1.56

$$\frac{2\,bdx + 2\,(bdx+bc)\tan(bx+a)^2 + i\,d\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right) - i\,d\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right) + 2\,(bdx+bc)\log\left(-\right.}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*d*x + 2*(b*d*x + b*c)*tan(b*x + a)^2 + I*d*dilog(2*(I*tan(b*x + a)
 - 1)/(tan(b*x + a)^2 + 1) + 1) - I*d*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*
x + a)^2 + 1) + 1) + 2*(b*d*x + b*c)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x +
 a)^2 + 1)) + 2*(b*d*x + b*c)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2
+ 1)) - 2*d*tan(b*x + a))/b^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\tan(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*tan(b*x + a)^3, x)
```

**maple [A]** time = 0.08, size = 183, normalized size = 1.69

$$-\frac{id\,x^2}{2}+icx+\frac{2bdx\,e^{2i(bx+a)}+2bc\,e^{2i(bx+a)}-id\,e^{2i(bx+a)}-id}{b^2\left(1+e^{2i(bx+a)}\right)^2}+\frac{c\ln\left(1+e^{2i(bx+a)}\right)}{b}-\frac{2c\ln\left(e^{i(bx+a)}\right)}{b}-\frac{2idax}{b}-\frac{id\,a^2}{b^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*tan(b*x+a)^3,x)
```

```
[Out] -1/2*I*d*x^2+I*c*x+(2*b*d*x*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d*exp
(2*I*(b*x+a))-I*d)/b^2/(1+exp(2*I*(b*x+a)))^2+1/b*c*ln(1+exp(2*I*(b*x+a)))-
2/b*c*ln(exp(I*(b*x+a)))-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(1+exp(2*I*(b*x+a)
))*x-1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b^2*d*a*ln(exp(I*(b*x+a)))
```

**maxima [B]** time = 0.58, size = 519, normalized size = 4.81

$$-\frac{b^2dx^2+2\,b^2cx-(2\,bdx+2\,bc+2\,(bdx+bc)\cos(4\,bx+4\,a)+4\,(bdx+bc)\cos(2\,bx+2\,a)+(2i\,bdx+2i\,bc)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -(b^2*d*x^2 + 2*b^2*c*x - (2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*
a) + 4*(b*d*x + b*c)*cos(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4
*a) + (4*I*b*d*x + 4*I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos
(2*b*x + 2*a) + 1) + (b^2*d*x^2 + 2*b^2*c*x)*cos(4*b*x + 4*a) + (2*b^2*d*x^
2 + 4*I*b*c + (4*b^2*c + 4*I*b*d)*x + 2*d)*cos(2*b*x + 2*a) + (d*cos(4*b*x
+ 4*a) + 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x + 2*
a) + d)*dilog(-e^(2*I*b*x + 2*I*a)) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c
)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c
)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (-I*b^2*d*x^2 - 2*I*b^2
*c*x)*sin(4*b*x + 4*a) - (-2*I*b^2*d*x^2 + 4*b*c - 4*(I*b^2*c - b*d)*x - 2*
I*d)*sin(2*b*x + 2*a) + 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) - 4*I*b^2*cos(2*b*x
 + 2*a) + 2*b^2*sin(4*b*x + 4*a) + 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{tan}\left(a+b\,x\right)}^3\,\left(c+d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3*(c + d*x),x)
```

```
[Out] int(tan(a + b*x)^3*(c + d*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\tan^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)**3, x)
```

# 3.307 $\int \frac{\tan^3(a+bx)}{c+dx} dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\frac{\tan^3(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^3/(d*x+c),x)
```

**Rubi [A]** time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]^3/(c + d*x),x]
[Out] Defer[Int][Tan[a + b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\tan^3(a+bx)}{c+dx} dx = \int \frac{\tan^3(a+bx)}{c+dx} dx$$

**Mathematica [A]** time = 6.53, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^3/(c + d*x),x]
[Out] Integrate[Tan[a + b*x]^3/(c + d*x), x]
```

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tan(bx+a)^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(tan(b*x + a)^3/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(bx+a)^3}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(tan(b*x + a)^3/(d*x + c), x)
```

**maple** [A] time = 1.76, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(tan(b*x+a)^3/(d*x+c),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
(2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b
*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*
b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(2*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2 - d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) + (d*cos(2*b*x + 2*a) + 2*(b*d*x + b*
c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + d*sin(2*b*x + 2*a))/(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x
 + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b
*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{{\tan\left(a+b\,x\right)}^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3/(c + d*x),x)
```

```
[Out] int(tan(a + b*x)^3/(c + d*x), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)**3/(c + d*x), x)
```

## 3.308 $\int \frac{\tan^3(a+bx)}{(c+dx)^2} \, dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\frac{\tan^3(a+bx)}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^3/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]^3/(c + d*x)^2,x]
[Out] Defer[Int][Tan[a + b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} \, dx = \int \frac{\tan^3(a+bx)}{(c+dx)^2} \, dx$$

**Mathematica [A]** time = 6.96, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^3/(c + d*x)^2,x]
[Out] Integrate[Tan[a + b*x]^3/(c + d*x)^2, x]
```

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tan(bx+a)^3}{d^2x^2 + 2\,cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(tan(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(bx+a)^3}{(dx+c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(tan(b*x + a)^3/(d*x + c)^2, x)
```

**maple** [A] time = 2.73, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(tan(b*x+a)^3/(d*x+c)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*inte
grate(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*d^2)*sin(2*b*x + 2*a)/(b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 +
(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^
4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2
+ 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^
3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)), x)
+ 2*(d*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4
*a) + 2*d*sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4
*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*
c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2
*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2
*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\tan(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3/(c + d*x)^2,x)
```

```
[Out] int(tan(a + b*x)^3/(c + d*x)^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)**3/(c + d*x)**2, x)
```

# 3.309 $\int (c+dx)^m \csc(a+bx) \sec^3(a+bx) \, dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\csc(a+bx)\sec^3(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

**Rubi** [A] time = 0.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc(a+bx) \sec^3(a+bx) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \csc(a+bx) \sec^3(a+bx) \, dx = \int (c+dx)^m \csc(a+bx) \sec^3(a+bx) \, dx$$

**Mathematica** [A] time = 13.37, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc(a+bx) \sec^3(a+bx) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]
```

**fricas** [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \csc(bx+a) \sec(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a) \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

**maple** [A] time = 0.10, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a) \left(\sec^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \csc (bx + a) \sec (bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c + d\,x)^m}{\cos (a + b\,x)^3 \, \sin (a + b\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)),x)
```

```
[Out] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.310 $\int (c+dx)^4 \csc(a+bx)\sec^3(a+bx)\,dx$

**Optimal**. Leaf size=399

$$-\frac{3d^4\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^5}+\frac{3d^4\text{Li}_5\left(-e^{2i(a+bx)}\right)}{2b^5}-\frac{3d^4\text{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}+\frac{6id^3(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^4}-\frac{3id^3(c+dx)\text{Li}_4\left(-e^{2i(a-}\right.}{b^4}$$

```
[Out] 2*I*d*(d*x+c)^3/b^2+1/2*(d*x+c)^4/b-2*(d*x+c)^4*arctanh(exp(2*I*(b*x+a)))/b
-6*d^2*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^3+6*I*d^3*(d*x+c)*polylog(2,-exp(
2*I*(b*x+a)))/b^4-2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2+3*I*d^3*(
d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4-3*d^4*polylog(3,-exp(2*I*(b*x+a)))/b
^5-3*d^2*(d*x+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+3*d^2*(d*x+c)^2*polylog
(3,exp(2*I*(b*x+a)))/b^3+2*I*d*(d*x+c)^3*polylog(2,-exp(2*I*(b*x+a)))/b^2-3
*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,-exp(2*I*
(b*x+a)))/b^5-3/2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5-2*d*(d*x+c)^3*tan(b*x
+a)/b^2+1/2*(d*x+c)^4*tan(b*x+a)^2/b
```

**Rubi [A]** time = 0.97, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2620, 14, 4420, 6741, 12, 6742, 2551, 4183, 2531, 6609, 2282, 6589, 3720, 3719, 2190, 32}

$$\frac{6id^3(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^4}-\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{3d^{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] ((2*I)*d*(c + d*x)^3)/b^2 + (c + d*x)^4/(2*b) - (2*(c + d*x)^4*ArcTanh[E^((
2*I)*(a + b*x))])/b - (6*d^2*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^3
+ ((6*I)*d^3*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + ((2*I)*d*(c
+ d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)^3*PolyL
og[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^4*PolyLog[3, -E^((2*I)*(a + b*x))])/
b^5 - (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 + (3*d^2*(c
+ d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLo
g[4, -E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*
(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (3*d^
4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5) - (2*d*(c + d*x)^3*Tan[a + b*x])
/b^2 + ((c + d*x)^4*Tan[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
```

```
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^4 \csc(a+bx)\sec^3(a+bx)\,dx = \frac{(c+dx)^4\log(\tan(a+bx))}{b} + \frac{(c+dx)^4\tan^2(a+bx)}{2b} - (4d)\int (c+d$$

$$= \frac{(c+dx)^4\log(\tan(a+bx))}{b} + \frac{(c+dx)^4\tan^2(a+bx)}{2b} - (4d)\int \frac{(c+d}{}$$

$$= \frac{(c+dx)^4\log(\tan(a+bx))}{b} + \frac{(c+dx)^4\tan^2(a+bx)}{2b} - \frac{(2d)\int (c+dx}{}$$

$$= \frac{(c+dx)^4\log(\tan(a+bx))}{b} + \frac{(c+dx)^4\tan^2(a+bx)}{2b} - \frac{(2d)\int \left(2(c+\right.}{}$$

$$= \frac{(c+dx)^4\log(\tan(a+bx))}{b} + \frac{(c+dx)^4\tan^2(a+bx)}{2b} - \frac{(2d)\int (c+dx}{}$$

$$= -\frac{2d(c+dx)^3\tan(a+bx)}{b^2} + \frac{(c+dx)^4\tan^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^4}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2d(c+dx)^3\tan(a+bx)}{b^2} + \frac{(c+dx)^4\tan^2}{2b}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+d}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+d}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+d}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+d}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+d}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+d}{}$$

**Mathematica [B]** time = 7.47, size = 2090, normalized size = 5.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] -((c^2*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*
I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*L
og[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)
*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E
^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a +
 b*x))]))/E^((2*I)*a)))/b^3) - (c*d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a)
 + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3
*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))
*(b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(
a + b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^(
(2*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^
((-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^4 -
(d^4*E^(I*a)*Csc[a]*((2*b^5*x^5)/E^((2*I)*a) + (5*I)*b^4*(1 - E^((-2*I)*a))
*x^4*Log[1 - E^((-I)*(a + b*x))] + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1 +
 E^((-I)*(a + b*x))] - (20*(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, -E^((-I)*
(a + b*x))] - (3*I)*b^2*x^2*PolyLog[3, -E^((-I)*(a + b*x))] - 6*b*x*PolyLog
```

```
[4, -E^((-I)*(a + b*x))] + (6*I)*PolyLog[5, -E^((-I)*(a + b*x))]))/E^((2*I)
*a) - (20*(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, E^((-I)*(a + b*x))] - (3*I
)*b^2*x^2*PolyLog[3, E^((-I)*(a + b*x))] - 6*b*x*PolyLog[4, E^((-I)*(a + b*
x))] + (6*I)*PolyLog[5, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(10*b^5) + (x*(
5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)*Csc[a]*Sec[a])
/5 - ((I/2)*c^2*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^(
(-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*
x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(
b^3*E^(I*a)) - ((I/2)*d^4*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1
 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(
a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec
[a])/(b^5*E^(I*a)) - (I/2)*c*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 +
 E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*
(2*b^2*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2
*I)*(a + b*x))] - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Se
c[a] + (d^4*((-4*I)*x^5 - (10*(1 + E^((2*I)*a))*x^4*Log[1 + E^((-2*I)*(a +
b*x))])/b + (5*(1 + E^((2*I)*a))*((-4*I)*b^3*x^3*PolyLog[2, -E^((-2*I)*(a +
 b*x))] - 6*b^2*x^2*PolyLog[3, -E^((-2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[4
, -E^((-2*I)*(a + b*x))] + 3*PolyLog[5, -E^((-2*I)*(a + b*x))]))/b^5)*Sec[a
])/(20*E^(I*a)) + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b) - (c^4*Sec[a]*(Cos[a]*
Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]
^2)) - (6*c^2*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b
*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (c^4*Csc[a]*(-(b*x*Cos[a]) + Log[
Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (2*
c^3*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcT
an[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 -
 E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Lo
g[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])
)]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)])
 - (6*c*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi -
2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*L
og[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[
a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Co
t[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a
]^2)]) - (2*Sec[a]*Sec[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] + 3*
c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2 - (2*c^3*d*Csc[a]*Sec[a]*(b^2*E
^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^(
(-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[
a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]
 + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^
2]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

**fricas [C]** time = 1.05, size = 3308, normalized size = 8.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4 - 24*d^4*cos(b*x + a)^2*polylog(5, cos(b*x + a) + I*sin(b*x + a)) - 2
4*d^4*cos(b*x + a)^2*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 24*d^4*cos
(b*x + a)^2*polylog(5, I*cos(b*x + a) + sin(b*x + a)) + 24*d^4*cos(b*x + a)
^2*polylog(5, I*cos(b*x + a) - sin(b*x + a)) + 24*d^4*cos(b*x + a)^2*polylo
g(5, -I*cos(b*x + a) + sin(b*x + a)) + 24*d^4*cos(b*x + a)^2*polylog(5, -I*
cos(b*x + a) - sin(b*x + a)) - 24*d^4*cos(b*x + a)^2*polylog(5, -cos(b*x +
a) + I*sin(b*x + a)) - 24*d^4*cos(b*x + a)^2*polylog(5, -cos(b*x + a) - I*s
in(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x
- 4*I*b^3*c^3*d)*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) + (4*I
*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*cos
```

```
(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I
*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d - 12*I*b*c*d^3 - 12*I*(b^3*c^2*d^2 + b*d^4)*
x)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (4*I*b^3*d^4*x^3 +
 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d + 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 + b*
d^4)*x)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (4*I*b^3*d^4*
x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d + 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2
 + b*d^4)*x)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-4*I*b
^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d - 12*I*b*c*d^3 - 12*I*(b^3*
c^2*d^2 + b*d^4)*x)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) +
(4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)
*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-4*I*b^3*d^4*x^3 -
 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*cos(b*x + a)^2*di
log(-cos(b*x + a) - I*sin(b*x + a)) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^
4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*cos(b*x + a)^2*log(cos(b*x + a) +
I*sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 -
4*(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4)*cos(b*x + a)^2*log(cos(b*x + a)
+ I*sin(b*x + a) + I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2
+ 4*b^4*c^3*d*x + b^4*c^4)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a)
 + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*(a^3 + 3*a)*
b*c*d^3 + (a^4 + 6*a^2)*d^4)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x +
a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^
2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^2 + b^2*d^4)*x
^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin
(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2
*c^2*d^2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^2 + b^2
*d^4)*x^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a
) - sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*
a^2*b^2*c^2*d^2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^
2 + b^2*d^4)*x^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*cos(b*x + a)^2*log(-I*cos
(b*x + a) + sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^
3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^
4*c^2*d^2 + b^2*d^4)*x^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*cos(b*x + a)^2*lo
g(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^
2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) +
 1/2*I*sin(b*x + a) + 1/2) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(
b*x + a) + 1/2) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^
4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*co
s(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c
^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4)
*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*d^4*x^4 + 4*
b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b
^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) - I*
sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*
(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) -
 I*sin(b*x + a) + I) + (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)^2*polylog
(4, cos(b*x + a) + I*sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x
 + a)^2*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*
c*d^3)*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + (-24*I*b*
d^4*x - 24*I*b*c*d^3)*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x +
a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x + a)^2*polylog(4, -I*cos(b*x +
 a) + sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)^2*polylog(
4, -I*cos(b*x + a) - sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x
 + a)^2*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b
*c*d^3)*cos(b*x + a)^2*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 12*(b^2
*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2*polylog(3, cos(b*x +
 a) + I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(
b*x + a)^2*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + b^2*c^2*d^2 + d^4)*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) +
```

```
 sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 + d^4)*cos(b
*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b
^2*c*d^3*x + b^2*c^2*d^2 + d^4)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) +
 sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 + d^4)*cos(b
*x + a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2*polylog(3, -cos(b*x + a) + I*sin(
b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2*p
olylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^
2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*cos(b*x + a)*sin(b*x + a))/(b^5*cos(b*x +
a)^2)
```

**giac [F]**   time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \csc(bx+a) \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a)^3, x)
```

**maple [B]**   time = 0.25, size = 1729, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x)
```

```
[Out] 3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-3*d^4*polylog(3,-exp(2*I*(b*x+a)))
/b^5+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)+12/b^3*c^2*d^2*polylog(3,-exp(I*(b*
x+a)))+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))-1/b^5*d^4*a^4*ln(1-exp(I*(b
*x+a)))+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2+12/b^3*d^4*polylog(3,-exp(
I*(b*x+a)))*x^2-6/b^3*d^2*c^2*ln(1+exp(2*I*(b*x+a)))-6/b^3*d^4*ln(1+exp(2*I
*(b*x+a)))*x^2+4*I/b^2*d^4*x^3-8*I/b^5*d^4*a^3-3/b^3*c^2*d^2*polylog(3,-exp
(2*I*(b*x+a)))-3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2+24*I/b^3*d^3*c*a*
x-24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^
5-12/b^3*d^3*c*ln(1+exp(2*I*(b*x+a)))*x+12*I/b^2*d^3*c*x^2+12*I/b^4*d^3*c*a
^2+6*I/b^4*d^4*polylog(2,-exp(2*I*(b*x+a)))*x-12*I/b^4*d^4*a^2*x+6*I/b^4*d^
3*c*polylog(2,-exp(2*I*(b*x+a)))+2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^
3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*
x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp
(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2*I*(b*x+a))-2*I*d^4*x^3-2*I*c^3*d*exp(2*I*
(b*x+a))-6*I*c*d^3*x^2-6*I*c^2*d^2*x-2*I*c^3*d)/b^2/(1+exp(2*I*(b*x+a)))^2-
4/b*c*d^3*ln(1+exp(2*I*(b*x+a)))*x^3+6*I/b^2*c*d^3*polylog(2,-exp(2*I*(b*x+
a)))*x^2+6*I/b^2*c^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-1/b*c^4*ln(1+exp(2*
I*(b*x+a)))+1/b*c^4*ln(exp(I*(b*x+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)+12/b^
5*d^4*a^2*ln(exp(I*(b*x+a)))+12/b^3*d^2*c^2*ln(exp(I*(b*x+a)))-1/b*d^4*ln(1
+exp(2*I*(b*x+a)))*x^4+4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+4/b*c^3*d*ln(1-exp(
I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a+6/b*c^2*d^2*ln(exp(I*(b*x+
a))+1)*x^2+24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x-6/b^3*c^2*d^2*a^2*ln(1
-exp(I*(b*x+a)))+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2+24/b^3*c*d^3*polylog(
3,exp(I*(b*x+a)))*x+24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))+24*I/b^4*c*d^
3*polylog(4,exp(I*(b*x+a)))-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3+24*I/
b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*
x^3+24*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x-4*I/b^2*c^3*d*polylog(2,-exp(
I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(
I*(b*x+a))-1)+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a))-1)-4/b^2*c^3*d*a*ln(exp(I
*(b*x+a))-1)+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b*d^4*ln(exp(I*(b*x+a))+1)*
x^4-12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(
2,-exp(I*(b*x+a)))*x-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I/b^2*
```

```
c*d^3*polylog(2,exp(I*(b*x+a)))*x^2+4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+4/b*
c*d^3*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3-24/b^4*
d^3*c*a*ln(exp(I*(b*x+a)))-4/b*c^3*d*ln(1+exp(2*I*(b*x+a)))*x-6/b^3*c*d^3*p
olylog(3,-exp(2*I*(b*x+a)))*x-6/b*c^2*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-3*I/b^
4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x+2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x+a)
))*x^3-3*I/b^4*c*d^3*polylog(4,-exp(2*I*(b*x+a)))+2*I/b^2*c^3*d*polylog(2,-
exp(2*I*(b*x+a)))
```

**maxima** [B] time = 6.05, size = 8770, normalized size = 21.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^4*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2)) - 4*a*c^3*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log
(sin(b*x + a)^2))/b + 6*a^2*c^2*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x +
 a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 4*a^3*c*d^3*(1/(sin(b*x + a)^2 - 1)
 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + a^4*d^4*(1/(sin(b*x
 + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^4 + 2*(24*b
^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 - 24*a^3*d^4 + (12*(b*x + a)^4
*d^4 + 36*b^2*c^2*d^2 - 72*a*b*c*d^3 + 36*a^2*d^4 + 32*(b*c*d^3 - a*d^4)*(b
*x + a)^3 + 36*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 24
*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x
 + a) + 4*(3*(b*x + a)^4*d^4 + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8
*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d
^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a
^3 + 3*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 8*(3*(b*x + a)^4*d^4 + 9*b^2*c
^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b
^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*cos(2*b*x +
2*a) + (12*I*(b*x + a)^4*d^4 + 36*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2
*d^4 + (32*I*b*c*d^3 - 32*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*d^2 - 72*I*a
*b*c*d^3 + (36*I*a^2 + 36*I)*d^4)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a*b^
2*c^2*d^2 + (72*I*a^2 + 72*I)*b*c*d^3 + (-24*I*a^3 - 72*I*a)*d^4)*(b*x + a)
)*sin(4*b*x + 4*a) + (24*I*(b*x + a)^4*d^4 + 72*I*b^2*c^2*d^2 - 144*I*a*b*c
*d^3 + 72*I*a^2*d^4 + (64*I*b*c*d^3 - 64*I*a*d^4)*(b*x + a)^3 + (72*I*b^2*c
^2*d^2 - 144*I*a*b*c*d^3 + (72*I*a^2 + 72*I)*d^4)*(b*x + a)^2 + (48*I*b^3*c
^3*d - 144*I*a*b^2*c^2*d^2 + (144*I*a^2 + 144*I)*b*c*d^3 + (-48*I*a^3 - 144
*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x
 + 2*a) + 1) - (6*(b*x + a)^4*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 36*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 24*(b^3*c^3*d - 3*a*b^2*
c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a) + 6*((b*x + a)^4*d^4 + 4*(b*c*
d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a
)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*
cos(4*b*x + 4*a) + 12*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 +
6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^
2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b
*x + a)^4*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a)^3 + (-36*I*b^2*c^2*d
^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^4)*(b*x + a)^2 + (-24*I*b^3*c^3*d + 72*I*a
*b^2*c^2*d^2 - 72*I*a^2*b*c*d^3 + 24*I*a^3*d^4)*(b*x + a))*sin(4*b*x + 4*a)
 - (-12*I*(b*x + a)^4*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a)^3 + (-72
*I*b^2*c^2*d^2 + 144*I*a*b*c*d^3 - 72*I*a^2*d^4)*(b*x + a)^2 + (-48*I*b^3*c
^3*d + 144*I*a*b^2*c^2*d^2 - 144*I*a^2*b*c*d^3 + 48*I*a^3*d^4)*(b*x + a))*s
in(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*(b*x + a)^4*d
^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 36*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2
*d^4)*(b*x + a)^2 + 24*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d
^4)*(b*x + a) + 6*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^
```

2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 12*((b*x + a
)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3
*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^4*d^4 + (24*I*b*c*d^3 -
24*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4
)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 2
4*I*a^3*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (12*I*(b*x + a)^4*d^4 + (48*I*b*
c*d^3 - 48*I*a*d^4)*(b*x + a)^3 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*
I*a^2*d^4)*(b*x + a)^2 + (48*I*b^3*c^3*d - 144*I*a*b^2*c^2*d^2 + 144*I*a^2*
b*c*d^3 - 48*I*a^3*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a),
-cos(b*x + a) + 1) - 24*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2
+ 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (12
*I*(b*x + a)^4*d^4 + 24*b^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 - 24*
a^3*d^4 + (48*I*b*c*d^3 - 24*(2*I*a + 1)*d^4)*(b*x + a)^3 + (72*I*b^2*c^2*d
^2 - 72*(2*I*a + 1)*b*c*d^3 + (72*I*a^2 + 72*a)*d^4)*(b*x + a)^2 + (48*I*b^
3*c^3*d - 72*(2*I*a + 1)*b^2*c^2*d^2 + (144*I*a^2 + 144*a)*b*c*d^3 + (-48*I
*a^3 - 72*a^2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (12*b^3*c^3*d - 36*a*b^2*
c^2*d^2 + 24*(b*x + a)^3*d^4 + 36*(a^2 + 1)*b*c*d^3 - 12*(a^3 + 3*a)*d^4 +
48*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 36*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1
)*d^4)*(b*x + a) + 12*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3*
(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 24
*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3*(a^2 + 1)*b*c*d^3 - (
a^3 + 3*a)*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c
*d^3 + (a^2 + 1)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-12*I*b^3*c^3*d + 36*I
*a*b^2*c^2*d^2 - 24*I*(b*x + a)^3*d^4 + (-36*I*a^2 - 36*I)*b*c*d^3 + (12*I*
a^3 + 36*I*a)*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a)^2 + (-36*I*b^2*c
^2*d^2 + 72*I*a*b*c*d^3 + (-36*I*a^2 - 36*I)*d^4)*(b*x + a))*sin(4*b*x + 4*
a) - (-24*I*b^3*c^3*d + 72*I*a*b^2*c^2*d^2 - 48*I*(b*x + a)^3*d^4 + (-72*I*
a^2 - 72*I)*b*c*d^3 + (24*I*a^3 + 72*I*a)*d^4 + (-96*I*b*c*d^3 + 96*I*a*d^4
)*(b*x + a)^2 + (-72*I*b^2*c^2*d^2 + 144*I*a*b*c*d^3 + (-72*I*a^2 - 72*I)*d
^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) + (24*b^3*c^3*
d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 + 24*(b*x + a)^3*d^4 - 24*a^3*d^4 + 7
2*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
(b*x + a) + 24*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d
^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d
^3 + a^2*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 48*(b^3*c^3*d - 3*a*b^2*c^2*d^2
 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a
)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) +
 (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 + 24*I*(b*x + a)^3
*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*I*a*d^4)*(b*x + a)^2 + (72*I*b^2*c
^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (48*
I*b^3*c^3*d - 144*I*a*b^2*c^2*d^2 + 144*I*a^2*b*c*d^3 + 48*I*(b*x + a)^3*d^
4 - 48*I*a^3*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a)^2 + (144*I*b^2*c
^2*d^2 - 288*I*a*b*c*d^3 + 144*I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilo
g(-e^(I*b*x + I*a)) + (24*b^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 + 2
4*(b*x + a)^3*d^4 - 24*a^3*d^4 + 72*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 72*(b^2
*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 24*(b^3*c^3*d - 3*a*b^2*c^2*d
^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x +
 a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(4*b*x + 4*a)
 + 48*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*
d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*
d^4)*(b*x + a))*cos(2*b*x + 2*a) + (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 7
2*I*a^2*b*c*d^3 + 24*I*(b*x + a)^3*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*
I*a*d^4)*(b*x + a)^2 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*
(b*x + a))*sin(4*b*x + 4*a) + (48*I*b^3*c^3*d - 144*I*a*b^2*c^2*d^2 + 144*I
*a^2*b*c*d^3 + 48*I*(b*x + a)^3*d^4 - 48*I*a^3*d^4 + (144*I*b*c*d^3 - 144*I
*a*d^4)*(b*x + a)^2 + (144*I*b^2*c^2*d^2 - 288*I*a*b*c*d^3 + 144*I*a^2*d^4)
*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-6*I*(b*x + a)^4*d^

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4 - 18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 - 18*I*a^2*d^4 + (-16*I*b*c*d^3 + 16*
I*a*d^4)*(b*x + a)^3 + (-18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 + (-18*I*a^2 - 1
8*I)*d^4)*(b*x + a)^2 + (-12*I*b^3*c^3*d + 36*I*a*b^2*c^2*d^2 + (-36*I*a^2
- 36*I)*b*c*d^3 + (12*I*a^3 + 36*I*a)*d^4)*(b*x + a) + (-6*I*(b*x + a)^4*d^
4 - 18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 - 18*I*a^2*d^4 + (-16*I*b*c*d^3 + 16*
I*a*d^4)*(b*x + a)^3 + (-18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 + (-18*I*a^2 - 1
8*I)*d^4)*(b*x + a)^2 + (-12*I*b^3*c^3*d + 36*I*a*b^2*c^2*d^2 + (-36*I*a^2
- 36*I)*b*c*d^3 + (12*I*a^3 + 36*I*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (-
12*I*(b*x + a)^4*d^4 - 36*I*b^2*c^2*d^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^4 + (
-32*I*b*c*d^3 + 32*I*a*d^4)*(b*x + a)^3 + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*d
^3 + (-36*I*a^2 - 36*I)*d^4)*(b*x + a)^2 + (-24*I*b^3*c^3*d + 72*I*a*b^2*c^
2*d^2 + (-72*I*a^2 - 72*I)*b*c*d^3 + (24*I*a^3 + 72*I*a)*d^4)*(b*x + a))*co
s(2*b*x + 2*a) + 2*(3*(b*x + a)^4*d^4 + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^
2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a
^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c
*d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) + 4*(3*(b*x + a)^4*d^4
+ 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^
3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3*
d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*sin
(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1) + (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^3
+ (18*I*b^2*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (12*I*b^
3*c^3*d - 36*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x + a) +
 (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^3 + (18*I*b^2
*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (12*I*b^3*c^3*d - 3
6*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x + a))*cos(4*b*x +
 4*a) + (6*I*(b*x + a)^4*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^3 + (3
6*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*b^3*c^
3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a))*cos(
2*b*x + 2*a) - 3*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2
*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 6*((b*x + a)^
4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d
^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*co
s(b*x + a) + 1) + (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x +
 a)^3 + (18*I*b^2*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (1
2*I*b^3*c^3*d - 36*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x
+ a) + (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^3 + (18
*I*b^2*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (12*I*b^3*c^3
*d - 36*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x + a))*cos(4
*b*x + 4*a) + (6*I*(b*x + a)^4*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^
3 + (36*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*
b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a)
)*cos(2*b*x + 2*a) - 3*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 +
 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b
^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 6*((b*x
 + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^
3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
- 2*cos(b*x + a) + 1) + (18*I*d^4*cos(4*b*x + 4*a) + 36*I*d^4*cos(2*b*x + 2
*a) - 18*d^4*sin(4*b*x + 4*a) - 36*d^4*sin(2*b*x + 2*a) + 18*I*d^4)*polylog
(5, -e^(2*I*b*x + 2*I*a)) + (-144*I*d^4*cos(4*b*x + 4*a) - 288*I*d^4*cos(2*
b*x + 2*a) + 144*d^4*sin(4*b*x + 4*a) + 288*d^4*sin(2*b*x + 2*a) - 144*I*d^
4)*polylog(5, -e^(I*b*x + I*a)) + (-144*I*d^4*cos(4*b*x + 4*a) - 288*I*d^4*
cos(2*b*x + 2*a) + 144*d^4*sin(4*b*x + 4*a) + 288*d^4*sin(2*b*x + 2*a) - 14
4*I*d^4)*polylog(5, e^(I*b*x + I*a)) + (24*b*c*d^3 + 36*(b*x + a)*d^4 - 24*
a*d^4 + 12*(2*b*c*d^3 + 3*(b*x + a)*d^4 - 2*a*d^4)*cos(4*b*x + 4*a) + 24*(2
*b*c*d^3 + 3*(b*x + a)*d^4 - 2*a*d^4)*cos(2*b*x + 2*a) + (24*I*b*c*d^3 + 36
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*I*(b*x + a)*d^4 - 24*I*a*d^4)*sin(4*b*x + 4*a) + (48*I*b*c*d^3 + 72*I*(b*x
 + a)*d^4 - 48*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, -e^(2*I*b*x + 2*I*a))
- (144*b*c*d^3 + 144*(b*x + a)*d^4 - 144*a*d^4 + 144*(b*c*d^3 + (b*x + a)*d
^4 - a*d^4)*cos(4*b*x + 4*a) + 288*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*
b*x + 2*a) - (-144*I*b*c*d^3 - 144*I*(b*x + a)*d^4 + 144*I*a*d^4)*sin(4*b*x
 + 4*a) - (-288*I*b*c*d^3 - 288*I*(b*x + a)*d^4 + 288*I*a*d^4)*sin(2*b*x +
2*a))*polylog(4, -e^(I*b*x + I*a)) - (144*b*c*d^3 + 144*(b*x + a)*d^4 - 144
*a*d^4 + 144*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) + 288*(b*c*
d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) - (-144*I*b*c*d^3 - 144*I*(b*
x + a)*d^4 + 144*I*a*d^4)*sin(4*b*x + 4*a) - (-288*I*b*c*d^3 - 288*I*(b*x +
 a)*d^4 + 288*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, e^(I*b*x + I*a)) + (-18
*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 - 36*I*(b*x + a)^2*d^4 + (-18*I*a^2 - 18*I)
*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a) + (-18*I*b^2*c^2*d^2 + 36*I*a
*b*c*d^3 - 36*I*(b*x + a)^2*d^4 + (-18*I*a^2 - 18*I)*d^4 + (-48*I*b*c*d^3 +
 48*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*
d^3 - 72*I*(b*x + a)^2*d^4 + (-36*I*a^2 - 36*I)*d^4 + (-96*I*b*c*d^3 + 96*I
*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 6*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(b
*x + a)^2*d^4 + 3*(a^2 + 1)*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x
+ 4*a) + 12*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(b*x + a)^2*d^4 + 3*(a^2 + 1)*
d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*polylog(3, -e^(2*I*b
*x + 2*I*a)) + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*(b*x + a)^2*d^4 +
 72*I*a^2*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a) + (72*I*b^2*c^2*d^2
 - 144*I*a*b*c*d^3 + 72*I*(b*x + a)^2*d^4 + 72*I*a^2*d^4 + (144*I*b*c*d^3 -
 144*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (144*I*b^2*c^2*d^2 - 288*I*a*b*
c*d^3 + 144*I*(b*x + a)^2*d^4 + 144*I*a^2*d^4 + (288*I*b*c*d^3 - 288*I*a*d^
4)*(b*x + a))*cos(2*b*x + 2*a) - 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^
2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 144*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(
b*x + a))*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) + (72*I*b^2*c^2*d^
2 - 144*I*a*b*c*d^3 + 72*I*(b*x + a)^2*d^4 + 72*I*a^2*d^4 + (144*I*b*c*d^3
- 144*I*a*d^4)*(b*x + a) + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*(b*x
+ a)^2*d^4 + 72*I*a^2*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a))*cos(4*
b*x + 4*a) + (144*I*b^2*c^2*d^2 - 288*I*a*b*c*d^3 + 144*I*(b*x + a)^2*d^4 +
 144*I*a^2*d^4 + (288*I*b*c*d^3 - 288*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a)
- 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 -
a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 144*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x
+ a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*pol
ylog(3, e^(I*b*x + I*a)) + (-24*I*(b*x + a)^3*d^4 + (-72*I*b*c*d^3 + 72*I*a
*d^4)*(b*x + a)^2 + (-72*I*b^2*c^2*d^2 + 144*I*a*b*c*d^3 - 72*I*a^2*d^4)*(b
*x + a))*sin(4*b*x + 4*a) - (12*(b*x + a)^4*d^4 - 24*I*b^3*c^3*d + 72*I*a*b
^2*c^2*d^2 - 72*I*a^2*b*c*d^3 + 24*I*a^3*d^4 + (48*b*c*d^3 - (48*a - 24*I)*
d^4)*(b*x + a)^3 + (72*b^2*c^2*d^2 - (144*a - 72*I)*b*c*d^3 + 72*(a^2 - I*a
)*d^4)*(b*x + a)^2 + (48*b^3*c^3*d - (144*a - 72*I)*b^2*c^2*d^2 + 144*(a^2
- I*a)*b*c*d^3 - 24*(2*a^3 - 3*I*a^2)*d^4)*(b*x + a))*sin(2*b*x + 2*a))/(-6
*I*b^4*cos(4*b*x + 4*a) - 12*I*b^4*cos(2*b*x + 2*a) + 6*b^4*sin(4*b*x + 4*a
) + 12*b^4*sin(2*b*x + 2*a) - 6*I*b^4))/b
```

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(cos(a + b*x)^3*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.311 $\int (c + dx)^3 \csc(a + bx) \sec^3(a + bx) \, dx$

**Optimal**. Leaf size=325

$$\frac{3id^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4} - \frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4} + \frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4} - \frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3} + \frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx\right.}}{2b^3}$$

```
[Out] 3/2*I*d*(d*x+c)^2/b^2+1/2*(d*x+c)^3/b-2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a)))
/b-3*d^2*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^3+3/2*I*d^3*polylog(2,-exp(2*I*(b
*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*I*d*(d*x
+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b
*x+a)))/b^3+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polyl
og(4,-exp(2*I*(b*x+a)))/b^4+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4-3/2*d
*(d*x+c)^2*tan(b*x+a)/b^2+1/2*(d*x+c)^3*tan(b*x+a)^2/b
```

**Rubi** [A] time = 0.64, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {2620, 14, 4420, 6741, 12, 6742, 2551, 4183, 2531, 6609, 2282, 6589, 3720, 3719, 2190, 2279, 2391, 32}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} + \frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3} + \frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} - 3id$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 + (c + d*x)^3/(2*b) - (2*(c + d*x)^3*ArcTanh[
E^((2*I)*(a + b*x))])/b - (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^
3 + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/2)*d*(c
+ d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (((3*I)/2)*d*(c + d*x)^2*P
olyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)
*(a + b*x))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(
2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/4)*
d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Tan[a + b*x])/(
2*b^2) + ((c + d*x)^3*Tan[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc(a+bx)\sec^3(a+bx)\,dx = \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - (3d)\int (c+d$$

$$= \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - (3d)\int \frac{(c+d}{}$$

$$= \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - \frac{(3d)\int (c+dx}{}$$

$$= \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - \frac{(3d)\int \left(2(c+\right.}{}$$

$$= \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - \frac{(3d)\int (c+dx}{}$$

$$= -\frac{3d(c+dx)^2\tan(a+bx)}{2b^2} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^3}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{3d(c+dx)^2\tan(a+bx)}{2b^2} + \frac{(c+dx)^3\tan^2}{2b}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c+d}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c+d}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c+d}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c+d}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c+d}{}$$

**Mathematica [B]** time = 6.91, size = 1486, normalized size = 4.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] -1/2*(c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2
*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*
Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I
)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 +
E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a
+ b*x))]))/E^((2*I)*a)))/b^3 - (d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) +
 (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(
1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(
b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a
+ b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2
*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^((
-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(4*b^4)
+ (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Csc[a]*Sec[a])/4 - ((I/4)*
c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*
x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(
1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) -
 (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))*x^3*Log
[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*PolyLog[2,
```

```
-E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))] - Poly
Log[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] + ((c + d*x)^3*Se
c[a + b*x]^2)/(2*b) - (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[
b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c*d^2*Sec[a]*(Cos[a]*Lo
g[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]
^2)) + (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*
Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan
[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*
b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))]
+ Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*Po
lyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2
*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^3*Csc[a]*((b^2*x^2)/E^(I*
ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((
-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a
]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]]
+ I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[
a])/(2*b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*Sec[a]*Sec[a + b*x]*(
c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) - (3*c^2*d
*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[
a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*
I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b
*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan
[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

**fricas [C]** time = 0.83, size = 2260, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 6*I*d^3*cos(
b*x + a)^2*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x + a)
^2*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*cos(b*x + a)^2*polyl
og(4, I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, I*
cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x
 + a) + sin(b*x + a)) + 6*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) -
 sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, -cos(b*x + a) + I*sin(b*
x + a)) + 6*I*d^3*cos(b*x + a)^2*polylog(4, -cos(b*x + a) - I*sin(b*x + a))
 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*cos(b*x + a)^2*dilo
g(cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I
*b^2*c^2*d)*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2
*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2*dilog(
I*cos(b*x + a) + sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b
^2*c^2*d + 3*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (
3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^2
*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*
x - 3*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x
 + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*cos(b*x + a)^2
*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*
x - 3*I*b^2*c^2*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2*log
(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1
)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x +
a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x
 + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d +
 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I
*sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2*log(I*
cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2
```

```
*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x
 + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d
^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(
b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1
) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2*log(
-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b
*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c
^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*
x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*
d^3)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3
)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b
^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2*log(-cos(b
*x + a) - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylo
g(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*
polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x +
 a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos
(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^
2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x +
 b*c*d^2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 6*(b*
d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, -cos(b*x + a) + I*sin(b*x + a))
+ 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, -cos(b*x + a) - I*sin(b*x
 + a)) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x +
 a))/(b^4*cos(b*x + a)^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \csc(bx+a) \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a)^3, x)
```

**maple [B]** time = 0.16, size = 1115, normalized size = 3.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x)
```

```
[Out] -6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,-exp(I*(
b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+3/2*I*d^3*polylog(2,-exp(2
*I*(b*x+a)))/b^4-3/b^3*d^2*c*ln(1+exp(2*I*(b*x+a)))-3/b^3*d^3*ln(1+exp(2*I*
(b*x+a)))*x-1/b*c^3*ln(1+exp(2*I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-
1)+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,exp(I*(b*x+
a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,-exp(I*(b*x+
a)))*x+6*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-3/2/b^3*c*d^2*polylog(3,-exp(
2*I*(b*x+a)))-3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-3/4*I*d^3*polylog(
4,-exp(2*I*(b*x+a)))/b^4+(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*
(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^
2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*c^3*exp(2
*I*(b*x+a))-6*I*c*d^2*x-3*I*c^2*d)/b^2/(1+exp(2*I*(b*x+a)))^2+1/b*c^3*ln(ex
p(I*(b*x+a))-1)+1/b*c^3*ln(exp(I*(b*x+a))+1)+3*I/b^2*c*d^2*polylog(2,-exp(2
*I*(b*x+a)))*x+3*I/b^2*d^3*x^2+3*I/b^4*d^3*a^2+6/b^3*d^2*c*ln(exp(I*(b*x+a)
))-6/b^4*d^3*a*ln(exp(I*(b*x+a)))+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-3*I/
```

```
b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a))
)-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b
*x+a)))*x^2+3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))
*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+
3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/b^2
*c^2*d*a*ln(exp(I*(b*x+a))-1)+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln
(1-exp(I*(b*x+a)))*a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-1/b*d^3*ln(1+exp(2*
I*(b*x+a)))*x^3+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+6*I/b^3*d^3*
a*x+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-3/b*c^2*d*ln(1+exp(2*I*(b*
x+a)))*x-3/b*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2
```

**maxima** [B] time = 2.02, size = 4918, normalized size = 15.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2)) - 3*a*c^2*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log
(sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a
)^2 - 1) - log(sin(b*x + a)^2))/b^2 - a^3*d^3*(1/(sin(b*x + a)^2 - 1) + log
(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + 2*(18*b^2*c^2*d - 36*a*b*
c*d^2 + 18*a^2*d^3 + (8*(b*x + a)^3*d^3 + 18*b*c*d^2 - 18*a*d^3 + 18*(b*c*d
^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x
 + a) + 2*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b
*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(4*b*
x + 4*a) + 4*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(2
*b*x + 2*a) + (8*I*(b*x + a)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (18*I*b*c*
d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (18*I*a^
2 + 18*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (16*I*(b*x + a)^3*d^3 + 36*I*b
*c*d^2 - 36*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c
^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 36*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))
*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (6*(b*x + a)^3*d^3 + 18*
(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x
 + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d
 - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*((b*x + a)^3*d^3
 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*
(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^3*d^3 + (-18*I*b*c*d^2 + 18*I
*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3)*(b*
x + a))*sin(4*b*x + 4*a) - (-12*I*(b*x + a)^3*d^3 + (-36*I*b*c*d^2 + 36*I*a
*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 - 36*I*a^2*d^3)*(b*x
+ a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*(b*x +
 a)^3*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2
+ a^2*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2
 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*(
(b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^3 + (18*I*b
*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*
a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (12*I*(b*x + a)^3*d^3 + (36*I*b*c*d^
2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*a^2*d
^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) -
 18*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (1
2*I*(b*x + a)^3*d^3 + 18*b^2*c^2*d - 36*a*b*c*d^2 + 18*a^2*d^3 + (36*I*b*c*
d^2 - 18*(2*I*a + 1)*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 36*(2*I*a + 1)*b*
c*d^2 + (36*I*a^2 + 36*a)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (9*b^2*c^2*d -
 18*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 9*(a^2 + 1)*d^3 + 18*(b*c*d^2 - a*d^3)
*(b*x + a) + 3*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 + 1)
*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*(3*b^2*c^2*d - 6
```

```
*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 + 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x
 + a))*cos(2*b*x + 2*a) - (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 - 12*I*(b*x + a)
^2*d^3 + (-9*I*a^2 - 9*I)*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a))*sin
(4*b*x + 4*a) - (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 24*I*(b*x + a)^2*d^3 +
(-18*I*a^2 - 18*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(2*b*x
+ 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) + (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*
x + a)^2*d^3 + 18*a^2*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(4*b*x + 4*a) + 36*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3
+ 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (18*I*b^2*c^2*d - 36*I*
a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^
3)*(b*x + a))*sin(4*b*x + 4*a) + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*(b
*x + a)^2*d^3 + 36*I*a^2*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b
*x + a)^2*d^3 + 18*a^2*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d
 - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*cos(4*b*x + 4*a) + 36*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (18*I*b^2*c^2*d - 36*I
*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d
^3)*(b*x + a))*sin(4*b*x + 4*a) + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*(
b*x + a)^2*d^3 + 36*I*a^2*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*sin(
2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-4*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2
+ 9*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18
*I*a*b*c*d^2 + (-9*I*a^2 - 9*I)*d^3)*(b*x + a) + (-4*I*(b*x + a)^3*d^3 - 9*
I*b*c*d^2 + 9*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*
c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 9*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a)
+ (-8*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*I
*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 18*I
)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d
^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 +
1)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*
a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2
 + 1)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (3*I*(b*x + a)^3*d^3 + (9*I*b*c*d^2 -
9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + 9*I*a^2*d^3)*(b*
x + a) + (3*I*(b*x + a)^3*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*
I*b^2*c^2*d - 18*I*a*b*c*d^2 + 9*I*a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (
6*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c
^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x
 + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2
+ a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 -
a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (3
*I*(b*x + a)^3*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d
 - 18*I*a*b*c*d^2 + 9*I*a^2*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + (9*I*b*
c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + 9*I*a^2*
d^3)*(b*x + a))*cos(4*b*x + 4*a) + (6*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 1
8*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(
b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
+ a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a)
- 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a
*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b
*x + a)^2 - 2*cos(b*x + a) + 1) + (6*d^3*cos(4*b*x + 4*a) + 12*d^3*cos(2*b*
x + 2*a) + 6*I*d^3*sin(4*b*x + 4*a) + 12*I*d^3*sin(2*b*x + 2*a) + 6*d^3)*po
lylog(4, -e^(2*I*b*x + 2*I*a)) - (36*d^3*cos(4*b*x + 4*a) + 72*d^3*cos(2*b*
x + 2*a) + 36*I*d^3*sin(4*b*x + 4*a) + 72*I*d^3*sin(2*b*x + 2*a) + 36*d^3)*
polylog(4, -e^(I*b*x + I*a)) - (36*d^3*cos(4*b*x + 4*a) + 72*d^3*cos(2*b*x
+ 2*a) + 36*I*d^3*sin(4*b*x + 4*a) + 72*I*d^3*sin(2*b*x + 2*a) + 36*d^3)*po
lylog(4, e^(I*b*x + I*a)) + (-9*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 9*I*a*d^3
```

```
+ (-9*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 9*I*a*d^3)*cos(4*b*x + 4*a) + (-18*I
*b*c*d^2 - 24*I*(b*x + a)*d^3 + 18*I*a*d^3)*cos(2*b*x + 2*a) + 3*(3*b*c*d^2
 + 4*(b*x + a)*d^3 - 3*a*d^3)*sin(4*b*x + 4*a) + 6*(3*b*c*d^2 + 4*(b*x + a)
*d^3 - 3*a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(2*I*b*x + 2*I*a)) + (36*I*
b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I*(b*x + a)*
d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + (72*I*b*c*d^2 + 72*I*(b*x + a)*d^3 - 7
2*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*
x + 4*a) - 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3
, -e^(I*b*x + I*a)) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (36
*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + (72*I*b*c*
d^2 + 72*I*(b*x + a)*d^3 - 72*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*
x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)
*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) + (-18*I*(b*x + a)^2*d^3 + (
-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (12*(b*x + a)^3*d
^3 - 18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3 + (36*b*c*d^2 - (36*a -
 18*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a - 36*I)*b*c*d^2 + 36*(a^2 -
 I*a)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(-6*I*b^3*cos(4*b*x + 4*a) - 12*I*b
^3*cos(2*b*x + 2*a) + 6*b^3*sin(4*b*x + 4*a) + 12*b^3*sin(2*b*x + 2*a) - 6*
I*b^3))/b
```

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.312 $\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx$

**Optimal**. Leaf size=201

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}-\frac{d^2\log(\cos(a+bx))}{b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}$$

```
[Out] c*d*x/b+1/2*d^2*x^2/b-2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d^2*ln(cos(b*
x+a))/b^3+I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*(d*x+c)*polylog(
2,exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/2*d^2*po
lylog(3,exp(2*I*(b*x+a)))/b^3-d*(d*x+c)*tan(b*x+a)/b^2+1/2*(d*x+c)^2*tan(b*
x+a)^2/b
```

**Rubi [A]** time = 0.41, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {2620, 14, 4420, 6741, 12, 6742, 2551, 4183, 2531, 2282, 6589, 3720, 3475}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{PolyLog}\left(3,\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (c*d*x)/b + (d^2*x^2)/(2*b) - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/
b - (d^2*Log[Cos[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a +
b*x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*Po
lyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E^((2*I)*(a + b*x
))])/(2*b^3) - (d*(c + d*x)*Tan[a + b*x])/b^2 + ((c + d*x)^2*Tan[a + b*x]^2
)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx = \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - (2d)\int (c+d$$
$$= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - (2d)\int \frac{(c+d}{}$$
$$= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{d\int (c+dx)\big(2}{}$$
$$= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{d\int \big(2(c+dx}{}$$
$$= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{d\int (c+dx)\,\mathrm{ta}}{}$$
$$= -\frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^2\csc}{b}$$
$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{d^2\log(\cos(a+bx))}{b^3} - \frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{(c+d}{}$$
$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} -$$
$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} +$$
$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} +$$
$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} +$$

**Mathematica [B]** time = 6.76, size = 875, normalized size = 4.35

$$-\frac{\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))c^2}{b\left(\cos^2(a)+\sin^2(a)\right)} + \frac{\csc(a)(\log(\cos(bx)\sin(a)+\cos(a)\sin(bx)}{b\left(\cos^2(a)+\sin^2(a)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] -1/6*(d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I
)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Lo
g[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*
(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^
((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a +
b*x))]))/E^((2*I)*a)))/b^3 + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Csc[a]*Sec[a])/
3 - ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*
I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
 - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*
E^(I*a)) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) - (c^2*Sec[a]*(Cos[a]*Log[Cos
[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) -
(d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(
b^3*(Cos[a]^2 + Sin[a]^2)) + (c^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[
a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*((b^
2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*
Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x -
ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTa
```

```
n[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]]))/Sqrt[1 + Cot
[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a
+ b*x]*(-(c*d*Sin[b*x]) - d^2*x*Sin[b*x]))/b^2 - (c*d*Csc[a]*Sec[a]*(b^2*E^
(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((
-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a
]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]
+ I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2
]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

**fricas [C]** time = 0.65, size = 1396, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*d^2*cos(b*x + a)^2*polylog(3, cos(b*x +
a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*polylog(3, cos(b*x + a) - I*sin
(b*x + a)) - 2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a))
 - 2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*d^2*c
os(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x +
a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*poly
log(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*polylog(3, -c
os(b*x + a) - I*sin(b*x + a)) + b^2*c^2 + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*
x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*c
os(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*
c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2
*I*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (2*I*b*d^2*
x + 2*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-2*I
*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))
+ (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*x
+ a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*s
in(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a)^2*log(cos
(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos
(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^
2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(
b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
 a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x +
 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1)
 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*
cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d -
 a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*
x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2*log(-1/2*cos
(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*
c*d - a^2*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^
2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin
(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x
+ a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^
2 + 1)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*(b*d
^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \csc(bx + a) \sec(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

[Out] integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a)^3, x)

**maple [B]** time = 0.13, size = 614, normalized size = 3.05

$$\frac{2b\,d^2x^2e^{2i(bx+a)} - 2id^2x\,e^{2i(bx+a)} + 4bcdx\,e^{2i(bx+a)} - 2icd\,e^{2i(bx+a)} + 2b\,c^2e^{2i(bx+a)} - 2id^2x - 2icd}{b^2\left(1+e^{2i(bx+a)}\right)^2} + \frac{d^2a^2\ln\left(e^{i(bx+}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x)

[Out] -1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1
)+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d
^2*ln(exp(I*(b*x+a))+1)*x^2+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*poly
log(3,exp(I*(b*x+a)))/b^3-1/b*c^2*ln(1+exp(2*I*(b*x+a)))+1/b*c^2*ln(exp(I*(
b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1)-1/b*d^2*ln(1+exp(2*I*(b*x+a)))*x^2+
2*(b*d^2*x^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b
*x+a))-I*c*d*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x-I*c*d)/b^2/(1+
exp(2*I*(b*x+a)))^2+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+I/b^2*c*d*poly
log(2,-exp(2*I*(b*x+a)))-2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x+2/b*c*d*ln(1-exp(
I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+2/b*c*d*ln(exp(I*(b*x+a))+1)
*x-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)-2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*
x-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,-exp(I*(b*x
+a)))-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-1/b^3*d^2*ln(1+exp(2*I*(b*x+a))
)+2/b^3*d^2*ln(exp(I*(b*x+a)))

**maxima [B]** time = 0.81, size = 2442, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")

[Out] -1/2*(c^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2)) - 2*a*c*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(s
in(b*x + a)^2))/b + a^2*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 -
1) - log(sin(b*x + a)^2))/b^2 - 2*(4*(b*x + a)*d^2*cos(4*b*x + 4*a) + 4*I*(
b*x + a)*d^2*sin(4*b*x + 4*a) - 4*b*c*d + 4*a*d^2 - (2*(b*x + a)^2*d^2 + 4*
(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*
(b*x + a) + d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*
(b*x + a) + d^2)*cos(2*b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I
*a*d^2)*(b*x + a) + 2*I*d^2)*sin(4*b*x + 4*a) + (4*I*(b*x + a)^2*d^2 + (8*I
*b*c*d - 8*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*b*
x + 2*a), cos(2*b*x + 2*a) + 1) + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b
*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a
) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-
2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a)
- (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*sin(2*b*x + 2
*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*(b*x + a)^2*d^2 + 4*(b*c*
d - a*d^2)*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*co
s(4*b*x + 4*a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*
x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(4*
b*x + 4*a) + (4*I*(b*x + a)^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a))*sin(
2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (4*I*(b*x + a)^2*d
^2 + 4*b*c*d - 4*a*d^2 + (8*I*b*c*d - 4*(2*I*a + 1)*d^2)*(b*x + a))*cos(2*b
*x + 2*a) + (2*b*c*d + 2*(b*x + a)*d^2 - 2*a*d^2 + 2*(b*c*d + (b*x + a)*d^2
 - a*d^2)*cos(4*b*x + 4*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x +
2*a) - (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*sin(4*b*x + 4*a) - (-4*
I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*
x + 2*I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d

```
^2 - a*d^2)*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x
+ 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (8*
I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x
+ I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 -
 a*d^2)*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*
a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (8*I*b*
c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a
)) - (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) - I*d^2 + (-I
*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) - I*d^2)*cos(4*b*x +
4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a) - 2*I*d^2
)*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*
sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*
sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
 + 2*a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) + (I*
(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a))*cos(4*b*x + 4*a) + (2*
I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (
(b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x +
 a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d
 - 2*I*a*d^2)*(b*x + a) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x
 + a))*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b
*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))
*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-
I*d^2*cos(4*b*x + 4*a) - 2*I*d^2*cos(2*b*x + 2*a) + d^2*sin(4*b*x + 4*a) +
2*d^2*sin(2*b*x + 2*a) - I*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) - (4*I*d^2
*cos(4*b*x + 4*a) + 8*I*d^2*cos(2*b*x + 2*a) - 4*d^2*sin(4*b*x + 4*a) - 8*d
^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, -e^(I*b*x + I*a)) - (4*I*d^2*cos(
4*b*x + 4*a) + 8*I*d^2*cos(2*b*x + 2*a) - 4*d^2*sin(4*b*x + 4*a) - 8*d^2*si
n(2*b*x + 2*a) + 4*I*d^2)*polylog(3, e^(I*b*x + I*a)) + (4*(b*x + a)^2*d^2
- 4*I*b*c*d + 4*I*a*d^2 + (8*b*c*d - (8*a - 4*I)*d^2)*(b*x + a))*sin(2*b*x
+ 2*a))/(-2*I*b^2*cos(4*b*x + 4*a) - 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4
*b*x + 4*a) + 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2))/b
```

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

**sympy** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \csc(a+bx) \sec^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x)**3, x)
```

# 3.313 $\int (c + dx) \csc(a + bx) \sec^3(a + bx)\, dx$

**Optimal**. Leaf size=139

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} - \frac{d\tan(a+bx)}{2b^2} + \frac{c\tan^2(a+bx)}{2b} + \frac{c\log(\tan(a+bx))}{b} + \frac{dx\tan^2(a+bx)}{2b} - \frac{2dx}{}$$

[Out] 1/2*d*x/b-2*d*x*arctanh(exp(2*I*a+2*I*b*x))/b+c*ln(tan(b*x+a))/b+1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*d*tan(b*x+a)/b^2+1/2*c*tan(b*x+a)^2/b+1/2*d*x*tan(b*x+a)^2/b

**Rubi [A]** time = 0.14, antiderivative size = 141, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2620, 14, 4420, 2548, 12, 4183, 2279, 2391, 3473, 8}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} - \frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c+dx)\tan^2(a+bx)}{2b} + \frac{(c+dx)\log(\tan(a}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^3,x]

[Out] (d*x)/(2*b) - (2*d*x*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*x*Log[Tan[a + b*x]])/b + ((c + d*x)*Log[Tan[a + b*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d*Tan[a + b*x])/(2*b^2) + ((c + d*x)*Tan[a + b*x]^2)/(2*b)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx &= \frac{(c+dx)\log(\tan(a+bx))}{b}+\frac{(c+dx)\tan^2(a+bx)}{2b}-d\int\Bigg(\frac{\log(\tan(a+b\\
&= \frac{(c+dx)\log(\tan(a+bx))}{b}+\frac{(c+dx)\tan^2(a+bx)}{2b}-\frac{d\int\tan^2(a+bx)\,dx}{2b}\\
&= -\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log(\tan(a+bx))}{b}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c\\
&= \frac{dx}{2b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log(\tan(a+bx))}{b}-\frac{d\tan(a+bx)}{2b^2}+\\
&= \frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log(\tan(a}{b}\\
&= \frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log(\tan(a}{b}\\
&= \frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log(\tan(a}{b}
\end{aligned}$$

**Mathematica [A]** time = 0.56, size = 212, normalized size = 1.53

$$\frac{d\left(\frac{1}{2}i\text{Li}_2\left(-e^{2i(a+bx)}\right)+\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)\right)}{b^2}+\frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2+\text{L}\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (a*d*Log[Cos[a + b*x]])/b^2 - (a*d*Log[Sin[a + b*x]])/b^2 + (d*((I/2)*(a +
b*x)^2 - (a + b*x)*Log[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I
)*(a + b*x))]))/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((
```

```
a + b*x)^2 + PolyLog[2, E^((2*I)*(a + b*x))]))))/b^2 + (d*x*Sec[a + b*x]^2)/
(2*b) - (c*(2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]] - Sec[a + b*x]^2))/(2
*b) - (d*Tan[a + b*x])/(2*b^2)
```

**fricas [B]** time = 0.56, size = 760, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*cos(b*x
 + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(I*c
os(b*x + a) + sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin
(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d
*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) + I*d*cos(b*x + a)^2*
dilog(-cos(b*x + a) + I*sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-cos(b*x +
 a) - I*sin(b*x + a)) + (b*d*x + b*c)*cos(b*x + a)^2*log(cos(b*x + a) + I*s
in(b*x + a) + 1) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x
+ a) + I) + (b*d*x + b*c)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a)
+ 1) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) -
(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*
x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a
*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*
cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*cos(b*
x + a)^2*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*co
s(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x + a
*d)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b*c - a*d)*co
s(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*d*x + a*d)*cos(b*
x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a*d)*cos(b*x + a)
^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) + b*d*x - d*cos(b*x + a)*sin(b*x
 + a) + b*c)/(b^2*cos(b*x + a)^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\csc(bx + a)\sec(bx + a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a)^3, x)
```

**maple [B]** time = 0.11, size = 270, normalized size = 1.94

$$\frac{2bdx\, e^{2i(bx+a)} + 2bc\, e^{2i(bx+a)} - id\, e^{2i(bx+a)} - id}{b^2\left(1 + e^{2i(bx+a)}\right)^2} + \frac{c\ln\left(e^{i(bx+a)} - 1\right)}{b} - \frac{c\ln\left(1 + e^{2i(bx+a)}\right)}{b} + \frac{c\ln\left(e^{i(bx+a)} + 1\right)}{b} - d\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x)
```

```
[Out] (2*b*d*x*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))-I*d)/
b^2/(1+exp(2*I*(b*x+a)))^2+1/b*c*ln(exp(I*(b*x+a))-1)-1/b*c*ln(1+exp(2*I*(b
*x+a)))+1/b*c*ln(exp(I*(b*x+a))+1)-1/b*d*ln(1+exp(2*I*(b*x+a)))*x+1/2*I*d*p
olylog(2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,
-exp(I*(b*x+a)))/b^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a
)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

**maxima [B]** time = 0.63, size = 1035, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) + 4*(b*d*x + b*c)*cos
(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*a) + (4*I*b*d*x + 4*I*b
*c)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (2*
b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) + 4*(b*d*x + b*c)*cos(2*b*
x + 2*a) - (-2*I*b*d*x - 2*I*b*c)*sin(4*b*x + 4*a) - (-4*I*b*d*x - 4*I*b*c)
*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos(4*b
*x + 4*a) + 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin(4*b*x + 4*a) + 4*I*b*c*sin
(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*c
os(4*b*x + 4*a) + 4*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*sin(4*b*x + 4*a) + 4
*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) +
1) + (4*I*b*d*x + 4*I*b*c + 2*d)*cos(2*b*x + 2*a) - (d*cos(4*b*x + 4*a) + 2
*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x + 2*a) + d)*di
log(-e^(2*I*b*x + 2*I*a)) + (2*d*cos(4*b*x + 4*a) + 4*d*cos(2*b*x + 2*a) +
2*I*d*sin(4*b*x + 4*a) + 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*
a)) + (2*d*cos(4*b*x + 4*a) + 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a)
 + 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c
 + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*cos(2*b*x +
 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*
log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*
b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*
cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b
*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*c
os(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*
x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (4*b*
d*x + 4*b*c - 2*I*d)*sin(2*b*x + 2*a) + 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) - 4
*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) + 4*b^2*sin(2*b*x + 2*a) -
 2*I*b^2)
```

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

**sympy** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \csc (a + bx) \sec^3 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)*sec(a + b*x)**3, x)
```

## 3.314 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\frac{\csc(a+bx)\sec^3(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

**Rubi [A]** time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x),x]
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

**Mathematica [A]** time = 9.08, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]
```

**fricas [A]** time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)\sec(bx+a)^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)*sec(b*x + a)^3/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\sec(bx+a)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)*sec(b*x + a)^3/(d*x + c), x)
```

**maple [A]** time = 3.35, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\left(\sec^3(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
(2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b
*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*
b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(2*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2 + d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*si
n(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*s
in(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/((d*x
+ c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x
+ a) + c), x) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x +
4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x
 + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d
*x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + (d*cos(2
*b*x + 2*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + d*si
n(2*b*x + 2*a))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x +
4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x
 + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a))
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)^3\,\sin(a+b\,x)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**3/(d*x+c),x)
[Out] Integral(csc(a + b*x)*sec(a + b*x)**3/(c + d*x), x)
```

## 3.315 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=25

$$\text{Int}\left(\frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2,x]
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

**Mathematica [A]** time = 6.65, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

**fricas [A]** time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)\sec(bx+a)^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
[Out] Timed out
```

**maple [A]** time = 5.62, size = 0, normalized size = 0.00

$$\int \frac{\csc (bx+a) \left(\sec^{3}(bx+a)\right)}{(dx+c)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*inte
grate(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 3*d^2)*sin(2*b*x + 2*a)/(b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 +
(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^
4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2
+ 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^
3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)), x)
- (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 +
 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2
*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 +
4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a
)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*co
s(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2
*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(d^2*x^2 + 2*c
*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)
*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) + (b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*
x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b
*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(d^2*x^2 + 2*c*d*x
+ (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(
b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) + 2*(d*cos
```

```
(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + 2*d*
sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
+ (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a
)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x
 + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4
*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c
*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc{\left(a + bx \right)} \sec^{3}{\left(a + bx \right)}}{\left(c + dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)**3/(c + d*x)**2, x)
```

# 3.316 $\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$

**Optimal**. Leaf size=27

$$\mathrm{Int}\left(\csc^2(a+bx)\sec^3(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

**Rubi** [A] time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$$

**Mathematica** [A] time = 30.51, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]
```

**fricas** [A] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

**maple** [A] time = 0.14, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^2(bx+a)\right)\left(\sec^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

**maxima** [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

**mupad** [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)^3 \sin(a+b\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

```
[Out] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^2), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.317 $\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx) \, dx$

**Optimal**. Leaf size=486

$$\frac{3id^3\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}-\frac{9id^3\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{9id^3\text{Li}_4\left(ie^{i(\cdots}\right.}{b^4}$$

```
[Out] -6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^3-3*I*(d*x+c)^3*arctan(exp(I*(
b*x+a)))/b-6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2-3/2*(d*x+c)^3*csc(b*x+
a)/b-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4+6*I*d^2*(d*x+c)*polylog(2,-exp
(I*(b*x+a)))/b^3+3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4+9/2*I*d*(d*x+c)^2
*polylog(2,-I*exp(I*(b*x+a)))/b^2-9/2*I*d*(d*x+c)^2*polylog(2,I*exp(I*(b*x+
a)))/b^2+9*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,-exp(I*(b*
x+a)))/b^4-9*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^3+9*d^2*(d*x+c)*pol
ylog(3,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4-9*I*d^3*po
lylog(4,-I*exp(I*(b*x+a)))/b^4-6*I*d^2*(d*x+c)*arctan(exp(I*(b*x+a)))/b^3-3
/2*d*(d*x+c)^2*sec(b*x+a)/b^2+1/2*(d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2/b
```

**Rubi [A]** time = 1.21, antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 19, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.792, Rules used = {2621, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4181, 2531, 6609, 2282, 6589, 4183, 2622, 6741, 2279, 2391}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{9d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{9d^{\cdots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 - ((3*I)*(c + d*x)^3*Arc
Tan[E^(I*(a + b*x))])/b - (6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 -
(3*(c + d*x)^3*Csc[a + b*x])/(2*b) + ((6*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*
(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 + (((9*
I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*Poly
Log[2, I*E^(I*(a + b*x))])/b^4 - (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E^(I
*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 -
 (6*d^3*PolyLog[3, -E^(I*(a + b*x))])/b^4 - (9*d^2*(c + d*x)*PolyLog[3, (-I
)*E^(I*(a + b*x))])/b^3 + (9*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b
^3 + (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4 - ((9*I)*d^3*PolyLog[4, (-I)*E
^(I*(a + b*x))])/b^4 + ((9*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 - (3*d
*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]
^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\,dx = \frac{3(c+dx)^3 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3 \csc(a+bx)}{2b} + \frac{(c+dx)^3 \mathrm{c}}{}$$

$$= \frac{3(c+dx)^3 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3 \csc(a+bx)}{2b} + \frac{(c+dx)^3 \mathrm{c}}{}$$

$$= \frac{3(c+dx)^3 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3 \csc(a+bx)}{2b} + \frac{(c+dx)^3 \mathrm{c}}{}$$

$$= \frac{3(c+dx)^3 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3 \csc(a+bx)}{2b} + \frac{(c+dx)^3 \mathrm{c}}{}$$

$$= \frac{3(c+dx)^3 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3 \csc(a+bx)}{2b} + \frac{(c+dx)^3 \mathrm{c}}{}$$

$$= \frac{3d(c+dx)^2 \tanh^{-1}(\cos(a+bx))}{2b^2} + \frac{3(c+dx)^3 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3}{}$$

$$= \frac{3d(c+dx)^2 \tanh^{-1}(\cos(a+bx))}{2b^2} + \frac{3(c+dx)^3 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3}{}$$

$$= -\frac{9d(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{3d(c+dx)^2 \tanh^{-1}(\cos(a+bx))}{2b^2} - \frac{3c}{}$$

$$= -\frac{9d(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{3d(c+dx)^2 \tanh^{-1}(\cos(a+bx))}{2b^2} - \frac{3c}{}$$

$$= -\frac{3i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{9d(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{3d(c+d}{}$$

$$= -\frac{6id^2(c+dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{9d(c+d}{}$$

$$= -\frac{6id^2(c+dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{9d(c+d}{}$$

$$= -\frac{6id^2(c+dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+d}{}$$

$$= -\frac{6id^2(c+dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+d}{}$$

$$= -\frac{6id^2(c+dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+d}{}$$

$$= -\frac{6id^2(c+dx) \tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+d}{}$$

**Mathematica** [A] time = 7.90, size = 819, normalized size = 1.69

$$-\frac{\csc(a+bx)\left(bc^3 + 3b\cos(2a+2bx)c^3 + 3bdxc^2 + 9bdx\cos(2a+2bx)c^2 + 3d\sin(2a+2bx)c^2 + 3bd^2x^2c + 9bd^2\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] (3*d*((c + d*x)^2*Log[1 - E^(I*(a + b*x))] - (c + d*x)^2*Log[1 + E^(I*(a +
b*x))] + ((2*I)*d*(b*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + I*d*PolyLog[3
, -E^(I*(a + b*x))]))/b^2 + (2*d*((-I)*b*(c + d*x)*PolyLog[2, E^(I*(a + b*x
))] + d*PolyLog[3, E^(I*(a + b*x))]))/b^2))/b^2 - (3*((2*I)*b^3*c^3*ArcTan[
```

```
E^(I*(a + b*x))] + (4*I)*b*c*d^2*ArcTan[E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Lo
g[1 - I*E^(I*(a + b*x))] - 2*b*d^3*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c*d
^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))]
+ 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] + 2*b*d^3*x*Log[1 + I*E^(I*(a +
b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 + I
*E^(I*(a + b*x))] - I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*PolyLog[2, (-I)*E^(I*(a
 + b*x))] + I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*PolyLog[2, I*E^(I*(a + b*x))] +
 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, (-I)*E^(
I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog
[3, I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] - (6*I)
*d^3*PolyLog[4, I*E^(I*(a + b*x))]))/(2*b^4) - (Csc[a + b*x]*Sec[a + b*x]^2
*(b*c^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2 + b*d^3*x^3 + 3*b*c^3*Cos[2*a + 2*b*x
] + 9*b*c^2*d*x*Cos[2*a + 2*b*x] + 9*b*c*d^2*x^2*Cos[2*a + 2*b*x] + 3*b*d^3
*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b*x] + 6*c*d^2*x*Sin[2*a + 2*b*
x] + 3*d^3*x^2*Sin[2*a + 2*b*x]))/(4*b^2)
```

**fricas [C]** time = 0.84, size = 2218, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 18*I*d^3*cos(b*x + a
)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 18*I*d^3*cos(b
*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 18*I*d^3
*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 1
8*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a))*sin(b*x +
 a) + 12*d^3*cos(b*x + a)^2*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b
*x + a) + 12*d^3*cos(b*x + a)^2*polylog(3, cos(b*x + a) - I*sin(b*x + a))*s
in(b*x + a) - 12*d^3*cos(b*x + a)^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a))*sin(b*x + a) - 12*d^3*cos(b*x + a)^2*polylog(3, -cos(b*x + a) - I*sin(b
*x + a))*sin(b*x + a) + 2*b^3*c^3 + (-12*I*b*d^3*x - 12*I*b*c*d^2)*cos(b*x
+ a)^2*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + (12*I*b*d^3*x +
12*I*b*c*d^2)*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x +
 a) + (-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b
*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + (-9*I*b^2*d^3
*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2*dilog(I*c
os(b*x + a) - sin(b*x + a))*sin(b*x + a) + (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^
2*x + 9*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b
*x + a))*sin(b*x + a) + (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d
 + 6*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x +
a) + (-12*I*b*d^3*x - 12*I*b*c*d^2)*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*
sin(b*x + a))*sin(b*x + a) + (12*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a)^2*d
ilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 6*(b^2*d^3*x^2 + 2*b^2*
c*d^2*x + b^2*c^2*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + 1)*
sin(b*x + a) + 3*(b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*
a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a)
- 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)^2*log(cos(b*x +
a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2
 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*
x + a) + I)*sin(b*x + a) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a
)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*
c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1)*s
in(b*x + a) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(-I*co
s(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*
x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*
```

```
d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a)
 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)^2*log(-1/2*cos(b*x +
a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*s
in(b*x + a) + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b
*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^3*c^3
 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^2*lo
g(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 6*(b^2*d^3*x^2 + 2*b^2
*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(
b*x + a) + 1)*sin(b*x + a) - 3*(b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d
^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I
)*sin(b*x + a) - 18*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*cos(b*x
 + a) + sin(b*x + a))*sin(b*x + a) + 18*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*
polylog(3, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 18*(b*d^3*x + b*c*
d^2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a)
 + 18*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) - sin(b
*x + a))*sin(b*x + a) - 6*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
b^3*c^3)*cos(b*x + a)^2 - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b
*x + a)*sin(b*x + a))/(b^4*cos(b*x + a)^2*sin(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \csc(bx+a)^2 \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

**maple [B]** time = 0.56, size = 1629, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

```
[Out] -9*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(2,-I*exp(I*(b*x+a
)))/b^4+9*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,-exp(I*(b*x
+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+3/b^2*c^2*d*ln(exp(I*(b*x+a))
-1)-3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)+3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-1)-3*I
*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-3/b^2*d^3*ln(exp(I*(b*x+a))+1)*x^2+3/b
^2*d^3*ln(1-exp(I*(b*x+a)))*x^2+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^2-9*I/b^3*
a^2*c*d^2*arctan(exp(I*(b*x+a)))+9*I/b^2*c*d^2*polylog(2,-I*exp(I*(b*x+a)))
*x+9*I/b^2*a*c^2*d*arctan(exp(I*(b*x+a)))-9*I/b^2*c*d^2*polylog(2,I*exp(I*(
b*x+a)))*x+6*I/b^4*d^3*a*arctan(exp(I*(b*x+a)))-6*I/b^3*c*d^2*arctan(exp(I*
(b*x+a)))-3/2/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)))+9/b^3*d^3*polylog(3,I*exp(
I*(b*x+a)))*x+3/2/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3-3/2/b*d^3*ln(1+I*exp(I*(
b*x+a)))*x^3-9/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x+9/b^3*d^2*c*polylog(3
,I*exp(I*(b*x+a)))-9/b^3*d^2*c*polylog(3,-I*exp(I*(b*x+a)))+3/2/b^4*a^3*d^3
*ln(1-I*exp(I*(b*x+a)))+9/2/b*d^2*c*ln(1-I*exp(I*(b*x+a)))*x^2-9/2/b*d^2*c*
ln(1+I*exp(I*(b*x+a)))*x^2+9/2/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x+9/2/b^2*c^2
*d*ln(1-I*exp(I*(b*x+a)))*a+9/2/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x+a)))-9/2/b*
c^2*d*ln(1+I*exp(I*(b*x+a)))*x-9/2/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a-9/2/b
^3*a^2*c*d^2*ln(1-I*exp(I*(b*x+a)))-I/b^2/(1+exp(2*I*(b*x+a)))^2/(exp(2*I*(
b*x+a))-1)*(3*d^3*x^3*b*exp(5*I*(b*x+a))+9*c*d^2*x^2*b*exp(5*I*(b*x+a))+9*c
^2*d*x*b*exp(5*I*(b*x+a))+2*d^3*x^3*b*exp(3*I*(b*x+a))+3*c^3*b*exp(5*I*(b*x
+a))+6*c*d^2*x^2*b*exp(3*I*(b*x+a))+6*I*c*d^2*x*exp(I*(b*x+a))+6*c^2*d*x*b*
exp(3*I*(b*x+a))+3*d^3*x^3*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(5*I*(b*x+a))+2*
c^3*b*exp(3*I*(b*x+a))+9*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(5*I*(b*
```

```
x+a))+9*c^2*d*x*b*exp(I*(b*x+a))+3*c^3*b*exp(I*(b*x+a))+3*I*c^2*d*exp(I*(b*
x+a))+3*I*d^3*x^2*exp(I*(b*x+a))-3*I*c^2*d*exp(5*I*(b*x+a)))+3/b^3*d^3*ln(1
-I*exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-I*exp(I*(b*x+a)))*a-3/b^3*d^3*ln(1+I*ex
p(I*(b*x+a)))*x-3/b^4*d^3*ln(1+I*exp(I*(b*x+a)))*a-6/b^3*c*d^2*a*ln(exp(I*(
b*x+a))-1)+6/b^3*d^3*ln(1-exp(I*(b*x+a)))*a*x+6*I/b^3*d^2*c*dilog(exp(I*(b*
x+a))+1)-6*I/b^4*d^3*a*dilog(exp(I*(b*x+a))+1)+6*I/b^4*d^3*polylog(2,-exp(I
*(b*x+a)))*a-6*I/b^4*d^3*polylog(2,exp(I*(b*x+a)))*a+6*I/b^3*dilog(exp(I*(b
*x+a)))*c*d^2-6*I/b^4*dilog(exp(I*(b*x+a)))*d^3*a-3*I/b*c^3*arctan(exp(I*(b
*x+a)))-6*I/b^3*d^3*polylog(2,exp(I*(b*x+a)))*x-6/b^2*d^2*c*ln(exp(I*(b*x+a
))+1)*x+6*I/b^3*d^3*polylog(2,-exp(I*(b*x+a)))*x+3*I/b^4*a^3*d^3*arctan(exp
(I*(b*x+a)))-9/2*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))+9/2*I/b^2*c^2*d*po
lylog(2,-I*exp(I*(b*x+a)))-9/2*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+9/
2*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2
```

**maxima [B]** time = 6.46, size = 8032, normalized size = 16.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c^3*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log
(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*(3*sin(b*x + a
)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(
sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^
3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^2
- a^3*d^3*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log
(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^3 - 4*((6*(b*x + a)^3*d^3 +
 12*b*c*d^2 - 12*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d
- 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2
 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 +
 (3*a^2 + 2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*((b*x + a)^3*d^3 + 2*b*c*
d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^
2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*((b*x + a)^3*d^3 + 2*b
*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c
*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-6*I*(b*x + a)^3*d^3
 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (
-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(6
*b*x + 6*a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*
c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*
I*a^2 - 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (6*I*(b*x + a)^3*d^3 + 12*
I*b*c*d^2 - 12*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^
2*c^2*d - 36*I*a*b*c*d^2 + (18*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(2*b*x + 2*
a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (6*(b*x + a)^3*d^3 + 12*b*c*d
^2 - 12*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d - 6*a*b*c
*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 +
 2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a
*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^
2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*((b*x + a)^3*d^3 + 2*b*c*d^2 -
2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3
*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b
*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2
*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(6*b*x + 6*
a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 1
8*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 1
2*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (6*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2
 - 12*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d -
 36*I*a*b*c*d^2 + (18*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arcta
n2(cos(b*x + a), -sin(b*x + a) + 1) + (12*b^2*c^2*d - 24*a*b*c*d^2 + 12*(b*
```

```
x + a)^2*d^3 + 12*a^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) - 12*(b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(6*b*x + 6*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3
+ 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*(b^2*c^2*d - 2*a*b*c
*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x
 + 2*a) + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a
^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-12*I*
b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b
*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (12*I*b^2*c^2*d - 24*I*a
*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3
)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (1
2*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 - 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3)*cos(6*b*x + 6*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(4*b*x +
 4*a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(2*b*x + 2*a) - (12*I*b^2
*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3)*sin(6*b*x + 6*a) - (12*I*b^2*c^2*d
- 24*I*a*b*c*d^2 + 12*I*a^2*d^3)*sin(4*b*x + 4*a) - (-12*I*b^2*c^2*d + 24*I
*a*b*c*d^2 - 12*I*a^2*d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x
+ a) - 1) + (12*(b*x + a)^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) - 12*((b*x
 + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 12*((b*x +
a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*((b*x + a)^
2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-12*I*(b*x + a)^
2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-12*I*(
b*x + a)^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) +
 (12*I*(b*x + a)^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x +
 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (12*(b*x + a)^3*d^3 - 12*
I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (36*b*c*d^2 - (36*a + 12*I)*d
^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a + 24*I)*b*c*d^2 + 12*(3*a^2 + 2*I*a
)*d^3)*(b*x + a))*cos(5*b*x + 5*a) - 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(3*b*x
 + 3*a) - (12*(b*x + a)^3*d^3 + 12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*
d^3 + (36*b*c*d^2 - (36*a - 12*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a
- 24*I)*b*c*d^2 + 12*(3*a^2 - 2*I*a)*d^3)*(b*x + a))*cos(b*x + a) + (18*b^2
*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^3 + 36*(b*c*d^
2 - a*d^3)*(b*x + a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (
3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*(3*b^2
*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a
*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x +
 a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*
a) + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2
- 12*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-
18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 12*I)
*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (18*I*b^2
*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 12*I)*d^3 + (3
6*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I
*a)) - (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^
3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x
+ a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6
*a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 +
6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*(3*b^2*c^2*d - 6*a*b*c*
d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(2*b*x + 2*a) - (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3
+ (18*I*a^2 + 12*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(6*b*x
+ 6*a) - (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^
2 + 12*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (
-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 12*I
)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I*
e^(I*b*x + I*a)) - (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 - 24*(b*c*d^2
+ (b*x + a)*d^3 - a*d^3)*cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a
*d^3)*cos(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2
```

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*a) - (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(6*b*x + 6*a) - (
24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(4*b*x + 4*a) - (-24*I*b
*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x
 + I*a)) + (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 - 24*(b*c*d^2 + (b*x +
 a)*d^3 - a*d^3)*cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*co
s(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (-
24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(6*b*x + 6*a) + (-24*I*b
*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(4*b*x + 4*a) + (24*I*b*c*d^2
+ 24*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a))
 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (
-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6
*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*c
os(6*b*x + 6*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6
*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-6*
I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b
*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d
^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(6*b*x +
 6*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2
 - a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x +
 a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(
cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (6*I*b^2*c^2*d - 12
*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d
^3)*(b*x + a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*
I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (-6*
I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b
*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (6*I*b^2*c^2*d - 12*I*a*
b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(
b*x + a))*cos(2*b*x + 2*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 +
 a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + 6*(b^2*c^2*d -
 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*s
in(4*b*x + 4*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 +
2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b
*x + a)^2 - 2*cos(b*x + a) + 1) + (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*
a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c
*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2
 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 1
8*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (-3*I*(
b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x
+ a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a)
)*cos(4*b*x + 4*a) + (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*
b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a
^2 + 6*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2
 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 +
 (3*a^2 + 2)*d^3)*(b*x + a))*sin(6*b*x + 6*a) + 3*((b*x + a)^3*d^3 + 2*b*c*
d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^
2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 3*((b*x + a)^3*d^3 + 2*b
*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c
*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + s
in(b*x + a)^2 + 2*sin(b*x + a) + 1) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 +
 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*
I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + 6*I*
b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*
d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (3*
I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*
x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a)
)*cos(4*b*x + 4*a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*
I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9
*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^3 + 2*b*c
*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d
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^2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(6*b*x + 6*a) - 3*((b*x + a)^3*d^3 + 2*
b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*
c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 3*((b*x + a)^3*d^3 +
 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a
*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (36*d^3*cos(6*b*x + 6*a) + 36*d^3
*cos(4*b*x + 4*a) - 36*d^3*cos(2*b*x + 2*a) + 36*I*d^3*sin(6*b*x + 6*a) + 3
6*I*d^3*sin(4*b*x + 4*a) - 36*I*d^3*sin(2*b*x + 2*a) - 36*d^3)*polylog(4, I
*e^(I*b*x + I*a)) - (36*d^3*cos(6*b*x + 6*a) + 36*d^3*cos(4*b*x + 4*a) - 36
*d^3*cos(2*b*x + 2*a) + 36*I*d^3*sin(6*b*x + 6*a) + 36*I*d^3*sin(4*b*x + 4*
a) - 36*I*d^3*sin(2*b*x + 2*a) - 36*d^3)*polylog(4, -I*e^(I*b*x + I*a)) + (
36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (-36*I*b*c*d^2 - 36*I*(b*x
 + a)*d^3 + 36*I*a*d^3)*cos(6*b*x + 6*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*
d^3 + 36*I*a*d^3)*cos(4*b*x + 4*a) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 3
6*I*a*d^3)*cos(2*b*x + 2*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(6*b*
x + 6*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 36*(b*c*
d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, I*e^(I*b*x + I*a)
) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I
*(b*x + a)*d^3 - 36*I*a*d^3)*cos(6*b*x + 6*a) + (36*I*b*c*d^2 + 36*I*(b*x +
 a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^
3 + 36*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin
(6*b*x + 6*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 36*
(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -I*e^(I*b*x
+ I*a)) + (24*I*d^3*cos(6*b*x + 6*a) + 24*I*d^3*cos(4*b*x + 4*a) - 24*I*d^3
*cos(2*b*x + 2*a) - 24*d^3*sin(6*b*x + 6*a) - 24*d^3*sin(4*b*x + 4*a) + 24*
d^3*sin(2*b*x + 2*a) - 24*I*d^3)*polylog(3, -e^(I*b*x + I*a)) + (-24*I*d^3*
cos(6*b*x + 6*a) - 24*I*d^3*cos(4*b*x + 4*a) + 24*I*d^3*cos(2*b*x + 2*a) +
24*d^3*sin(6*b*x + 6*a) + 24*d^3*sin(4*b*x + 4*a) - 24*d^3*sin(2*b*x + 2*a)
 + 24*I*d^3)*polylog(3, e^(I*b*x + I*a)) + (-12*I*(b*x + a)^3*d^3 - 12*b^2*
c^2*d + 24*a*b*c*d^2 - 12*a^2*d^3 - 12*(3*I*b*c*d^2 + (-3*I*a + 1)*d^3)*(b*
x + a)^2 + (-36*I*b^2*c^2*d - 24*(-3*I*a + 1)*b*c*d^2 + (-36*I*a^2 + 24*a)*
d^3)*(b*x + a))*sin(5*b*x + 5*a) + (-8*I*(b*x + a)^3*d^3 + (-24*I*b*c*d^2 +
 24*I*a*d^3)*(b*x + a)^2 + (-24*I*b^2*c^2*d + 48*I*a*b*c*d^2 - 24*I*a^2*d^3
)*(b*x + a))*sin(3*b*x + 3*a) + (-12*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*
a*b*c*d^2 + 12*a^2*d^3 + (-36*I*b*c*d^2 - 12*(-3*I*a - 1)*d^3)*(b*x + a)^2
+ (-36*I*b^2*c^2*d - 24*(-3*I*a - 1)*b*c*d^2 + (-36*I*a^2 - 24*a)*d^3)*(b*x
 + a))*sin(b*x + a))/(-4*I*b^3*cos(6*b*x + 6*a) - 4*I*b^3*cos(4*b*x + 4*a)
+ 4*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(6*b*x + 6*a) + 4*b^3*sin(4*b*x + 4*a
) - 4*b^3*sin(2*b*x + 2*a) + 4*I*b^3))/b
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

```
[Out] \text{Hanged}
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.318 $\int (c+dx)^2 \csc^2(a+bx)\sec^3(a+bx)\,dx$

**Optimal**. Leaf size=341

$$\frac{2id^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{3d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{3d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}+\frac{3id(c+\cdots}{}$$

```
[Out] -3*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+2*d^2*x*arctanh(exp(I*(b*x+a)))/b^2
-6*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-d^2*x*arctanh(cos(b*x+a))/b^2+d*(d
*x+c)*arctanh(cos(b*x+a))/b^2+d^2*arctanh(sin(b*x+a))/b^3-3/2*(d*x+c)^2*csc
(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3+3*I*d*(d*x+c)*polylog(2,-I
*exp(I*(b*x+a)))/b^2-3*I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^2-2*I*d^2*
polylog(2,exp(I*(b*x+a)))/b^3-3*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+3*d^2*
polylog(3,I*exp(I*(b*x+a)))/b^3-d*(d*x+c)*sec(b*x+a)/b^2+1/2*(d*x+c)^2*csc(
b*x+a)*sec(b*x+a)^2/b
```

**Rubi [A]** time = 0.65, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 19, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.792, Rules used = {2621, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4181, 2531, 2282, 6589, 4183, 2279, 2391, 2622, 6271, 3770}

$$\frac{3id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}+\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLo}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-3*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (2*d^2*x*ArcTanh[E^(I*(a +
 b*x))])/b^2 - (6*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - (d^2*x*ArcTan
h[Cos[a + b*x]])/b^2 + (d*(c + d*x)*ArcTanh[Cos[a + b*x]])/b^2 + (d^2*ArcTa
nh[Sin[a + b*x]])/b^3 - (3*(c + d*x)^2*Csc[a + b*x])/(2*b) + ((2*I)*d^2*Pol
yLog[2, -E^(I*(a + b*x))])/b^3 + ((3*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^2 - ((3*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((
2*I)*d^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (3*d^2*PolyLog[3, (-I)*E^(I*(a
+ b*x))])/b^3 + (3*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Se
c[a + b*x])/b^2 + ((c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc^2(a+bx)\sec^3(a+bx)\,dx = \frac{3(c+dx)^2\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2\csc(a+bx)}{2b} + \frac{(c+dx)^2 \mathrm{c}}{}$$

$$= \frac{3(c+dx)^2\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2\csc(a+bx)}{2b} + \frac{(c+dx)^2 \mathrm{c}}{}$$

$$= \frac{3(c+dx)^2\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2\csc(a+bx)}{2b} + \frac{(c+dx)^2 \mathrm{c}}{}$$

$$= \frac{3(c+dx)^2\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2\csc(a+bx)}{2b} + \frac{(c+dx)^2 \mathrm{c}}{}$$

$$= \frac{3(c+dx)^2\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2\csc(a+bx)}{2b} + \frac{(c+dx)^2 \mathrm{c}}{}$$

$$= \frac{d(c+dx)\tanh^{-1}(\cos(a+bx))}{b^2} + \frac{3(c+dx)^2\tanh^{-1}(\sin(a+bx))}{2b} - 3(c$$

$$= \frac{d(c+dx)\tanh^{-1}(\cos(a+bx))}{b^2} + \frac{3(c+dx)^2\tanh^{-1}(\sin(a+bx))}{2b} - 3(c$$

$$= -\frac{6d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{d^2x\tanh^{-1}(\cos(a+bx))}{b^2} + \frac{d(c+dx)\,\mathrm{t}}{}$$

$$= -\frac{6d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{d^2x\tanh^{-1}(\cos(a+bx))}{b^2} + \frac{d(c+dx)\,\mathrm{t}}{}$$

$$= -\frac{3i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+dx)\,\mathrm{tan}}{b^2}$$

$$= -\frac{3i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+dx)\,\mathrm{tan}}{b^2}$$

$$= -\frac{3i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+dx)\,\mathrm{tan}}{b^2}$$

$$= -\frac{3i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+dx)\,\mathrm{tan}}{b^2}$$

**Mathematica [B]** time = 7.53, size = 889, normalized size = 2.61

$$\frac{2\left(\frac{\left(\left(bx+\tan^{-1}(\tan(a))\right)\left(\log\left(1-e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)-\log\left(1+e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)\right)+i\left(\mathrm{Li}_2\left(-e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)-\mathrm{Li}_2\left(e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)\right)\right)\sec(a)}{\sqrt{\tan^2(a)+1}} - \frac{2\,\mathrm{ta}}{}\right.}{b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] -1/2*((6*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + (4*I)*d^2*ArcTan[E^(I*(a + b*
x))] - 6*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^2*d^2*x^2*Log[1 - I*E^(
I*(a + b*x))] + 6*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^2*d^2*x^2*Log[
1 + I*E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x)
)] + (6*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + 6*d^2*PolyLog[3, (
-I)*E^(I*(a + b*x))] - 6*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((c + d*x
)*Csc[a]*Sec[a]*(b*c*Cos[a] + b*d*x*Cos[a] + d*Sin[a]))/b^2 + ((4*I)*c*d*Ar
cTan[(I*Cos[a] - I*Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/(b^2*Sq
rt[Cos[a]^2 + Sin[a]^2]) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^2*Sin[(b*x)/2]
```

```
) - 2*c*d*x*Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2]))/(2*b) + (Csc[a/2]*Csc[a/2
 + (b*x)/2]*(c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2] + d^2*x^2*Sin[(b*x)/2]
))/(2*b) + (c^2 + 2*c*d*x + d^2*x^2)/(4*b*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (
b*x)/2])^2) + (-(c*d*Sin[(b*x)/2]) - d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] - S
in[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-c^2 - 2*c*d*x - d^2
*x^2)/(4*b*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])^2) + (c*d*Sin[(b*x)/2]
 + d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin
[a/2 + (b*x)/2])) + (2*d^2*((-2*ArcTan[Tan[a]]*ArcTanh[(-Cos[a] + Sin[a]*Ta
n[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^2 + Sin[a]^2] + (((b*x
+ ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b*
x + ArcTan[Tan[a]]))]) + I*(PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - Pol
yLog[2, E^(I*(b*x + ArcTan[Tan[a]]))]))*Sec[a])/Sqrt[1 + Tan[a]^2]))/b^3
```

**fricas [C]** time = 0.65, size = 1362, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d^2*x^2 - 4*I*d^2*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x
+ a))*sin(b*x + a) + 4*I*d^2*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x
+ a))*sin(b*x + a) - 4*I*d^2*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*x
 + a))*sin(b*x + a) + 4*I*d^2*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*sin(b*
x + a))*sin(b*x + a) - 6*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin
(b*x + a))*sin(b*x + a) + 6*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) -
sin(b*x + a))*sin(b*x + a) - 6*d^2*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a
) + sin(b*x + a))*sin(b*x + a) + 6*d^2*cos(b*x + a)^2*polylog(3, -I*cos(b*x
 + a) - sin(b*x + a))*sin(b*x + a) + 4*b^2*c*d*x + (-6*I*b*d^2*x - 6*I*b*c*
d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + (-6*I
*b*d^2*x - 6*I*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a))*s
in(b*x + a) + (6*I*b*d^2*x + 6*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a
) + sin(b*x + a))*sin(b*x + a) + (6*I*b*d^2*x + 6*I*b*c*d)*cos(b*x + a)^2*d
ilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 4*(b*d^2*x + b*c*d)*cos
(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (3*b^2*c^
2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*
x + a) + I)*sin(b*x + a) - 4*(b*d^2*x + b*c*d)*cos(b*x + a)^2*log(cos(b*x +
 a) - I*sin(b*x + a) + 1)*sin(b*x + a) - (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 +
2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a)
+ 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*
cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) +
 1)*sin(b*x + a) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(
b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^
2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x +
a) - sin(b*x + a) + 1)*sin(b*x + a) + 4*(b*c*d - a*d^2)*cos(b*x + a)^2*log(
-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 4*(b*c*d - a*d
^2)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*
x + a) + 4*(b*d^2*x + a*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x +
 a) + 1)*sin(b*x + a) + (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x +
 a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 4*(b*d^2*x + a
*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) -
 (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a)
 - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b^2*c^2 - 6*(b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x +
a))/(b^3*cos(b*x + a)^2*sin(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc(bx+a)^2 \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

**maple [B]** time = 0.32, size = 770, normalized size = 2.26

$$-\frac{3cd\ln\left(1+ie^{i(bx+a)}\right)x}{b}-\frac{3icd\operatorname{polylog}\left(2,ie^{i(bx+a)}\right)}{b^2}-\frac{3id^2\operatorname{polylog}\left(2,ie^{i(bx+a)}\right)x}{b^2}+\frac{3cd\ln\left(1-ie^{i(bx+a)}\right)a}{b^2}+\frac{3cd\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

```
[Out] -3*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+3*d^2*polylog(3,I*exp(I*(b*x+a)))/b
^3-2/b^2*d^2*ln(exp(I*(b*x+a))+1)*x+2*I/b^3*d^2*dilog(exp(I*(b*x+a))+1)+2/b
^2*d*c*ln(exp(I*(b*x+a))-1)-2/b^2*d*c*ln(exp(I*(b*x+a))+1)-2/b^3*d^2*a*ln(e
xp(I*(b*x+a))-1)-3*I/b^3*a^2*d^2*arctan(exp(I*(b*x+a)))+3*I/b^2*d^2*polylog
(2,-I*exp(I*(b*x+a)))*x-3*I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))+3*I/b^2*c*d
*polylog(2,-I*exp(I*(b*x+a)))-3*I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x-3/b
*c*d*ln(1+I*exp(I*(b*x+a)))*x+3/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a+3/b*c*d*ln
(1-I*exp(I*(b*x+a)))*x-3/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a+6*I/b^2*a*c*d*arc
tan(exp(I*(b*x+a)))+2*I/b^3*dilog(exp(I*(b*x+a)))*d^2-3/2/b*d^2*ln(1+I*exp(
I*(b*x+a)))*x^2+3/2/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))+3/2/b*d^2*ln(1-I*exp
(I*(b*x+a)))*x^2-3/2/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))-3*I/b*c^2*arctan(ex
p(I*(b*x+a)))-2*I/b^3*d^2*arctan(exp(I*(b*x+a)))-I/b^2/(1+exp(2*I*(b*x+a)))
^2/(exp(2*I*(b*x+a))-1)*(3*d^2*x^2*b*exp(5*I*(b*x+a))+6*c*d*x*b*exp(5*I*(b*
x+a))+3*c^2*b*exp(5*I*(b*x+a))+2*d^2*x^2*b*exp(3*I*(b*x+a))+4*c*d*x*b*exp(3
*I*(b*x+a))-2*I*d^2*x*exp(5*I*(b*x+a))+2*c^2*b*exp(3*I*(b*x+a))+3*d^2*x^2*b
*exp(I*(b*x+a))-2*I*c*d*exp(5*I*(b*x+a))+6*c*d*x*b*exp(I*(b*x+a))+3*c^2*b*e
xp(I*(b*x+a))+2*I*d^2*x*exp(I*(b*x+a))+2*I*d*c*exp(I*(b*x+a)))
```

**maxima [B]** time = 1.62, size = 3819, normalized size = 11.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log
(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)) - 2*a*c*d*(2*(3*sin(b*x + a)^
2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(si
n(b*x + a) - 1))/b + a^2*d^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - si
n(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^2 - 4*((
6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) + 4*d^2 - 2*(3*(b*x + a)^2
*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - 2*(3*(b*x +
a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + 2*(3*(b*
x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) + (-6*
I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(6*b
*x + 6*a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) -
4*I*d^2)*sin(4*b*x + 4*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2
)*(b*x + a) + 4*I*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a)
 + 1) + (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) + 4*d^2 - 2*(3*(b
*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - 2*(
3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) +
 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*
a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) - 4*I*d^2
)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*
x + a) - 4*I*d^2)*sin(4*b*x + 4*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 1
```

```
2*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -si
n(b*x + a) + 1) + (8*b*c*d + 8*(b*x + a)*d^2 - 8*a*d^2 - 8*(b*c*d + (b*x +
a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*
b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (-8*I*b*c
*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(6*b*x + 6*a) + (-8*I*b*c*d - 8*I*(b
*x + a)*d^2 + 8*I*a*d^2)*sin(4*b*x + 4*a) + (8*I*b*c*d + 8*I*(b*x + a)*d^2
- 8*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (8
*b*c*d - 8*a*d^2 - 8*(b*c*d - a*d^2)*cos(6*b*x + 6*a) - 8*(b*c*d - a*d^2)*c
os(4*b*x + 4*a) + 8*(b*c*d - a*d^2)*cos(2*b*x + 2*a) - (8*I*b*c*d - 8*I*a*d
^2)*sin(6*b*x + 6*a) - (8*I*b*c*d - 8*I*a*d^2)*sin(4*b*x + 4*a) - (-8*I*b*c
*d + 8*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) -
 (8*(b*x + a)*d^2*cos(6*b*x + 6*a) + 8*(b*x + a)*d^2*cos(4*b*x + 4*a) - 8*(
b*x + a)*d^2*cos(2*b*x + 2*a) + 8*I*(b*x + a)*d^2*sin(6*b*x + 6*a) + 8*I*(b
*x + a)*d^2*sin(4*b*x + 4*a) - 8*I*(b*x + a)*d^2*sin(2*b*x + 2*a) - 8*(b*x
+ a)*d^2)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (12*(b*x + a)^2*d^2 -
8*I*b*c*d + 8*I*a*d^2 + (24*b*c*d - (24*a + 8*I)*d^2)*(b*x + a))*cos(5*b*x
+ 5*a) - 8*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(3*b*x + 3*a)
 - (12*(b*x + a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (24*b*c*d - (24*a - 8*I)*d
^2)*(b*x + a))*cos(b*x + a) + (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d^2 - 12*
(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 12*(b*c*d + (b*x + a)*d^
2 - a*d^2)*cos(4*b*x + 4*a) + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x
+ 2*a) + (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*sin(6*b*x + 6*a) +
 (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*sin(4*b*x + 4*a) + (12*I*b
*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x
+ I*a)) - (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d^2 - 12*(b*c*d + (b*x + a)*d
^2 - a*d^2)*cos(6*b*x + 6*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x
 + 4*a) + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (12*I*b*c*d
 + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(6*b*x + 6*a) - (12*I*b*c*d + 12*I*(
b*x + a)*d^2 - 12*I*a*d^2)*sin(4*b*x + 4*a) - (-12*I*b*c*d - 12*I*(b*x + a)
*d^2 + 12*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + (8*d^2*cos
(6*b*x + 6*a) + 8*d^2*cos(4*b*x + 4*a) - 8*d^2*cos(2*b*x + 2*a) + 8*I*d^2*s
in(6*b*x + 6*a) + 8*I*d^2*sin(4*b*x + 4*a) - 8*I*d^2*sin(2*b*x + 2*a) - 8*d
^2)*dilog(-e^(I*b*x + I*a)) - (8*d^2*cos(6*b*x + 6*a) + 8*d^2*cos(4*b*x + 4
*a) - 8*d^2*cos(2*b*x + 2*a) + 8*I*d^2*sin(6*b*x + 6*a) + 8*I*d^2*sin(4*b*x
 + 4*a) - 8*I*d^2*sin(2*b*x + 2*a) - 8*d^2)*dilog(e^(I*b*x + I*a)) + (-4*I*
b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2 + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*
I*a*d^2)*cos(6*b*x + 6*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos
(4*b*x + 4*a) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*cos(2*b*x + 2*
a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(6*b*x + 6*a) - 4*(b*c*d + (b*x +
 a)*d^2 - a*d^2)*sin(4*b*x + 4*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (4
*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2 + (-4*I*b*c*d - 4*I*(b*x + a)*d^2
+ 4*I*a*d^2)*cos(6*b*x + 6*a) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2
)*cos(4*b*x + 4*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos(2*b*x
+ 2*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(6*b*x + 6*a) + 4*(b*c*d + (b
*x + a)*d^2 - a*d^2)*sin(4*b*x + 4*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*s
in(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
+ (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*I*d^2 + (-3*
I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(6*b*x
 + 6*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*
d^2)*cos(4*b*x + 4*a) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x
 + a) + 2*I*d^2)*cos(2*b*x + 2*a) + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*
(b*x + a) + 2*d^2)*sin(6*b*x + 6*a) + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2
)*(b*x + a) + 2*d^2)*sin(4*b*x + 4*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d
^2)*(b*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*sin(b*x + a) + 1) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*
(b*x + a) - 2*I*d^2 + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x +
 a) + 2*I*d^2)*cos(6*b*x + 6*a) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a
*d^2)*(b*x + a) + 2*I*d^2)*cos(4*b*x + 4*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I
```

```
*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) - (3*(b*x + a)^2*
d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(6*b*x + 6*a) - (3*(b*x + a)^
2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(4*b*x + 4*a) + (3*(b*x + a
)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*
x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (-12*I*d^2*cos(6*b*x + 6*
a) - 12*I*d^2*cos(4*b*x + 4*a) + 12*I*d^2*cos(2*b*x + 2*a) + 12*d^2*sin(6*b
*x + 6*a) + 12*d^2*sin(4*b*x + 4*a) - 12*d^2*sin(2*b*x + 2*a) + 12*I*d^2)*p
olylog(3, I*e^(I*b*x + I*a)) + (12*I*d^2*cos(6*b*x + 6*a) + 12*I*d^2*cos(4*
b*x + 4*a) - 12*I*d^2*cos(2*b*x + 2*a) - 12*d^2*sin(6*b*x + 6*a) - 12*d^2*s
in(4*b*x + 4*a) + 12*d^2*sin(2*b*x + 2*a) - 12*I*d^2)*polylog(3, -I*e^(I*b*
x + I*a)) - 4*(3*I*(b*x + a)^2*d^2 + 2*b*c*d - 2*a*d^2 + 2*(3*I*b*c*d + (-3
*I*a + 1)*d^2)*(b*x + a))*sin(5*b*x + 5*a) + (-8*I*(b*x + a)^2*d^2 + (-16*I
*b*c*d + 16*I*a*d^2)*(b*x + a))*sin(3*b*x + 3*a) + (-12*I*(b*x + a)^2*d^2 +
 8*b*c*d - 8*a*d^2 + (-24*I*b*c*d - 8*(-3*I*a - 1)*d^2)*(b*x + a))*sin(b*x
+ a))/(-4*I*b^2*cos(6*b*x + 6*a) - 4*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2
*b*x + 2*a) + 4*b^2*sin(6*b*x + 6*a) + 4*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2
*b*x + 2*a) + 4*I*b^2))/b
```

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)^2),x)
[Out] \text{Hanged}
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a)**3,x)
[Out] Timed out
```

# 3.319 $\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$

**Optimal**. Leaf size=162

$$\frac{3id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}-\frac{3id\text{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{3(c+dx)\csc(a+bx)}{2b}+\frac{(c+dx)\text{c}}{}$$

```
[Out] -3*I*d*x*arctan(exp(I*(b*x+a)))/b-d*arctanh(cos(b*x+a))/b^2+3/2*c*arctanh(s
in(b*x+a))/b-3/2*(d*x+c)*csc(b*x+a)/b+3/2*I*d*polylog(2,-I*exp(I*(b*x+a)))/
b^2-3/2*I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-1/2*d*sec(b*x+a)/b^2+1/2*(d*x+c
)*csc(b*x+a)*sec(b*x+a)^2/b
```

**Rubi [A]** time = 0.20, antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2621, 288, 321, 207, 4420, 6271, 12, 4181, 2279, 2391, 3770, 2622}

$$\frac{3id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}-\frac{3id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{3(c+dx)\csc(a+}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-3*I)*d*x*ArcTan[E^(I*(a + b*x))])/b - (d*ArcTanh[Cos[a + b*x]])/b^2 - (3
*d*x*ArcTanh[Sin[a + b*x]])/(2*b) + (3*(c + d*x)*ArcTanh[Sin[a + b*x]])/(2*
b) - (3*(c + d*x)*Csc[a + b*x])/(2*b) + (((3*I)/2)*d*PolyLog[2, (-I)*E^(I*(
a + b*x))])/b^2 - (((3*I)/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[
a + b*x])/(2*b^2) + ((c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx = \frac{3(c+dx)\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)\csc(a+bx)}{2b} + \frac{(c+dx)\operatorname{cs}}{}$$

$$= \frac{3(c+dx)\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)\csc(a+bx)}{2b} + \frac{(c+dx)\operatorname{cs}}{}$$

$$= -\frac{3d\tanh^{-1}(\cos(a+bx))}{2b^2} - \frac{3dx\tanh^{-1}(\sin(a+bx))}{2b} + \frac{3(c+dx)\operatorname{tan}}{}$$

$$= -\frac{3d\tanh^{-1}(\cos(a+bx))}{2b^2} - \frac{3dx\tanh^{-1}(\sin(a+bx))}{2b} + \frac{3(c+dx)\operatorname{tan}}{}$$

$$= -\frac{3idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{3dx\tanh^{-1}(\sin(a}{2b}$$

$$= -\frac{3idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{3dx\tanh^{-1}(\sin(a}{2b}$$

$$= -\frac{3idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{3dx\tanh^{-1}(\sin(a}{2b}$$

**Mathematica [C]** time = 6.59, size = 669, normalized size = 4.13

$$\frac{d\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} - \frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} - \frac{d\sin\left(\frac{1}{2}(a+bx)\right)}{2b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)} + \frac{d\sin\left(\right.}{2b^2\left(\sin\left(\frac{1}{2}(a+bx\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] (d*(a*Cos[(a + b*x)/2] - (a + b*x)*Cos[(a + b*x)/2])*Csc[(a + b*x)/2])/(2*b
^2) - (c*Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b -
(d*Log[Cos[(a + b*x)/2]])/b^2 + (d*Log[Sin[(a + b*x)/2]])/b^2 - (3*d*x*(a*L
og[1 - Tan[(a + b*x)/2]] - a*Log[1 + Tan[(a + b*x)/2]] - I*(Log[1 + I*Tan[(
a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, ((1 + I)
- (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(1/2 +
 I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/
2])]) - I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)
/2])] + PolyLog[2, ((1 + I) + (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 + I*
Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] + PolyLog[2, ((
1 - I) + (1 + I)*Tan[(a + b*x)/2])/2])))/(2*b*(a - I*Log[1 - I*Tan[(a + b*x
)/2]] + I*Log[1 + I*Tan[(a + b*x)/2]])) + (d*x)/(4*b*(Cos[(a + b*x)/2] - Si
n[(a + b*x)/2])^2) - (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] - Sin[(a
 + b*x)/2])) - (d*x)/(4*b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])^2) + (d*Sin
[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (d*Sec[(a +
b*x)/2]*(a*Sin[(a + b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/(2*b^2)
```

**fricas [B]** time = 0.54, size = 592, normalized size = 3.65

$$\frac{-3i\,d\cos(bx+a)^2\operatorname{Li}_2(i\cos(bx+a)+\sin(bx+a))\sin(bx+a) - 3i\,d\cos(bx+a)^2\operatorname{Li}_2(i\cos(bx+a)-\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(-3*I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a
) - 3*I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a)
```

```
+ 3*I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) +
 3*I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) +
3*(b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x
 + a) - 3*(b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I)
*sin(b*x + a) - 2*d*cos(b*x + a)^2*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a)
 + 3*(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1)*si
n(b*x + a) - 3*(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x +
a) + 1)*sin(b*x + a) + 3*(b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) +
 sin(b*x + a) + 1)*sin(b*x + a) - 3*(b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos
(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + 2*d*cos(b*x + a)^2*log(-1/2*co
s(b*x + a) + 1/2)*sin(b*x + a) + 3*(b*c - a*d)*cos(b*x + a)^2*log(-cos(b*x
+ a) + I*sin(b*x + a) + I)*sin(b*x + a) - 3*(b*c - a*d)*cos(b*x + a)^2*log(
-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b*d*x - 6*(b*d*x + b*c
)*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) + 2*b*c)/(b^2*cos(b*x + a)
^2*sin(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c) \csc (bx + a)^2 \sec (bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

**maple [B]** time = 0.16, size = 344, normalized size = 2.12

$$-\frac{i\left(3bdx\,e^{5i(bx+a)} + 3cb\,e^{5i(bx+a)} + 2bdx\,e^{3i(bx+a)} + 2cb\,e^{3i(bx+a)} - id\,e^{5i(bx+a)} + 3bdx\,e^{i(bx+a)} + 3cb\,e^{i(bx+a)} + id\,e^{i(bx+}\right.}{b^2\left(1 + e^{2i(bx+a)}\right)^2\left(e^{2i(bx+a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

```
[Out] -I/b^2/(1+exp(2*I*(b*x+a)))^2/(exp(2*I*(b*x+a))-1)*(3*b*d*x*exp(5*I*(b*x+a)
)+3*c*b*exp(5*I*(b*x+a))+2*b*d*x*exp(3*I*(b*x+a))+2*c*b*exp(3*I*(b*x+a))-I*
d*exp(5*I*(b*x+a))+3*b*d*x*exp(I*(b*x+a))+3*c*b*exp(I*(b*x+a))+I*d*exp(I*(b
*x+a)))-3*I/b*c*arctan(exp(I*(b*x+a)))+3*I/b^2*d*a*arctan(exp(I*(b*x+a)))+d
/b^2*ln(exp(I*(b*x+a))-1)-d/b^2*ln(exp(I*(b*x+a))+1)-3/2*I/b^2*d*dilog(1-I*
exp(I*(b*x+a)))+3/2/b*d*ln(1-I*exp(I*(b*x+a)))*x+3/2/b^2*d*ln(1-I*exp(I*(b*
x+a)))*a+3/2*I/b^2*d*dilog(1+I*exp(I*(b*x+a)))-3/2/b*d*ln(1+I*exp(I*(b*x+a)
))*x-3/2/b^2*d*ln(1+I*exp(I*(b*x+a)))*a
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

```
[Out] \text{Hanged}
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \csc^2 (a + bx) \sec^3 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a)**3,x)
[Out] Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x)**3, x)
```

# 3.320 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=27

$$\text{Int}\left(\frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

**Rubi [A]** time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x),x]
[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

**Mathematica [A]** time = 20.74, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]
```

**fricas [A]** time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^3/(d*x + c), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] Timed out
```

**maple [A]** time = 3.25, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2\left(bx+a\right)\right)\left(\sec^3\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(a+bx\right)\sec^3\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**3/(c + d*x), x)
```

## 3.321 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=27

$$\text{Int}\left(\frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2,x]
[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

**Mathematica [A]** time = 25.88, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

**fricas [A]** time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^2\sec(bx+a)^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
[Out] Timed out
```

**maple** [A] time = 4.29, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2(bx+a)\right)\left(\sec^3(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
[Out] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)^3\,\sin(a+b\,x)^2\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**3/(d*x+c)**2,x)
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**3/(c + d*x)**2, x)
```

# 3.322 $\int (c+dx)^m \csc^3(a+bx) \sec^3(a+bx)\,dx$

**Optimal**. Leaf size=27

$$\text{Int}\left(\csc^3(a+bx)\sec^3(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

**Rubi [A]** time = 0.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$$

**Mathematica [A]** time = 32.37, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]
```

**fricas [A]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \csc(bx+a)^3 \sec(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

**maple [A]** time = 0.12, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^3(bx+a)\right)\left(\sec^3(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)^3 \sin(a+b\,x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

```
[Out] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^3), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.323 $\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx) \, dx$

**Optimal**. Leaf size=318

$$\frac{3id^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{2b^4}+\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^2}{}$$

```
[Out] -6*d^2*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b^3-4*(d*x+c)^3*arctanh(exp(2*I*(b
*x+a)))/b-3*d*(d*x+c)^2*csc(2*b*x+2*a)/b^2-2*(d*x+c)^3*cot(2*b*x+2*a)*csc(2
*b*x+2*a)/b+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/b^4+3*I*d*(d*x+c)^2*poly
log(2,-exp(2*I*(b*x+a)))/b^2-3/2*I*d^3*polylog(2,exp(2*I*(b*x+a)))/b^4-3*I*
d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x+c)*polylog(3,-exp(2*
I*(b*x+a)))/b^3+3*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/2*I*d^3*pol
ylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

**Rubi [A]** time = 0.32, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4419, 4186, 4183, 2279, 2391, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{3i}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b^3 - (4*(c + d*x)^3*ArcTan
h[E^((2*I)*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[2*a + 2*b*x])/b^2 - (2*(c
+ d*x)^3*Cot[2*a + 2*b*x]*Csc[2*a + 2*b*x])/b + (((3*I)/2)*d^3*PolyLog[2, -
E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a +
b*x))])/b^2 - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 - ((3*I)*
d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyL
og[3, -E^((2*I)*(a + b*x))])/b^3 + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a
+ b*x))])/b^3 - (((3*I)/2)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3
*I)/2)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \csc^3(a+bx)\sec^3(a+bx)\,dx = 8\int (c+dx)^3 \csc^3(2a+2bx)\,dx$$

$$= -\frac{3d(c+dx)^2\csc(2a+2bx)}{b^2} - \frac{2(c+dx)^3\cot(2a+2bx)\csc(2a+2bx)}{b}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

**Mathematica [A]** time = 8.63, size = 483, normalized size = 1.52

$$-\frac{8b^3c^3\tanh^{-1}\left(e^{2i(a+bx)}\right) - 12b^3c^2dx\log\left(1-e^{2i(a+bx)}\right) + 12b^3c^2dx\log\left(1+e^{2i(a+bx)}\right) - 12b^3cd^2x^2\log\left(1-e^{2i(a+bx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] -1/2*(8*b^3*c^3*ArcTanh[E^((2*I)*(a + b*x))] + 12*b*c*d^2*ArcTanh[E^((2*I)*
(a + b*x))] + 2*b^2*(c + d*x)^2*(3*d + 2*b*(c + d*x)*Cot[2*(a + b*x)])*Csc[
2*(a + b*x)] - 12*b^3*c^2*d*x*Log[1 - E^((2*I)*(a + b*x))] - 6*b*d^3*x*Log[
1 - E^((2*I)*(a + b*x))] - 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] -
4*b^3*d^3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 + E^((2*I
)*(a + b*x))] + 6*b*d^3*x*Log[1 + E^((2*I)*(a + b*x))] + 12*b^3*c*d^2*x^2*L
og[1 + E^((2*I)*(a + b*x))] + 4*b^3*d^3*x^3*Log[1 + E^((2*I)*(a + b*x))] -
(3*I)*d*(d^2 + 2*b^2*(c + d*x)^2)*PolyLog[2, -E^((2*I)*(a + b*x))] + (3*I)*
d*(d^2 + 2*b^2*(c + d*x)^2)*PolyLog[2, E^((2*I)*(a + b*x))] + 6*b*c*d^2*Pol
yLog[3, -E^((2*I)*(a + b*x))] + 6*b*d^3*x*PolyLog[3, -E^((2*I)*(a + b*x))]
- 6*b*c*d^2*PolyLog[3, E^((2*I)*(a + b*x))] - 6*b*d^3*x*PolyLog[3, E^((2*I)
*(a + b*x))] + (3*I)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] - (3*I)*d^3*PolyL
og[4, E^((2*I)*(a + b*x))])/b^4
```

**fricas [C]** time = 0.95, size = 4193, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 2*(b^3*d^3*
x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2 - 3*(b^2*d^
3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) - ((-6*I*b^2*d
^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^4 + (6*I*
b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^2)*d
ilog(cos(b*x + a) + I*sin(b*x + a)) - ((6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x
+ 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^4 + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*
d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin
(b*x + a)) - ((-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d^
```

```
3)*cos(b*x + a)^4 + (6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d + 3
*I*d^3)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) - ((6*I*b^2*d^
3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^4 + (-6*I*
b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*d
ilog(I*cos(b*x + a) - sin(b*x + a)) - ((6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x
+ 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^4 + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*
d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + si
n(b*x + a)) - ((-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d
^3)*cos(b*x + a)^4 + (6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d +
3*I*d^3)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - ((6*I*b^2*
d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^4 + (-6*
I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)
*dilog(-cos(b*x + a) + I*sin(b*x + a)) - ((-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^
2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^4 + (6*I*b^2*d^3*x^2 + 12*I*b^2
*c*d^2*x + 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I
*sin(b*x + a)) - ((2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2
+ 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*
x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2)*lo
g(cos(b*x + a) + I*sin(b*x + a) + 1) + ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a
^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*
c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^2)*log(cos(
b*x + a) + I*sin(b*x + a) + I) - ((2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*
c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^
3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*d^3)*x)*co
s(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + ((2*b^3*c^3 - 6*a*b^
2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^4 - (2*b^
3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x
+ a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) + ((2*b^3*d^3*x^3 + 6*b^3*c*
d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*
d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c
^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b
*x + a)^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + ((2*b^3*d^3*x^3 + 6*b^3
*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c
^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^
2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*co
s(b*x + a)^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + ((2*b^3*d^3*x^3 + 6*
b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^
3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a
*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)
*cos(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + ((2*b^3*d^3*x^3
+ 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(
2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 +
 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3
)*x)*cos(b*x + a)^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - ((2*b^3*c^3
- 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^4
 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*
cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - ((2*b^3
*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x +
 a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*
d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - ((
2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 +
3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b
^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3
*c^2*d + b*d^3)*x)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)
+ ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*
cos(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^
3 + 3*a)*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - ((2
*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3
*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^
```

```
3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*
c^2*d + b*d^3)*x)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) +
 ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*c
os(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3
 + 3*a)*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - (12*
I*d^3*cos(b*x + a)^4 - 12*I*d^3*cos(b*x + a)^2)*polylog(4, cos(b*x + a) + I
*sin(b*x + a)) - (-12*I*d^3*cos(b*x + a)^4 + 12*I*d^3*cos(b*x + a)^2)*polyl
og(4, cos(b*x + a) - I*sin(b*x + a)) - (12*I*d^3*cos(b*x + a)^4 - 12*I*d^3*
cos(b*x + a)^2)*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - (-12*I*d^3*cos(
b*x + a)^4 + 12*I*d^3*cos(b*x + a)^2)*polylog(4, I*cos(b*x + a) - sin(b*x +
 a)) - (-12*I*d^3*cos(b*x + a)^4 + 12*I*d^3*cos(b*x + a)^2)*polylog(4, -I*c
os(b*x + a) + sin(b*x + a)) - (12*I*d^3*cos(b*x + a)^4 - 12*I*d^3*cos(b*x +
 a)^2)*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - (-12*I*d^3*cos(b*x + a)
^4 + 12*I*d^3*cos(b*x + a)^2)*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) -
(12*I*d^3*cos(b*x + a)^4 - 12*I*d^3*cos(b*x + a)^2)*polylog(4, -cos(b*x + a
) - I*sin(b*x + a)) - 12*((b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3*x + b
*c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 12*((b*
d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polyl
og(3, cos(b*x + a) - I*sin(b*x + a)) + 12*((b*d^3*x + b*c*d^2)*cos(b*x + a)
^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, I*cos(b*x + a) + sin(b*
x + a)) + 12*((b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(
b*x + a)^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 12*((b*d^3*x + b*c*
d^2)*cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -I*cos
(b*x + a) + sin(b*x + a)) + 12*((b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3
*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) -
12*((b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2
)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12*((b*d^3*x + b*c*d^2)*cos(
b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) -
 I*sin(b*x + a)))/(b^4*cos(b*x + a)^4 - b^4*cos(b*x + a)^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \csc(bx + a)^3 \sec(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

**maple [B]** time = 0.22, size = 1329, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

```
[Out] -3/2*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*I*d^3*polylog(2,-exp(2*I*(b
*x+a)))/b^4-3/b^3*d^2*c*ln(1+exp(2*I*(b*x+a)))-3/b^3*d^3*ln(1+exp(2*I*(b*x+
a)))*x-2/b*c^3*ln(1+exp(2*I*(b*x+a)))-2/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+12
/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*c*d^2*polylog(3,exp(I*(b*x+a))
)+12/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+12/b^3*d^3*polylog(3,-exp(I*(b*x+a
)))*x-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-3/b^3*c*d^2*polylog(3,-exp(2*I*
(b*x+a)))-3/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+2/b*c^3*ln(exp(I*(b*x+a)
)-1)+2/b*c^3*ln(exp(I*(b*x+a))+1)+3/b^3*d^2*c*ln(exp(I*(b*x+a))-1)+3/b^3*d^
2*c*ln(exp(I*(b*x+a))+1)+3/b^3*d^3*ln(exp(I*(b*x+a))+1)*x+3/b^3*d^3*ln(1-ex
p(I*(b*x+a)))*x+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a-3/b^4*d^3*a*ln(exp(I*(b*x+
a))-1)-3*I/b^4*d^3*polylog(2,-exp(I*(b*x+a)))+2/b^2/(1+exp(2*I*(b*x+a)))^2/
(exp(2*I*(b*x+a))-1)^2*(2*d^3*x^3*b*exp(6*I*(b*x+a))+6*c*d^2*x^2*b*exp(6*I*
(b*x+a))+6*c^2*d*x*b*exp(6*I*(b*x+a))+2*c^3*b*exp(6*I*(b*x+a))-3*I*d^3*x^2*
```

```
exp(6*I*(b*x+a))+2*b*d^3*x^3*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(6*I*(b*x+a))+
6*b*c*d^2*x^2*exp(2*I*(b*x+a))-3*I*c^2*d*exp(6*I*(b*x+a))+6*b*c^2*d*x*exp(2
*I*(b*x+a))+2*b*c^3*exp(2*I*(b*x+a))+3*I*d^3*x^2*exp(2*I*(b*x+a))+6*I*c*d^2
*x*exp(2*I*(b*x+a))+3*I*c^2*d*exp(2*I*(b*x+a)))+12*I/b^4*d^3*polylog(4,exp(
I*(b*x+a)))+12*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-6*I/b^2*d^3*polylog(2,e
xp(I*(b*x+a)))*x^2-6*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+3*I/b^2*c^2*d
*polylog(2,-exp(2*I*(b*x+a)))+3*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2-
6*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-6*I/b^2*c^2*d*polylog(2,-exp(I*(b*x
+a)))+6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)+6/b*c^2*d*ln(exp(I*(b*x+a))+1)*x
+6/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+6/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-6/b^3*
c*d^2*a^2*ln(1-exp(I*(b*x+a)))+6/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+6/b*c*d^2
*ln(exp(I*(b*x+a))+1)*x^2-6/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+2/b*d^3*ln(1-e
xp(I*(b*x+a)))*x^3+2/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+2/b*d^3*ln(exp(I*(b*x
+a))+1)*x^3-2/b*d^3*ln(1+exp(2*I*(b*x+a)))*x^3-6/b*c^2*d*ln(1+exp(2*I*(b*x+
a)))*x-6/b*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2+6*I/b^2*polylog(2,-exp(2*I*(b*x
+a)))*c*d^2*x-12*I/b^2*polylog(2,-exp(I*(b*x+a)))*c*d^2*x-12*I/b^2*polylog(
2,exp(I*(b*x+a)))*c*d^2*x
```

**maxima [B]** time = 3.73, size = 5610, normalized size = 17.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log
(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2)) - 3*a*c^2*d*((2*sin(b*x + a)^
2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*lo
g(sin(b*x + a)^2))/b + 3*a^2*c*d^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4
- sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^2
- a^3*d^3*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log
(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^3 + 2*((16*(b*x + a)^3*d^3
+ 18*b*c*d^2 - 18*a*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(2*b^2*c^2*
d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a) + 2*(8*(b*x + a)^3*d^3 + 9*b*c
*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d - 4*a*b*
c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 4*(8*(b*x + a)^3*d^3
 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d
- 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (16*I*(b*x +
 a)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x +
a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 18*I)*d^3)*(b*x + a))
*sin(8*b*x + 8*a) + (-32*I*(b*x + a)^3*d^3 - 36*I*b*c*d^2 + 36*I*a*d^3 + (-
72*I*b*c*d^2 + 72*I*a*d^3)*(b*x + a)^2 + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2
 + (-72*I*a^2 - 36*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(2*b*x +
 2*a), cos(2*b*x + 2*a) + 1) - (12*(b*x + a)^3*d^3 + 18*b*c*d^2 - 18*a*d^3
+ 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2
 + 1)*d^3)*(b*x + a) + 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*
d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*
(b*x + a))*cos(8*b*x + 8*a) - 12*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 +
 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 +
1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (-12*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2
 + 18*I*a*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d
 + 72*I*a*b*c*d^2 + (-36*I*a^2 - 18*I)*d^3)*(b*x + a))*sin(8*b*x + 8*a) - (
24*I*(b*x + a)^3*d^3 + 36*I*b*c*d^2 - 36*I*a*d^3 + (72*I*b*c*d^2 - 72*I*a*d
^3)*(b*x + a)^2 + (72*I*b^2*c^2*d - 144*I*a*b*c*d^2 + (72*I*a^2 + 36*I)*d^3
)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (1
8*b*c*d^2 - 18*a*d^3 + 18*(b*c*d^2 - a*d^3)*cos(8*b*x + 8*a) - 36*(b*c*d^2
- a*d^3)*cos(4*b*x + 4*a) - (-18*I*b*c*d^2 + 18*I*a*d^3)*sin(8*b*x + 8*a) -
 (36*I*b*c*d^2 - 36*I*a*d^3)*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), cos(b*
x + a) - 1) + (12*(b*x + a)^3*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(
2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a) + 6*(2*(b*x + a)^3*d
```

^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 12*(2*(b*x + a)^3*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (12*I*(b*x + a)^3*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 18*I)*d^3)*(b*x + a))*sin(8*b*x + 8*a) + (-24*I*(b*x + a)^3*d^3 + (-72*I*b*c*d^2 + 72*I*a*d^3)*(b*x + a)^2 + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2 + (-72*I*a^2 - 36*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (24*I*(b*x + a)^3*d^3 + 36*b^2*c^2*d - 72*a*b*c*d^2 + 36*a^2*d^3 - 36*(-2*I*b*c*d^2 + (2*I*a - 1)*d^3)*(b*x + a)^2 + (72*I*b^2*c^2*d - 72*(2*I*a - 1)*b*c*d^2 + (72*I*a^2 - 72*a)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (24*I*(b*x + a)^3*d^3 - 36*b^2*c^2*d + 72*a*b*c*d^2 - 36*a^2*d^3 + (72*I*b*c*d^2 - 36*(2*I*a + 1)*d^3)*(b*x + a)^2 + (72*I*b^2*c^2*d - 72*(2*I*a + 1)*b*c*d^2 + (72*I*a^2 + 72*a)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (18*b^2*c^2*d - 36*a*b*c*d^2 + 24*(b*x + a)^2*d^3 + 9*(2*a^2 + 1)*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 3*(6*b^2*c^2*d - 12*a*b*c*d^2 + 8*(b*x + a)^2*d^3 + 3*(2*a^2 + 1)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 6*(6*b^2*c^2*d - 12*a*b*c*d^2 + 8*(b*x + a)^2*d^3 + 3*(2*a^2 + 1)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 24*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 9*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(8*b*x + 8*a) - (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 48*I*(b*x + a)^2*d^3 + (36*I*a^2 + 18*I)*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*dilog(-e^(2*I*b*x + 2*I*a)) + (36*b^2*c^2*d - 72*a*b*c*d^2 + 36*(b*x + a)^2*d^3 + 18*(2*a^2 + 1)*d^3 + 72*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 36*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*(b*x + a)^2*d^3 + (36*I*a^2 + 18*I)*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*sin(8*b*x + 8*a) + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2 - 72*I*(b*x + a)^2*d^3 + (-72*I*a^2 - 36*I)*d^3 + (-144*I*b*c*d^2 + 144*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*dilog(-e^(I*b*x + I*a)) + (36*b^2*c^2*d - 72*a*b*c*d^2 + 36*(b*x + a)^2*d^3 + 18*(2*a^2 + 1)*d^3 + 72*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 36*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*(b*x + a)^2*d^3 + (36*I*a^2 + 18*I)*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*sin(8*b*x + 8*a) + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2 - 72*I*(b*x + a)^2*d^3 + (-72*I*a^2 - 36*I)*d^3 + (-144*I*b*c*d^2 + 144*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*dilog(e^(I*b*x + I*a)) + (-8*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 9*I)*d^3)*(b*x + a) + (-8*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 9*I)*d^3)*(b*x + a))*cos(8*b*x + 8*a) + (16*I*(b*x + a)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 18*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (8*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*sin(8*b*x + 8*a) - 2*(8*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (6*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (18*I*a^2 + 9*I)*d^3)*(b*x + a) + (6*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (18*I*a^2 + 9*I)*d^3)*(b*x + a))*cos(8*b*x + 8*a) + (-12*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 + (-36*I*a^2 - 18*I)*d^3)*(b*x + a)

```
)*cos(4*b*x + 4*a) - 3*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b
*x + a))*sin(8*b*x + 8*a) + 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*
(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*
d^3)*(b*x + a))*sin(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*c
os(b*x + a) + 1) + (6*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (18*I*b
*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (18*I
*a^2 + 9*I)*d^3)*(b*x + a) + (6*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3
 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d
^2 + (18*I*a^2 + 9*I)*d^3)*(b*x + a))*cos(8*b*x + 8*a) + (-12*I*(b*x + a)^3
*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2
 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 + (-36*I*a^2 - 18*I)*d^3)*(b*x + a))*c
os(4*b*x + 4*a) - 3*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*d^2 -
 a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x
+ a))*sin(8*b*x + 8*a) + 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*
c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3
)*(b*x + a))*sin(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(
b*x + a) + 1) + (12*d^3*cos(8*b*x + 8*a) - 24*d^3*cos(4*b*x + 4*a) + 12*I*d
^3*sin(8*b*x + 8*a) - 24*I*d^3*sin(4*b*x + 4*a) + 12*d^3)*polylog(4, -e^(2*
I*b*x + 2*I*a)) - (72*d^3*cos(8*b*x + 8*a) - 144*d^3*cos(4*b*x + 4*a) + 72*
I*d^3*sin(8*b*x + 8*a) - 144*I*d^3*sin(4*b*x + 4*a) + 72*d^3)*polylog(4, -e
^(I*b*x + I*a)) - (72*d^3*cos(8*b*x + 8*a) - 144*d^3*cos(4*b*x + 4*a) + 72*
I*d^3*sin(8*b*x + 8*a) - 144*I*d^3*sin(4*b*x + 4*a) + 72*d^3)*polylog(4, e^
(I*b*x + I*a)) + (-18*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 18*I*a*d^3 + (-18*I*
b*c*d^2 - 24*I*(b*x + a)*d^3 + 18*I*a*d^3)*cos(8*b*x + 8*a) + (36*I*b*c*d^2
 + 48*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + 6*(3*b*c*d^2 + 4*(b*
x + a)*d^3 - 3*a*d^3)*sin(8*b*x + 8*a) - 12*(3*b*c*d^2 + 4*(b*x + a)*d^3 -
3*a*d^3)*sin(4*b*x + 4*a))*polylog(3, -e^(2*I*b*x + 2*I*a)) + (72*I*b*c*d^2
 + 72*I*(b*x + a)*d^3 - 72*I*a*d^3 + (72*I*b*c*d^2 + 72*I*(b*x + a)*d^3 - 7
2*I*a*d^3)*cos(8*b*x + 8*a) + (-144*I*b*c*d^2 - 144*I*(b*x + a)*d^3 + 144*I
*a*d^3)*cos(4*b*x + 4*a) - 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(8*b*x +
 8*a) + 144*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a))*polylog(3,
-e^(I*b*x + I*a)) + (72*I*b*c*d^2 + 72*I*(b*x + a)*d^3 - 72*I*a*d^3 + (72*I
*b*c*d^2 + 72*I*(b*x + a)*d^3 - 72*I*a*d^3)*cos(8*b*x + 8*a) + (-144*I*b*c*
d^2 - 144*I*(b*x + a)*d^3 + 144*I*a*d^3)*cos(4*b*x + 4*a) - 72*(b*c*d^2 + (
b*x + a)*d^3 - a*d^3)*sin(8*b*x + 8*a) + 144*(b*c*d^2 + (b*x + a)*d^3 - a*d
^3)*sin(4*b*x + 4*a))*polylog(3, e^(I*b*x + I*a)) - (24*(b*x + a)^3*d^3 - 3
6*I*b^2*c^2*d + 72*I*a*b*c*d^2 - 36*I*a^2*d^3 + (72*b*c*d^2 - (72*a + 36*I)
*d^3)*(b*x + a)^2 + (72*b^2*c^2*d - (144*a + 72*I)*b*c*d^2 + 72*(a^2 + I*a)
*d^3)*(b*x + a))*sin(6*b*x + 6*a) - (24*(b*x + a)^3*d^3 + 36*I*b^2*c^2*d -
72*I*a*b*c*d^2 + 36*I*a^2*d^3 + (72*b*c*d^2 - (72*a - 36*I)*d^3)*(b*x + a)^
2 + (72*b^2*c^2*d - (144*a - 72*I)*b*c*d^2 + 72*(a^2 - I*a)*d^3)*(b*x + a))
*sin(2*b*x + 2*a))/(-6*I*b^3*cos(8*b*x + 8*a) + 12*I*b^3*cos(4*b*x + 4*a) +
 6*b^3*sin(8*b*x + 8*a) - 12*b^3*sin(4*b*x + 4*a) - 6*I*b^3))/b
```

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.324 $\int (c+dx)^2 \csc^3(a+bx)\sec^3(a+bx)\,dx$

**Optimal**. Leaf size=190

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^3}-\frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(\right.}{b^2}$$

```
[Out] -4*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d^2*arctanh(cos(2*b*x+2*a))/b^3-2*
d*(d*x+c)*csc(2*b*x+2*a)/b^2-2*(d*x+c)^2*cot(2*b*x+2*a)*csc(2*b*x+2*a)/b+2*
I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,exp(2*
I*(b*x+a)))/b^2-d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+d^2*polylog(3,exp(2*I*
(b*x+a)))/b^3
```

**Rubi [A]** time = 0.21, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4419, 4186, 3770, 4183, 2531, 2282, 6589}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{d^2\text{PolyLog}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] (-4*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b - (d^2*ArcTanh[Cos[2*a + 2*
b*x]])/b^3 - (2*d*(c + d*x)*Csc[2*a + 2*b*x])/b^2 - (2*(c + d*x)^2*Cot[2*a
+ 2*b*x]*Csc[2*a + 2*b*x])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a +
 b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d
^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 + (d^2*PolyLog[3, E^((2*I)*(a + b*
x))])/b^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \csc^3(a+bx)\sec^3(a+bx)\,dx = 8\int (c+dx)^2\csc^3(2a+2bx)\,dx$$

$$= -\frac{2d(c+dx)\csc(2a+2bx)}{b^2} - \frac{2(c+dx)^2\cot(2a+2bx)\csc(2a+2bx)}{b} +$$

$$= -\frac{4(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3} - \frac{2d(c+d}{}$$

$$= -\frac{4(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3} - \frac{2d(c+d}{}$$

$$= -\frac{4(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3} - \frac{2d(c+d}{}$$

$$= -\frac{4(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3} - \frac{2d(c+d}{}$$

**Mathematica [B]** time = 8.18, size = 381, normalized size = 2.01

$$8\left(\frac{\csc(a)\csc(a+bx)\left(cd\sin(bx)+d^2x\sin(bx)\right)}{8b^2} + \frac{\sec(a)\sec(a+bx)\left(d^2(-x)\sin(bx)-cd\sin(bx)\right)}{8b^2} - \frac{d\csc(2a)(c}{4b^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] 8*(-1/4*(d*(c + d*x)*Csc[2*a])/b^2 + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a + b*
x]^2)/(16*b) - (4*b^2*c^2*ArcTanh[E^((2*I)*(a + b*x))] + 2*d^2*ArcTanh[E^((
2*I)*(a + b*x))] - 4*b^2*c*d*x*Log[1 - E^((2*I)*(a + b*x))] - 2*b^2*d^2*x^2
*Log[1 - E^((2*I)*(a + b*x))] + 4*b^2*c*d*x*Log[1 + E^((2*I)*(a + b*x))] +
2*b^2*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2,
 -E^((2*I)*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
] + d^2*PolyLog[3, -E^((2*I)*(a + b*x))] - d^2*PolyLog[3, E^((2*I)*(a + b*x
))])/(8*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a + b*x]^2)/(16*b) + (Sec[a]*
```

```
Sec[a + b*x]*(-(c*d*Sin[b*x]) - d^2*x*Sin[b*x]))/(8*b^2) + (Csc[a]*Csc[a +
b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x]))/(8*b^2))
```

**fricas [C]** time = 0.70, size = 2387, normalized size = 12.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - (
(-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^4 + (4*I*b*d^2*x + 4*I*b*c*d)*cos(b
*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) - ((4*I*b*d^2*x + 4*I*b*c*d
)*cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x
 + a) - I*sin(b*x + a)) - ((-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^4 + (4*I
*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a))
- ((4*I*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*d)*co
s(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) - ((4*I*b*d^2*x + 4*I*b*
c*d)*cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^2)*dilog(-I*c
os(b*x + a) + sin(b*x + a)) - ((-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^4 +
(4*I*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x +
 a)) - ((4*I*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*
d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - ((-4*I*b*d^2*x -
 4*I*b*c*d)*cos(b*x + a)^4 + (4*I*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^2)*dilo
g(-cos(b*x + a) - I*sin(b*x + a)) - ((2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c
^2 + d^2)*cos(b*x + a)^4 - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*
cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + ((2*b^2*c^2 - 4*a*
b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 +
 1)*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) - ((2*b^2*d
^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cos(b*x + a)^4 - (2*b^2*d^2*x^2 + 4
*b^2*c*d*x + 2*b^2*c^2 + d^2)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x
+ a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*
b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) - I
*sin(b*x + a) + I) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*c
os(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
 a)^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*((b^2*d^2*x^2 + 2*b^2*c*d
*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a
*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) +
2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^4 - (b^2*
d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-I*cos(b*x
 + a) + sin(b*x + a) + 1) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2
*d^2)*cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*co
s(b*x + a)^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - ((2*b^2*c^2 - 4*a*b
*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 +
1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) -
 ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4
*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*s
in(b*x + a) + 1/2) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*c
os(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
 a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d +
(2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)
*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - 2*((b^2*d^2*x^2
+ 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*
c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x
+ a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*
b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) -
I*sin(b*x + a) + I) - 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*polylog(3
, cos(b*x + a) + I*sin(b*x + a)) - 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)
^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 4*(d^2*cos(b*x + a)^4 - d^2
```

```
*cos(b*x + a)^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 4*(d^2*cos(b*x
 + a)^4 - d^2*cos(b*x + a)^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 4
*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*polylog(3, -I*cos(b*x + a) + sin
(b*x + a)) + 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*polylog(3, -I*cos(
b*x + a) - sin(b*x + a)) - 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*poly
log(3, -cos(b*x + a) + I*sin(b*x + a)) - 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*
x + a)^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^3*cos(b*x + a)^4 -
 b^3*cos(b*x + a)^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc(bx+a)^3 \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

**maple [B]** time = 0.16, size = 716, normalized size = 3.77

$$\frac{2d^2a^2\ln\left(e^{i(bx+a)}-1\right)}{b^3}+\frac{2d^2\ln\left(1-e^{i(bx+a)}\right)x^2}{b}-\frac{2d^2\ln\left(1-e^{i(bx+a)}\right)a^2}{b^3}+\frac{2d^2\ln\left(e^{i(bx+a)}+1\right)x^2}{b}-\frac{2c^2\ln\left(1+e^{2i(bx+}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

```
[Out] -d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+2/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+2/
b*d^2*ln(1-exp(I*(b*x+a)))*x^2-2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+2/b*d^2*l
n(exp(I*(b*x+a))+1)*x^2+4*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+4*d^2*polylog(
3,exp(I*(b*x+a)))/b^3-2/b*c^2*ln(1+exp(2*I*(b*x+a)))+2/b*c^2*ln(exp(I*(b*x+
a))-1)+2/b*c^2*ln(exp(I*(b*x+a))+1)-2/b*d^2*ln(1+exp(2*I*(b*x+a)))*x^2+4/b^
2/(1+exp(2*I*(b*x+a)))^2/(exp(2*I*(b*x+a))-1)^2*(d^2*x^2*b*exp(6*I*(b*x+a))
+2*c*d*x*b*exp(6*I*(b*x+a))+c^2*b*exp(6*I*(b*x+a))-I*d^2*x*exp(6*I*(b*x+a))
+b*d^2*x^2*exp(2*I*(b*x+a))-I*c*d*exp(6*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a
))+b*c^2*exp(2*I*(b*x+a))+I*d^2*x*exp(2*I*(b*x+a))+I*c*d*exp(2*I*(b*x+a)))-
4/b*c*d*ln(1+exp(2*I*(b*x+a)))*x+4/b*c*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c*d*l
n(1-exp(I*(b*x+a)))*a+4/b*c*d*ln(exp(I*(b*x+a))+1)*x-4/b^2*c*d*a*ln(exp(I*(
b*x+a))-1)-1/b^3*d^2*ln(1+exp(2*I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(b*x+a))+1)+
1/b^3*d^2*ln(exp(I*(b*x+a))-1)+2*I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-4
*I/b^2*polylog(2,-exp(I*(b*x+a)))*d^2*x-4*I/b^2*polylog(2,exp(I*(b*x+a)))*d
^2*x+2*I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))-4*I/b^2*c*d*polylog(2,-exp(I*
(b*x+a)))-4*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))
```

**maxima [B]** time = 1.10, size = 2722, normalized size = 14.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log
(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2)) - 2*a*c*d*((2*sin(b*x + a)^2
- 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(
sin(b*x + a)^2))/b + a^2*d^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(
b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^2 + 2*((
4*(b*x + a)^2*d^2 + 8*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*(2*(b*x + a)^2*
d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(8*b*x + 8*a) - 4*(2*(b*x + a)^
2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(4*b*x + 4*a) + (4*I*(b*x + a
```

```
)^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(8*b*x + 8*a) + (
-8*I*(b*x + a)^2*d^2 + (-16*I*b*c*d + 16*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(
4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (4*(b*x + a
)^2*d^2 + 8*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*(2*(b*x + a)^2*d^2 + 4*(b
*c*d - a*d^2)*(b*x + a) + d^2)*cos(8*b*x + 8*a) - 4*(2*(b*x + a)^2*d^2 + 4*
(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(4*b*x + 4*a) - (-4*I*(b*x + a)^2*d^2 +
 (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(8*b*x + 8*a) - (8*I*(b*x
 + a)^2*d^2 + (16*I*b*c*d - 16*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(4*b*x + 4*
a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*d^2*cos(8*b*x + 8*a) - 4*d
^2*cos(4*b*x + 4*a) + 2*I*d^2*sin(8*b*x + 8*a) - 4*I*d^2*sin(4*b*x + 4*a) +
 2*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (4*(b*x + a)^2*d^2 + 8*(b
*c*d - a*d^2)*(b*x + a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))
*cos(8*b*x + 8*a) - 8*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4
*b*x + 4*a) + (4*I*(b*x + a)^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a))*sin
(8*b*x + 8*a) + (-8*I*(b*x + a)^2*d^2 + (-16*I*b*c*d + 16*I*a*d^2)*(b*x + a
))*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 8*(-I*(b*x
+ a)^2*d^2 - b*c*d + a*d^2 + (-2*I*b*c*d + (2*I*a - 1)*d^2)*(b*x + a))*cos(
6*b*x + 6*a) + (8*I*(b*x + a)^2*d^2 - 8*b*c*d + 8*a*d^2 + (16*I*b*c*d - 8*(
2*I*a + 1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (4*b*c*d + 4*(b*x + a)*d^2 -
4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(8*b*x + 8*a) - 8*(b*c*d + (
b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 +
4*I*a*d^2)*sin(8*b*x + 8*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*s
in(4*b*x + 4*a))*dilog(-e^(2*I*b*x + 2*I*a)) + (8*b*c*d + 8*(b*x + a)*d^2 -
 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(8*b*x + 8*a) - 16*(b*c*d +
 (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + (8*I*b*c*d + 8*I*(b*x + a)*d^2 -
 8*I*a*d^2)*sin(8*b*x + 8*a) + (-16*I*b*c*d - 16*I*(b*x + a)*d^2 + 16*I*a*d
^2)*sin(4*b*x + 4*a))*dilog(-e^(I*b*x + I*a)) + (8*b*c*d + 8*(b*x + a)*d^2
- 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(8*b*x + 8*a) - 16*(b*c*d
+ (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + (8*I*b*c*d + 8*I*(b*x + a)*d^2
- 8*I*a*d^2)*sin(8*b*x + 8*a) + (-16*I*b*c*d - 16*I*(b*x + a)*d^2 + 16*I*a*
d^2)*sin(4*b*x + 4*a))*dilog(e^(I*b*x + I*a)) + (-2*I*(b*x + a)^2*d^2 + (-4
*I*b*c*d + 4*I*a*d^2)*(b*x + a) - I*d^2 + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c
*d + 4*I*a*d^2)*(b*x + a) - I*d^2)*cos(8*b*x + 8*a) + (4*I*(b*x + a)^2*d^2
+ (8*I*b*c*d - 8*I*a*d^2)*(b*x + a) + 2*I*d^2)*cos(4*b*x + 4*a) + (2*(b*x +
 a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(8*b*x + 8*a) - 2*(2*(b*x
 + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(4*b*x + 4*a))*log(cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (2*I*(b*x +
 a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2 + (2*I*(b*x + a)^2*d^
2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2)*cos(8*b*x + 8*a) + (-4*I*(b*
x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(4*b*x + 4*
a) - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(8*b*x + 8*
a) + 2*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(4*b*x +
4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*I*(b*x
 + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2 + (2*I*(b*x + a)^2*
d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2)*cos(8*b*x + 8*a) + (-4*I*(
b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(4*b*x +
4*a) - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(8*b*x +
8*a) + 2*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(4*b*x
+ 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (-2*I*d
^2*cos(8*b*x + 8*a) + 4*I*d^2*cos(4*b*x + 4*a) + 2*d^2*sin(8*b*x + 8*a) - 4
*d^2*sin(4*b*x + 4*a) - 2*I*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + (8*I*d^
2*cos(8*b*x + 8*a) - 16*I*d^2*cos(4*b*x + 4*a) - 8*d^2*sin(8*b*x + 8*a) + 1
6*d^2*sin(4*b*x + 4*a) + 8*I*d^2)*polylog(3, -e^(I*b*x + I*a)) + (8*I*d^2*c
os(8*b*x + 8*a) - 16*I*d^2*cos(4*b*x + 4*a) - 8*d^2*sin(8*b*x + 8*a) + 16*d
^2*sin(4*b*x + 4*a) + 8*I*d^2)*polylog(3, e^(I*b*x + I*a)) - (8*(b*x + a)^2
*d^2 - 8*I*b*c*d + 8*I*a*d^2 + (16*b*c*d - (16*a + 8*I)*d^2)*(b*x + a))*sin
(6*b*x + 6*a) - (8*(b*x + a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (16*b*c*d - (1
6*a - 8*I)*d^2)*(b*x + a))*sin(2*b*x + 2*a))/(-2*I*b^2*cos(8*b*x + 8*a) + 4
*I*b^2*cos(4*b*x + 4*a) + 2*b^2*sin(8*b*x + 8*a) - 4*b^2*sin(4*b*x + 4*a) -
```

```
2*I*b^2))/b
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)^3),x)
[Out] \text{Hanged}
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a)**3,x)
[Out] Timed out
```

## 3.325 $\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx$

**Optimal**. Leaf size=110

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{d\csc(2a+2bx)}{b^2}-\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{2(c+dx)\cot(2a+2bx)\csc(}{b}$$

```
[Out] -4*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b-d*csc(2*b*x+2*a)/b^2-2*(d*x+c)*cot(2
*b*x+2*a)*csc(2*b*x+2*a)/b+I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*polylog
(2,exp(2*I*(b*x+a)))/b^2
```

**Rubi** [A] time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4419, 4185, 4183, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d\csc(2a+2bx)}{b^2}-\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{2(c+dx)\text{cc}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] (-4*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*Csc[2*a + 2*b*x])/b^2 -
(2*(c + d*x)*Cot[2*a + 2*b*x]*Csc[2*a + 2*b*x])/b + (I*d*PolyLog[2, -E^((2*
I)*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx = 8\int (c+dx)\csc^3(2a+2bx)\,dx$$
$$= -\frac{d\csc(2a+2bx)}{b^2} - \frac{2(c+dx)\cot(2a+2bx)\csc(2a+2bx)}{b} + 4\int (c+d$$
$$= -\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\csc(2a+2bx)}{b^2} - \frac{2(c+dx)\cot(2a+2b}{b}$$
$$= -\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\csc(2a+2bx)}{b^2} - \frac{2(c+dx)\cot(2a+2b}{b}$$
$$= -\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\csc(2a+2bx)}{b^2} - \frac{2(c+dx)\cot(2a+2b}{b}$$

**Mathematica [B]** time = 2.09, size = 236, normalized size = 2.15

$$\frac{d\left(i\left(\text{Li}_2\left(-e^{2i(a+bx)}\right)-\text{Li}_2\left(e^{2i(a+bx)}\right)\right)+2(a+bx)\left(\log\left(1-e^{2i(a+bx)}\right)-\log\left(1+e^{2i(a+bx)}\right)\right)\right)}{b^2}-\frac{d\tan(a+bx)}{2b^2}-\frac{d\cot(a}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] -1/2*(d*Cot[a + b*x])/b^2 - (c*Csc[a + b*x]^2)/(2*b) + (d*(2*a - 2*(a + b*x
))*Csc[a + b*x]^2)/(4*b^2) - (2*c*Log[Cos[a + b*x]])/b + (2*c*Log[Sin[a + b
*x]])/b - (2*a*d*Log[Tan[a + b*x]])/b^2 + (d*(2*(a + b*x)*(Log[1 - E^((2*I)
*(a + b*x))] - Log[1 + E^((2*I)*(a + b*x))]) + I*(PolyLog[2, -E^((2*I)*(a +
 b*x))] - PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 + (c*Sec[a + b*x]^2)/(2*b)
 + (d*(-2*a + 2*(a + b*x))*Sec[a + b*x]^2)/(4*b^2) - (d*Tan[a + b*x])/(2*b^
2)
```

**fricas [B]** time = 0.60, size = 1193, normalized size = 10.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*sin(b*x + a)
+ b*c - (-2*I*d*cos(b*x + a)^4 + 2*I*d*cos(b*x + a)^2)*dilog(cos(b*x + a) +
 I*sin(b*x + a)) - (2*I*d*cos(b*x + a)^4 - 2*I*d*cos(b*x + a)^2)*dilog(cos(
b*x + a) - I*sin(b*x + a)) - (-2*I*d*cos(b*x + a)^4 + 2*I*d*cos(b*x + a)^2)
*dilog(I*cos(b*x + a) + sin(b*x + a)) - (2*I*d*cos(b*x + a)^4 - 2*I*d*cos(b
*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) - (2*I*d*cos(b*x + a)^4 - 2
*I*d*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - (-2*I*d*cos(b*
x + a)^4 + 2*I*d*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (2
*I*d*cos(b*x + a)^4 - 2*I*d*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x
 + a)) - (-2*I*d*cos(b*x + a)^4 + 2*I*d*cos(b*x + a)^2)*dilog(-cos(b*x + a)
 - I*sin(b*x + a)) - 2*((b*d*x + b*c)*cos(b*x + a)^4 - (b*d*x + b*c)*cos(b*
x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x +
 a)^4 - (b*c - a*d)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I)
- 2*((b*d*x + b*c)*cos(b*x + a)^4 - (b*d*x + b*c)*cos(b*x + a)^2)*log(cos(b
*x + a) - I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^4 - (b*c - a*d)
*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 2*((b*d*x + a*d)*
cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(I*cos(b*x + a) + sin(b*x
 + a) + 1) + 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 2*((b*d*x + a*d)*cos(b*x + a)^4
- (b*d*x + a*d)*cos(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2
```

```
*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(-I*cos(b
*x + a) - sin(b*x + a) + 1) - 2*((b*c - a*d)*cos(b*x + a)^4 - (b*c - a*d)*c
os(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*((b*c
- a*d)*cos(b*x + a)^4 - (b*c - a*d)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) -
 1/2*I*sin(b*x + a) + 1/2) - 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d
)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 2*((b*c - a*d)*
cos(b*x + a)^4 - (b*c - a*d)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x
+ a) + I) - 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)
*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^4 -
(b*c - a*d)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/(b^2*c
os(b*x + a)^4 - b^2*cos(b*x + a)^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\csc(bx+a)^3\sec(bx+a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

**maple [B]** time = 0.17, size = 325, normalized size = 2.95

$$\frac{4bdx\,e^{6i(bx+a)} + 4cb\,e^{6i(bx+a)} - 2id\,e^{6i(bx+a)} + 4bdx\,e^{2i(bx+a)} + 4bc\,e^{2i(bx+a)} + 2id\,e^{2i(bx+a)}}{b^2\left(1+e^{2i(bx+a)}\right)^2\left(e^{2i(bx+a)}-1\right)^2} + \frac{2c\ln\left(e^{i(bx+a)}-1\right)}{b} - \frac{2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

```
[Out] 2/b^2/(1+exp(2*I*(b*x+a)))^2/(exp(2*I*(b*x+a))-1)^2*(2*b*d*x*exp(6*I*(b*x+a
))+2*c*b*exp(6*I*(b*x+a))-I*d*exp(6*I*(b*x+a))+2*b*d*x*exp(2*I*(b*x+a))+2*b
*c*exp(2*I*(b*x+a))+I*d*exp(2*I*(b*x+a)))+2/b*c*ln(exp(I*(b*x+a))-1)-2/b*c*
ln(1+exp(2*I*(b*x+a)))+2/b*c*ln(exp(I*(b*x+a))+1)-2/b*d*ln(1+exp(2*I*(b*x+a
)))*x+I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b*d*ln(exp(I*(b*x+a))+1)*x-2*I
*d*polylog(2,-exp(I*(b*x+a)))/b^2+2/b*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*d*ln(1
-exp(I*(b*x+a)))*a-2*I/b^2*d*polylog(2,exp(I*(b*x+a)))-2/b^2*d*a*ln(exp(I*(
b*x+a))-1)
```

**maxima [B]** time = 0.75, size = 1078, normalized size = 9.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(8*b*x + 8*a) - 4*(b*d*x + b*c)*cos
(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*sin(8*b*x + 8*a) + (-4*I*b*d*x - 4*I*
b*c)*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (2
*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(8*b*x + 8*a) - 4*(b*d*x + b*c)*cos(4*b
*x + 4*a) - (-2*I*b*d*x - 2*I*b*c)*sin(8*b*x + 8*a) - (4*I*b*d*x + 4*I*b*c)
*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos(8*b
*x + 8*a) - 4*b*c*cos(4*b*x + 4*a) + 2*I*b*c*sin(8*b*x + 8*a) - 4*I*b*c*sin
(4*b*x + 4*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*c
os(8*b*x + 8*a) - 4*b*d*x*cos(4*b*x + 4*a) + 2*I*b*d*x*sin(8*b*x + 8*a) - 4
*I*b*d*x*sin(4*b*x + 4*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) +
1) + (4*I*b*d*x + 4*I*b*c + 2*d)*cos(6*b*x + 6*a) + (4*I*b*d*x + 4*I*b*c -
2*d)*cos(2*b*x + 2*a) - (d*cos(8*b*x + 8*a) - 2*d*cos(4*b*x + 4*a) + I*d*si
n(8*b*x + 8*a) - 2*I*d*sin(4*b*x + 4*a) + d)*dilog(-e^(2*I*b*x + 2*I*a)) +
```

```
(2*d*cos(8*b*x + 8*a) - 4*d*cos(4*b*x + 4*a) + 2*I*d*sin(8*b*x + 8*a) - 4*I
*d*sin(4*b*x + 4*a) + 2*d)*dilog(-e^(I*b*x + I*a)) + (2*d*cos(8*b*x + 8*a)
- 4*d*cos(4*b*x + 4*a) + 2*I*d*sin(8*b*x + 8*a) - 4*I*d*sin(4*b*x + 4*a) +
2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(8*
b*x + 8*a) + (2*I*b*d*x + 2*I*b*c)*cos(4*b*x + 4*a) + (b*d*x + b*c)*sin(8*b
*x + 8*a) - 2*(b*d*x + b*c)*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*
b*c)*cos(8*b*x + 8*a) + (-2*I*b*d*x - 2*I*b*c)*cos(4*b*x + 4*a) - (b*d*x +
b*c)*sin(8*b*x + 8*a) + 2*(b*d*x + b*c)*sin(4*b*x + 4*a))*log(cos(b*x + a)^
2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*
b*c)*cos(8*b*x + 8*a) + (-2*I*b*d*x - 2*I*b*c)*cos(4*b*x + 4*a) - (b*d*x +
b*c)*sin(8*b*x + 8*a) + 2*(b*d*x + b*c)*sin(4*b*x + 4*a))*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (4*b*d*x + 4*b*c - 2*I*d)*sin(6*
b*x + 6*a) - (4*b*d*x + 4*b*c + 2*I*d)*sin(2*b*x + 2*a))/(-I*b^2*cos(8*b*x
+ 8*a) + 2*I*b^2*cos(4*b*x + 4*a) + b^2*sin(8*b*x + 8*a) - 2*b^2*sin(4*b*x
+ 4*a) - I*b^2)
```

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.326 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

**Optimal**. Leaf size=24

$$8\text{Int}\left(\frac{\csc^3(2a+2bx)}{c+dx},x\right)$$

```
[Out] 8*Unintegrable(csc(2*b*x+2*a)^3/(d*x+c),x)
```

**Rubi [A]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x),x]
[Out] 8*Defer[Int][Csc[2*a + 2*b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = 8\int \frac{\csc^3(2a+2bx)}{c+dx}\,dx$$

**Mathematica [A]** time = 28.70, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x), x]
```

**fricas [A]** time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^3\sec(bx+a)^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^3\sec(bx+a)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c), x)
```

**maple** [**A**] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^3(bx+a)\right)\left(\sec^3(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x)
```

**maxima** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad** [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)^3\,\sin(a+b\,x)^3\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)), x)
```

**sympy** [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**3/(c + d*x), x)
```

## 3.327 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=24

$$8\text{Int}\left(\frac{\csc^3(2a+2bx)}{(c+dx)^2},x\right)$$

```
[Out] 8*Unintegrable(csc(2*b*x+2*a)^3/(d*x+c)^2,x)
```

**Rubi** [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2,x]
[Out] 8*Defer[Int][Csc[2*a + 2*b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = 8\int \frac{\csc^3(2a+2bx)}{(c+dx)^2}\,dx$$

**Mathematica** [A] time = 32.47, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

**fricas** [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
[Out] Timed out
```

**maple** [A] time = 3.79, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^3(bx+a)\right)\left(\sec^3(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x)
[Out] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)^3\,\sin(a+b\,x)^3\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**3/(d*x+c)**2,x)
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**3/(c + d*x)**2, x)
```

# 3.328 $\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=83

$$\frac{20F\left(\frac{1}{2}(a+bx)\middle|2\right)}{147b^2}+\frac{4\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{20\sin(a+bx)\sqrt{\cos(a+bx)}}{147b^2}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

```
[Out] -2/7*x*cos(b*x+a)^(7/2)/b+20/147*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1
/2*a)*EllipticF(sin(1/2*b*x+1/2*a),2^(1/2))/b^2+4/49*cos(b*x+a)^(5/2)*sin(b
*x+a)/b^2+20/147*sin(b*x+a)*cos(b*x+a)^(1/2)/b^2
```

**Rubi** [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2635, 2641}

$$\frac{20F\left(\frac{1}{2}(a+bx)\middle|2\right)}{147b^2}+\frac{4\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{20\sin(a+bx)\sqrt{\cos(a+bx)}}{147b^2}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]^(5/2)*Sin[a + b*x],x]
```

```
[Out] (-2*x*Cos[a + b*x]^(7/2))/(7*b) + (20*EllipticF[(a + b*x)/2, 2])/(147*b^2)
+ (20*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(147*b^2) + (4*Cos[a + b*x]^(5/2)*Si
n[a + b*x])/(49*b^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx &= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\int\cos^{\frac{7}{2}}(a+bx)\,dx}{7b}\\ &= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}+\frac{4\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{49b^2}+\frac{10\int\cos^{\frac{3}{2}}(a+bx)\,dx}{49b}\\ &= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}+\frac{20\sqrt{\cos(a+bx)}\,\sin(a+bx)}{147b^2}+\frac{4\cos^{\frac{5}{2}}(a+bx)\sin(a+\\ &= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}+\frac{20F\left(\frac{1}{2}(a+bx)\middle|2\right)}{147b^2}+\frac{20\sqrt{\cos(a+bx)}\,\sin(a+bx)}{147b^2}+\frac{4}{}\end{aligned}$$

**Mathematica** [A] time = 0.33, size = 73, normalized size = 0.88

$$\frac{40F\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}\,(46\sin(a+bx)+6\sin(3(a+bx))-63bx\cos(a+bx)-21bx\cos(3(a+bx)))}{294b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]^(5/2)*Sin[a + b*x],x]
```

```
[Out] (40*EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*(-63*b*x*Cos[a + b*x] -
21*b*x*Cos[3*(a + b*x)] + 46*Sin[a + b*x] + 6*Sin[3*(a + b*x)]))/(294*b^2)
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cos\left(bx+a\right)^{\frac{5}{2}} \sin\left(bx+a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)^(5/2)*sin(b*x + a), x)
```

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x \left(\cos^{\frac{5}{2}}\left(bx+a\right)\right) \sin\left(bx+a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^(5/2)*sin(b*x+a),x)
```

```
[Out] int(x*cos(b*x+a)^(5/2)*sin(b*x+a),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cos\left(bx+a\right)^{\frac{5}{2}} \sin\left(bx+a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)^(5/2)*sin(b*x + a), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\cos\left(a+b\,x\right)}^{5/2}\,\sin\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)^(5/2)*sin(a + b*x),x)
```

```
[Out] int(x*cos(a + b*x)^(5/2)*sin(a + b*x), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(5/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

# 3.329 $\int x\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=60

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}+\frac{4\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{25b^2}-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] -2/5*x*cos(b*x+a)^(5/2)/b+12/25*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/
2*a)*EllipticE(sin(1/2*b*x+1/2*a),2^(1/2))/b^2+4/25*cos(b*x+a)^(3/2)*sin(b*
x+a)/b^2
```

**Rubi [A]** time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2635, 2639}

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}+\frac{4\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{25b^2}-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]^(3/2)*Sin[a + b*x],x]
```

```
[Out] (-2*x*Cos[a + b*x]^(5/2))/(5*b) + (12*EllipticE[(a + b*x)/2, 2])/(25*b^2) +
 (4*Cos[a + b*x]^(3/2)*Sin[a + b*x])/(25*b^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx&=-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int\cos^{\frac{5}{2}}(a+bx)\,dx}{5b}\\&=-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{25b^2}+\frac{6\int\sqrt{\cos(a+bx)}\,dx}{25b}\\&=-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}+\frac{4\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{25b^2}\end{aligned}$$

**Mathematica [A]** time = 0.37, size = 51, normalized size = 0.85

$$-\frac{2\left(\cos^{\frac{3}{2}}(a+bx)(5bx\cos(a+bx)-2\sin(a+bx))-6E\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{25b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]^(3/2)*Sin[a + b*x],x]
```

```
[Out] (-2*(-6*EllipticE[(a + b*x)/2, 2] + Cos[a + b*x]^(3/2)*(5*b*x*Cos[a + b*x]
- 2*Sin[a + b*x])))/(25*b^2)
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(bx+a\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2)*sin(b*x + a), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int x\left(\cos^{\frac{3}{2}}\left(bx+a\right)\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^(3/2)*sin(b*x+a),x)
```

```
[Out] int(x*cos(b*x+a)^(3/2)*sin(b*x+a),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(bx+a\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2)*sin(b*x + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x\,{\cos\left(a+b\,x\right)}^{3/2}\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)^(3/2)*sin(a + b*x),x)
```

```
[Out] int(x*cos(a + b*x)^(3/2)*sin(a + b*x), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(3/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

# 3.330 $\int x\sqrt{\cos(a+bx)}\,\sin(a+bx)\,dx$

**Optimal**. Leaf size=60

$$\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2}+\frac{4\sin(a+bx)\sqrt{\cos(a+bx)}}{9b^2}-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] -2/3*x*cos(b*x+a)^(3/2)/b+4/9*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*
a)*EllipticF(sin(1/2*b*x+1/2*a),2^(1/2))/b^2+4/9*sin(b*x+a)*cos(b*x+a)^(1/2
)/b^2
```

**Rubi** [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2635, 2641}

$$\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2}+\frac{4\sin(a+bx)\sqrt{\cos(a+bx)}}{9b^2}-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Cos[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (-2*x*Cos[a + b*x]^(3/2))/(3*b) + (4*EllipticF[(a + b*x)/2, 2])/(9*b^2) + (
4*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(9*b^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{\cos(a+bx)}\,\sin(a+bx)\,dx&=-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int\cos^{\frac{3}{2}}(a+bx)\,dx}{3b}\\&=-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}+\frac{4\sqrt{\cos(a+bx)}\,\sin(a+bx)}{9b^2}+\frac{2\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{9b}\\&=-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}+\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2}+\frac{4\sqrt{\cos(a+bx)}\,\sin(a+bx)}{9b^2}\end{aligned}$$

**Mathematica** [**A**] time = 0.15, size = 52, normalized size = 0.87

$$\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)+2\sqrt{\cos(a+bx)}\,(2\sin(a+bx)-3bx\cos(a+bx))}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Cos[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (4*EllipticF[(a + b*x)/2, 2] + 2*Sqrt[Cos[a + b*x]]*(-3*b*x*Cos[a + b*x] +
2*Sin[a + b*x]))/(9*b^2)
```

**fricas** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\cos(bx+a)}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cos(b*x + a))*sin(b*x + a), x)
```

**maple** [**F**] time = 0.04, size = 0, normalized size = 0.00

$$\int x\sin(bx+a)\left(\sqrt{\cos}\,(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)*cos(b*x+a)^(1/2),x)
```

```
[Out] int(x*sin(b*x+a)*cos(b*x+a)^(1/2),x)
```

**maxima** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\cos(bx+a)}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(cos(b*x + a))*sin(b*x + a), x)
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int x\sqrt{\cos(a+b\,x)}\,\sin(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)^(1/2)*sin(a + b*x),x)
```

```
[Out] int(x*cos(a + b*x)^(1/2)*sin(a + b*x), x)
```

**sympy [F]**     time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin\left(a+bx\right)\sqrt{\cos\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sin(a + b*x)*sqrt(cos(a + b*x)), x)
```

# 3.331 $\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx$

**Optimal**. Leaf size=33

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x\sqrt{\cos(a+bx)}}{b}$$

```
[Out] 4*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*EllipticE(sin(1/2*b*x+1/2
*a),2^(1/2))/b^2-2*x*cos(b*x+a)^(1/2)/b
```

**Rubi [A]** time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3444, 2639}

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x\sqrt{\cos(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (-2*x*Sqrt[Cos[a + b*x]])/b + (4*EllipticE[(a + b*x)/2, 2])/b^2
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx &= -\frac{2x\sqrt{\cos(a+bx)}}{b}+\frac{2\int \sqrt{\cos(a+bx)}\,dx}{b}\\ &= -\frac{2x\sqrt{\cos(a+bx)}}{b}+\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}\end{aligned}$$

**Mathematica [B]** time = 1.76, size = 181, normalized size = 5.48

$$\frac{4\cos^2\left(\frac{1}{2}(a+bx)\right)^{3/2}\sqrt{\frac{\cos(a+bx)}{(\cos(a+bx)+1)^2}}\sqrt{\frac{1}{\cos(a+bx)+1}}\left(\left(2\tan\left(\frac{1}{2}(a+bx)\right)-bx\right)\sqrt{\cos(a+bx)\sec^2\left(\frac{1}{2}(a+bx)\right)}-2\sqrt{\ldots}\right.}{b^2\sqrt{\frac{\cos(a+\ldots}{\cos(a+b\ldots}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sin[a + b*x])/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (4*(Cos[(a + b*x)/2]^2)^(3/2)*Sqrt[Cos[a + b*x]/(1 + Cos[a + b*x])^2]*Sqrt[
(1 + Cos[a + b*x])^(-1)]*(2*EllipticE[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Se
```

```
c[(a + b*x)/2]^2] - 2*EllipticF[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Sec[(a +
 b*x)/2]^2] + Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^2]*(-(b*x) + 2*Tan[(a + b*
x)/2])))/(b^2*Sqrt[Cos[a + b*x]/(1 + Cos[a + b*x])])
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\sqrt{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(cos(b*x + a)), x)
```

**maple [C]** time = 0.16, size = 310, normalized size = 9.39

$$-\frac{(bx+2i)\left(1+e^{2i(bx+a)}\right)\sqrt{2}\,e^{-i(bx+a)}}{b^{2}\sqrt{\left(1+e^{2i(bx+a)}\right)e^{-i(bx+a)}}}-\frac{2i\left(-\frac{2\left(1+e^{2i(bx+a)}\right)}{\sqrt{\left(1+e^{2i(bx+a)}\right)e^{i(bx+a)}}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(bx+a)}-i\right)}\,\sqrt{ie^{i(bx+a)}}\left(-2i\,\text{Elliptic}\right.}{\sqrt{e^{3i(bx+a)}}}\right.}{b^{2}\sqrt{\left(1\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(1/2),x)
```

```
[Out] -(b*x+2*I)*(exp(I*(b*x+a))^2+1)/b^2*2^(1/2)/((exp(I*(b*x+a))^2+1)/exp(I*(b*
x+a)))^(1/2)/exp(I*(b*x+a))-2*I/b^2*(-2*(exp(I*(b*x+a))^2+1)/((exp(I*(b*x+a
))^2+1)*exp(I*(b*x+a)))^(1/2)+I*(-I*(exp(I*(b*x+a))+I))^(1/2)*2^(1/2)*(I*(e
xp(I*(b*x+a))-I))^(1/2)*(I*exp(I*(b*x+a)))^(1/2)/(exp(I*(b*x+a))^3+exp(I*(b
*x+a)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))+I*
EllipticF((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))))*2^(1/2)/((exp(I*(b*x
+a))^2+1)/exp(I*(b*x+a)))^(1/2)*((exp(I*(b*x+a))^2+1)*exp(I*(b*x+a)))^(1/2)
/exp(I*(b*x+a))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\sqrt{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(cos(b*x + a)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,\sin(a+b\,x)}{\sqrt{\cos(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(1/2),x)
```

```
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/sqrt(cos(a + b*x)), x)
```

# 3.332 $\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

**Optimal**. Leaf size=33

$$\frac{2x}{b\sqrt{\cos(a+bx)}}-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

```
[Out] -4*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*EllipticF(sin(1/2*b*x+1/
2*a),2^(1/2))/b^2+2*x/b/cos(b*x+a)^(1/2)
```

**Rubi** [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3444, 2641}

$$\frac{2x}{b\sqrt{\cos(a+bx)}}-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(3/2),x]
[Out] (2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2x}{b\sqrt{\cos(a+bx)}}-\frac{2\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{b}\\ &= \frac{2x}{b\sqrt{\cos(a+bx)}}-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}\end{aligned}$$

**Mathematica** [A] time = 0.16, size = 33, normalized size = 1.00

$$\frac{2x}{b\sqrt{\cos(a+bx)}}-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(3/2),x]
[Out] (2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(3/2), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)
```

```
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(3/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,\sin(a+b\,x)}{{\cos(a+b\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(3/2),x)
```

```
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/cos(a + b*x)**(3/2), x)
```

## 3.333 $\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx$

**Optimal**. Leaf size=60

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}-\frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}+\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}$$

```
[Out] 2/3*x/b/cos(b*x+a)^(3/2)+4/3*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a
)*EllipticE(sin(1/2*b*x+1/2*a),2^(1/2))/b^2-4/3*sin(b*x+a)/b^2/cos(b*x+a)^(
1/2)
```

**Rubi [A]** time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2636, 2639}

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}-\frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}+\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(5/2),x]
```

```
[Out] (2*x)/(3*b*Cos[a + b*x]^(3/2)) + (4*EllipticE[(a + b*x)/2, 2])/(3*b^2) - (4
*Sin[a + b*x])/(3*b^2*Sqrt[Cos[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx &= \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\
&= \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}-\frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}+\frac{2\int \sqrt{\cos(a+bx)}\,dx}{3b}\\
&= \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}+\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}-\frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.19, size = 54, normalized size = 0.90

$$\frac{2\left(-\sin(2(a+bx))+2\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)+bx\right)}{3b^2\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(5/2),x]
```

```
[Out] (2*(b*x + 2*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2] - Sin[2*(a + b*x)]
))/(3*b^2*Cos[a + b*x]^(3/2))
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(5/2), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(5/2),x)
```

```
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(5/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sin\left(a+b\,x\right)}{{\cos\left(a+b\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(5/2),x)
```

```
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(5/2), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

## 3.334 $\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx$

**Optimal**. Leaf size=60

$$-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2}-\frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}+\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}$$

```
[Out] 2/5*x/b/cos(b*x+a)^(5/2)-4/15*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*
a)*EllipticF(sin(1/2*b*x+1/2*a),2^(1/2))/b^2-4/15*sin(b*x+a)/b^2/cos(b*x+a)
^(3/2)
```

**Rubi [A]** time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2636, 2641}

$$-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2}-\frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}+\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(7/2),x]
```

```
[Out] (2*x)/(5*b*Cos[a + b*x]^(5/2)) - (4*EllipticF[(a + b*x)/2, 2])/(15*b^2) - (
4*Sin[a + b*x])/(15*b^2*Cos[a + b*x]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx}{5b}$$

$$= \frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{15b}$$

$$= \frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2} - \frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}$$

**Mathematica** [A] time = 0.23, size = 53, normalized size = 0.88

$$-\frac{2\left(\sin(2(a+bx)) + 2\cos^{\frac{5}{2}}(a+bx)F\left(\frac{1}{2}(a+bx)\middle|2\right) - 3bx\right)}{15b^2\cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(7/2),x]
```

```
[Out] (-2*(-3*b*x + 2*Cos[a + b*x]^(5/2)*EllipticF[(a + b*x)/2, 2] + Sin[2*(a + b
*x)]))/(15*b^2*Cos[a + b*x]^(5/2))
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(7/2), x)
```

**maple** [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(7/2),x)
```

```
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(7/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(7/2), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sin(a+b\,x)}{\cos(a+b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(7/2),x)
```

```
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(7/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

# 3.335 $\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx$

**Optimal**. Leaf size=83

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2}-\frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)}-\frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}}+\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}$$

```
[Out] 2/7*x/b/cos(b*x+a)^(7/2)+12/35*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2
*a)*EllipticE(sin(1/2*b*x+1/2*a),2^(1/2))/b^2-4/35*sin(b*x+a)/b^2/cos(b*x+a
)^(5/2)-12/35*sin(b*x+a)/b^2/cos(b*x+a)^(1/2)
```

**Rubi** [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2636, 2639}

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2}-\frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)}-\frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}}+\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(9/2),x]
```

```
[Out] (2*x)/(7*b*Cos[a + b*x]^(7/2)) + (12*EllipticE[(a + b*x)/2, 2])/(35*b^2) -
(4*Sin[a + b*x])/(35*b^2*Cos[a + b*x]^(5/2)) - (12*Sin[a + b*x])/(35*b^2*Sq
rt[Cos[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx = \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$
$$= \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{6\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx}{35b}$$
$$= \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}} + \frac{6\int\sqrt{\cos(a+bx)}\,dx}{35b}$$
$$= \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} + \frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}}$$

**Mathematica [A]** time = 0.26, size = 65, normalized size = 0.78

$$\frac{-10\sin(2(a+bx)) - 3\sin(4(a+bx)) + 24\cos^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right) + 20bx}{70b^2\cos^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(9/2),x]
```

```
[Out] (20*b*x + 24*Cos[a + b*x]^(7/2)*EllipticE[(a + b*x)/2, 2] - 10*Sin[2*(a + b
*x)] - 3*Sin[4*(a + b*x)])/(70*b^2*Cos[a + b*x]^(7/2))
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(9/2), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(9/2),x)
```

```
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(9/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="maxima")
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(9/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin(a+b\,x)}{\cos(a+b\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(9/2),x)
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(9/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(9/2),x)
[Out] Timed out
```

# 3.336 $\int x \sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=103

$$-\frac{4\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{12\sin(a+bx)\sqrt{\sec(a+bx)}}{35b^2}+\frac{12\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2}+\frac{2x\,\mathrm{se}}{}$$

[Out] 2/7*x*sec(b*x+a)^(7/2)/b-4/35*sec(b*x+a)^(5/2)*sin(b*x+a)/b^2-12/35*sin(b*x+a)*sec(b*x+a)^(1/2)/b^2+12/35*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*EllipticE(sin(1/2*b*x+1/2*a),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2

**Rubi** [A] time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4212, 3768, 3771, 2639}

$$-\frac{4\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{12\sin(a+bx)\sqrt{\sec(a+bx)}}{35b^2}+\frac{12\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2}+\frac{2x\,\mathrm{se}}{}$$

Antiderivative was successfully verified.

[In] Int[x*Sec[a + b*x]^(9/2)*Sin[a + b*x],x]

[Out] (12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(35*b^2) + (2*x*Sec[a + b*x]^(7/2))/(7*b) - (12*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/(35*b^2) - (4*Sec[a + b*x]^(5/2)*Sin[a + b*x])/(35*b^2)

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]
```

Rubi steps

$$\int x\sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx = \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\int\sec^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$= \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{4\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{35b^2} - \frac{6\int\sec^{\frac{3}{2}}(a+bx)\,dx}{35b}$$
$$= \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{12\sqrt{\sec(a+bx)}\sin(a+bx)}{35b^2} - \frac{4\sec^{\frac{5}{2}}(a+bx)\sin(a+bx}{35b^2}$$
$$= \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{12\sqrt{\sec(a+bx)}\sin(a+bx)}{35b^2} - \frac{4\sec^{\frac{5}{2}}(a+bx)\sin(a+bx}{35b^2}$$
$$= \frac{12\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{35b^2} + \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - 12\sqrt{se}$$

**Mathematica [A]** time = 0.28, size = 65, normalized size = 0.63

$$\frac{\sec^{\frac{7}{2}}(a+bx)\left(-10\sin(2(a+bx))-3\sin(4(a+bx))+24\cos^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)+20bx\right)}{70b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(9/2)*Sin[a + b*x],x]
```

```
[Out] (Sec[a + b*x]^(7/2)*(20*b*x + 24*Cos[a + b*x]^(7/2)*EllipticE[(a + b*x)/2,
2] - 10*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)]))/(70*b^2)
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec(bx+a)^{\frac{9}{2}}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(9/2)*sin(b*x + a), x)
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int x\left(\sec^{\frac{9}{2}}(bx+a)\right)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)
```

```
[Out] int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \sec (bx + a)^{\frac{9}{2}} \sin (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sec(b*x + a)^(9/2)*sin(b*x + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x \sin (a + b\,x) \left(\frac{1}{\cos (a + b\,x)}\right)^{9/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(9/2),x)
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(9/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(9/2)*sin(b*x+a),x)
[Out] Timed out
```

## 3.337 $\int x \sec^{\frac{7}{2}}(a+bx) \sin(a+bx) \, dx$

**Optimal**. Leaf size=80

$$-\frac{4\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2}+\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] 2/5*x*sec(b*x+a)^(5/2)/b-4/15*sec(b*x+a)^(3/2)*sin(b*x+a)/b^2-4/15*(cos(1/2
*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*EllipticF(sin(1/2*b*x+1/2*a),2^(1/2
))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

**Rubi** [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3768, 3771, 2641}

$$-\frac{4\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2}+\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(7/2)*Sin[a + b*x],x]
```

```
[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(15*b^
2) + (2*x*Sec[a + b*x]^(5/2))/(5*b) - (4*Sec[a + b*x]^(3/2)*Sin[a + b*x])/(
15*b^2)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\int x\sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx = \frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\int\sec^{\frac{5}{2}}(a+bx)\,dx}{5b}$$
$$= \frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{4\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{15b^2} - \frac{2\int\sqrt{\sec(a+bx)}\,dx}{15b}$$
$$= \frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{4\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{15b^2} - \frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right.}{15b}$$
$$= -\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{15b^2} + \frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{4\sec^{\frac{3}{2}}(a}{}$$

**Mathematica** [A] time = 0.23, size = 61, normalized size = 0.76

$$\frac{2\sqrt{\sec(a+bx)}\left(-2\tan(a+bx)+3bx\sec^2(a+bx)-2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(7/2)*Sin[a + b*x],x]
```

```
[Out] (2*Sqrt[Sec[a + b*x]]*(-2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 3*
b*x*Sec[a + b*x]^2 - 2*Tan[a + b*x]))/(15*b^2)
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec(bx+a)^{\frac{7}{2}}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(7/2)*sin(b*x + a), x)
```

**maple** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x\left(\sec^{\frac{7}{2}}(bx+a)\right)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(7/2)*sin(b*x+a),x)
```

```
[Out] int(x*sec(b*x+a)^(7/2)*sin(b*x+a),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec(bx+a)^{\frac{7}{2}}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sec(b*x + a)^(7/2)*sin(b*x + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x \sin(a+bx)\left(\frac{1}{\cos(a+bx)}\right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(7/2),x)
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(7/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(7/2)*sin(b*x+a),x)
[Out] Timed out
```

# 3.338 $\int x \sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$

**Optimal**. Leaf size=80

$$-\frac{4\sin(a+bx)\sqrt{\sec(a+bx)}}{3b^2}+\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}+\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] 2/3*x*sec(b*x+a)^(3/2)/b-4/3*sin(b*x+a)*sec(b*x+a)^(1/2)/b^2+4/3*(cos(1/2*b
*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*EllipticE(sin(1/2*b*x+1/2*a),2^(1/2))
*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

**Rubi [A]** time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3768, 3771, 2639}

$$-\frac{4\sin(a+bx)\sqrt{\sec(a+bx)}}{3b^2}+\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}+\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(5/2)*Sin[a + b*x],x]
```

```
[Out] (4*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b^2)
 + (2*x*Sec[a + b*x]^(3/2))/(3*b) - (4*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/(3*
b^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\int x\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int\sec^{\frac{3}{2}}(a+bx)\,dx}{3b}$$
$$= \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{4\sqrt{\sec(a+bx)}\sin(a+bx)}{3b^2} + \frac{2\int\frac{1}{\sqrt{\sec(a+bx)}}dx}{3b}$$
$$= \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{4\sqrt{\sec(a+bx)}\sin(a+bx)}{3b^2} + \frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+\right.}{3b}$$
$$= \frac{4\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{3b^2} + \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{4\sqrt{\sec(}{}$$

**Mathematica [A]** time = 0.18, size = 54, normalized size = 0.68

$$\frac{2\sec^{\frac{3}{2}}(a+bx)\left(-\sin(2(a+bx))+2\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)+bx\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(5/2)*Sin[a + b*x],x]
```

```
[Out] (2*Sec[a + b*x]^(3/2)*(b*x + 2*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2]
 - Sin[2*(a + b*x)]))/(3*b^2)
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec\left(bx+a\right)^{\frac{5}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(5/2)*sin(b*x + a), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int x\left(\sec^{\frac{5}{2}}\left(bx+a\right)\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)
```

```
[Out] int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec\left(bx+a\right)^{\frac{5}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sec(b*x + a)^(5/2)*sin(b*x + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sin(a+b\,x)\,\left(\frac{1}{\cos(a+b\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(5/2),x)
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(5/2)*sin(b*x+a),x)
[Out] Timed out
```

# 3.339 $\int x \sec^{\frac{3}{2}}(a + bx) \sin(a + bx) \, dx$

**Optimal**. Leaf size=53

$$\frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

```
[Out] 2*x*sec(b*x+a)^(1/2)/b-4*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*El
lipticF(sin(1/2*b*x+1/2*a),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

**Rubi [A]** time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{4212, 3771, 2641\}$

$$\frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(3/2)*Sin[a + b*x],x]
```

```
[Out] (2*x*Sqrt[Sec[a + b*x]])/b - (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2
]*Sqrt[Sec[a + b*x]])/b^2
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\begin{aligned}
\int x \sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx &= \frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{2\int\sqrt{\sec(a+bx)}\,dx}{b}\\
&= \frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{b}\\
&= \frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 42, normalized size = 0.79

$$\frac{2\sqrt{\sec(a+bx)}\left(bx-2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(3/2)*Sin[a + b*x],x]
```

```
[Out] (2*(b*x - 2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])*Sqrt[Sec[a + b*x]
])/b^2
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \sec(bx+a)^{\frac{3}{2}} \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(3/2)*sin(b*x + a), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int x \left(\sec^{\frac{3}{2}}(bx+a)\right) \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(3/2)*sin(b*x+a),x)
```

```
[Out] int(x*sec(b*x+a)^(3/2)*sin(b*x+a),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \sec(bx+a)^{\frac{3}{2}} \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*sec(b*x + a)^(3/2)*sin(b*x + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x \sin(a+b\,x) \left(\frac{1}{\cos(a+b\,x)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(3/2),x)
```

```
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(3/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

## 3.340 $\int x\sqrt{\sec(a+bx)}\,\sin(a+bx)\,dx$

**Optimal**. Leaf size=53

$$\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sec(a+bx)}}$$

```
[Out] -2*x/b/sec(b*x+a)^(1/2)+4*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*E
llipticE(sin(1/2*b*x+1/2*a),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

**Rubi [A]** time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4212, 3771, 2639}

$$\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Sec[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (-2*x)/(b*Sqrt[Sec[a + b*x]]) + (4*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2
, 2]*Sqrt[Sec[a + b*x]])/b^2
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{\sec(a+bx)}\,\sin(a+bx)\,dx &= -\frac{2x}{b\sqrt{\sec(a+bx)}}+\frac{2\int\frac{1}{\sqrt{\sec(a+bx)}}\,dx}{b}\\
&= -\frac{2x}{b\sqrt{\sec(a+bx)}}+\frac{\left(2\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,dx}{b}\\
&= -\frac{2x}{b\sqrt{\sec(a+bx)}}+\frac{4\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b^2}
\end{aligned}$$

**Mathematica [B]** time = 2.15, size = 132, normalized size = 2.49

$$\frac{2\left(2\tan\left(\frac{1}{2}(a+bx)\right)-\frac{2\sec^2\left(\frac{1}{2}(a+bx)\right)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)|-1\right)}{\sqrt{\cos(a+bx)\sec^4\left(\frac{1}{2}(a+bx)\right)}}+\frac{2\sec^2\left(\frac{1}{2}(a+bx)\right)E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)|-1\right)}{\sqrt{\cos(a+bx)\sec^4\left(\frac{1}{2}(a+bx)\right)}}-bx\right)}{b^2\sqrt{\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Sec[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (2*(-(b*x) + (2*EllipticE[ArcSin[Tan[(a + b*x)/2]], -1]*Sec[(a + b*x)/2]^2)
/Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^4] - (2*EllipticF[ArcSin[Tan[(a + b*x)/
2]], -1]*Sec[(a + b*x)/2]^2)/Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^4] + 2*Tan[
(a + b*x)/2]))/(b^2*Sqrt[Sec[a + b*x]])
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\sec(bx+a)}\,\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(sec(b*x + a))*sin(b*x + a), x)
```

**maple [C]** time = 0.08, size = 310, normalized size = 5.85

$$-\frac{(bx+2i)\left(1+e^{2i(bx+a)}\right)\sqrt{2}\sqrt{\frac{e^{i(bx+a)}}{1+e^{2i(bx+a)}}}\;e^{-i(bx+a)}}{b^2}-\frac{2i\left(-\frac{2\left(1+e^{2i(bx+a)}\right)}{\sqrt{\left(1+e^{2i(bx+a)}\right)e^{i(bx+a)}}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\;\sqrt{2}\;\sqrt{i\left(e^{i(bx+a)}-i\right)}\;\sqrt{ie^{i(bx+a)}}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)*sec(b*x+a)^(1/2),x)
```

```
[Out] -(b*x+2*I)*(exp(I*(b*x+a))^2+1)/b^2*2^(1/2)*(exp(I*(b*x+a))/(exp(I*(b*x+a))
^2+1))^(1/2)/exp(I*(b*x+a))-2*I/b^2*(-2*(exp(I*(b*x+a))^2+1)/((exp(I*(b*x+a
))^2+1)*exp(I*(b*x+a)))^(1/2)+I*(-I*(exp(I*(b*x+a))+I))^(1/2)*2^(1/2)*(I*(e
xp(I*(b*x+a))-I))^(1/2)*(I*exp(I*(b*x+a)))^(1/2)/(exp(I*(b*x+a))^3+exp(I*(b
*x+a)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))+I*
EllipticF((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))))*2^(1/2)*(exp(I*(b*x+
a))/(exp(I*(b*x+a))^2+1))^(1/2)*((exp(I*(b*x+a))^2+1)*exp(I*(b*x+a)))^(1/2)
/exp(I*(b*x+a))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\sec(bx+a)}\,\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(sec(b*x + a))*sin(b*x + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x \sin(a+bx)\sqrt{\frac{1}{\cos(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(1/2),x)
```

```
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin(a+bx)\sqrt{\sec(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sin(a + b*x)*sqrt(sec(a + b*x)), x)
```

## 3.341 $\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx$

Optimal. Leaf size=80

$$\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}+\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

```
[Out] -2/3*x/b/sec(b*x+a)^(3/2)+4/9*sin(b*x+a)/b^2/sec(b*x+a)^(1/2)+4/9*(cos(1/2*
b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*EllipticF(sin(1/2*b*x+1/2*a),2^(1/2)
)*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

**Rubi [A]** time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4212, 3769, 3771, 2641}

$$\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}+\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sqrt[Sec[a + b*x]],x]
```

```
[Out] (-2*x)/(3*b*Sec[a + b*x]^(3/2)) + (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)
/2, 2]*Sqrt[Sec[a + b*x]])/(9*b^2) + (4*Sin[a + b*x])/(9*b^2*Sqrt[Sec[a + b
*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)} + \frac{2\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx}{3b}$$

$$= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)} + \frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}} + \frac{2\int \sqrt{\sec(a+bx)}\,dx}{9b}$$

$$= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)} + \frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}} + \frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{9b}$$

$$= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)} + \frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{9b^2} + \frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}$$

**Mathematica** [A] time = 0.24, size = 63, normalized size = 0.79

$$\frac{\sqrt{\sec(a+bx)}\left(2\sin(2(a+bx)) - 6bx\cos^2(a+bx) + 4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Sqrt[Sec[a + b*x]],x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(-6*b*x*Cos[a + b*x]^2 + 4*Sqrt[Cos[a + b*x]]*EllipticF
[(a + b*x)/2, 2] + 2*Sin[2*(a + b*x)]))/(9*b^2)
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\sqrt{\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(sec(b*x + a)), x)
```

**maple** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\sqrt{\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/sec(b*x+a)^(1/2),x)
```

```
[Out] int(x*sin(b*x+a)/sec(b*x+a)^(1/2),x)
```

**maximua** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\sqrt{\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(sec(b*x + a)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\ \sin(a+b\,x)}{\sqrt{\frac{1}{\cos(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/(1/cos(a + b*x))^(1/2),x)
```

```
[Out] int((x*sin(a + b*x))/(1/cos(a + b*x))^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/sqrt(sec(a + b*x)), x)
```

# 3.342 $\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx$

**Optimal**. Leaf size=80

$$\frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}+\frac{12\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

```
[Out] -2/5*x/b/sec(b*x+a)^(5/2)+4/25*sin(b*x+a)/b^2/sec(b*x+a)^(3/2)+12/25*(cos(1
/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*EllipticE(sin(1/2*b*x+1/2*a),2^(1
/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

**Rubi [A]** time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3769, 3771, 2639}

$$\frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}+\frac{12\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sec[a + b*x]^(3/2),x]
```

```
[Out] (-2*x)/(5*b*Sec[a + b*x]^(5/2)) + (12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x
)/2, 2]*Sqrt[Sec[a + b*x]])/(25*b^2) + (4*Sin[a + b*x])/(25*b^2*Sec[a + b*x
]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)} + \frac{2\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}dx}{5b}$$

$$= -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)} + \frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)} + \frac{6\int \frac{1}{\sqrt{\sec(a+bx)}}dx}{25b}$$

$$= -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)} + \frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)} + \frac{\left(6\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,d}{25b}$$

$$= -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)} + \frac{12\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{25b^2} + \frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}$$

**Mathematica [B]** time = 8.10, size = 212, normalized size = 2.65

$$\frac{\cos^2\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec(a+bx)}\left(\left(5(a+bx)-12\tan\left(\frac{1}{2}(a+bx)\right)-5a\right)\left(\tan^2\left(\frac{1}{2}(a+bx)\right)-1\right)-12\sqrt{\cos(a+bx)\,s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Sec[a + b*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(-1/10*(x*Cos[a + b*x]) - (x*Cos[3*(a + b*x)])/10 + Sin
[a + b*x]/(25*b) + Sin[3*(a + b*x)]/(25*b)))/b + (Cos[(a + b*x)/2]^2*Sqrt[S
ec[a + b*x]]*(12*EllipticE[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Cos[a + b*x]*
Sec[(a + b*x)/2]^4] - 12*EllipticF[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Cos[a
 + b*x]*Sec[(a + b*x)/2]^4] + (-5*a + 5*(a + b*x) - 12*Tan[(a + b*x)/2])*(-
1 + Tan[(a + b*x)/2]^2)))/(25*b^2)
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\sec(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(3/2), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\sec(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)
```

```
[Out] int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx+a)}{\sec(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(3/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin(a+b\,x)}{\left(\frac{1}{\cos(a+b\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/(1/cos(a + b*x))^(3/2),x)
```

```
[Out] int((x*sin(a + b*x))/(1/cos(a + b*x))^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/sec(a + b*x)**(3/2), x)
```

# 3.343 $\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx$

**Optimal**. Leaf size=103

$$\frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)}+\frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}}+\frac{20\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{147b^2}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

```
[Out] -2/7*x/b/sec(b*x+a)^(7/2)+4/49*sin(b*x+a)/b^2/sec(b*x+a)^(5/2)+20/147*sin(b
*x+a)/b^2/sec(b*x+a)^(1/2)+20/147*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+
1/2*a)*EllipticF(sin(1/2*b*x+1/2*a),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1
/2)/b^2
```

**Rubi [A]** time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3769, 3771, 2641}

$$\frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)}+\frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}}+\frac{20\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{147b^2}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sec[a + b*x]^(5/2),x]
```

```
[Out] (-2*x)/(7*b*Sec[a + b*x]^(7/2)) + (20*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x
)/2, 2]*Sqrt[Sec[a + b*x]])/(147*b^2) + (4*Sin[a + b*x])/(49*b^2*Sec[a + b*
x]^(5/2)) + (20*Sin[a + b*x])/(147*b^2*Sqrt[Sec[a + b*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{2\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)} + \frac{10\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx}{49b}$$

$$= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)} + \frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}} + \frac{10\int \sqrt{\sec(a+bx)}\,dx}{147b}$$

$$= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)} + \frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}} + \frac{\left(10\sqrt{\cos(a+bx)}\,\sqrt{\sec(\right.}{14}$$

$$= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{20\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{147b^2} + \frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)}$$

**Mathematica** [A] time = 0.34, size = 89, normalized size = 0.86

$$\frac{\sqrt{\sec(a+bx)}\left(52\sin(2(a+bx))+6\sin(4(a+bx))-84bx\cos(2(a+bx))-21bx\cos(4(a+bx))+80\sqrt{\cos(a\right.}{588b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Sec[a + b*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(-63*b*x - 84*b*x*Cos[2*(a + b*x)] - 21*b*x*Cos[4*(a +
b*x)] + 80*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 52*Sin[2*(a + b*x
)] + 6*Sin[4*(a + b*x)]))/(588*b^2)
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\sec(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(5/2), x)
```

**maple** [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\sec(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/sec(b*x+a)^(5/2),x)
```

```
[Out] int(x*sin(b*x+a)/sec(b*x+a)^(5/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx+a)}{\sec(bx+a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin(a+b\,x)}{\left(\frac{1}{\cos(a+b\,x)}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/(1/cos(a + b*x))^(5/2),x)
```

```
[Out] int((x*sin(a + b*x))/(1/cos(a + b*x))^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(5/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/sec(a + b*x)**(5/2), x)
```

# 3.344 $\int x \cos(a+bx) \sin^{\frac{5}{2}}(a+bx)\,dx$

**Optimal**. Leaf size=88

$$-\frac{20F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{4\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{49b^2}+\frac{20\sqrt{\sin(a+bx)}\,\cos(a+bx)}{147b^2}+\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}$$

```
[Out] 20/147*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipt
icF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+4/49*cos(b*x+a)*sin(b*x+a)^(5/2)
/b^2+2/7*x*sin(b*x+a)^(7/2)/b+20/147*cos(b*x+a)*sin(b*x+a)^(1/2)/b^2
```

**Rubi [A]** time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3443, 2635, 2641}

$$-\frac{20F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{4\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{49b^2}+\frac{20\sqrt{\sin(a+bx)}\,\cos(a+bx)}{147b^2}+\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sin[a + b*x]^(5/2),x]
```

```
[Out] (-20*EllipticF[(a - Pi/2 + b*x)/2, 2])/(147*b^2) + (20*Cos[a + b*x]*Sqrt[Si
n[a + b*x]])/(147*b^2) + (4*Cos[a + b*x]*Sin[a + b*x]^(5/2))/(49*b^2) + (2*
x*Sin[a + b*x]^(7/2))/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx &= \frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\int\sin^{\frac{7}{2}}(a+bx)\,dx}{7b}\\
&= \frac{4\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}-\frac{10\int\sin^{\frac{3}{2}}(a+bx)\,dx}{49b}\\
&= \frac{20\cos(a+bx)\sqrt{\sin(a+bx)}}{147b^2}+\frac{4\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{2x\sin^{\frac{7}{2}}(a+b
\end{aligned}$$

$$= -\frac{20F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{147b^2}+\frac{20\cos(a+bx)\sqrt{\sin(a+bx)}}{147b^2}+\frac{4\cos(a+bx)\,\mathrm{s}}{49b^2}$$

**Mathematica** [A] time = 0.54, size = 67, normalized size = 0.76

$$\frac{40F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+\sqrt{\sin(a+bx)}\left(84bx\sin^3(a+bx)+46\cos(a+bx)-6\cos(3(a+bx))\right)}{294b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sin[a + b*x]^(5/2),x]
```

```
[Out] (40*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt[Sin[a + b*x]]*(46*Cos[a + b*
x] - 6*Cos[3*(a + b*x)] + 84*b*x*Sin[a + b*x]^3))/(294*b^2)
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\sin(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(5/2), x)
```

**maple** [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\sin^{\frac{5}{2}}(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*sin(b*x+a)^(5/2),x)
```

```
[Out] int(x*cos(b*x+a)*sin(b*x+a)^(5/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\sin(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(5/2), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\cos(a+b\,x)\,\sin(a+b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*sin(a + b*x)^(5/2),x)
```

```
[Out] int(x*cos(a + b*x)*sin(a + b*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

# 3.345 $\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx$

**Optimal**. Leaf size=65

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2}+\frac{4\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{25b^2}+\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] 12/25*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipti
cE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+4/25*cos(b*x+a)*sin(b*x+a)^(3/2)/
b^2+2/5*x*sin(b*x+a)^(5/2)/b
```

**Rubi [A]** time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2635, 2639}

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2}+\frac{4\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{25b^2}+\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sin[a + b*x]^(3/2),x]
```

```
[Out] (-12*EllipticE[(a - Pi/2 + b*x)/2, 2])/(25*b^2) + (4*Cos[a + b*x]*Sin[a + b
*x]^(3/2))/(25*b^2) + (2*x*Sin[a + b*x]^(5/2))/(5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx &= \frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int\sin^{\frac{5}{2}}(a+bx)\,dx}{5b}\\
&= \frac{4\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{25b^2}+\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}-\frac{6\int\sqrt{\sin(a+bx)}\,dx}{25b}\\
&= -\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{25b^2}+\frac{4\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{25b^2}+\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}
\end{aligned}$$

**Mathematica [C]** time = 0.90, size = 108, normalized size = 1.66

$$\frac{\sqrt{\sin(a+bx)}\left(4\tan\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+2\sin(2(a+bx))-5bx\right.}{25b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sin[a + b*x]^(3/2),x]
```

```
[Out] (Sqrt[Sin[a + b*x]]*(5*b*x - 5*b*x*Cos[2*(a + b*x)] + 2*Sin[2*(a + b*x)] -
12*Tan[(a + b*x)/2] + 4*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^
2]*Sqrt[Sec[(a + b*x)/2]^2]*Tan[(a + b*x)/2]))/(25*b^2)
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\sin(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(3/2), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\sin^{\frac{3}{2}}(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*sin(b*x+a)^(3/2),x)
```

```
[Out] int(x*cos(b*x+a)*sin(b*x+a)^(3/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\sin(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(3/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\cos(a+b\,x)\,\sin(a+b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*sin(a + b*x)^(3/2),x)
```

```
[Out] int(x*cos(a + b*x)*sin(a + b*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*sin(a + b*x)**(3/2)*cos(a + b*x), x)
```

# 3.346 $\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx$

**Optimal**. Leaf size=65

$$-\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2}+\frac{4\sqrt{\sin(a+bx)}\,\cos(a+bx)}{9b^2}+\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] 4/9*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF
(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+2/3*x*sin(b*x+a)^(3/2)/b+4/9*cos(b*
x+a)*sin(b*x+a)^(1/2)/b^2
```

**Rubi** [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2635, 2641}

$$-\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2}+\frac{4\sqrt{\sin(a+bx)}\,\cos(a+bx)}{9b^2}+\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sqrt[Sin[a + b*x]],x]
```

```
[Out] (-4*EllipticF[(a - Pi/2 + b*x)/2, 2])/(9*b^2) + (4*Cos[a + b*x]*Sqrt[Sin[a
+ b*x]])/(9*b^2) + (2*x*Sin[a + b*x]^(3/2))/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx &= \frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int \sin^{\frac{3}{2}}(a+bx)\,dx}{3b}\\
&= \frac{4\cos(a+bx)\sqrt{\sin(a+bx)}}{9b^2}+\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{9b}\\
&= -\frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{9b^2}+\frac{4\cos(a+bx)\sqrt{\sin(a+bx)}}{9b^2}+\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}
\end{aligned}$$

**Mathematica [A]** time = 0.19, size = 56, normalized size = 0.86

$$\frac{4F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+2\sqrt{\sin(a+bx)}\,(3bx\sin(a+bx)+2\cos(a+bx))}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sqrt[Sin[a + b*x]],x]
```

```
[Out] (4*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + 2*Sqrt[Sin[a + b*x]]*(2*Cos[a + b*
x] + 3*b*x*Sin[a + b*x]))/(9*b^2)
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\sqrt{\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sqrt(sin(b*x + a)), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\sqrt{\sin}\,(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)
```

```
[Out] int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\sqrt{\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*sqrt(sin(b*x + a)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\cos(a+b\,x)\,\sqrt{\sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*sin(a + b*x)^(1/2),x)
```

```
[Out] int(x*cos(a + b*x)*sin(a + b*x)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\sin(a+bx)}\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(1/2),x)
[Out] Integral(x*sqrt(sin(a + b*x))*cos(a + b*x), x)
```

## 3.347 $\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx$

**Optimal**. Leaf size=38

$$\frac{2x\sqrt{\sin(a+bx)}}{b}-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

```
[Out] 4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(c
os(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+2*x*sin(b*x+a)^(1/2)/b
```

**Rubi [A]** time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3443, 2639}

$$\frac{2x\sqrt{\sin(a+bx)}}{b}-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sqrt[Sin[a + b*x]],x]
```

```
[Out] (-4*EllipticE[(a - Pi/2 + b*x)/2, 2])/b^2 + (2*x*Sqrt[Sin[a + b*x]])/b
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx &= \frac{2x\sqrt{\sin(a+bx)}}{b}-\frac{2\int \sqrt{\sin(a+bx)}\,dx}{b}\\ &= -\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b^2}+\frac{2x\sqrt{\sin(a+bx)}}{b}\end{aligned}$$

**Mathematica [C]** time = 1.16, size = 86, normalized size = 2.26

$$\frac{2\sqrt{\sin(a+bx)}\left(2\tan\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-6\tan\left(\frac{1}{2}(a+bx)\right)+3bx\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sqrt[Sin[a + b*x]],x]
```

```
[Out] (2*Sqrt[Sin[a + b*x]]*(3*b*x - 6*Tan[(a + b*x)/2] + 2*Hypergeometric2F1[1/2
, 3/4, 7/4, -Tan[(a + b*x)/2]^2]*Sqrt[Sec[(a + b*x)/2]^2]*Tan[(a + b*x)/2])
)/(3*b^2)
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sqrt{\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(sin(b*x + a)), x)
```

**maple [C]** time = 0.12, size = 308, normalized size = 8.11

$$-\frac{i(bx+2i)\left(e^{2i(bx+a)}-1\right)\sqrt{2}\,e^{-i(bx+a)}}{b^2\sqrt{-i\left(e^{2i(bx+a)}-1\right)e^{-i(bx+a)}}}-\frac{2\Bigg(\frac{2i\left(i-ie^{2i(bx+a)}\right)}{\sqrt{e^{i(bx+a)}\left(i-ie^{2i(bx+a)}\right)}}-\frac{\sqrt{e^{i(bx+a)}+1}\,\sqrt{-2\,e^{i(bx+a)}+2}\,\sqrt{-e^{i(bx+a)}}\,\Big(-2\,\mathrm{EllipticE}\Big(\sqrt{e^{i(bx\cdots}}}{\sqrt{-ie^{3i(bx+a)}+ie^{i(bx+a\cdots}}}}{b^2\sqrt{-i\,\Big(e^{2i(bx+\cdots}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(1/2),x)
```

```
[Out] -I*(b*x+2*I)*(exp(I*(b*x+a))^2-1)/b^2*2^(1/2)/(-I*(exp(I*(b*x+a))^2-1)/exp(
I*(b*x+a)))^(1/2)/exp(I*(b*x+a))-2/b^2*(2*I*(I-I*exp(I*(b*x+a))^2)/(exp(I*(
b*x+a))*(I-I*exp(I*(b*x+a))^2))^(1/2)-(exp(I*(b*x+a))+1)^(1/2)*(-2*exp(I*(b
*x+a))+2)^(1/2)*(-exp(I*(b*x+a)))^(1/2)/(-I*exp(I*(b*x+a))^3+I*exp(I*(b*x+a
)))^(1/2)*(-2*EllipticE((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))+EllipticF((ex
p(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))))*2^(1/2)/(-I*(exp(I*(b*x+a))^2-1)/exp(I
*(b*x+a)))^(1/2)*(-I*(exp(I*(b*x+a))^2-1)*exp(I*(b*x+a)))^(1/2)/exp(I*(b*x+
a))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sqrt{\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(sin(b*x + a)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,\cos(a+b\,x)}{\sqrt{\sin(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(1/2),x)
```

```
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/sqrt(sin(a + b*x)), x)
```

# 3.348 $\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$

**Optimal**. Leaf size=38

$$\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sin(a+bx)}}$$

```
[Out] -4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(
cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2*x/b/sin(b*x+a)^(1/2)
```

**Rubi** [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3443, 2641}

$$\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(3/2),x]
```

```
[Out] (4*EllipticF[(a - Pi/2 + b*x)/2, 2])/b^2 - (2*x)/(b*Sqrt[Sin[a + b*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx &= -\frac{2x}{b\sqrt{\sin(a+bx)}}+\frac{2\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{b}\\ &= \frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sin(a+bx)}}\end{aligned}$$

**Mathematica** [A] time = 0.18, size = 37, normalized size = 0.97

$$\frac{2\left(-\frac{bx}{\sqrt{\sin(a+bx)}}-2F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(3/2),x]
```

```
[Out] (2*(-2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] - (b*x)/Sqrt[Sin[a + b*x]]))/b^2
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(3/2), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)
```

```
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(3/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,\cos(a+b\,x)}{{\sin(a+b\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(3/2),x)
```

```
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/sin(a + b*x)**(3/2), x)
```

# 3.349 $\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$

**Optimal**. Leaf size=65

$$-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2}-\frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

```
[Out] 4/3*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE
(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2/3*x/b/sin(b*x+a)^(3/2)-4/3*cos(b*
x+a)/b^2/sin(b*x+a)^(1/2)
```

**Rubi [A]** time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2636, 2639}

$$-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2}-\frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(5/2),x]
```

```
[Out] (-4*EllipticE[(a - Pi/2 + b*x)/2, 2])/(3*b^2) - (2*x)/(3*b*Sin[a + b*x]^(3/
2)) - (4*Cos[a + b*x])/(3*b^2*Sqrt[Sin[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx &= -\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}+\frac{2\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\
&= -\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}-\frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}-\frac{2\int \sqrt{\sin(a+bx)}\,dx}{3b}\\
&= -\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{3b^2}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}-\frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 56, normalized size = 0.86

$$-\frac{2\left(\sin(2(a+bx))-2\sin^{\frac{3}{2}}(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+bx\right)}{3b^2\sin^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(5/2),x]
```

```
[Out] (-2*(b*x - 2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2) + Sin[2
*(a + b*x)]))/(3*b^2*Sin[a + b*x]^(3/2))
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(5/2), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(5/2),x)
```

```
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(5/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\cos(a+b\,x)}{{\sin(a+b\,x)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(5/2),x)
```

```
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(5/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/sin(a + b*x)**(5/2), x)
```

# 3.350 $\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$

**Optimal**. Leaf size=65

$$\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)}-\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

```
[Out] -4/15*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2/5*x/b/sin(b*x+a)^(5/2)-4/15*cos(b*x+a)/b^2/sin(b*x+a)^(3/2)
```

**Rubi [A]** time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2636, 2641}

$$\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)}-\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(7/2),x]
```

```
[Out] (4*EllipticF[(a - Pi/2 + b*x)/2, 2])/(15*b^2) - (2*x)/(5*b*Sin[a + b*x]^(5/2)) - (4*Cos[a + b*x])/(15*b^2*Sin[a + b*x]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}dx}{5b}$$

$$= -\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)} + \frac{2\int \frac{1}{\sqrt{\sin(a+bx)}}dx}{15b}$$

$$= \frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{15b^2} - \frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)}$$

**Mathematica [A]** time = 0.23, size = 57, normalized size = 0.88

$$-\frac{2\left(\sin(2(a+bx)) + 2\sin^{\frac{5}{2}}(a+bx)F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right) + 3bx\right)}{15b^2\sin^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(7/2),x]
```

```
[Out] (-2*(3*b*x + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(5/2) + Sin
[2*(a + b*x)]))/(15*b^2*Sin[a + b*x]^(5/2))
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(7/2), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(7/2),x)
```

```
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(7/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(bx+a\right)}{\sin\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(7/2), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\cos\left(a+b\,x\right)}{{\sin\left(a+b\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(7/2),x)
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(7/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(7/2),x)
[Out] Timed out
```

# 3.351 $\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx$

**Optimal**. Leaf size=88

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2}-\frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)}-\frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

```
[Out] 12/35*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipti
cE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2/7*x/b/sin(b*x+a)^(7/2)-4/35*cos
(b*x+a)/b^2/sin(b*x+a)^(5/2)-12/35*cos(b*x+a)/b^2/sin(b*x+a)^(1/2)
```

**Rubi [A]** time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2636, 2639}

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2}-\frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)}-\frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(9/2),x]
```

```
[Out] (-12*EllipticE[(a - Pi/2 + b*x)/2, 2])/(35*b^2) - (2*x)/(7*b*Sin[a + b*x]^(
7/2)) - (4*Cos[a + b*x])/(35*b^2*Sin[a + b*x]^(5/2)) - (12*Cos[a + b*x])/(3
5*b^2*Sqrt[Sin[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = -\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)} + \frac{2\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= -\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)} - \frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)} + \frac{6\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{35b}$$

$$= -\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)} - \frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)} - \frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}} - \frac{6\int \sqrt{\sin(a+bx)}\,dx}{35b}$$

$$= -\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{35b^2} - \frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)} - \frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)} - \frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}$$

**Mathematica [A]** time = 0.29, size = 73, normalized size = 0.83

$$-\frac{2\left(\sin(2(a+bx)) + 6\sin^3(a+bx)\cos(a+bx) - 6\sin^{\frac{7}{2}}(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right) + 5bx\right)}{35b^2\sin^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(9/2),x]
```

```
[Out] (-2*(5*b*x + 6*Cos[a + b*x]*Sin[a + b*x]^3 - 6*EllipticE[(-2*a + Pi - 2*b*x
)/4, 2]*Sin[a + b*x]^(7/2) + Sin[2*(a + b*x)]))/(35*b^2*Sin[a + b*x]^(7/2))
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(9/2), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)
```

```
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(9/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\cos(a+b\,x)}{\sin(a+b\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(9/2),x)
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(9/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(9/2),x)
[Out] Timed out
```

# 3.352 $\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx$

**Optimal**. Leaf size=108

$$-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{12\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2}-$$

```
[Out] -4/35*cos(b*x+a)*csc(b*x+a)^(5/2)/b^2-2/7*x*csc(b*x+a)^(7/2)/b-12/35*cos(b*
x+a)*csc(b*x+a)^(1/2)/b^2+12/35*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2
*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^
(1/2)*sin(b*x+a)^(1/2)/b^2
```

**Rubi** [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3768, 3771, 2639}

$$-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{12\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(9/2),x]
```

```
[Out] (-12*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/(35*b^2) - (4*Cos[a + b*x]*Csc[a + b*
x]^(5/2))/(35*b^2) - (2*x*Csc[a + b*x]^(7/2))/(7*b) - (12*Sqrt[Csc[a + b*x]
]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(35*b^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx = -\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b} + \frac{2\int\csc^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$= -\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2} - \frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b} + \frac{6\int\csc^{\frac{3}{2}}(a+bx)\,dx}{35b}$$
$$= -\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2} - \frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2} - \frac{2x\csc^{\frac{7}{2}}(a+$$
$$= -\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2} - \frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2} - \frac{2x\csc^{\frac{7}{2}}(a+$$
$$= -\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2} - \frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2} - \frac{2x\csc^{\frac{7}{2}}(a+$$

**Mathematica [A]** time = 0.27, size = 73, normalized size = 0.68

$$-\frac{2\csc^{\frac{7}{2}}(a+bx)\left(\sin(2(a+bx))+6\sin^3(a+bx)\cos(a+bx)-6\sin^{\frac{7}{2}}(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+5bx\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(9/2),x]
```

```
[Out] (-2*Csc[a + b*x]^(7/2)*(5*b*x + 6*Cos[a + b*x]*Sin[a + b*x]^3 - 6*EllipticE
[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(7/2) + Sin[2*(a + b*x)]))/(35*b^2)
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(9/2), x)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\csc^{\frac{9}{2}}(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)
```

```
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(9/2), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\cos(a+b\,x)\,\left(\frac{1}{\sin(a+b\,x)}\right)^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(9/2),x)
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(9/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(9/2),x)
[Out] Timed out
```

## 3.353 $\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx$

**Optimal**. Leaf size=85

$$-\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}+\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] -4/15*cos(b*x+a)*csc(b*x+a)^(3/2)/b^2-2/5*x*csc(b*x+a)^(5/2)/b-4/15*(sin(1/
2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+
1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

**Rubi** [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3768, 3771, 2641}

$$-\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}+\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(7/2),x]
```

```
[Out] (-4*Cos[a + b*x]*Csc[a + b*x]^(3/2))/(15*b^2) - (2*x*Csc[a + b*x]^(5/2))/(5
*b) + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b
*x]])/(15*b^2)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx = -\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int\csc^{\frac{5}{2}}(a+bx)\,dx}{5b}$$
$$= -\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int\sqrt{\csc(a+bx)}\,dx}{15b}$$
$$= -\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx}}{15b}$$
$$= -\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}-\right.\right.}{15b^2}$$

**Mathematica** [A] time = 0.29, size = 65, normalized size = 0.76

$$-\frac{2\sqrt{\csc(a+bx)}\left(2\cot(a+bx)+3bx\csc^2(a+bx)+2\sqrt{\sin(a+bx)}\,F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(7/2),x]
[Out] (-2*Sqrt[Csc[a + b*x]]*(2*Cot[a + b*x] + 3*b*x*Csc[a + b*x]^2 + 2*EllipticF
[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]]))/(15*b^2)
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(7/2), x)
```

**maple** [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\csc^{\frac{7}{2}}(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(7/2),x)
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(7/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(7/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x\cos(a+bx)\left(\frac{1}{\sin(a+bx)}\right)^{7/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(7/2),x)
```

```
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(7/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

## 3.354 $\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx$

**Optimal**. Leaf size=85

$$-\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] -2/3*x*csc(b*x+a)^(3/2)/b-4/3*cos(b*x+a)*csc(b*x+a)^(1/2)/b^2+4/3*(sin(1/2*
a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/
4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

**Rubi [A]** time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3768, 3771, 2639}

$$-\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(5/2),x]
```

```
[Out] (-4*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/(3*b^2) - (2*x*Csc[a + b*x]^(3/2))/(3*
b) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*
x]])/(3*b^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx = -\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b} + \frac{2\int\csc^{\frac{3}{2}}(a+bx)\,dx}{3b}$$
$$= -\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2} - \frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int\frac{1}{\sqrt{\csc(a+bx)}}dx}{3b}$$
$$= -\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2} - \frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b} - \frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a -}\right.}{3}$$
$$= -\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2} - \frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b} - \frac{4\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a -\right.\right.}{3}$$

**Mathematica [A]** time = 0.18, size = 56, normalized size = 0.66

$$-\frac{2\csc^{\frac{3}{2}}(a+bx)\left(\sin(2(a+bx))-2\sin^{\frac{3}{2}}(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+bx\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(5/2),x]
[Out] (-2*Csc[a + b*x]^(3/2)*(b*x - 2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a +
 b*x]^(3/2) + Sin[2*(a + b*x)]))/(3*b^2)
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(5/2), x)
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\csc^{\frac{5}{2}}(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x \cos(a+bx) \left(\frac{1}{\sin(a+bx)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(5/2),x)
```

```
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

# 3.355 $\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx$

**Optimal**. Leaf size=58

$$\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}-\frac{2x\sqrt{\csc(a+bx)}}{b}$$

```
[Out] -2*x*csc(b*x+a)^(1/2)/b-4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4
*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*
sin(b*x+a)^(1/2)/b^2
```

**Rubi [A]** time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4213, 3771, 2641}

$$\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}-\frac{2x\sqrt{\csc(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(3/2),x]
```

```
[Out] (-2*x*Sqrt[Csc[a + b*x]])/b + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b
*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\begin{aligned}\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx&=-\frac{2x\sqrt{\csc(a+bx)}}{b}+\frac{2\int\sqrt{\csc(a+bx)}\,dx}{b}\\&=-\frac{2x\sqrt{\csc(a+bx)}}{b}+\frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int\frac{1}{\sqrt{\sin(a+bx)}}\,dx}{b}\\&=-\frac{2x\sqrt{\csc(a+bx)}}{b}+\frac{4\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b^2}\end{aligned}$$

**Mathematica** [A] time = 0.14, size = 46, normalized size = 0.79

$$-\frac{2\sqrt{\csc(a+bx)}\left(2\sqrt{\sin(a+bx)}F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+bx\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[Csc[a + b*x]]*(b*x + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Si
n[a + b*x]]))/b^2
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(3/2), x)
```

**maple** [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\csc^{\frac{3}{2}}(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)
```

```
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(3/2), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\cos(a+b\,x)\,{\left(\frac{1}{\sin(a+b\,x)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(3/2),x)
```

```
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(3/2), x)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

# 3.356 $\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx$

**Optimal**. Leaf size=58

$$\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

```
[Out] 2*x/b/csc(b*x+a)^(1/2)+4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*
Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*s
in(b*x+a)^(1/2)/b^2
```

**Rubi [A]** time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4213, 3771, 2639}

$$\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*x)/(b*Sqrt[Csc[a + b*x]]) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 +
b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\begin{aligned}
\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx &= \frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{2\int\frac{1}{\sqrt{\csc(a+bx)}}\,dx}{b}\\
&= \frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int\sqrt{\sin(a+bx)}\,dx}{b}\\
&= \frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b^2}
\end{aligned}$$

**Mathematica [C]** time = 0.72, size = 106, normalized size = 1.83

$$\frac{4\sin\left(\frac{1}{2}(a+bx)\right)\cos\left(\frac{1}{2}(a+bx)\right)\sqrt{\csc(a+bx)}\left(2\tan\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2\left(\frac{1}{2}(a\right.\right.\right.}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (4*Cos[(a + b*x)/2]*Sqrt[Csc[a + b*x]]*Sin[(a + b*x)/2]*(3*b*x - 6*Tan[(a +
 b*x)/2] + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2]*Sqrt[Sec
[(a + b*x)/2]^2]*Tan[(a + b*x)/2]))/(3*b^2)
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\sqrt{\csc(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sqrt(csc(b*x + a)), x)
```

**maple [C]** time = 0.12, size = 308, normalized size = 5.31

$$-\frac{i\,(bx+2i)\left(e^{2i(bx+a)}-1\right)\sqrt{2}\,\sqrt{\frac{i e^{i(bx+a)}}{e^{2i(bx+a)}-1}}\;e^{-i(bx+a)}}{b^2}-\frac{2\left(-\frac{2i\left(-i+ie^{2i(bx+a)}\right)}{\sqrt{e^{i(bx+a)}\left(-i+ie^{2i(bx+a)}\right)}}-\frac{\sqrt{e^{i(bx+a)}+1}\,\sqrt{-2\,e^{i(bx+a)}+2}\,\sqrt{-e^{i(bx+a)}}\,\left(\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(1/2),x)
```

```
[Out] -I*(b*x+2*I)*(exp(I*(b*x+a))^2-1)/b^2*2^(1/2)*(I*exp(I*(b*x+a))/(exp(I*(b*x
+a))^2-1))^(1/2)/exp(I*(b*x+a))-2/b^2*(-2*I*(-I+I*exp(I*(b*x+a))^2)/(exp(I*
(b*x+a))*(-I+I*exp(I*(b*x+a))^2))^(1/2)-(exp(I*(b*x+a))+1)^(1/2)*(-2*exp(I*
(b*x+a))+2)^(1/2)*(-exp(I*(b*x+a)))^(1/2)/(I*exp(I*(b*x+a))^3-I*exp(I*(b*x+
a)))^(1/2)*(-2*EllipticE((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))+EllipticF((e
xp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))))*2^(1/2)*(I*exp(I*(b*x+a))/(exp(I*(b*x
+a))^2-1))^(1/2)*(I*(exp(I*(b*x+a))^2-1)*exp(I*(b*x+a)))^(1/2)/exp(I*(b*x+a
))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\sqrt{\csc(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*sqrt(csc(b*x + a)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\cos\left(a+b\,x\right)\,\sqrt{\frac{1}{\sin\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(1/2),x)
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(a+bx\right)\sqrt{\csc\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(1/2),x)
[Out] Integral(x*cos(a + b*x)*sqrt(csc(a + b*x)), x)
```

# 3.357 $\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

**Optimal**. Leaf size=85

$$\frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2}+\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}$$

```
[Out] 2/3*x/b/csc(b*x+a)^(3/2)+4/9*cos(b*x+a)/b^2/csc(b*x+a)^(1/2)+4/9*(sin(1/2*a
+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4
*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

**Rubi [A]** time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3769, 3771, 2641}

$$\frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2}+\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*x)/(3*b*Csc[a + b*x]^(3/2)) + (4*Cos[a + b*x])/(9*b^2*Sqrt[Csc[a + b*x]]
) - (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x
]])/(9*b^2)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)}\,dx}{3b}$$
$$= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{2\int \sqrt{\csc(a+bx)}\,dx}{9b}$$
$$= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{9b}$$
$$= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{4\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{9b^2}$$

**Mathematica [A]** time = 0.23, size = 65, normalized size = 0.76

$$\frac{2\sqrt{\csc(a+bx)}\left(3bx\sin^2(a+bx)+\sin(2(a+bx))+2\sqrt{\sin(a+bx)}\,F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*Sqrt[Csc[a + b*x]]*(2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b
*x]] + 3*b*x*Sin[a + b*x]^2 + Sin[2*(a + b*x)]))/(9*b^2)
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(csc(b*x + a)), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/csc(b*x+a)^(1/2),x)
```

```
[Out] int(x*cos(b*x+a)/csc(b*x+a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(csc(b*x + a)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\cos(a+b\,x)}{\sqrt{\frac{1}{\sin(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/(1/sin(a + b*x))^(1/2),x)
```

```
[Out] int((x*cos(a + b*x))/(1/sin(a + b*x))^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/sqrt(csc(a + b*x)), x)
```

# 3.358 $\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx$

**Optimal**. Leaf size=85

$$\frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)}-\frac{12\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2}+\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}$$

```
[Out] 2/5*x/b/csc(b*x+a)^(5/2)+4/25*cos(b*x+a)/b^2/csc(b*x+a)^(3/2)+12/25*(sin(1/
2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+
1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

**Rubi [A]** time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3769, 3771, 2639}

$$\frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)}-\frac{12\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2}+\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Csc[a + b*x]^(3/2),x]
```

```
[Out] (2*x)/(5*b*Csc[a + b*x]^(5/2)) + (4*Cos[a + b*x])/(25*b^2*Csc[a + b*x]^(3/2
)) - (12*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b
*x]])/(25*b^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc^{\frac{5}{2}}(a+bx)}dx}{5b}$$

$$= \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} + \frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)} - \frac{6\int \frac{1}{\sqrt{\csc(a+bx)}}dx}{25b}$$

$$= \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} + \frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)} - \frac{\left(6\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int\sqrt{\sin(a+bx)}\,d}{25b}$$

$$= \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} + \frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)} - \frac{12\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+}}{25b^2}$$

**Mathematica [C]** time = 0.98, size = 114, normalized size = 1.34

$$\frac{\tan\left(\frac{1}{2}(a+bx)\right)\left(4\sqrt{2}\sqrt{\frac{1}{\cos(a+bx)+1}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+10bx\sin(a+bx)+5bx\sin(2(a+bx))+}{25b^2\sqrt{\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Csc[a + b*x]^(3/2),x]
```

```
[Out] ((-10 + 4*Cos[a + b*x] + 2*Cos[2*(a + b*x)] + 4*Sqrt[2]*Sqrt[(1 + Cos[a + b
*x])^(-1)]*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2] + 10*b*x*S
in[a + b*x] + 5*b*x*Sin[2*(a + b*x)])*Tan[(a + b*x)/2])/(25*b^2*Sqrt[Csc[a
+ b*x]])
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\csc(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(3/2), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\csc(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/csc(b*x+a)^(3/2),x)
```

```
[Out] int(x*cos(b*x+a)/csc(b*x+a)^(3/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(bx+a\right)}{\csc\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(3/2), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\cos\left(a+b\,x\right)}{{\left(\frac{1}{\sin(a+b\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/(1/sin(a + b*x))^(3/2),x)
```

```
[Out] int((x*cos(a + b*x))/(1/sin(a + b*x))^(3/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(a+bx\right)}{\csc^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/csc(a + b*x)**(3/2), x)
```

# 3.359 $\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$

**Optimal**. Leaf size=108

$$\frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)}+\frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}}-\frac{20\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}$$

```
[Out] 2/7*x/b/csc(b*x+a)^(7/2)+4/49*cos(b*x+a)/b^2/csc(b*x+a)^(5/2)+20/147*cos(b*
x+a)/b^2/csc(b*x+a)^(1/2)+20/147*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/
2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)
^(1/2)*sin(b*x+a)^(1/2)/b^2
```

**Rubi [A]** time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3769, 3771, 2641}

$$\frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)}+\frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}}-\frac{20\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Csc[a + b*x]^(5/2),x]
```

```
[Out] (2*x)/(7*b*Csc[a + b*x]^(7/2)) + (4*Cos[a + b*x])/(49*b^2*Csc[a + b*x]^(5/2
)) + (20*Cos[a + b*x])/(147*b^2*Sqrt[Csc[a + b*x]]) - (20*Sqrt[Csc[a + b*x]
]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(147*b^2)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} - \frac{10\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)}\,dx}{49b}$$

$$= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{10\int \sqrt{\csc(a+bx)}\,dx}{147b}$$

$$= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{\left(10\sqrt{\csc(a+bx)}\sqrt{\sin(a+b}\right.}{147b}$$

$$= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{20\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a-\frac{\pi}{2}\right.\right.}{147b^2}$$

**Mathematica [A]** time = 0.39, size = 93, normalized size = 0.86

$$\frac{\sqrt{\csc(a+bx)}\left(52\sin(2(a+bx)) - 6\sin(4(a+bx)) - 84bx\cos(2(a+bx)) + 21bx\cos(4(a+bx)) + 80\sqrt{\sin(a+bx}\right.}{588b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Csc[a + b*x]^(5/2),x]
```

```
[Out] (Sqrt[Csc[a + b*x]]*(63*b*x - 84*b*x*Cos[2*(a + b*x)] + 21*b*x*Cos[4*(a + b
*x)] + 80*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]] + 52*Sin[2
*(a + b*x)] - 6*Sin[4*(a + b*x)]))/(588*b^2)
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\csc(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(5/2), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\csc(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/csc(b*x+a)^(5/2),x)
```

```
[Out] int(x*cos(b*x+a)/csc(b*x+a)^(5/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(bx+a\right)}{\csc\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\cos\left(a+b\,x\right)}{{\left(\frac{1}{\sin\left(a+b\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/(1/sin(a + b*x))^(5/2),x)
```

```
[Out] int((x*cos(a + b*x))/(1/sin(a + b*x))^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(a+bx\right)}{\csc^{\frac{5}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(5/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/csc(a + b*x)**(5/2), x)
```

## 3.360 $\int x \csc(x) \sin(3x)\,dx$

**Optimal**. Leaf size=31

$$\frac{x^2}{2} - \frac{\sin^2(x)}{4} + \frac{3\cos^2(x)}{4} + 2x\sin(x)\cos(x)$$

```
[Out] 1/2*x^2+3/4*cos(x)^2+2*x*cos(x)*sin(x)-1/4*sin(x)^2
```

**Rubi [A]** time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4431, 3310, 30}

$$\frac{x^2}{2} - \frac{\sin^2(x)}{4} + \frac{3\cos^2(x)}{4} + 2x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]*Sin[3*x],x]
```

```
[Out] x^2/2 + (3*Cos[x]^2)/4 + 2*x*Cos[x]*Sin[x] - Sin[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int x\csc(x)\sin(3x)\,dx &= \int \left(3x\cos^2(x) - x\sin^2(x)\right)\,dx\\
&= 3\int x\cos^2(x)\,dx - \int x\sin^2(x)\,dx\\
&= \frac{3\cos^2(x)}{4} + 2x\cos(x)\sin(x) - \frac{\sin^2(x)}{4} - \frac{\int x\,dx}{2} + \frac{3\int x\,dx}{2}\\
&= \frac{x^2}{2} + \frac{3\cos^2(x)}{4} + 2x\cos(x)\sin(x) - \frac{\sin^2(x)}{4}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 22, normalized size = 0.71

$$\frac{x^2}{2} + x\sin(2x) + \frac{1}{2}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]*Sin[3*x],x]
[Out] x^2/2 + Cos[2*x]/2 + x*Sin[2*x]
```

**fricas** [A] time = 0.45, size = 17, normalized size = 0.55

$$2\,x\cos(x)\sin(x) + \frac{1}{2}\,x^2 + \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x, algorithm="fricas")
[Out] 2*x*cos(x)*sin(x) + 1/2*x^2 + cos(x)^2
```

**giac** [A] time = 0.15, size = 18, normalized size = 0.58

$$\frac{1}{2}\,x^2 + x\sin\left(2\,x\right) + \frac{1}{2}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x, algorithm="giac")
[Out] 1/2*x^2 + x*sin(2*x) + 1/2*cos(2*x)
```

**maple** [A] time = 0.06, size = 26, normalized size = 0.84

$$4x\left(\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}\right) - \frac{3x^2}{2} - \left(\sin^2(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sin(3*x),x)
[Out] 4*x*(1/2*cos(x)*sin(x)+1/2*x)-3/2*x^2-sin(x)^2
```

**maxima** [A] time = 0.32, size = 18, normalized size = 0.58

$$\frac{1}{2}\,x^2 + x\sin\left(2\,x\right) + \frac{1}{2}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x, algorithm="maxima")
[Out] 1/2*x^2 + x*sin(2*x) + 1/2*cos(2*x)
```

**mupad** [B] time = 0.09, size = 18, normalized size = 0.58

$$\frac{\cos\left(2\,x\right)}{2} + x\,\sin\left(2\,x\right) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(3*x))/sin(x),x)
[Out] cos(2*x)/2 + x*sin(2*x) + x^2/2
```

**sympy** [A] time = 1.88, size = 37, normalized size = 1.19

$$-x^2\sin^2\left(x\right) - x^2\cos^2\left(x\right) + \frac{3x^2}{2} + 2x\sin\left(x\right)\cos\left(x\right) + \cos^2\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x)
[Out] -x**2*sin(x)**2 - x**2*cos(x)**2 + 3*x**2/2 + 2*x*sin(x)*cos(x) + cos(x)**2
```

# 3.361 $\int (c+dx)^4 \csc(x)\sin(3x)\,dx$

**Optimal**. Leaf size=131

$$\frac{3}{2}d^3\sin^2(x)(c+dx)-\frac{9}{2}d^3\cos^2(x)(c+dx)-6d^2\sin(x)\cos(x)(c+dx)^2+\frac{(c+dx)^5}{5d}-d(c+dx)^3-d\sin^2(x)(c+dx)^3+3d\cos$$

```
[Out] 3/2*d^4*x-d*(d*x+c)^3+1/5*(d*x+c)^5/d-9/2*d^3*(d*x+c)*cos(x)^2+3*d*(d*x+c)^
3*cos(x)^2+3*d^4*cos(x)*sin(x)-6*d^2*(d*x+c)^2*cos(x)*sin(x)+2*(d*x+c)^4*co
s(x)*sin(x)+3/2*d^3*(d*x+c)*sin(x)^2-d*(d*x+c)^3*sin(x)^2
```

**Rubi [A]** time = 0.19, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4431, 3311, 32, 2635, 8}

$$\frac{3}{2}d^3\sin^2(x)(c+dx)-\frac{9}{2}d^3\cos^2(x)(c+dx)-6d^2\sin(x)\cos(x)(c+dx)^2+\frac{(c+dx)^5}{5d}-d(c+dx)^3-d\sin^2(x)(c+dx)^3+3d\cos$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[x]*Sin[3*x],x]
```

```
[Out] (3*d^4*x)/2 - d*(c + d*x)^3 + (c + d*x)^5/(5*d) - (9*d^3*(c + d*x)*Cos[x]^2
)/2 + 3*d*(c + d*x)^3*Cos[x]^2 + 3*d^4*Cos[x]*Sin[x] - 6*d^2*(c + d*x)^2*Co
s[x]*Sin[x] + 2*(c + d*x)^4*Cos[x]*Sin[x] + (3*d^3*(c + d*x)*Sin[x]^2)/2 -
d*(c + d*x)^3*Sin[x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)^4 \csc(x)\sin(3x)\,dx = \int \left(3(c+dx)^4\cos^2(x) - (c+dx)^4\sin^2(x)\right) dx$$
$$= 3\int (c+dx)^4\cos^2(x)\,dx - \int (c+dx)^4\sin^2(x)\,dx$$
$$= 3d(c+dx)^3\cos^2(x) + 2(c+dx)^4\cos(x)\sin(x) - d(c+dx)^3\sin^2(x) - \frac{1}{2}\int (c+$$
$$= \frac{(c+dx)^5}{5d} - \frac{9}{2}d^3(c+dx)\cos^2(x) + 3d(c+dx)^3\cos^2(x) - 6d^2(c+dx)^2\cos(x)\mathrm{si}$$
$$= -d(c+dx)^3 + \frac{(c+dx)^5}{5d} - \frac{9}{2}d^3(c+dx)\cos^2(x) + 3d(c+dx)^3\cos^2(x) + 3d^4\cos$$
$$= \frac{3d^4x}{2} - d(c+dx)^3 + \frac{(c+dx)^5}{5d} - \frac{9}{2}d^3(c+dx)\cos^2(x) + 3d(c+dx)^3\cos^2(x) + $$

**Mathematica [A]** time = 0.22, size = 154, normalized size = 1.18

$$c^4x+2c^3dx^2+2c^2d^2x^3+d\cos(2x)\left(2c^3+6c^2dx+3cd^2\left(2x^2-1\right)+d^3x\left(2x^2-3\right)\right)+\frac{1}{2}\sin(2x)\left(2c^4+8c^3dx+6c^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[x]*Sin[3*x],x]
```

```
[Out] c^4*x + 2*c^3*d*x^2 + 2*c^2*d^2*x^3 + c*d^3*x^4 + (d^4*x^5)/5 + d*(2*c^3 +
6*c^2*d*x + d^3*x*(-3 + 2*x^2) + 3*c*d^2*(-1 + 2*x^2))*Cos[2*x] + ((2*c^4 +
 8*c^3*d*x + 4*c*d^3*x*(-3 + 2*x^2) + 6*c^2*d^2*(-1 + 2*x^2) + d^4*(3 - 6*x
^2 + 2*x^4))*Sin[2*x])/2
```

**fricas [A]** time = 0.45, size = 200, normalized size = 1.53

$$\frac{1}{5}d^4x^5+cd^3x^4+2\left(c^2d^2-d^4\right)x^3+2\left(c^3d-3cd^3\right)x^2+2\left(2d^4x^3+6cd^3x^2+2c^3d-3cd^3+3\left(2c^2d^2-d^4\right)x\right)\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 1/5*d^4*x^5 + c*d^3*x^4 + 2*(c^2*d^2 - d^4)*x^3 + 2*(c^3*d - 3*c*d^3)*x^2 +
 2*(2*d^4*x^3 + 6*c*d^3*x^2 + 2*c^3*d - 3*c*d^3 + 3*(2*c^2*d^2 - d^4)*x)*co
s(x)^2 + (2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 - 6*c^2*d^2 + 3*d^4 + 6*(2*c^2*d^
2 - d^4)*x^2 + 4*(2*c^3*d - 3*c*d^3)*x)*cos(x)*sin(x) + (c^4 - 6*c^2*d^2 +
3*d^4)*x
```

**giac [A]** time = 2.64, size = 167, normalized size = 1.27

$$\frac{1}{5}d^4x^5+cd^3x^4+2c^2d^2x^3+2c^3dx^2+c^4x+\left(2d^4x^3+6cd^3x^2+6c^2d^2x-3d^4x+2c^3d-3cd^3\right)\cos(2x)+\frac{1}{2}\left(2d^4x^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/5*d^4*x^5 + c*d^3*x^4 + 2*c^2*d^2*x^3 + 2*c^3*d*x^2 + c^4*x + (2*d^4*x^3
+ 6*c*d^3*x^2 + 6*c^2*d^2*x - 3*d^4*x + 2*c^3*d - 3*c*d^3)*cos(2*x) + 1/2*(
2*d^4*x^4 + 8*c*d^3*x^3 + 12*c^2*d^2*x^2 - 6*d^4*x^2 + 8*c^3*d*x - 12*c*d^3
*x + 2*c^4 - 6*c^2*d^2 + 3*d^4)*sin(2*x)
```

**maple [B]** time = 0.08, size = 260, normalized size = 1.98

$$4d^4\left(x^4\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+x^3\left(\cos^2(x)\right)-3x^2\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x\left(\cos^2(x)\right)}{2}+\frac{3\cos(x)\sin(x)}{4}+\frac{3x}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(x)*sin(3*x),x)
```

```
[Out] 4*d^4*(x^4*(1/2*cos(x)*sin(x)+1/2*x)+x^3*cos(x)^2-3*x^2*(1/2*cos(x)*sin(x)+
1/2*x)-3/2*x*cos(x)^2+3/4*cos(x)*sin(x)+3/4*x+x^3-2/5*x^5)+16*d^3*c*(x^3*(1
/2*cos(x)*sin(x)+1/2*x)+3/4*x^2*cos(x)^2-3/2*x*(1/2*cos(x)*sin(x)+1/2*x)+3/
8*x^2+3/8*sin(x)^2-3/8*x^4)+24*c^2*d^2*(x^2*(1/2*cos(x)*sin(x)+1/2*x)+1/2*x
*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x-1/3*x^3)-1/5*d^4*x^5+16*c^3*d*(x*(1/2*cos
(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)-c*d^3*x^4+4*c^4*(1/2*cos(x)*sin(x)+
1/2*x)-2*c^2*d^2*x^3-2*c^3*d*x^2-c^4*x
```

**maxima [A]** time = 0.35, size = 146, normalized size = 1.11

$$2\left(x^{2}+2\,x\sin\left(2\,x\right)+\cos\left(2\,x\right)\right)c^{3}d+\left(2\,x^{3}+6\,x\cos\left(2\,x\right)+3\left(2\,x^{2}-1\right)\sin\left(2\,x\right)\right)c^{2}d^{2}+\left(x^{4}+3\left(2\,x^{2}-1\right)\cos\left(2\,x\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 2*(x^2 + 2*x*sin(2*x) + cos(2*x))*c^3*d + (2*x^3 + 6*x*cos(2*x) + 3*(2*x^2
- 1)*sin(2*x))*c^2*d^2 + (x^4 + 3*(2*x^2 - 1)*cos(2*x) + 2*(2*x^3 - 3*x)*si
n(2*x))*c*d^3 + 1/10*(2*x^5 + 10*(2*x^3 - 3*x)*cos(2*x) + 5*(2*x^4 - 6*x^2
+ 3)*sin(2*x))*d^4 + c^4*(x + sin(2*x))
```

**mupad [B]** time = 2.26, size = 212, normalized size = 1.62

$$c^{4}\,\sin\left(2\,x\right)+\frac{3\,d^{4}\,\sin\left(2\,x\right)}{2}+c^{4}\,x+\frac{d^{4}\,x^{5}}{5}-3\,c^{2}\,d^{2}\,\sin\left(2\,x\right)+2\,d^{4}\,x^{3}\,\cos\left(2\,x\right)-3\,d^{4}\,x^{2}\,\sin\left(2\,x\right)+d^{4}\,x^{4}\,\sin\left(2\,x\right)+2\,c^{3}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*x)*(c + d*x)^4)/sin(x),x)
```

```
[Out] c^4*sin(2*x) + (3*d^4*sin(2*x))/2 + c^4*x + (d^4*x^5)/5 - 3*c^2*d^2*sin(2*x
) + 2*d^4*x^3*cos(2*x) - 3*d^4*x^2*sin(2*x) + d^4*x^4*sin(2*x) + 2*c^3*d*x^
2 + c*d^3*x^4 + 2*c^2*d^2*x^3 - 3*c*d^3*cos(2*x) + 2*c^3*d*cos(2*x) - 3*d^4
*x*cos(2*x) + 6*c^2*d^2*x^2*sin(2*x) - 6*c*d^3*x*sin(2*x) + 4*c^3*d*x*sin(2
*x) + 6*c^2*d^2*x*cos(2*x) + 6*c*d^3*x^2*cos(2*x) + 4*c*d^3*x^3*sin(2*x)
```

**sympy [B]** time = 25.35, size = 440, normalized size = 3.36

$$c^{4}x+c^{4}\sin\left(2x\right)-4c^{3}dx^{2}\sin^{2}\left(x\right)-4c^{3}dx^{2}\cos^{2}\left(x\right)+6c^{3}dx^{2}+8c^{3}dx\sin\left(x\right)\cos\left(x\right)+4c^{3}d\cos^{2}\left(x\right)-4c^{2}d^{2}x^{3}\sin^{2}\left(x\right)-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(x)*sin(3*x),x)
```

```
[Out] c**4*x + c**4*sin(2*x) - 4*c**3*d*x**2*sin(x)**2 - 4*c**3*d*x**2*cos(x)**2
+ 6*c**3*d*x**2 + 8*c**3*d*x*sin(x)*cos(x) + 4*c**3*d*cos(x)**2 - 4*c**2*d*
*2*x**3*sin(x)**2 - 4*c**2*d**2*x**3*cos(x)**2 + 6*c**2*d**2*x**3 + 12*c**2
*d**2*x**2*sin(x)*cos(x) - 6*c**2*d**2*x*sin(x)**2 + 6*c**2*d**2*x*cos(x)**
2 - 6*c**2*d**2*sin(x)*cos(x) - 2*c*d**3*x**4*sin(x)**2 - 2*c*d**3*x**4*cos
(x)**2 + 3*c*d**3*x**4 + 8*c*d**3*x**3*sin(x)*cos(x) - 6*c*d**3*x**2*sin(x)
**2 + 6*c*d**3*x**2*cos(x)**2 - 12*c*d**3*x*sin(x)*cos(x) - 6*c*d**3*cos(x)
**2 - 2*d**4*x**5*sin(x)**2/5 - 2*d**4*x**5*cos(x)**2/5 + 3*d**4*x**5/5 + 2
*d**4*x**4*sin(x)*cos(x) - 2*d**4*x**3*sin(x)**2 + 2*d**4*x**3*cos(x)**2 -
6*d**4*x**2*sin(x)*cos(x) + 3*d**4*x*sin(x)**2 - 3*d**4*x*cos(x)**2 + 3*d**
4*sin(x)*cos(x)
```

# 3.362 $\int (c+dx)^3 \csc(x)\sin(3x)\,dx$

**Optimal**. Leaf size=115

$$-\frac{3}{2}cd^2x-3d^2\sin(x)\cos(x)(c+dx)+\frac{(c+dx)^4}{4d}-\frac{3}{4}d\sin^2(x)(c+dx)^2+\frac{9}{4}d\cos^2(x)(c+dx)^2+2\sin(x)\cos(x)(c+dx)^3-$$

```
[Out] -3/2*c*d^2*x-3/4*d^3*x^2+1/4*(d*x+c)^4/d-9/8*d^3*cos(x)^2+9/4*d*(d*x+c)^2*c
os(x)^2-3*d^2*(d*x+c)*cos(x)*sin(x)+2*(d*x+c)^3*cos(x)*sin(x)+3/8*d^3*sin(x
)^2-3/4*d*(d*x+c)^2*sin(x)^2
```

**Rubi [A]** time = 0.14, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4431, 3311, 32, 3310}

$$-\frac{3}{2}cd^2x-3d^2\sin(x)\cos(x)(c+dx)+\frac{(c+dx)^4}{4d}-\frac{3}{4}d\sin^2(x)(c+dx)^2+\frac{9}{4}d\cos^2(x)(c+dx)^2+2\sin(x)\cos(x)(c+dx)^3-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[x]*Sin[3*x],x]
```

```
[Out] (-3*c*d^2*x)/2 - (3*d^3*x^2)/4 + (c + d*x)^4/(4*d) - (9*d^3*Cos[x]^2)/8 + (
9*d*(c + d*x)^2*Cos[x]^2)/4 - 3*d^2*(c + d*x)*Cos[x]*Sin[x] + 2*(c + d*x)^3
*Cos[x]*Sin[x] + (3*d^3*Sin[x]^2)/8 - (3*d*(c + d*x)^2*Sin[x]^2)/4
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)^3 \csc(x)\sin(3x)\,dx = \int \left(3(c+dx)^3\cos^2(x) - (c+dx)^3\sin^2(x)\right)\,dx$$
$$= 3\int (c+dx)^3\cos^2(x)\,dx - \int (c+dx)^3\sin^2(x)\,dx$$
$$= \frac{9}{4}d(c+dx)^2\cos^2(x) + 2(c+dx)^3\cos(x)\sin(x) - \frac{3}{4}d(c+dx)^2\sin^2(x) - \frac{1}{2}\int (c+d$$
$$= \frac{(c+dx)^4}{4d} - \frac{9}{8}d^3\cos^2(x) + \frac{9}{4}d(c+dx)^2\cos^2(x) - 3d^2(c+dx)\cos(x)\sin(x) + 2(c$$
$$= -\frac{3}{2}cd^2x - \frac{3d^3x^2}{4} + \frac{(c+dx)^4}{4d} - \frac{9}{8}d^3\cos^2(x) + \frac{9}{4}d(c+dx)^2\cos^2(x) - 3d^2(c+dx)$$

**Mathematica** [A] time = 0.16, size = 109, normalized size = 0.95

$$\frac{1}{4}\left(3d\cos(2x)\left(2c^2+4cdx+d^2\left(2x^2-1\right)\right)+2\sin(2x)\left(2c^3+6c^2dx+3cd^2\left(2x^2-1\right)+d^3x\left(2x^2-3\right)\right)+x\left(4c^3+6\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[x]*Sin[3*x],x]
```

```
[Out] (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(2*c^2 + 4*c*d*x + d^2
*(-1 + 2*x^2))*Cos[2*x] + 2*(2*c^3 + 6*c^2*d*x + d^3*x*(-3 + 2*x^2) + 3*c*d
^2*(-1 + 2*x^2))*Sin[2*x])/4
```

**fricas** [A] time = 0.47, size = 127, normalized size = 1.10

$$\frac{1}{4}d^3x^4+cd^2x^3+\frac{3}{2}\left(c^2d-d^3\right)x^2+\frac{3}{2}\left(2d^3x^2+4cd^2x+2c^2d-d^3\right)\cos(x)^2+\left(2d^3x^3+6cd^2x^2+2c^3-3cd^2+3\left(2c^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*(c^2*d - d^3)*x^2 + 3/2*(2*d^3*x^2 + 4*c*d^2*
x + 2*c^2*d - d^3)*cos(x)^2 + (2*d^3*x^3 + 6*c*d^2*x^2 + 2*c^3 - 3*c*d^2 +
3*(2*c^2*d - d^3)*x)*cos(x)*sin(x) + (c^3 - 3*c*d^2)*x
```

**giac** [A] time = 0.29, size = 112, normalized size = 0.97

$$\frac{1}{4}d^3x^4+cd^2x^3+\frac{3}{2}c^2dx^2+c^3x+\frac{3}{4}\left(2d^3x^2+4cd^2x+2c^2d-d^3\right)\cos(2x)+\frac{1}{2}\left(2d^3x^3+6cd^2x^2+6c^2dx-3d^3x+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x + 3/4*(2*d^3*x^2 + 4*c*d^2*
x + 2*c^2*d - d^3)*cos(2*x) + 1/2*(2*d^3*x^3 + 6*c*d^2*x^2 + 6*c^2*d*x - 3*
d^3*x + 2*c^3 - 3*c*d^2)*sin(2*x)
```

**maple** [A] time = 0.06, size = 179, normalized size = 1.56

$$4d^3\left(x^3\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+\frac{3x^2\left(\cos^2(x)\right)}{4}-\frac{3x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)}{2}+\frac{3x^2}{8}+\frac{3\left(\sin^2(x)\right)}{8}-\frac{3x^4}{8}\right)+12cd^2\left(x^2\left(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(x)*sin(3*x),x)
```

[Out] 4*d^3*(x^3*(1/2*cos(x)*sin(x)+1/2*x)+3/4*x^2*cos(x)^2-3/2*x*(1/2*cos(x)*sin(x)+1/2*x)+3/8*x^2+3/8*sin(x)^2-3/8*x^4)+12*c*d^2*(x^2*(1/2*cos(x)*sin(x)+1/2*x)+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x-1/3*x^3)+12*c^2*d*(x*(1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)-1/4*d^3*x^4+4*c^3*(1/2*cos(x)*sin(x)+1/2*x)-c*d^2*x^3-3/2*c^2*d*x^2-c^3*x

**maxima** [A] time = 0.34, size = 101, normalized size = 0.88

$$\frac{3}{2}\left(x^2+2\,x\sin\left(2\,x\right)+\cos\left(2\,x\right)\right)c^2d+\frac{1}{2}\left(2\,x^3+6\,x\cos\left(2\,x\right)+3\left(2\,x^2-1\right)\sin\left(2\,x\right)\right)cd^2+\frac{1}{4}\left(x^4+3\left(2\,x^2-1\right)\text{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 3/2*(x^2 + 2*x*sin(2*x) + cos(2*x))*c^2*d + 1/2*(2*x^3 + 6*x*cos(2*x) + 3*(2*x^2 - 1)*sin(2*x))*c*d^2 + 1/4*(x^4 + 3*(2*x^2 - 1)*cos(2*x) + 2*(2*x^3 - 3*x)*sin(2*x))*d^3 + c^3*(x + sin(2*x))
```

**mupad** [B] time = 0.34, size = 136, normalized size = 1.18

$$c^3\,\sin\left(2\,x\right)-\frac{3\,d^3\,\cos\left(2\,x\right)}{4}+c^3\,x+\frac{d^3\,x^4}{4}+\frac{3\,d^3\,x^2\,\cos\left(2\,x\right)}{2}+d^3\,x^3\,\sin\left(2\,x\right)+\frac{3\,c^2\,d\,x^2}{2}+c\,d^2\,x^3+\frac{3\,c^2\,d\,\cos\left(2\,x\right)}{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*x)*(c + d*x)^3)/sin(x),x)
```

```
[Out] c^3*sin(2*x) - (3*d^3*cos(2*x))/4 + c^3*x + (d^3*x^4)/4 + (3*d^3*x^2*cos(2*x))/2 + d^3*x^3*sin(2*x) + (3*c^2*d*x^2)/2 + c*d^2*x^3 + (3*c^2*d*cos(2*x))/2 - (3*c*d^2*sin(2*x))/2 - (3*d^3*x*sin(2*x))/2 + 3*c*d^2*x*cos(2*x) + 3*c^2*d*x*sin(2*x) + 3*c*d^2*x^2*sin(2*x)
```

**sympy** [B] time = 13.05, size = 289, normalized size = 2.51

$$c^3x+c^3\sin\left(2x\right)-3c^2dx^2\sin^2\left(x\right)-3c^2dx^2\cos^2\left(x\right)+\frac{9c^2dx^2}{2}+6c^2dx\sin\left(x\right)\cos\left(x\right)+3c^2d\cos^2\left(x\right)-2cd^2x^3\sin^2\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(x)*sin(3*x),x)
```

```
[Out] c**3*x + c**3*sin(2*x) - 3*c**2*d*x**2*sin(x)**2 - 3*c**2*d*x**2*cos(x)**2 + 9*c**2*d*x**2/2 + 6*c**2*d*x*sin(x)*cos(x) + 3*c**2*d*cos(x)**2 - 2*c*d**2*x**3*sin(x)**2 - 2*c*d**2*x**3*cos(x)**2 + 3*c*d**2*x**3 + 6*c*d**2*x**2*sin(x)*cos(x) - 3*c*d**2*x*sin(x)**2 + 3*c*d**2*x*cos(x)**2 - 3*c*d**2*sin(x)*cos(x) - d**3*x**4*sin(x)**2/2 - d**3*x**4*cos(x)**2/2 + 3*d**3*x**4/4 + 2*d**3*x**3*sin(x)*cos(x) - 3*d**3*x**2*sin(x)**2/2 + 3*d**3*x**2*cos(x)**2/2 - 3*d**3*x*sin(x)*cos(x) - 3*d**3*cos(x)**2/2
```

# 3.363 $\int (c + dx)^2 \csc(x) \sin(3x)\, dx$

**Optimal**. Leaf size=73

$$\frac{(c+dx)^3}{3d} - \frac{1}{2}d\sin^2(x)(c+dx) + \frac{3}{2}d\cos^2(x)(c+dx) + 2\sin(x)\cos(x)(c+dx)^2 - \frac{d^2x}{2} - d^2\sin(x)\cos(x)$$

```
[Out] -1/2*d^2*x+1/3*(d*x+c)^3/d+3/2*d*(d*x+c)*cos(x)^2-d^2*cos(x)*sin(x)+2*(d*x+
c)^2*cos(x)*sin(x)-1/2*d*(d*x+c)*sin(x)^2
```

**Rubi [A]** time = 0.10, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4431, 3311, 32, 2635, 8}

$$\frac{(c+dx)^3}{3d} - \frac{1}{2}d\sin^2(x)(c+dx) + \frac{3}{2}d\cos^2(x)(c+dx) + 2\sin(x)\cos(x)(c+dx)^2 - \frac{d^2x}{2} - d^2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[x]*Sin[3*x],x]
```

```
[Out] -(d^2*x)/2 + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*Cos[x]^2)/2 - d^2*Cos[x]*Si
n[x] + 2*(c + d*x)^2*Cos[x]*Sin[x] - (d*(c + d*x)*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\csc(x)\sin(3x)\,dx &= \int \left(3(c+dx)^2\cos^2(x)-(c+dx)^2\sin^2(x)\right)\,dx\\ &= 3\int (c+dx)^2\cos^2(x)\,dx-\int (c+dx)^2\sin^2(x)\,dx\\ &= \frac{3}{2}d(c+dx)\cos^2(x)+2(c+dx)^2\cos(x)\sin(x)-\frac{1}{2}d(c+dx)\sin^2(x)-\frac{1}{2}\int (c+\\ &= \frac{(c+dx)^3}{3d}+\frac{3}{2}d(c+dx)\cos^2(x)-d^2\cos(x)\sin(x)+2(c+dx)^2\cos(x)\sin(x)-\\ &= -\frac{d^2x}{2}+\frac{(c+dx)^3}{3d}+\frac{3}{2}d(c+dx)\cos^2(x)-d^2\cos(x)\sin(x)+2(c+dx)^2\cos(x)\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 60, normalized size = 0.82

$$\sin(x)\cos(x)\left(2c^2+4cdx+d^2\left(2x^2-1\right)\right)+c^2x+cdx^2+d\cos(2x)(c+dx)+\frac{d^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[x]*Sin[3*x],x]
```

```
[Out] c^2*x + c*d*x^2 + (d^2*x^3)/3 + d*(c + d*x)*Cos[2*x] + (2*c^2 + 4*c*d*x + d
^2*(-1 + 2*x^2))*Cos[x]*Sin[x]
```

**fricas [A]** time = 0.45, size = 70, normalized size = 0.96

$$\frac{1}{3}d^2x^3+cdx^2+2\left(d^2x+cd\right)\cos(x)^2+\left(2\,d^2x^2+4\,cdx+2\,c^2-d^2\right)\cos(x)\sin(x)+\left(c^2-d^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + 2*(d^2*x + c*d)*cos(x)^2 + (2*d^2*x^2 + 4*c*d*x + 2
*c^2 - d^2)*cos(x)*sin(x) + (c^2 - d^2)*x
```

**giac [A]** time = 0.16, size = 64, normalized size = 0.88

$$\frac{1}{3}d^2x^3+cdx^2+c^2x+\left(d^2x+cd\right)\cos\left(2\,x\right)+\frac{1}{2}\left(2\,d^2x^2+4\,cdx+2\,c^2-d^2\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + c^2*x + (d^2*x + c*d)*cos(2*x) + 1/2*(2*d^2*x^2 + 4
*c*d*x + 2*c^2 - d^2)*sin(2*x)
```

**maple [A]** time = 0.06, size = 107, normalized size = 1.47

$$4d^2\left(x^2\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+\frac{x\left(\cos^2(x)\right)}{2}-\frac{\cos(x)\sin(x)}{4}-\frac{x}{4}-\frac{x^3}{3}\right)+8cd\left(x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{x^2}{4}-\frac{(\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(x)*sin(3*x),x)
```

```
[Out] 4*d^2*(x^2*(1/2*cos(x)*sin(x)+1/2*x)+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x
-1/3*x^3)+8*c*d*(x*(1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)+4*c^2*(1
/2*cos(x)*sin(x)+1/2*x)-1/3*d^2*x^3-c*d*x^2-c^2*x
```

**maxima [A]** time = 0.33, size = 60, normalized size = 0.82

$$\left(x^{2}+2\,x\sin\left(2\,x\right)+\cos\left(2\,x\right)\right)cd+\frac{1}{6}\left(2\,x^{3}+6\,x\cos\left(2\,x\right)+3\left(2\,x^{2}-1\right)\sin\left(2\,x\right)\right)d^{2}+c^{2}\left(x+\sin\left(2\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] (x^2 + 2*x*sin(2*x) + cos(2*x))*c*d + 1/6*(2*x^3 + 6*x*cos(2*x) + 3*(2*x^2
- 1)*sin(2*x))*d^2 + c^2*(x + sin(2*x))
```

**mupad [B]** time = 1.82, size = 73, normalized size = 1.00

$$c^{2}\,\sin\left(2\,x\right)-\frac{d^{2}\,\sin\left(2\,x\right)}{2}+c^{2}\,x+\frac{d^{2}\,x^{3}}{3}+d^{2}\,x^{2}\,\sin\left(2\,x\right)+c\,d\,\cos\left(2\,x\right)+d^{2}\,x\,\cos\left(2\,x\right)+c\,d\,x^{2}+2\,c\,d\,x\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*x)*(c + d*x)^2)/sin(x),x)
```

```
[Out] c^2*sin(2*x) - (d^2*sin(2*x))/2 + c^2*x + (d^2*x^3)/3 + d^2*x^2*sin(2*x) +
c*d*cos(2*x) + d^2*x*cos(2*x) + c*d*x^2 + 2*c*d*x*sin(2*x)
```

**sympy [B]** time = 6.97, size = 155, normalized size = 2.12

$$c^{2}x+c^{2}\sin\left(2x\right)-2cdx^{2}\sin^{2}\left(x\right)-2cdx^{2}\cos^{2}\left(x\right)+3cdx^{2}+4cdx\sin\left(x\right)\cos\left(x\right)+2cd\cos^{2}\left(x\right)-\frac{2d^{2}x^{3}\sin^{2}\left(x\right)}{3}-\frac{2d^{2}x^{3}\cos}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(x)*sin(3*x),x)
```

```
[Out] c**2*x + c**2*sin(2*x) - 2*c*d*x**2*sin(x)**2 - 2*c*d*x**2*cos(x)**2 + 3*c*
d*x**2 + 4*c*d*x*sin(x)*cos(x) + 2*c*d*cos(x)**2 - 2*d**2*x**3*sin(x)**2/3
- 2*d**2*x**3*cos(x)**2/3 + d**2*x**3 + 2*d**2*x**2*sin(x)*cos(x) - d**2*x*
sin(x)**2 + d**2*x*cos(x)**2 - d**2*sin(x)*cos(x)
```

# 3.364 $\int (c+dx)\csc(x)\sin(3x)\,dx$

**Optimal**. Leaf size=41

$$2\sin(x)\cos(x)(c+dx)+cx+\frac{dx^2}{2}-\frac{1}{4}d\sin^2(x)+\frac{3}{4}d\cos^2(x)$$

[Out] c*x+1/2*d*x^2+3/4*d*cos(x)^2+2*(d*x+c)*cos(x)*sin(x)-1/4*d*sin(x)^2

**Rubi** [A] time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4431, 3310}

$$2\sin(x)\cos(x)(c+dx)+cx+\frac{dx^2}{2}-\frac{1}{4}d\sin^2(x)+\frac{3}{4}d\cos^2(x)$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*Csc[x]*Sin[3*x],x]

[Out] c*x + (d*x^2)/2 + (3*d*Cos[x]^2)/4 + 2*(c + d*x)*Cos[x]*Sin[x] - (d*Sin[x]^2)/4

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\csc(x)\sin(3x)\,dx &= \int \left(3(c+dx)\cos^2(x)-(c+dx)\sin^2(x)\right)\,dx\\
&= 3\int (c+dx)\cos^2(x)\,dx-\int (c+dx)\sin^2(x)\,dx\\
&= \frac{3}{4}d\cos^2(x)+2(c+dx)\cos(x)\sin(x)-\frac{1}{4}d\sin^2(x)-\frac{1}{2}\int (c+dx)\,dx+\frac{3}{2}\int (c+\\
&= cx+\frac{dx^2}{2}+\frac{3}{4}d\cos^2(x)+2(c+dx)\cos(x)\sin(x)-\frac{1}{4}d\sin^2(x)
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 34, normalized size = 0.83

$$cx+c\sin(2x)+\frac{dx^2}{2}+dx\sin(2x)+\frac{1}{2}d\cos(2x)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Csc[x]*Sin[3*x],x]

```
[Out] c*x + (d*x^2)/2 + (d*Cos[2*x])/2 + c*Sin[2*x] + d*x*Sin[2*x]
```

**fricas** [A] time = 0.44, size = 27, normalized size = 0.66

$$\frac{1}{2}\,dx^2 + d\cos(x)^2 + 2\,(dx + c)\cos(x)\sin(x) + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 1/2*d*x^2 + d*cos(x)^2 + 2*(d*x + c)*cos(x)*sin(x) + c*x
```

**giac** [A] time = 1.51, size = 27, normalized size = 0.66

$$\frac{1}{2}\,dx^2 + cx + \frac{1}{2}\,d\cos\left(2\,x\right) + (dx + c)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + c*x + 1/2*d*cos(2*x) + (d*x + c)*sin(2*x)
```

**maple** [A] time = 0.06, size = 52, normalized size = 1.27

$$4d\left(x\left(\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}\right) - \frac{x^2}{4} - \frac{\left(\sin^2(x)\right)}{4}\right) + 4c\left(\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}\right) - \frac{d\,x^2}{2} - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(x)*sin(3*x),x)
```

```
[Out] 4*d*(x*(1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)+4*c*(1/2*cos(x)*sin(
x)+1/2*x)-1/2*d*x^2-c*x
```

**maxima** [A] time = 0.34, size = 27, normalized size = 0.66

$$\frac{1}{2}\left(x^2 + 2\,x\sin\left(2\,x\right) + \cos\left(2\,x\right)\right)d + c(x + \sin\left(2\,x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 + 2*x*sin(2*x) + cos(2*x))*d + c*(x + sin(2*x))
```

**mupad** [B] time = 1.72, size = 30, normalized size = 0.73

$$c\,\sin\left(2\,x\right) + c\,x + \frac{d\,x^2}{2} + \frac{d\,\cos\left(2\,x\right)}{2} + d\,x\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*x)*(c + d*x))/sin(x),x)
```

```
[Out] c*sin(2*x) + c*x + (d*x^2)/2 + (d*cos(2*x))/2 + d*x*sin(2*x)
```

**sympy** [A] time = 3.85, size = 56, normalized size = 1.37

$$cx + c\sin\left(2x\right) - dx^2\sin^2\left(x\right) - dx^2\cos^2\left(x\right) + \frac{3dx^2}{2} + 2dx\sin\left(x\right)\cos\left(x\right) + d\cos^2\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x)
```

```
[Out] c*x + c*sin(2*x) - d*x**2*sin(x)**2 - d*x**2*cos(x)**2 + 3*d*x**2/2 + 2*d*x
*sin(x)*cos(x) + d*cos(x)**2
```

## 3.365 $\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx$

**Optimal**. Leaf size=57

$$\frac{2\cos\left(\frac{2c}{d}\right)\text{Ci}\left(\frac{2c}{d}+2x\right)}{d}+\frac{2\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d}+\frac{\log(c+dx)}{d}$$

```
[Out] 2*Ci(2*c/d+2*x)*cos(2*c/d)/d+ln(d*x+c)/d+2*Si(2*c/d+2*x)*sin(2*c/d)/d
```

**Rubi [A]** time = 0.25, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4431, 3312, 3303, 3299, 3302}

$$\frac{2\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d}+\frac{2\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d}+\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sin[3*x])/(c + d*x),x]
```

```
[Out] (2*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*x])/d + Log[c + d*x]/d + (2*Sin[(2*
c)/d]*SinIntegral[(2*c)/d + 2*x])/d
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx &= \int \left(\frac{3\cos^2(x)}{c+dx} - \frac{\sin^2(x)}{c+dx}\right) dx \\
&= 3\int \frac{\cos^2(x)}{c+dx}\,dx - \int \frac{\sin^2(x)}{c+dx}\,dx \\
&= 3\int \left(\frac{1}{2(c+dx)} + \frac{\cos(2x)}{2(c+dx)}\right) dx - \int \left(\frac{1}{2(c+dx)} - \frac{\cos(2x)}{2(c+dx)}\right) dx \\
&= \frac{\log(c+dx)}{d} + \frac{1}{2}\int \frac{\cos(2x)}{c+dx}\,dx + \frac{3}{2}\int \frac{\cos(2x)}{c+dx}\,dx \\
&= \frac{\log(c+dx)}{d} + \frac{1}{2}\cos\left(\frac{2c}{d}\right)\int \frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx + \frac{1}{2}\left(3\cos\left(\frac{2c}{d}\right)\right)\int \frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx + \frac{1}{2}\ldots \\
&= \frac{2\cos\left(\frac{2c}{d}\right)\text{Ci}\left(\frac{2c}{d}+2x\right)}{d} + \frac{\log(c+dx)}{d} + \frac{2\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d}
\end{aligned}
$$

**Mathematica** [A] time = 0.06, size = 49, normalized size = 0.86

$$
\frac{2\cos\left(\frac{2c}{d}\right)\text{Ci}\left(2\left(\frac{c}{d}+x\right)\right) + 2\sin\left(\frac{2c}{d}\right)\text{Si}\left(2\left(\frac{c}{d}+x\right)\right) + \log(c+dx)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x),x]
```

```
[Out] (2*Cos[(2*c)/d]*CosIntegral[2*(c/d + x)] + Log[c + d*x] + 2*Sin[(2*c)/d]*Si
nIntegral[2*(c/d + x)])/d
```

**fricas** [A] time = 0.44, size = 62, normalized size = 1.09

$$
\frac{\left(\text{Ci}\left(\frac{2(dx+c)}{d}\right) + \text{Ci}\left(-\frac{2(dx+c)}{d}\right)\right)\cos\left(\frac{2c}{d}\right) + 2\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2(dx+c)}{d}\right) + \log(dx+c)}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="fricas")
```

```
[Out] ((cos_integral(2*(d*x + c)/d) + cos_integral(-2*(d*x + c)/d))*cos(2*c/d) +
2*sin(2*c/d)*sin_integral(2*(d*x + c)/d) + log(d*x + c))/d
```

**giac** [A] time = 2.95, size = 51, normalized size = 0.89

$$
\frac{2\cos\left(\frac{2c}{d}\right)\text{Ci}\left(\frac{2(dx+c)}{d}\right) + 2\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2(dx+c)}{d}\right) + \log(dx+c)}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="giac")
```

```
[Out] (2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 2*sin(2*c/d)*sin_integral(2*(d*
x + c)/d) + log(d*x + c))/d
```

**maple** [A] time = 0.06, size = 58, normalized size = 1.02

$$
\frac{2\,\text{Ci}\left(\frac{2c}{d}+2x\right)\cos\left(\frac{2c}{d}\right)}{d} + \frac{\ln(dx+c)}{d} + \frac{2\,\text{Si}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(3*x)/(d*x+c),x)
```

```
[Out] 2*Ci(2*c/d+2*x)*cos(2*c/d)/d+ln(d*x+c)/d+2*Si(2*c/d+2*x)*sin(2*c/d)/d
```

**maxima [C]** time = 0.38, size = 95, normalized size = 1.67

$$-\frac{\left(E_1\left(\frac{2i\,dx+2i\,c}{d}\right)+E_1\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)-\left(-i\,E_1\left(\frac{2i\,dx+2i\,c}{d}\right)+i\,E_1\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\sin\left(\frac{2\,c}{d}\right)-\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((exp_integral_e(1, (2*I*d*x + 2*I*c)/d) + exp_integral_e(1, -(2*I*d*x + 2
*I*c)/d))*cos(2*c/d) - (-I*exp_integral_e(1, (2*I*d*x + 2*I*c)/d) + I*exp_i
ntegral_e(1, -(2*I*d*x + 2*I*c)/d))*sin(2*c/d) - log(d*x + c))/d
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(3\,x\right)}{\sin(x)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)/(sin(x)*(c + d*x)),x)
```

```
[Out] int(sin(3*x)/(sin(x)*(c + d*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(3x\right)\csc\left(x\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x)
```

```
[Out] Integral(sin(3*x)*csc(x)/(c + d*x), x)
```

# 3.366 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=78

$$\frac{4\sin\left(\frac{2c}{d}\right)\mathrm{Ci}\left(\frac{2c}{d}+2x\right)}{d^2}-\frac{4\cos\left(\frac{2c}{d}\right)\mathrm{Si}\left(\frac{2c}{d}+2x\right)}{d^2}+\frac{\sin^2(x)}{d(c+dx)}-\frac{3\cos^2(x)}{d(c+dx)}$$

```
[Out] -3*cos(x)^2/d/(d*x+c)-4*cos(2*c/d)*Si(2*c/d+2*x)/d^2+4*Ci(2*c/d+2*x)*sin(2*
c/d)/d^2+sin(x)^2/d/(d*x+c)
```

**Rubi [A]** time = 0.24, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4431, 3313, 12, 3303, 3299, 3302}

$$\frac{4\sin\left(\frac{2c}{d}\right)\mathrm{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^2}-\frac{4\cos\left(\frac{2c}{d}\right)\mathrm{Si}\left(\frac{2c}{d}+2x\right)}{d^2}+\frac{\sin^2(x)}{d(c+dx)}-\frac{3\cos^2(x)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sin[3*x])/(c + d*x)^2,x]
```

```
[Out] (-3*Cos[x]^2)/(d*(c + d*x)) + (4*CosIntegral[(2*c)/d + 2*x]*Sin[(2*c)/d])/d
^2 + Sin[x]^2/(d*(c + d*x)) - (4*Cos[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d
^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx &= \int \left(\frac{3\cos^2(x)}{(c+dx)^2}-\frac{\sin^2(x)}{(c+dx)^2}\right)dx\\
&= 3\int \frac{\cos^2(x)}{(c+dx)^2}\,dx-\int \frac{\sin^2(x)}{(c+dx)^2}\,dx\\
&= -\frac{3\cos^2(x)}{d(c+dx)}+\frac{\sin^2(x)}{d(c+dx)}-\frac{2\int \frac{\sin(2x)}{2(c+dx)}\,dx}{d}+\frac{6\int -\frac{\sin(2x)}{2(c+dx)}\,dx}{d}\\
&= -\frac{3\cos^2(x)}{d(c+dx)}+\frac{\sin^2(x)}{d(c+dx)}-\frac{\int \frac{\sin(2x)}{c+dx}\,dx}{d}-\frac{3\int \frac{\sin(2x)}{c+dx}\,dx}{d}\\
&= -\frac{3\cos^2(x)}{d(c+dx)}+\frac{\sin^2(x)}{d(c+dx)}-\frac{\cos\left(\frac{2c}{d}\right)\int \frac{\sin\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx}{d}-\frac{\left(3\cos\left(\frac{2c}{d}\right)\right)\int \frac{\sin\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx}{d}+\frac{\sin}{}\\
&= -\frac{3\cos^2(x)}{d(c+dx)}+\frac{4\text{Ci}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d^2}+\frac{\sin^2(x)}{d(c+dx)}-\frac{4\cos\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 61, normalized size = 0.78

$$\frac{4\sin\left(\frac{2c}{d}\right)\text{Ci}\left(2\left(\frac{c}{d}+x\right)\right)-4\cos\left(\frac{2c}{d}\right)\text{Si}\left(2\left(\frac{c}{d}+x\right)\right)-\frac{d(2\cos(2x)+1)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x)^2,x]
```

```
[Out] (-((d*(1 + 2*Cos[2*x]))/(c + d*x)) + 4*CosIntegral[2*(c/d + x)]*Sin[(2*c)/d
] - 4*Cos[(2*c)/d]*SinIntegral[2*(c/d + x)])/d^2
```

**fricas [A]** time = 0.48, size = 95, normalized size = 1.22

$$-\frac{4\,d\cos(x)^2+4\,(dx+c)\cos\left(\frac{2\,c}{d}\right)\text{Si}\left(\frac{2\,(dx+c)}{d}\right)-2\left((dx+c)\,\text{Ci}\left(\frac{2\,(dx+c)}{d}\right)+(dx+c)\,\text{Ci}\left(-\frac{2\,(dx+c)}{d}\right)\right)\sin\left(\frac{2\,c}{d}\right)-d}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(4*d*cos(x)^2 + 4*(d*x + c)*cos(2*c/d)*sin_integral(2*(d*x + c)/d) - 2*((d
*x + c)*cos_integral(2*(d*x + c)/d) + (d*x + c)*cos_integral(-2*(d*x + c)/d
))*sin(2*c/d) - d)/(d^3*x + c*d^2)
```

**giac [A]** time = 0.20, size = 111, normalized size = 1.42

$$\frac{4\,dx\,\text{Ci}\left(\frac{2\,(dx+c)}{d}\right)\sin\left(\frac{2\,c}{d}\right)-4\,dx\cos\left(\frac{2\,c}{d}\right)\text{Si}\left(\frac{2\,(dx+c)}{d}\right)+4\,c\,\text{Ci}\left(\frac{2\,(dx+c)}{d}\right)\sin\left(\frac{2\,c}{d}\right)-4\,c\cos\left(\frac{2\,c}{d}\right)\text{Si}\left(\frac{2\,(dx+c)}{d}\right)-2\,c}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (4*d*x*cos_integral(2*(d*x + c)/d)*sin(2*c/d) - 4*d*x*cos(2*c/d)*sin_integr
al(2*(d*x + c)/d) + 4*c*cos_integral(2*(d*x + c)/d)*sin(2*c/d) - 4*c*cos(2*
c/d)*sin_integral(2*(d*x + c)/d) - 2*d*cos(2*x) - d)/(d^3*x + c*d^2)
```

**maple [A]** time = 0.05, size = 82, normalized size = 1.05

$$-\frac{2\cos(2x)}{(dx+c)\,d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(\frac{2c}{d}+2x\right)\cos\left(\frac{2c}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d}\right)}{d}-\frac{1}{d\,(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(3*x)/(d*x+c)^2,x)
```

```
[Out] -2*cos(2*x)/(d*x+c)/d-2*(2*Si(2*c/d+2*x)*cos(2*c/d)/d-2*Ci(2*c/d+2*x)*sin(2
*c/d)/d)/d-1/d/(d*x+c)
```

**maxima [C]** time = 0.40, size = 324, normalized size = 4.15

$$-\frac{\left(E_2\left(\frac{2i\,dx+2i\,c}{d}\right)+E_2\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)^3+\left(i\,E_2\left(\frac{2i\,dx+2i\,c}{d}\right)-i\,E_2\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\sin\left(\frac{2\,c}{d}\right)^3+\left(\left(E_2\left(\frac{2i\,dx+2i\,c}{d}\right)+E_2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((exp_integral_e(2, (2*I*d*x + 2*I*c)/d) + exp_integral_e(2, -(2*I*d*x
 + 2*I*c)/d))*cos(2*c/d)^3 + (I*exp_integral_e(2, (2*I*d*x + 2*I*c)/d) - I*
exp_integral_e(2, -(2*I*d*x + 2*I*c)/d))*sin(2*c/d)^3 + ((exp_integral_e(2,
 (2*I*d*x + 2*I*c)/d) + exp_integral_e(2, -(2*I*d*x + 2*I*c)/d))*cos(2*c/d)
 + 2)*sin(2*c/d)^2 + (exp_integral_e(2, (2*I*d*x + 2*I*c)/d) + exp_integral
_e(2, -(2*I*d*x + 2*I*c)/d))*cos(2*c/d) + 2*cos(2*c/d)^2 + ((I*exp_integral
_e(2, (2*I*d*x + 2*I*c)/d) - I*exp_integral_e(2, -(2*I*d*x + 2*I*c)/d))*cos
(2*c/d)^2 + I*exp_integral_e(2, (2*I*d*x + 2*I*c)/d) - I*exp_integral_e(2,
-(2*I*d*x + 2*I*c)/d))*sin(2*c/d))/((cos(2*c/d)^2 + sin(2*c/d)^2)*d^2*x + (
c*cos(2*c/d)^2 + c*sin(2*c/d)^2)*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,x)}{\sin(x)\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)/(sin(x)*(c + d*x)^2),x)
```

```
[Out] int(sin(3*x)/(sin(x)*(c + d*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(3x)\csc(x)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(3*x)*csc(x)/(c + d*x)**2, x)
```

# 3.367 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=99

$$-\frac{4\cos\left(\frac{2c}{d}\right)\text{Ci}\left(\frac{2c}{d}+2x\right)}{d^3}-\frac{4\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^3}+\frac{4\sin(x)\cos(x)}{d^2(c+dx)}+\frac{\sin^2(x)}{2d(c+dx)^2}-\frac{3\cos^2(x)}{2d(c+dx)^2}$$

```
[Out] -4*Ci(2*c/d+2*x)*cos(2*c/d)/d^3-3/2*cos(x)^2/d/(d*x+c)^2-4*Si(2*c/d+2*x)*si
n(2*c/d)/d^3+4*cos(x)*sin(x)/d^2/(d*x+c)+1/2*sin(x)^2/d/(d*x+c)^2
```

**Rubi [A]** time = 0.33, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4431, 3314, 31, 3312, 3303, 3299, 3302}

$$-\frac{4\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^3}-\frac{4\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^3}+\frac{4\sin(x)\cos(x)}{d^2(c+dx)}+\frac{\sin^2(x)}{2d(c+dx)^2}-\frac{3\cos^2(x)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sin[3*x])/(c + d*x)^3,x]
```

```
[Out] (-3*Cos[x]^2)/(2*d*(c + d*x)^2) - (4*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*x
])/d^3 + (4*Cos[x]*Sin[x])/(d^2*(c + d*x)) + Sin[x]^2/(2*d*(c + d*x)^2) - (
4*Sin[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
```

```
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx = \int \left(\frac{3\cos^2(x)}{(c+dx)^3} - \frac{\sin^2(x)}{(c+dx)^3}\right)dx$$
$$= 3\int \frac{\cos^2(x)}{(c+dx)^3}\,dx - \int \frac{\sin^2(x)}{(c+dx)^3}\,dx$$
$$= -\frac{3\cos^2(x)}{2d(c+dx)^2} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{\int \frac{1}{c+dx}\,dx}{d^2} + \frac{2\int \frac{\sin^2(x)}{c+dx}\,dx}{d^2} + \frac{3\int \frac{1}{c+dx}\,dx}{d^2}$$
$$= -\frac{3\cos^2(x)}{2d(c+dx)^2} + \frac{2\log(c+dx)}{d^3} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} + \frac{2\int \left(\frac{1}{2(c+dx)} - \frac{\cos(2x)}{2(c+dx)}\right)dx}{d^2}$$
$$= -\frac{3\cos^2(x)}{2d(c+dx)^2} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{\int \frac{\cos(2x)}{c+dx}\,dx}{d^2} - \frac{3\int \frac{\cos(2x)}{c+dx}\,dx}{d^2}$$
$$= -\frac{3\cos^2(x)}{2d(c+dx)^2} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{\cos\left(\frac{2c}{d}\right)\int \frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx}{d^2} - \frac{\left(3\cos\left(\frac{2c}{d}\right)\right)\int}{d^2}$$
$$= -\frac{3\cos^2(x)}{2d(c+dx)^2} - \frac{4\cos\left(\frac{2c}{d}\right)\text{Ci}\left(\frac{2c}{d}+2x\right)}{d^3} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{4\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2}{}\right.}{d^3}$$

**Mathematica [A]** time = 0.24, size = 77, normalized size = 0.78

$$\frac{-8\cos\left(\frac{2c}{d}\right)\text{Ci}\left(2\left(\frac{c}{d}+x\right)\right) - 8\sin\left(\frac{2c}{d}\right)\text{Si}\left(2\left(\frac{c}{d}+x\right)\right) + \frac{d(4\sin(2x)(c+dx)-2d\cos(2x)-d)}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x)^3,x]
```

```
[Out] (-8*Cos[(2*c)/d]*CosIntegral[2*(c/d + x)] + (d*(-d - 2*d*Cos[2*x] + 4*(c +
d*x)*Sin[2*x]))/(c + d*x)^2 - 8*Sin[(2*c)/d]*SinIntegral[2*(c/d + x)])/(2*d
^3)
```

**fricas [A]** time = 0.44, size = 158, normalized size = 1.60

$$-\frac{4\,d^2\cos(x)^2 - 8\left(d^2x + cd\right)\cos(x)\sin(x) + 8\left(d^2x^2 + 2\,cdx + c^2\right)\sin\left(\frac{2\,c}{d}\right)\text{Si}\left(\frac{2\,(dx+c)}{d}\right) - d^2 + 4\left(\left(d^2x^2 + 2\,cdx + c\right.\right.}{2\left(d^5x^2 + 2\,cd^4x + c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*d^2*cos(x)^2 - 8*(d^2*x + c*d)*cos(x)*sin(x) + 8*(d^2*x^2 + 2*c*d*x
 + c^2)*sin(2*c/d)*sin_integral(2*(d*x + c)/d) - d^2 + 4*((d^2*x^2 + 2*c*d*
x + c^2)*cos_integral(2*(d*x + c)/d) + (d^2*x^2 + 2*c*d*x + c^2)*cos_integr
al(-2*(d*x + c)/d))*cos(2*c/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**giac [B]** time = 2.73, size = 201, normalized size = 2.03

$$-\frac{8\,d^2x^2\cos\left(\frac{2\,c}{d}\right)\operatorname{Ci}\left(\frac{2\,(dx+c)}{d}\right)+8\,d^2x^2\sin\left(\frac{2\,c}{d}\right)\operatorname{Si}\left(\frac{2\,(dx+c)}{d}\right)+16\,cdx\cos\left(\frac{2\,c}{d}\right)\operatorname{Ci}\left(\frac{2\,(dx+c)}{d}\right)+16\,cdx\sin\left(\frac{2\,c}{d}\right)\operatorname{Si}\left(\right.}{2\left(d^5x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(8*d^2*x^2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 8*d^2*x^2*sin(2*c/
d)*sin_integral(2*(d*x + c)/d) + 16*c*d*x*cos(2*c/d)*cos_integral(2*(d*x +
c)/d) + 16*c*d*x*sin(2*c/d)*sin_integral(2*(d*x + c)/d) + 8*c^2*cos(2*c/d)*
cos_integral(2*(d*x + c)/d) - 4*d^2*x*sin(2*x) + 8*c^2*sin(2*c/d)*sin_integ
ral(2*(d*x + c)/d) + 2*d^2*cos(2*x) - 4*c*d*sin(2*x) + d^2)/(d^5*x^2 + 2*c*
d^4*x + c^2*d^3)
```

**maple [A]** time = 0.06, size = 104, normalized size = 1.05

$$-\frac{\cos\left(2x\right)}{\left(dx+c\right)^2d}-\frac{-\frac{2\sin(2x)}{(dx+c)d}+\frac{\frac{4\operatorname{Si}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d}+\frac{4\operatorname{Ci}\left(\frac{2c}{d}+2x\right)\cos\left(\frac{2c}{d}\right)}{d}}{d}}{d}-\frac{1}{2d\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(3*x)/(d*x+c)^3,x)
```

```
[Out] -cos(2*x)/(d*x+c)^2/d-(-2*sin(2*x)/(d*x+c)/d+2*(2*Si(2*c/d+2*x)*sin(2*c/d)/
d+2*Ci(2*c/d+2*x)*cos(2*c/d)/d)/d)/d-1/2/d/(d*x+c)^2
```

**maxima [C]** time = 0.42, size = 362, normalized size = 3.66

$$-\frac{2\left(E_3\left(\frac{2i\,dx+2i\,c}{d}\right)+E_3\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)^3+\left(2i\,E_3\left(\frac{2i\,dx+2i\,c}{d}\right)-2i\,E_3\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\sin\left(\frac{2\,c}{d}\right)^3+2\left(\left(E_3\left(\frac{2i\,dx+\right.\right.\right.}{d}}{4\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(exp_integral_e(3, (2*I*d*x + 2*I*c)/d) + exp_integral_e(3, -(2*I*d
*x + 2*I*c)/d))*cos(2*c/d)^3 + (2*I*exp_integral_e(3, (2*I*d*x + 2*I*c)/d)
- 2*I*exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*sin(2*c/d)^3 + 2*((exp_integ
ral_e(3, (2*I*d*x + 2*I*c)/d) + exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*co
s(2*c/d) + 1)*sin(2*c/d)^2 + 2*(exp_integral_e(3, (2*I*d*x + 2*I*c)/d) + ex
p_integral_e(3, -(2*I*d*x + 2*I*c)/d))*cos(2*c/d) + 2*cos(2*c/d)^2 + ((2*I*
exp_integral_e(3, (2*I*d*x + 2*I*c)/d) - 2*I*exp_integral_e(3, -(2*I*d*x +
2*I*c)/d))*cos(2*c/d)^2 + 2*I*exp_integral_e(3, (2*I*d*x + 2*I*c)/d) - 2*I*
exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*sin(2*c/d))/((cos(2*c/d)^2 + sin(2
*c/d)^2)*d^3*x^2 + 2*(c*cos(2*c/d)^2 + c*sin(2*c/d)^2)*d^2*x + (c^2*cos(2*c
/d)^2 + c^2*sin(2*c/d)^2)*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,x\right)}{\sin(x)\,\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)/(sin(x)*(c + d*x)^3),x)
```

```
[Out] int(sin(3*x)/(sin(x)*(c + d*x)^3), x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(3x)\csc(x)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(3*x)*csc(x)/(c + d*x)**3, x)
```

# 3.368 $\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx$

**Optimal**. Leaf size=198

$$\frac{3d^4\sin(a+bx)\cos(a+bx)}{b^5}+\frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}-\frac{6d^2(c+dx)^2\sin(a+bx)\cos}{b^3}$$

```
[Out] 3/2*d^4*x/b^4-d*(d*x+c)^3/b^2+1/5*(d*x+c)^5/d-9/2*d^3*(d*x+c)*cos(b*x+a)^2/
b^4+3*d*(d*x+c)^3*cos(b*x+a)^2/b^2+3*d^4*cos(b*x+a)*sin(b*x+a)/b^5-6*d^2*(d
*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^3+2*(d*x+c)^4*cos(b*x+a)*sin(b*x+a)/b+3/2*d
^3*(d*x+c)*sin(b*x+a)^2/b^4-d*(d*x+c)^3*sin(b*x+a)^2/b^2
```

**Rubi [A]** time = 0.25, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4431, 3311, 32, 2635, 8}

$$\frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}-\frac{6d^2(c+dx)^2\sin(a+bx)\cos(a+bx)}{b^3}-\frac{d(c+dx)^3\sin^2(a+}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (3*d^4*x)/(2*b^4) - (d*(c + d*x)^3)/b^2 + (c + d*x)^5/(5*d) - (9*d^3*(c + d
*x)*Cos[a + b*x]^2)/(2*b^4) + (3*d*(c + d*x)^3*Cos[a + b*x]^2)/b^2 + (3*d^4
*Cos[a + b*x]*Sin[a + b*x])/b^5 - (6*d^2*(c + d*x)^2*Cos[a + b*x]*Sin[a + b
*x])/b^3 + (2*(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/b + (3*d^3*(c + d*x)*S
in[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Sin[a + b*x]^2)/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
```

```
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^4\cos^2(a+bx)-(c+dx)^4\sin^2(a+bx)\right)dx$$
$$= 3\int (c+dx)^4\cos^2(a+bx)\,dx - \int (c+dx)^4\sin^2(a+bx)\,dx$$
$$= \frac{3d(c+dx)^3\cos^2(a+bx)}{b^2} + \frac{2(c+dx)^4\cos(a+bx)\sin(a+bx)}{b} - \frac{d(c+$$
$$= \frac{(c+dx)^5}{5d} - \frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4} + \frac{3d(c+dx)^3\cos^2(a+bx)}{b^2} - \frac{6c}{}$$
$$= -\frac{d(c+dx)^3}{b^2} + \frac{(c+dx)^5}{5d} - \frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4} + \frac{3d(c+dx)^3\cos}{b^2}$$
$$= \frac{3d^4x}{2b^4} - \frac{d(c+dx)^3}{b^2} + \frac{(c+dx)^5}{5d} - \frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4} + \frac{3d(c+d}{}$$

**Mathematica [A]** time = 0.68, size = 128, normalized size = 0.65

$$\frac{d(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)}{b^4}+\frac{\sin(2(a+bx))\left(2b^4(c+dx)^4-6b^2d^2(c+dx)^2+3d^4\right)}{2b^5}+c^4x+2c^3dx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] c^4*x + 2*c^3*d*x^2 + 2*c^2*d^2*x^3 + c*d^3*x^4 + (d^4*x^5)/5 + (d*(c + d*x
)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])/b^4 + ((3*d^4 - 6*b^2*d^2*
(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sin[2*(a + b*x)])/(2*b^5)
```

**fricas [A]** time = 0.45, size = 283, normalized size = 1.43

$$\frac{b^5d^4x^5+5\,b^5cd^3x^4+10\left(b^5c^2d^2-b^3d^4\right)x^3+10\left(b^5c^3d-3\,b^3cd^3\right)x^2+10\left(2\,b^3d^4x^3+6\,b^3cd^3x^2+2\,b^3c^3d-3\,bcd^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/5*(b^5*d^4*x^5 + 5*b^5*c*d^3*x^4 + 10*(b^5*c^2*d^2 - b^3*d^4)*x^3 + 10*(b
^5*c^3*d - 3*b^3*c*d^3)*x^2 + 10*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c
^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*(2*b^4*d
^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2
*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)*sin(b*x
 + a) + 5*(b^5*c^4 - 6*b^3*c^2*d^2 + 3*b*d^4)*x)/b^5
```

**giac [B]** time = 0.55, size = 4684, normalized size = 23.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/5*(b^5*d^4*x^5*tan(1/2*b*x)^4*tan(1/2*a)^4 + 5*b^5*c*d^3*x^4*tan(1/2*b*x)
^4*tan(1/2*a)^4 + 2*b^5*d^4*x^5*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^5*d^4*x^5
*tan(1/2*b*x)^2*tan(1/2*a)^4 + 10*b^5*c^2*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)
```

^4 + 10*b^5*c*d^3*x^4*tan(1/2*b*x)^4*tan(1/2*a)^2 - 20*b^4*d^4*x^4*tan(1/2*
b*x)^4*tan(1/2*a)^3 + 10*b^5*c*d^3*x^4*tan(1/2*b*x)^2*tan(1/2*a)^4 - 20*b^4
*d^4*x^4*tan(1/2*b*x)^3*tan(1/2*a)^4 + 10*b^5*c^3*d*x^2*tan(1/2*b*x)^4*tan(
1/2*a)^4 + b^5*d^4*x^5*tan(1/2*b*x)^4 + 4*b^5*d^4*x^5*tan(1/2*b*x)^2*tan(1/
2*a)^2 + 20*b^5*c^2*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 80*b^4*c*d^3*x^3*
tan(1/2*b*x)^4*tan(1/2*a)^3 + b^5*d^4*x^5*tan(1/2*a)^4 + 20*b^5*c^2*d^2*x^3
*tan(1/2*b*x)^2*tan(1/2*a)^4 - 80*b^4*c*d^3*x^3*tan(1/2*b*x)^3*tan(1/2*a)^4
 + 5*b^5*c^4*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 10*b^3*d^4*x^3*tan(1/2*b*x)^4*
tan(1/2*a)^4 + 5*b^5*c*d^3*x^4*tan(1/2*b*x)^4 + 20*b^4*d^4*x^4*tan(1/2*b*x)
^4*tan(1/2*a) + 20*b^5*c*d^3*x^4*tan(1/2*b*x)^2*tan(1/2*a)^2 + 120*b^4*d^4*
x^4*tan(1/2*b*x)^3*tan(1/2*a)^2 + 20*b^5*c^3*d*x^2*tan(1/2*b*x)^4*tan(1/2*a
)^2 + 120*b^4*d^4*x^4*tan(1/2*b*x)^2*tan(1/2*a)^3 - 120*b^4*c^2*d^2*x^2*tan
(1/2*b*x)^4*tan(1/2*a)^3 + 5*b^5*c*d^3*x^4*tan(1/2*a)^4 + 20*b^4*d^4*x^4*ta
n(1/2*b*x)*tan(1/2*a)^4 + 20*b^5*c^3*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 12
0*b^4*c^2*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + 30*b^3*c*d^3*x^2*tan(1/2*b*
x)^4*tan(1/2*a)^4 + 2*b^5*d^4*x^5*tan(1/2*b*x)^2 + 10*b^5*c^2*d^2*x^3*tan(1
/2*b*x)^4 + 80*b^4*c*d^3*x^3*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^5*d^4*x^5*tan(
1/2*a)^2 + 40*b^5*c^2*d^2*x^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 480*b^4*c*d^3*x
^3*tan(1/2*b*x)^3*tan(1/2*a)^2 + 10*b^5*c^4*x*tan(1/2*b*x)^4*tan(1/2*a)^2 -
 60*b^3*d^4*x^3*tan(1/2*b*x)^4*tan(1/2*a)^2 + 480*b^4*c*d^3*x^3*tan(1/2*b*x
)^2*tan(1/2*a)^3 - 160*b^3*d^4*x^3*tan(1/2*b*x)^3*tan(1/2*a)^3 - 80*b^4*c^3
*d*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 10*b^5*c^2*d^2*x^3*tan(1/2*a)^4 + 80*b^4
*c*d^3*x^3*tan(1/2*b*x)*tan(1/2*a)^4 + 10*b^5*c^4*x*tan(1/2*b*x)^2*tan(1/2*
a)^4 - 60*b^3*d^4*x^3*tan(1/2*b*x)^2*tan(1/2*a)^4 - 80*b^4*c^3*d*x*tan(1/2*
b*x)^3*tan(1/2*a)^4 + 30*b^3*c^2*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 10*b^5
*c*d^3*x^4*tan(1/2*b*x)^2 - 20*b^4*d^4*x^4*tan(1/2*b*x)^3 + 10*b^5*c^3*d*x^
2*tan(1/2*b*x)^4 - 120*b^4*d^4*x^4*tan(1/2*b*x)^2*tan(1/2*a) + 120*b^4*c^2*
d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a) + 10*b^5*c*d^3*x^4*tan(1/2*a)^2 - 120*b^4
*d^4*x^4*tan(1/2*b*x)*tan(1/2*a)^2 + 40*b^5*c^3*d*x^2*tan(1/2*b*x)^2*tan(1/
2*a)^2 + 720*b^4*c^2*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^2 - 180*b^3*c*d^3*x^
2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 20*b^4*d^4*x^4*tan(1/2*a)^3 + 720*b^4*c^2*d
^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^3 - 480*b^3*c*d^3*x^2*tan(1/2*b*x)^3*tan(1
/2*a)^3 - 20*b^4*c^4*tan(1/2*b*x)^4*tan(1/2*a)^3 + 60*b^2*d^4*x^2*tan(1/2*b
*x)^4*tan(1/2*a)^3 + 10*b^5*c^3*d*x^2*tan(1/2*a)^4 + 120*b^4*c^2*d^2*x^2*ta
n(1/2*b*x)*tan(1/2*a)^4 - 180*b^3*c*d^3*x^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2
0*b^4*c^4*tan(1/2*b*x)^3*tan(1/2*a)^4 + 60*b^2*d^4*x^2*tan(1/2*b*x)^3*tan(1
/2*a)^4 + 10*b^3*c^3*d*tan(1/2*b*x)^4*tan(1/2*a)^4 + b^5*d^4*x^5 + 20*b^5*c
^2*d^2*x^3*tan(1/2*b*x)^2 - 80*b^4*c*d^3*x^3*tan(1/2*b*x)^3 + 5*b^5*c^4*x*t
an(1/2*b*x)^4 + 10*b^3*d^4*x^3*tan(1/2*b*x)^4 - 480*b^4*c*d^3*x^3*tan(1/2*b
*x)^2*tan(1/2*a) + 160*b^3*d^4*x^3*tan(1/2*b*x)^3*tan(1/2*a) + 80*b^4*c^3*d
*x*tan(1/2*b*x)^4*tan(1/2*a) + 20*b^5*c^2*d^2*x^3*tan(1/2*a)^2 - 480*b^4*c*
d^3*x^3*tan(1/2*b*x)*tan(1/2*a)^2 + 20*b^5*c^4*x*tan(1/2*b*x)^2*tan(1/2*a)^
2 + 360*b^3*d^4*x^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 480*b^4*c^3*d*x*tan(1/2*b
*x)^3*tan(1/2*a)^2 - 180*b^3*c^2*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 80*b^4
*c*d^3*x^3*tan(1/2*a)^3 + 160*b^3*d^4*x^3*tan(1/2*b*x)*tan(1/2*a)^3 + 480*b
^4*c^3*d*x*tan(1/2*b*x)^2*tan(1/2*a)^3 - 480*b^3*c^2*d^2*x*tan(1/2*b*x)^3*t
an(1/2*a)^3 + 120*b^2*c*d^3*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 5*b^5*c^4*x*tan
(1/2*a)^4 + 10*b^3*d^4*x^3*tan(1/2*a)^4 + 80*b^4*c^3*d*x*tan(1/2*b*x)*tan(1
/2*a)^4 - 180*b^3*c^2*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 120*b^2*c*d^3*x*t
an(1/2*b*x)^3*tan(1/2*a)^4 - 15*b*d^4*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 5*b^5
*c*d^3*x^4 + 20*b^4*d^4*x^4*tan(1/2*b*x) + 20*b^5*c^3*d*x^2*tan(1/2*b*x)^2
- 120*b^4*c^2*d^2*x^2*tan(1/2*b*x)^3 + 30*b^3*c*d^3*x^2*tan(1/2*b*x)^4 + 20
*b^4*d^4*x^4*tan(1/2*a) - 720*b^4*c^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a) + 4
80*b^3*c*d^3*x^2*tan(1/2*b*x)^3*tan(1/2*a) + 20*b^4*c^4*tan(1/2*b*x)^4*tan(
1/2*a) - 60*b^2*d^4*x^2*tan(1/2*b*x)^4*tan(1/2*a) + 20*b^5*c^3*d*x^2*tan(1/
2*a)^2 - 720*b^4*c^2*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^2 + 1080*b^3*c*d^3*x^2
*tan(1/2*b*x)^2*tan(1/2*a)^2 + 120*b^4*c^4*tan(1/2*b*x)^3*tan(1/2*a)^2 - 36
0*b^2*d^4*x^2*tan(1/2*b*x)^3*tan(1/2*a)^2 - 60*b^3*c^3*d*tan(1/2*b*x)^4*tan
(1/2*a)^2 - 120*b^4*c^2*d^2*x^2*tan(1/2*a)^3 + 480*b^3*c*d^3*x^2*tan(1/2*b*

x)*tan(1/2*a)^3 + 120*b^4*c^4*tan(1/2*b*x)^2*tan(1/2*a)^3 - 360*b^2*d^4*x^2
*tan(1/2*b*x)^2*tan(1/2*a)^3 - 160*b^3*c^3*d*tan(1/2*b*x)^3*tan(1/2*a)^3 +
60*b^2*c^2*d^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 30*b^3*c*d^3*x^2*tan(1/2*a)^4
+ 20*b^4*c^4*tan(1/2*b*x)*tan(1/2*a)^4 - 60*b^2*d^4*x^2*tan(1/2*b*x)*tan(1/
2*a)^4 - 60*b^3*c^3*d*tan(1/2*b*x)^2*tan(1/2*a)^4 + 60*b^2*c^2*d^2*tan(1/2*
b*x)^3*tan(1/2*a)^4 - 15*b*c*d^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 10*b^5*c^2*d
^2*x^3 + 80*b^4*c*d^3*x^3*tan(1/2*b*x) + 10*b^5*c^4*x*tan(1/2*b*x)^2 - 60*b
^3*d^4*x^3*tan(1/2*b*x)^2 - 80*b^4*c^3*d*x*tan(1/2*b*x)^3 + 30*b^3*c^2*d^2*
x*tan(1/2*b*x)^4 + 80*b^4*c*d^3*x^3*tan(1/2*a) - 160*b^3*d^4*x^3*tan(1/2*b*
x)*tan(1/2*a) - 480*b^4*c^3*d*x*tan(1/2*b*x)^2*tan(1/2*a) + 480*b^3*c^2*d^2
*x*tan(1/2*b*x)^3*tan(1/2*a) - 120*b^2*c*d^3*x*tan(1/2*b*x)^4*tan(1/2*a) +
10*b^5*c^4*x*tan(1/2*a)^2 - 60*b^3*d^4*x^3*tan(1/2*a)^2 - 480*b^4*c^3*d*x*t
an(1/2*b*x)*tan(1/2*a)^2 + 1080*b^3*c^2*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^2 -
720*b^2*c*d^3*x*tan(1/2*b*x)^3*tan(1/2*a)^2 + 90*b*d^4*x*tan(1/2*b*x)^4*ta
n(1/2*a)^2 - 80*b^4*c^3*d*x*tan(1/2*a)^3 + 480*b^3*c^2*d^2*x*tan(1/2*b*x)*t
an(1/2*a)^3 - 720*b^2*c*d^3*x*tan(1/2*b*x)^2*tan(1/2*a)^3 + 240*b*d^4*x*tan
(1/2*b*x)^3*tan(1/2*a)^3 + 30*b^3*c^2*d^2*x*tan(1/2*a)^4 - 120*b^2*c*d^3*x*
tan(1/2*b*x)*tan(1/2*a)^4 + 90*b*d^4*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 10*b^5
*c^3*d*x^2 + 120*b^4*c^2*d^2*x^2*tan(1/2*b*x) - 180*b^3*c*d^3*x^2*tan(1/2*b
*x)^2 - 20*b^4*c^4*tan(1/2*b*x)^3 + 60*b^2*d^4*x^2*tan(1/2*b*x)^3 + 10*b^3*
c^3*d*tan(1/2*b*x)^4 + 120*b^4*c^2*d^2*x^2*tan(1/2*a) - 480*b^3*c*d^3*x^2*t
an(1/2*b*x)*tan(1/2*a) - 120*b^4*c^4*tan(1/2*b*x)^2*tan(1/2*a) + 360*b^2*d^
4*x^2*tan(1/2*b*x)^2*tan(1/2*a) + 160*b^3*c^3*d*tan(1/2*b*x)^3*tan(1/2*a) -
60*b^2*c^2*d^2*tan(1/2*b*x)^4*tan(1/2*a) - 180*b^3*c*d^3*x^2*tan(1/2*a)^2
- 120*b^4*c^4*tan(1/2*b*x)*tan(1/2*a)^2 + 360*b^2*d^4*x^2*tan(1/2*b*x)*tan(
1/2*a)^2 + 360*b^3*c^3*d*tan(1/2*b*x)^2*tan(1/2*a)^2 - 360*b^2*c^2*d^2*tan(
1/2*b*x)^3*tan(1/2*a)^2 + 90*b*c*d^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 20*b^4*c
^4*tan(1/2*a)^3 + 60*b^2*d^4*x^2*tan(1/2*a)^3 + 160*b^3*c^3*d*tan(1/2*b*x)*
tan(1/2*a)^3 - 360*b^2*c^2*d^2*tan(1/2*b*x)^2*tan(1/2*a)^3 + 240*b*c*d^3*ta
n(1/2*b*x)^3*tan(1/2*a)^3 - 30*d^4*tan(1/2*b*x)^4*tan(1/2*a)^3 + 10*b^3*c^3
*d*tan(1/2*a)^4 - 60*b^2*c^2*d^2*tan(1/2*b*x)*tan(1/2*a)^4 + 90*b*c*d^3*tan
(1/2*b*x)^2*tan(1/2*a)^4 - 30*d^4*tan(1/2*b*x)^3*tan(1/2*a)^4 + 5*b^5*c^4*x
+ 10*b^3*d^4*x^3 + 80*b^4*c^3*d*x*tan(1/2*b*x) - 180*b^3*c^2*d^2*x*tan(1/2
*b*x)^2 + 120*b^2*c*d^3*x*tan(1/2*b*x)^3 - 15*b*d^4*x*tan(1/2*b*x)^4 + 80*b
^4*c^3*d*x*tan(1/2*a) - 480*b^3*c^2*d^2*x*tan(1/2*b*x)*tan(1/2*a) + 720*b^2
*c*d^3*x*tan(1/2*b*x)^2*tan(1/2*a) - 240*b*d^4*x*tan(1/2*b*x)^3*tan(1/2*a)
- 180*b^3*c^2*d^2*x*tan(1/2*a)^2 + 720*b^2*c*d^3*x*tan(1/2*b*x)*tan(1/2*a)^
2 - 540*b*d^4*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 120*b^2*c*d^3*x*tan(1/2*a)^3
- 240*b*d^4*x*tan(1/2*b*x)*tan(1/2*a)^3 - 15*b*d^4*x*tan(1/2*a)^4 + 30*b^3*
c*d^3*x^2 + 20*b^4*c^4*tan(1/2*b*x) - 60*b^2*d^4*x^2*tan(1/2*b*x) - 60*b^3*
c^3*d*tan(1/2*b*x)^2 + 60*b^2*c^2*d^2*tan(1/2*b*x)^3 - 15*b*c*d^3*tan(1/2*b
*x)^4 + 20*b^4*c^4*tan(1/2*a) - 60*b^2*d^4*x^2*tan(1/2*a) - 160*b^3*c^3*d*t
an(1/2*b*x)*tan(1/2*a) + 360*b^2*c^2*d^2*tan(1/2*b*x)^2*tan(1/2*a) - 240*b*
c*d^3*tan(1/2*b*x)^3*tan(1/2*a) + 30*d^4*tan(1/2*b*x)^4*tan(1/2*a) - 60*b^3
*c^3*d*tan(1/2*a)^2 + 360*b^2*c^2*d^2*tan(1/2*b*x)*tan(1/2*a)^2 - 540*b*c*d
^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 180*d^4*tan(1/2*b*x)^3*tan(1/2*a)^2 + 60*b
^2*c^2*d^2*tan(1/2*a)^3 - 240*b*c*d^3*tan(1/2*b*x)*tan(1/2*a)^3 + 180*d^4*t
an(1/2*b*x)^2*tan(1/2*a)^3 - 15*b*c*d^3*tan(1/2*a)^4 + 30*d^4*tan(1/2*b*x)*
tan(1/2*a)^4 + 30*b^3*c^2*d^2*x - 120*b^2*c*d^3*x*tan(1/2*b*x) + 90*b*d^4*x
*tan(1/2*b*x)^2 - 120*b^2*c*d^3*x*tan(1/2*a) + 240*b*d^4*x*tan(1/2*b*x)*tan
(1/2*a) + 90*b*d^4*x*tan(1/2*a)^2 + 10*b^3*c^3*d - 60*b^2*c^2*d^2*tan(1/2*b
*x) + 90*b*c*d^3*tan(1/2*b*x)^2 - 30*d^4*tan(1/2*b*x)^3 - 60*b^2*c^2*d^2*ta
n(1/2*a) + 240*b*c*d^3*tan(1/2*b*x)*tan(1/2*a) - 180*d^4*tan(1/2*b*x)^2*tan
(1/2*a) + 90*b*c*d^3*tan(1/2*a)^2 - 180*d^4*tan(1/2*b*x)*tan(1/2*a)^2 - 30*
d^4*tan(1/2*a)^3 - 15*b*d^4*x - 15*b*c*d^3 + 30*d^4*tan(1/2*b*x) + 30*d^4*t
an(1/2*a))/(b^5*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^5*tan(1/2*b*x)^4*tan(1/2*
a)^2 + 2*b^5*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^5*tan(1/2*b*x)^4 + 4*b^5*tan(1
/2*b*x)^2*tan(1/2*a)^2 + b^5*tan(1/2*a)^4 + 2*b^5*tan(1/2*b*x)^2 + 2*b^5*ta
n(1/2*a)^2 + b^5)

**maple [B]** time = 0.06, size = 1000, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -c^4*x-1/5*d^4*x^5+4*c^4/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-2*c^3*
d*x^2-2*c^2*d^2*x^3-c*d^3*x^4+4*d^4/b^5*((b*x+a)^4*(1/2*cos(b*x+a)*sin(b*x+
a)+1/2*b*x+1/2*a)+(b*x+a)^3*cos(b*x+a)^2-3*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*
x+a)+1/2*b*x+1/2*a)-3/2*(b*x+a)*cos(b*x+a)^2+3/4*cos(b*x+a)*sin(b*x+a)+3/4*
b*x+3/4*a+(b*x+a)^3-2/5*(b*x+a)^5-4*a*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)
+1/2*b*x+1/2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(
b*x+a)+1/2*b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4)+6*a^2*(
(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^
2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)-4*a^3*((b*x+a)*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+a^4*
(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+16*c^3*d/b^2*((b*x+a)*(1/2*cos(b
*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2-a*(1/2*cos(b
*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+24*c^2*d^2/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*
sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a
)-1/4*b*x-1/4*a-1/3*(b*x+a)^3-2*a*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b
*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/
2*b*x+1/2*a))+16*d^3*c/b^4*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/
2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*
b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4-3*a*((b*x+a)^2*(1/2
*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+
a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)+3*a^2*((b*x+a)*(1/2*cos(b*x+a)*s
in(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-a^3*(1/2*cos(b*x+a
)*sin(b*x+a)+1/2*b*x+1/2*a))
```

**maxima [A]** time = 0.40, size = 244, normalized size = 1.23

$$\frac{(bx+\sin(2bx+2a))c^4}{b}+\frac{2\left(b^2x^2+2bx\sin(2bx+2a)+\cos(2bx+2a)\right)c^3d}{b^2}+\frac{\left(2b^3x^3+6bx\cos(2bx+2a)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c^4/b + 2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(
2*b*x + 2*a))*c^3*d/b^2 + (2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*b^2*x^
2 - 1)*sin(2*b*x + 2*a))*c^2*d^2/b^3 + (b^4*x^4 + 3*(2*b^2*x^2 - 1)*cos(2*b
*x + 2*a) + 2*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a))*c*d^3/b^4 + 1/10*(2*b^5
*x^5 + 10*(2*b^3*x^3 - 3*b*x)*cos(2*b*x + 2*a) + 5*(2*b^4*x^4 - 6*b^2*x^2 +
 3)*sin(2*b*x + 2*a))*d^4/b^5
```

**mupad [B]** time = 0.65, size = 344, normalized size = 1.74

$$\frac{\frac{3d^4\sin(2a+2bx)}{2}+b^5c^4x+b^4c^4\sin(2a+2bx)+\frac{b^5d^4x^5}{5}+2b^3c^3d\cos(2a+2bx)+2b^5c^3dx^2+b^5cd^3x^4-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^4)/sin(a + b*x),x)
```

```
[Out] ((3*d^4*sin(2*a + 2*b*x))/2 + b^5*c^4*x + b^4*c^4*sin(2*a + 2*b*x) + (b^5*d
^4*x^5)/5 + 2*b^3*c^3*d*cos(2*a + 2*b*x) + 2*b^5*c^3*d*x^2 + b^5*c*d^3*x^4
- 3*b^2*c^2*d^2*sin(2*a + 2*b*x) + 2*b^3*d^4*x^3*cos(2*a + 2*b*x) + 2*b^5*c
^2*d^2*x^3 - 3*b^2*d^4*x^2*sin(2*a + 2*b*x) + b^4*d^4*x^4*sin(2*a + 2*b*x)
```

```
- 3*b*c*d^3*cos(2*a + 2*b*x) - 3*b*d^4*x*cos(2*a + 2*b*x) + 6*b^4*c^2*d^2*x
^2*sin(2*a + 2*b*x) - 6*b^2*c*d^3*x*sin(2*a + 2*b*x) + 4*b^4*c^3*d*x*sin(2*
a + 2*b*x) + 6*b^3*c^2*d^2*x*cos(2*a + 2*b*x) + 6*b^3*c*d^3*x^2*cos(2*a + 2
*b*x) + 4*b^4*c*d^3*x^3*sin(2*a + 2*b*x))/b^5
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

# 3.369 $\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx$

**Optimal**. Leaf size=171

$$\frac{3d^3\sin^2(a+bx)}{8b^4}-\frac{9d^3\cos^2(a+bx)}{8b^4}-\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{b^3}-\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}+\frac{9d(c+dx}{}$$

```
[Out] -3/2*c*d^2*x/b^2-3/4*d^3*x^2/b^2+1/4*(d*x+c)^4/d-9/8*d^3*cos(b*x+a)^2/b^4+9
/4*d*(d*x+c)^2*cos(b*x+a)^2/b^2-3*d^2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^3+2*(
d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b+3/8*d^3*sin(b*x+a)^2/b^4-3/4*d*(d*x+c)^2*s
in(b*x+a)^2/b^2
```

**Rubi [A]** time = 0.18, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4431, 3311, 32, 3310}

$$-\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{b^3}-\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}+\frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{3d^3\sin^2(a+bx)}{8b^4}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (-3*c*d^2*x)/(2*b^2) - (3*d^3*x^2)/(4*b^2) + (c + d*x)^4/(4*d) - (9*d^3*Cos
[a + b*x]^2)/(8*b^4) + (9*d*(c + d*x)^2*Cos[a + b*x]^2)/(4*b^2) - (3*d^2*(c
 + d*x)*Cos[a + b*x]*Sin[a + b*x])/b^3 + (2*(c + d*x)^3*Cos[a + b*x]*Sin[a
+ b*x])/b + (3*d^3*Sin[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Sin[a + b*x]^
2)/(4*b^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^3\cos^2(a+bx)-(c+dx)^3\sin^2(a+bx)\right)dx$$
$$= 3\int (c+dx)^3\cos^2(a+bx)\,dx - \int (c+dx)^3\sin^2(a+bx)\,dx$$
$$= \frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2} + \frac{2(c+dx)^3\cos(a+bx)\sin(a+bx)}{b} - \frac{3d(c}{}$$
$$= \frac{(c+dx)^4}{4d} - \frac{9d^3\cos^2(a+bx)}{8b^4} + \frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2} - \frac{3d^2(c+d}{}$$
$$= -\frac{3cd^2x}{2b^2} - \frac{3d^3x^2}{4b^2} + \frac{(c+dx)^4}{4d} - \frac{9d^3\cos^2(a+bx)}{8b^4} + \frac{9d(c+dx)^2\cos^2(a}{4b^2}$$

**Mathematica [A]** time = 0.42, size = 105, normalized size = 0.61

$$\frac{2b(c+dx)\sin(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)+3d\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+b^4x\left(4c^3+6c^2dx+4cd^2\right.}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(-d^2 + 2*b^2*(c +
 d*x)^2)*Cos[2*(a + b*x)] + 2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Sin[
2*(a + b*x)])/(4*b^4)
```

**fricas [A]** time = 0.45, size = 188, normalized size = 1.10

$$\frac{b^4d^3x^4+4\,b^4cd^2x^3+6\left(b^4c^2d-b^2d^3\right)x^2+6\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3\right)\cos\left(bx+a\right)^2+4\left(2\,b^3d^3x^3+6\,\right.}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*(b^4*c^2*d - b^2*d^3)*x^2 + 6*(2*b^2
*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 4*(2*b^3*d^3
*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)
*cos(b*x + a)*sin(b*x + a) + 4*(b^4*c^3 - 3*b^2*c*d^2)*x)/b^4
```

**giac [B]** time = 4.43, size = 3139, normalized size = 18.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/4*(b^4*d^3*x^4*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^4*c*d^2*x^3*tan(1/2*b*x)
^4*tan(1/2*a)^4 + 2*b^4*d^3*x^4*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^4*d^3*x^4
*tan(1/2*b*x)^2*tan(1/2*a)^4 + 6*b^4*c^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4
+ 8*b^4*c*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 16*b^3*d^3*x^3*tan(1/2*b*x)
^4*tan(1/2*a)^3 + 8*b^4*c*d^2*x^3*tan(1/2*b*x)^2*tan(1/2*a)^4 - 16*b^3*d^3*
x^3*tan(1/2*b*x)^3*tan(1/2*a)^4 + 4*b^4*c^3*x*tan(1/2*b*x)^4*tan(1/2*a)^4 +
 b^4*d^3*x^4*tan(1/2*b*x)^4 + 4*b^4*d^3*x^4*tan(1/2*b*x)^2*tan(1/2*a)^2 + 1
2*b^4*c^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 48*b^3*c*d^2*x^2*tan(1/2*b*x)
^4*tan(1/2*a)^3 + b^4*d^3*x^4*tan(1/2*a)^4 + 12*b^4*c^2*d*x^2*tan(1/2*b*x)^
2*tan(1/2*a)^4 - 48*b^3*c*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + 6*b^2*d^3*x
^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^4*c*d^2*x^3*tan(1/2*b*x)^4 + 16*b^3*d^
3*x^3*tan(1/2*b*x)^4*tan(1/2*a) + 16*b^4*c*d^2*x^3*tan(1/2*b*x)^2*tan(1/2*a
```

)^2 + 96*b^3*d^3*x^3*tan(1/2*b*x)^3*tan(1/2*a)^2 + 8*b^4*c^3*x*tan(1/2*b*x)
^4*tan(1/2*a)^2 + 96*b^3*d^3*x^3*tan(1/2*b*x)^2*tan(1/2*a)^3 - 48*b^3*c^2*d
*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*b^4*c*d^2*x^3*tan(1/2*a)^4 + 16*b^3*d^3*
x^3*tan(1/2*b*x)*tan(1/2*a)^4 + 8*b^4*c^3*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 4
8*b^3*c^2*d*x*tan(1/2*b*x)^3*tan(1/2*a)^4 + 12*b^2*c*d^2*x*tan(1/2*b*x)^4*t
an(1/2*a)^4 + 2*b^4*d^3*x^4*tan(1/2*b*x)^2 + 6*b^4*c^2*d*x^2*tan(1/2*b*x)^4
 + 48*b^3*c*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^4*d^3*x^4*tan(1/2*a)^2
+ 24*b^4*c^2*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 288*b^3*c*d^2*x^2*tan(1/2*
b*x)^3*tan(1/2*a)^2 - 36*b^2*d^3*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 288*b^3*
c*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^3 - 96*b^2*d^3*x^2*tan(1/2*b*x)^3*tan(1
/2*a)^3 - 16*b^3*c^3*tan(1/2*b*x)^4*tan(1/2*a)^3 + 6*b^4*c^2*d*x^2*tan(1/2*
a)^4 + 48*b^3*c*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^4 - 36*b^2*d^3*x^2*tan(1/2*
b*x)^2*tan(1/2*a)^4 - 16*b^3*c^3*tan(1/2*b*x)^3*tan(1/2*a)^4 + 6*b^2*c^2*d*
tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*b^4*c*d^2*x^3*tan(1/2*b*x)^2 - 16*b^3*d^3*x
^3*tan(1/2*b*x)^3 + 4*b^4*c^3*x*tan(1/2*b*x)^4 - 96*b^3*d^3*x^3*tan(1/2*b*x
)^2*tan(1/2*a) + 48*b^3*c^2*d*x*tan(1/2*b*x)^4*tan(1/2*a) + 8*b^4*c*d^2*x^3
*tan(1/2*a)^2 - 96*b^3*d^3*x^3*tan(1/2*b*x)*tan(1/2*a)^2 + 16*b^4*c^3*x*tan
(1/2*b*x)^2*tan(1/2*a)^2 + 288*b^3*c^2*d*x*tan(1/2*b*x)^3*tan(1/2*a)^2 - 72
*b^2*c*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 16*b^3*d^3*x^3*tan(1/2*a)^3 + 28
8*b^3*c^2*d*x*tan(1/2*b*x)^2*tan(1/2*a)^3 - 192*b^2*c*d^2*x*tan(1/2*b*x)^3*
tan(1/2*a)^3 + 24*b*d^3*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*b^4*c^3*x*tan(1/2
*a)^4 + 48*b^3*c^2*d*x*tan(1/2*b*x)*tan(1/2*a)^4 - 72*b^2*c*d^2*x*tan(1/2*b
*x)^2*tan(1/2*a)^4 + 24*b*d^3*x*tan(1/2*b*x)^3*tan(1/2*a)^4 + b^4*d^3*x^4 +
 12*b^4*c^2*d*x^2*tan(1/2*b*x)^2 - 48*b^3*c*d^2*x^2*tan(1/2*b*x)^3 + 6*b^2*
d^3*x^2*tan(1/2*b*x)^4 - 288*b^3*c*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a) + 96*b
^2*d^3*x^2*tan(1/2*b*x)^3*tan(1/2*a) + 16*b^3*c^3*tan(1/2*b*x)^4*tan(1/2*a)
 + 12*b^4*c^2*d*x^2*tan(1/2*a)^2 - 288*b^3*c*d^2*x^2*tan(1/2*b*x)*tan(1/2*a
)^2 + 216*b^2*d^3*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 96*b^3*c^3*tan(1/2*b*x)
^3*tan(1/2*a)^2 - 36*b^2*c^2*d*tan(1/2*b*x)^4*tan(1/2*a)^2 - 48*b^3*c*d^2*x
^2*tan(1/2*a)^3 + 96*b^2*d^3*x^2*tan(1/2*b*x)*tan(1/2*a)^3 + 96*b^3*c^3*tan
(1/2*b*x)^2*tan(1/2*a)^3 - 96*b^2*c^2*d*tan(1/2*b*x)^3*tan(1/2*a)^3 + 24*b*
c*d^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 6*b^2*d^3*x^2*tan(1/2*a)^4 + 16*b^3*c^3
*tan(1/2*b*x)*tan(1/2*a)^4 - 36*b^2*c^2*d*tan(1/2*b*x)^2*tan(1/2*a)^4 + 24*
b*c*d^2*tan(1/2*b*x)^3*tan(1/2*a)^4 - 3*d^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4
*b^4*c*d^2*x^3 + 16*b^3*d^3*x^3*tan(1/2*b*x) + 8*b^4*c^3*x*tan(1/2*b*x)^2 -
 48*b^3*c^2*d*x*tan(1/2*b*x)^3 + 12*b^2*c*d^2*x*tan(1/2*b*x)^4 + 16*b^3*d^3
*x^3*tan(1/2*a) - 288*b^3*c^2*d*x*tan(1/2*b*x)^2*tan(1/2*a) + 192*b^2*c*d^2
*x*tan(1/2*b*x)^3*tan(1/2*a) - 24*b*d^3*x*tan(1/2*b*x)^4*tan(1/2*a) + 8*b^4
*c^3*x*tan(1/2*a)^2 - 288*b^3*c^2*d*x*tan(1/2*b*x)*tan(1/2*a)^2 + 432*b^2*c
*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 144*b*d^3*x*tan(1/2*b*x)^3*tan(1/2*a)^
2 - 48*b^3*c^2*d*x*tan(1/2*a)^3 + 192*b^2*c*d^2*x*tan(1/2*b*x)*tan(1/2*a)^3
 - 144*b*d^3*x*tan(1/2*b*x)^2*tan(1/2*a)^3 + 12*b^2*c*d^2*x*tan(1/2*a)^4 -
24*b*d^3*x*tan(1/2*b*x)*tan(1/2*a)^4 + 6*b^4*c^2*d*x^2 + 48*b^3*c*d^2*x^2*t
an(1/2*b*x) - 36*b^2*d^3*x^2*tan(1/2*b*x)^2 - 16*b^3*c^3*tan(1/2*b*x)^3 + 6
*b^2*c^2*d*tan(1/2*b*x)^4 + 48*b^3*c*d^2*x^2*tan(1/2*a) - 96*b^2*d^3*x^2*ta
n(1/2*b*x)*tan(1/2*a) - 96*b^3*c^3*tan(1/2*b*x)^2*tan(1/2*a) + 96*b^2*c^2*d
*tan(1/2*b*x)^3*tan(1/2*a) - 24*b*c*d^2*tan(1/2*b*x)^4*tan(1/2*a) - 36*b^2*
d^3*x^2*tan(1/2*a)^2 - 96*b^3*c^3*tan(1/2*b*x)*tan(1/2*a)^2 + 216*b^2*c^2*d
*tan(1/2*b*x)^2*tan(1/2*a)^2 - 144*b*c*d^2*tan(1/2*b*x)^3*tan(1/2*a)^2 + 18
*d^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 16*b^3*c^3*tan(1/2*a)^3 + 96*b^2*c^2*d*t
an(1/2*b*x)*tan(1/2*a)^3 - 144*b*c*d^2*tan(1/2*b*x)^2*tan(1/2*a)^3 + 48*d^3
*tan(1/2*b*x)^3*tan(1/2*a)^3 + 6*b^2*c^2*d*tan(1/2*a)^4 - 24*b*c*d^2*tan(1/
2*b*x)*tan(1/2*a)^4 + 18*d^3*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b^4*c^3*x + 48
*b^3*c^2*d*x*tan(1/2*b*x) - 72*b^2*c*d^2*x*tan(1/2*b*x)^2 + 24*b*d^3*x*tan(
1/2*b*x)^3 + 48*b^3*c^2*d*x*tan(1/2*a) - 192*b^2*c*d^2*x*tan(1/2*b*x)*tan(1
/2*a) + 144*b*d^3*x*tan(1/2*b*x)^2*tan(1/2*a) - 72*b^2*c*d^2*x*tan(1/2*a)^2
 + 144*b*d^3*x*tan(1/2*b*x)*tan(1/2*a)^2 + 24*b*d^3*x*tan(1/2*a)^3 + 6*b^2*
d^3*x^2 + 16*b^3*c^3*tan(1/2*b*x) - 36*b^2*c^2*d*tan(1/2*b*x)^2 + 24*b*c*d^
2*tan(1/2*b*x)^3 - 3*d^3*tan(1/2*b*x)^4 + 16*b^3*c^3*tan(1/2*a) - 96*b^2*c^

```
2*d*tan(1/2*b*x)*tan(1/2*a) + 144*b*c*d^2*tan(1/2*b*x)^2*tan(1/2*a) - 48*d^
3*tan(1/2*b*x)^3*tan(1/2*a) - 36*b^2*c^2*d*tan(1/2*a)^2 + 144*b*c*d^2*tan(1
/2*b*x)*tan(1/2*a)^2 - 108*d^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 24*b*c*d^2*tan
(1/2*a)^3 - 48*d^3*tan(1/2*b*x)*tan(1/2*a)^3 - 3*d^3*tan(1/2*a)^4 + 12*b^2*
c*d^2*x - 24*b*d^3*x*tan(1/2*b*x) - 24*b*d^3*x*tan(1/2*a) + 6*b^2*c^2*d - 2
4*b*c*d^2*tan(1/2*b*x) + 18*d^3*tan(1/2*b*x)^2 - 24*b*c*d^2*tan(1/2*a) + 48
*d^3*tan(1/2*b*x)*tan(1/2*a) + 18*d^3*tan(1/2*a)^2 - 3*d^3)/(b^4*tan(1/2*b*
x)^4*tan(1/2*a)^4 + 2*b^4*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^4*tan(1/2*b*x)^
2*tan(1/2*a)^4 + b^4*tan(1/2*b*x)^4 + 4*b^4*tan(1/2*b*x)^2*tan(1/2*a)^2 + b
^4*tan(1/2*a)^4 + 2*b^4*tan(1/2*b*x)^2 + 2*b^4*tan(1/2*a)^2 + b^4)
```

**maple** [B] time = 0.04, size = 580, normalized size = 3.39

$$-c^3x-\frac{d^3x^4}{4}+\frac{4c^3\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{3c^2d\,x^2}{2}-c\,d^2x^3+\frac{4d^3\left((bx+a)^3\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{3(bx+a}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -c^3*x-1/4*d^3*x^4+4*c^3/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/2*c^
2*d*x^2-c*d^2*x^3+4*d^3/b^4*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1
/2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2
*b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4-3*a*((b*x+a)^2*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x
+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)+3*a^2*((b*x+a)*(1/2*cos(b*x+a)*
sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-a^3*(1/2*cos(b*x+
a)*sin(b*x+a)+1/2*b*x+1/2*a))+12*c^2*d/b^2*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x
+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2-a*(1/2*cos(b*x+a)*sin(b*x
+a)+1/2*b*x+1/2*a))+12*c*d^2/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*
b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a
-1/3*(b*x+a)^3-2*a*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(
b*x+a)^2-1/4*sin(b*x+a)^2)+a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

**maxima** [A] time = 0.37, size = 173, normalized size = 1.01

$$\frac{(bx+\sin(2\,bx+2\,a))c^3}{b}+\frac{3\left(b^2x^2+2\,bx\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)c^2d}{2\,b^2}+\frac{\left(2\,b^3x^3+6\,bx\cos(2\,bx+2\,a)+\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c^3/b + 3/2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + co
s(2*b*x + 2*a))*c^2*d/b^2 + 1/2*(2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*
b^2*x^2 - 1)*sin(2*b*x + 2*a))*c*d^2/b^3 + 1/4*(b^4*x^4 + 3*(2*b^2*x^2 - 1)
*cos(2*b*x + 2*a) + 2*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a))*d^3/b^4
```

**mupad** [B] time = 2.18, size = 216, normalized size = 1.26

$$c^3\,x+\frac{d^3\,x^4}{4}+\frac{3\,c^2\,d\,x^2}{2}+c\,d^2\,x^3-\frac{3\,d^3\,\cos\left(2\,a+2\,b\,x\right)}{4\,b^4}+\frac{c^3\,\sin\left(2\,a+2\,b\,x\right)}{b}+\frac{3\,c^2\,d\,\cos\left(2\,a+2\,b\,x\right)}{2\,b^2}-\frac{3\,c\,d^2\,\sin\left(2\,a+\right.}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^3)/sin(a + b*x),x)
```

```
[Out] c^3*x + (d^3*x^4)/4 + (3*c^2*d*x^2)/2 + c*d^2*x^3 - (3*d^3*cos(2*a + 2*b*x)
)/(4*b^4) + (c^3*sin(2*a + 2*b*x))/b + (3*c^2*d*cos(2*a + 2*b*x))/(2*b^2) -
 (3*c*d^2*sin(2*a + 2*b*x))/(2*b^3) - (3*d^3*x*sin(2*a + 2*b*x))/(2*b^3) +
```

```
(3*d^3*x^2*cos(2*a + 2*b*x))/(2*b^2) + (d^3*x^3*sin(2*a + 2*b*x))/b + (3*c*
d^2*x*cos(2*a + 2*b*x))/b^2 + (3*c^2*d*x*sin(2*a + 2*b*x))/b + (3*c*d^2*x^2
*sin(2*a + 2*b*x))/b
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

# 3.370 $\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx$

**Optimal**. Leaf size=112

$$-\frac{d^2\sin(a+bx)\cos(a+bx)}{b^3}-\frac{d(c+dx)\sin^2(a+bx)}{2b^2}+\frac{3d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{2(c+dx)^2\sin(a+bx)\cos(a+bx)}{b}$$

```
[Out] -1/2*d^2*x/b^2+1/3*(d*x+c)^3/d+3/2*d*(d*x+c)*cos(b*x+a)^2/b^2-d^2*cos(b*x+a
)*sin(b*x+a)/b^3+2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b-1/2*d*(d*x+c)*sin(b*x+
a)^2/b^2
```

**Rubi [A]** time = 0.14, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4431, 3311, 32, 2635, 8}

$$-\frac{d(c+dx)\sin^2(a+bx)}{2b^2}+\frac{3d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\sin(a+bx)\cos(a+bx)}{b^3}+\frac{2(c+dx)^2\sin(a+bx)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] -(d^2*x)/(2*b^2) + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*Cos[a + b*x]^2)/(2*b^
2) - (d^2*Cos[a + b*x]*Sin[a + b*x])/b^3 + (2*(c + d*x)^2*Cos[a + b*x]*Sin[
a + b*x])/b - (d*(c + d*x)*Sin[a + b*x]^2)/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^2\cos^2(a+bx) - (c+dx)^2\sin^2(a+bx)\right)\,dx$$
$$= 3\int (c+dx)^2\cos^2(a+bx)\,dx - \int (c+dx)^2\sin^2(a+bx)\,dx$$
$$= \frac{3d(c+dx)\cos^2(a+bx)}{2b^2} + \frac{2(c+dx)^2\cos(a+bx)\sin(a+bx)}{b} - \frac{d(c \cdots}{\cdots}$$
$$= \frac{(c+dx)^3}{3d} + \frac{3d(c+dx)\cos^2(a+bx)}{2b^2} - \frac{d^2\cos(a+bx)\sin(a+bx)}{b^3} - \cdots$$
$$= -\frac{d^2x}{2b^2} + \frac{(c+dx)^3}{3d} + \frac{3d(c+dx)\cos^2(a+bx)}{2b^2} - \frac{d^2\cos(a+bx)\sin(\cdots}{b^3}$$

**Mathematica [A]** time = 0.40, size = 73, normalized size = 0.65

$$\frac{d(c+dx)\cos(2(a+bx))}{b^2} + \frac{\sin(2(a+bx))\left(2b^2(c+dx)^2 - d^2\right)}{2b^3} + c^2x + cdx^2 + \frac{d^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] c^2*x + c*d*x^2 + (d^2*x^3)/3 + (d*(c + d*x)*Cos[2*(a + b*x)])/b^2 + ((-d^2
 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(2*b^3)
```

**fricas [A]** time = 0.45, size = 111, normalized size = 0.99

$$\frac{b^3d^2x^3 + 3\,b^3cdx^2 + 6\left(bd^2x + bcd\right)\cos\left(bx+a\right)^2 + 3\left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(bx+a\right)\sin\left(bx+\cdots\right.}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/3*(b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 3*(
2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)*sin(b*x + a) +
3*(b^3*c^2 - b*d^2)*x)/b^3
```

**giac [B]** time = 5.77, size = 1880, normalized size = 16.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/3*(b^3*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 3*b^3*c*d*x^2*tan(1/2*b*x)^4
*tan(1/2*a)^4 + 2*b^3*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^3*d^2*x^3*t
an(1/2*b*x)^2*tan(1/2*a)^4 + 3*b^3*c^2*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 6*b^
3*c*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 12*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1
/2*a)^3 + 6*b^3*c*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 12*b^2*d^2*x^2*tan(1/
2*b*x)^3*tan(1/2*a)^4 + b^3*d^2*x^3*tan(1/2*b*x)^4 + 4*b^3*d^2*x^3*tan(1/2*
b*x)^2*tan(1/2*a)^2 + 6*b^3*c^2*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 24*b^2*c*d*
x*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^3*d^2*x^3*tan(1/2*a)^4 + 6*b^3*c^2*x*tan(
1/2*b*x)^2*tan(1/2*a)^4 - 24*b^2*c*d*x*tan(1/2*b*x)^3*tan(1/2*a)^4 + 3*b*d^
2*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 3*b^3*c*d*x^2*tan(1/2*b*x)^4 + 12*b^2*d^2
*x^2*tan(1/2*b*x)^4*tan(1/2*a) + 12*b^3*c*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2
 + 72*b^2*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^2 + 72*b^2*d^2*x^2*tan(1/2*b*x)
^2*tan(1/2*a)^3 - 12*b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 3*b^3*c*d*x^2*ta
n(1/2*a)^4 + 12*b^2*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^4 - 12*b^2*c^2*tan(1/2*
```

```
b*x)^3*tan(1/2*a)^4 + 3*b*c*d*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^3*d^2*x^3*t
an(1/2*b*x)^2 + 3*b^3*c^2*x*tan(1/2*b*x)^4 + 24*b^2*c*d*x*tan(1/2*b*x)^4*ta
n(1/2*a) + 2*b^3*d^2*x^3*tan(1/2*a)^2 + 12*b^3*c^2*x*tan(1/2*b*x)^2*tan(1/2
*a)^2 + 144*b^2*c*d*x*tan(1/2*b*x)^3*tan(1/2*a)^2 - 18*b*d^2*x*tan(1/2*b*x)
^4*tan(1/2*a)^2 + 144*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^3 - 48*b*d^2*x*ta
n(1/2*b*x)^3*tan(1/2*a)^3 + 3*b^3*c^2*x*tan(1/2*a)^4 + 24*b^2*c*d*x*tan(1/2
*b*x)*tan(1/2*a)^4 - 18*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 6*b^3*c*d*x^2
*tan(1/2*b*x)^2 - 12*b^2*d^2*x^2*tan(1/2*b*x)^3 - 72*b^2*d^2*x^2*tan(1/2*b*
x)^2*tan(1/2*a) + 12*b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a) + 6*b^3*c*d*x^2*tan(
1/2*a)^2 - 72*b^2*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^2 + 72*b^2*c^2*tan(1/2*b*
x)^3*tan(1/2*a)^2 - 18*b*c*d*tan(1/2*b*x)^4*tan(1/2*a)^2 - 12*b^2*d^2*x^2*t
an(1/2*a)^3 + 72*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^3 - 48*b*c*d*tan(1/2*b*x
)^3*tan(1/2*a)^3 + 6*d^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 12*b^2*c^2*tan(1/2*b
*x)*tan(1/2*a)^4 - 18*b*c*d*tan(1/2*b*x)^2*tan(1/2*a)^4 + 6*d^2*tan(1/2*b*x
)^3*tan(1/2*a)^4 + b^3*d^2*x^3 + 6*b^3*c^2*x*tan(1/2*b*x)^2 - 24*b^2*c*d*x*
tan(1/2*b*x)^3 + 3*b*d^2*x*tan(1/2*b*x)^4 - 144*b^2*c*d*x*tan(1/2*b*x)^2*ta
n(1/2*a) + 48*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a) + 6*b^3*c^2*x*tan(1/2*a)^2
- 144*b^2*c*d*x*tan(1/2*b*x)*tan(1/2*a)^2 + 108*b*d^2*x*tan(1/2*b*x)^2*tan(
1/2*a)^2 - 24*b^2*c*d*x*tan(1/2*a)^3 + 48*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^3
 + 3*b*d^2*x*tan(1/2*a)^4 + 3*b^3*c*d*x^2 + 12*b^2*d^2*x^2*tan(1/2*b*x) - 1
2*b^2*c^2*tan(1/2*b*x)^3 + 3*b*c*d*tan(1/2*b*x)^4 + 12*b^2*d^2*x^2*tan(1/2*
a) - 72*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a) + 48*b*c*d*tan(1/2*b*x)^3*tan(1/2
*a) - 6*d^2*tan(1/2*b*x)^4*tan(1/2*a) - 72*b^2*c^2*tan(1/2*b*x)*tan(1/2*a)^
2 + 108*b*c*d*tan(1/2*b*x)^2*tan(1/2*a)^2 - 36*d^2*tan(1/2*b*x)^3*tan(1/2*a
)^2 - 12*b^2*c^2*tan(1/2*a)^3 + 48*b*c*d*tan(1/2*b*x)*tan(1/2*a)^3 - 36*d^2
*tan(1/2*b*x)^2*tan(1/2*a)^3 + 3*b*c*d*tan(1/2*a)^4 - 6*d^2*tan(1/2*b*x)*ta
n(1/2*a)^4 + 3*b^3*c^2*x + 24*b^2*c*d*x*tan(1/2*b*x) - 18*b*d^2*x*tan(1/2*b
*x)^2 + 24*b^2*c*d*x*tan(1/2*a) - 48*b*d^2*x*tan(1/2*b*x)*tan(1/2*a) - 18*b
*d^2*x*tan(1/2*a)^2 + 12*b^2*c^2*tan(1/2*b*x) - 18*b*c*d*tan(1/2*b*x)^2 + 6
*d^2*tan(1/2*b*x)^3 + 12*b^2*c^2*tan(1/2*a) - 48*b*c*d*tan(1/2*b*x)*tan(1/2
*a) + 36*d^2*tan(1/2*b*x)^2*tan(1/2*a) - 18*b*c*d*tan(1/2*a)^2 + 36*d^2*tan
(1/2*b*x)*tan(1/2*a)^2 + 6*d^2*tan(1/2*a)^3 + 3*b*d^2*x + 3*b*c*d - 6*d^2*t
an(1/2*b*x) - 6*d^2*tan(1/2*a))/(b^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^3*ta
n(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^3*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^3*tan(1/2
*b*x)^4 + 4*b^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + b^3*tan(1/2*a)^4 + 2*b^3*tan(
1/2*b*x)^2 + 2*b^3*tan(1/2*a)^2 + b^3)
```

**maple [B]** time = 0.03, size = 294, normalized size = 2.62

$$-c^2x-\frac{d^2x^3}{3}+\frac{4c^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-cd\,x^2+\frac{4d^2\left((bx+a)^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{(bx+a)\left(\cos^2(bx+a)\right)}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -c^2*x-1/3*d^2*x^3+4*c^2/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-c*d*x^
2+4*d^2/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a
)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3-2*a*((
b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+
a)^2)+a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+8*c*d/b^2*((b*x+a)*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2-a*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

**maxima [A]** time = 0.35, size = 108, normalized size = 0.96

$$\frac{(bx+\sin(2\,bx+2\,a))c^2}{b}+\frac{\left(b^2x^2+2\,bx\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)cd}{b^2}+\frac{\left(2\,b^3x^3+6\,bx\cos(2\,bx+2\,a)+3\,(\right.}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c^2/b + (b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(2*
b*x + 2*a))*c*d/b^2 + 1/6*(2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*b^2*x^
2 - 1)*sin(2*b*x + 2*a))*d^2/b^3
```

**mupad [B]** time = 0.31, size = 121, normalized size = 1.08

$$c^2\,x+\frac{d^2\,x^3}{3}+\frac{c^2\,\sin\left(2\,a+2\,b\,x\right)}{b}-\frac{d^2\,\sin\left(2\,a+2\,b\,x\right)}{2\,b^3}+c\,d\,x^2+\frac{d^2\,x\,\cos\left(2\,a+2\,b\,x\right)}{b^2}+\frac{d^2\,x^2\,\sin\left(2\,a+2\,b\,x\right)}{b}+\frac{c\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^2)/sin(a + b*x),x)
```

```
[Out] c^2*x + (d^2*x^3)/3 + (c^2*sin(2*a + 2*b*x))/b - (d^2*sin(2*a + 2*b*x))/(2*
b^3) + c*d*x^2 + (d^2*x*cos(2*a + 2*b*x))/b^2 + (d^2*x^2*sin(2*a + 2*b*x))/
b + (c*d*cos(2*a + 2*b*x))/b^2 + (2*c*d*x*sin(2*a + 2*b*x))/b
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

# 3.371 $\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx$

**Optimal**. Leaf size=66

$$-\frac{d\sin^2(a+bx)}{4b^2}+\frac{3d\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)\sin(a+bx)\cos(a+bx)}{b}+cx+\frac{dx^2}{2}$$

```
[Out] c*x+1/2*d*x^2+3/4*d*cos(b*x+a)^2/b^2+2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b-1/4*
d*sin(b*x+a)^2/b^2
```

**Rubi** [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4431, 3310}

$$-\frac{d\sin^2(a+bx)}{4b^2}+\frac{3d\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)\sin(a+bx)\cos(a+bx)}{b}+cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] c*x + (d*x^2)/2 + (3*d*Cos[a + b*x]^2)/(4*b^2) + (2*(c + d*x)*Cos[a + b*x]*
Sin[a + b*x])/b - (d*Sin[a + b*x]^2)/(4*b^2)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx &= \int \left(3(c+dx)\cos^2(a+bx)-(c+dx)\sin^2(a+bx)\right)dx\\ &= 3\int (c+dx)\cos^2(a+bx)\,dx-\int (c+dx)\sin^2(a+bx)\,dx\\ &= \frac{3d\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)\cos(a+bx)\sin(a+bx)}{b}-\frac{d\sin^2(a+bx)}{4b^2}-\\ &= cx+\frac{dx^2}{2}+\frac{3d\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)\cos(a+bx)\sin(a+bx)}{b}-\frac{d\sin}{}\end{aligned}$$

**Mathematica** [A] time = 0.14, size = 46, normalized size = 0.70

$$\frac{b(2(c+dx)\sin(2(a+bx))+bx(2c+dx))+d\cos(2(a+bx))}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (d*Cos[2*(a + b*x)] + b*(b*x*(2*c + d*x) + 2*(c + d*x)*Sin[2*(a + b*x)]))/(
2*b^2)
```

**fricas [A]** time = 0.44, size = 54, normalized size = 0.82

$$\frac{b^2dx^2 + 2\,b^2cx + 2\,d\cos\left(bx + a\right)^2 + 4\,\left(bdx + bc\right)\cos\left(bx + a\right)\sin\left(bx + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d*x^2 + 2*b^2*c*x + 2*d*cos(b*x + a)^2 + 4*(b*d*x + b*c)*cos(b*x +
 a)*sin(b*x + a))/b^2
```

**giac [B]** time = 1.99, size = 920, normalized size = 13.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*x*tan(1/2*b*x)^4*tan(1
/2*a)^4 + 2*b^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d*x^2*tan(1/2*b*x
)^2*tan(1/2*a)^4 + 4*b^2*c*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 8*b*d*x*tan(1/2*
b*x)^4*tan(1/2*a)^3 + 4*b^2*c*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 8*b*d*x*tan(1
/2*b*x)^3*tan(1/2*a)^4 + b^2*d*x^2*tan(1/2*b*x)^4 + 4*b^2*d*x^2*tan(1/2*b*x
)^2*tan(1/2*a)^2 - 8*b*c*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*d*x^2*tan(1/2*a)
^4 - 8*b*c*tan(1/2*b*x)^3*tan(1/2*a)^4 + d*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*
b^2*c*x*tan(1/2*b*x)^4 + 8*b*d*x*tan(1/2*b*x)^4*tan(1/2*a) + 8*b^2*c*x*tan(
1/2*b*x)^2*tan(1/2*a)^2 + 48*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^2 + 48*b*d*x*t
an(1/2*b*x)^2*tan(1/2*a)^3 + 2*b^2*c*x*tan(1/2*a)^4 + 8*b*d*x*tan(1/2*b*x)*
tan(1/2*a)^4 + 2*b^2*d*x^2*tan(1/2*b*x)^2 + 8*b*c*tan(1/2*b*x)^4*tan(1/2*a)
 + 2*b^2*d*x^2*tan(1/2*a)^2 + 48*b*c*tan(1/2*b*x)^3*tan(1/2*a)^2 - 6*d*tan(
1/2*b*x)^4*tan(1/2*a)^2 + 48*b*c*tan(1/2*b*x)^2*tan(1/2*a)^3 - 16*d*tan(1/2
*b*x)^3*tan(1/2*a)^3 + 8*b*c*tan(1/2*b*x)*tan(1/2*a)^4 - 6*d*tan(1/2*b*x)^2
*tan(1/2*a)^4 + 4*b^2*c*x*tan(1/2*b*x)^2 - 8*b*d*x*tan(1/2*b*x)^3 - 48*b*d*
x*tan(1/2*b*x)^2*tan(1/2*a) + 4*b^2*c*x*tan(1/2*a)^2 - 48*b*d*x*tan(1/2*b*x
)*tan(1/2*a)^2 - 8*b*d*x*tan(1/2*a)^3 + b^2*d*x^2 - 8*b*c*tan(1/2*b*x)^3 +
d*tan(1/2*b*x)^4 - 48*b*c*tan(1/2*b*x)^2*tan(1/2*a) + 16*d*tan(1/2*b*x)^3*t
an(1/2*a) - 48*b*c*tan(1/2*b*x)*tan(1/2*a)^2 + 36*d*tan(1/2*b*x)^2*tan(1/2*
a)^2 - 8*b*c*tan(1/2*a)^3 + 16*d*tan(1/2*b*x)*tan(1/2*a)^3 + d*tan(1/2*a)^4
 + 2*b^2*c*x + 8*b*d*x*tan(1/2*b*x) + 8*b*d*x*tan(1/2*a) + 8*b*c*tan(1/2*b*
x) - 6*d*tan(1/2*b*x)^2 + 8*b*c*tan(1/2*a) - 16*d*tan(1/2*b*x)*tan(1/2*a) -
 6*d*tan(1/2*a)^2 + d)/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*tan(1/2*b*x
)^4*tan(1/2*a)^2 + 2*b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^2*tan(1/2*b*x)^4 +
 4*b^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + b^2*tan(1/2*a)^4 + 2*b^2*tan(1/2*b*x)^
2 + 2*b^2*tan(1/2*a)^2 + b^2)
```

**maple [A]** time = 0.03, size = 119, normalized size = 1.80

$$-cx-\frac{d\,x^2}{2}+\frac{4c\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{4d\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\frac{\left(\sin^2(bx+a)\right)}{4}-a\left(\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -c*x-1/2*d*x^2+4*c/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+4*d/b^2*((b*
x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)
^2-a*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

**maxima** [A] time = 0.34, size = 55, normalized size = 0.83

$$\frac{(bx + \sin(2\,bx + 2\,a))c}{b} + \frac{\left(b^2x^2 + 2\,bx\sin(2\,bx + 2\,a) + \cos(2\,bx + 2\,a)\right)d}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c/b + 1/2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(
2*b*x + 2*a))*d/b^2
```

**mupad** [B] time = 0.21, size = 53, normalized size = 0.80

$$c\,x + \frac{d\,x^2}{2} + \frac{\frac{d\,\cos(2\,a+2\,b\,x)}{2} + b\,\left(c\,\sin(2\,a + 2\,b\,x) + d\,x\,\sin(2\,a + 2\,b\,x)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x))/sin(a + b*x),x)
```

```
[Out] c*x + (d*x^2)/2 + ((d*cos(2*a + 2*b*x))/2 + b*(c*sin(2*a + 2*b*x) + d*x*sin
(2*a + 2*b*x)))/b^2
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)\sin(3a + 3bx)\csc(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Integral((c + d*x)*sin(3*a + 3*b*x)*csc(a + b*x), x)
```

# 3.372 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx} \, dx$

**Optimal**. Leaf size=71

$$\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\log(c+dx)}{d}$$

```
[Out] 2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d+ln(d*x+c)/d-2*Si(2*b*c/d+2*b*x)*sin(
2*a-2*b*c/d)/d
```

**Rubi [A]** time = 0.28, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4431, 3312, 3303, 3299, 3302}

$$\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d + Log[c + d*x]/d
- (2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \left(\frac{3\cos^2(a+bx)}{c+dx} - \frac{\sin^2(a+bx)}{c+dx}\right)dx$$
$$= 3\int \frac{\cos^2(a+bx)}{c+dx}\,dx - \int \frac{\sin^2(a+bx)}{c+dx}\,dx$$
$$= 3\int \left(\frac{1}{2(c+dx)} + \frac{\cos(2a+2bx)}{2(c+dx)}\right)dx - \int \left(\frac{1}{2(c+dx)} - \frac{\cos(2a+2bx)}{2(c+dx)}\right)dx$$
$$= \frac{\log(c+dx)}{d} + \frac{1}{2}\int \frac{\cos(2a+2bx)}{c+dx}\,dx + \frac{3}{2}\int \frac{\cos(2a+2bx)}{c+dx}\,dx$$
$$= \frac{\log(c+dx)}{d} + \frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx + \frac{1}{2}\left(3\cos\left(2a-\frac{2bc}{d}\right)\right)$$
$$= \frac{2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{d} + \frac{\log(c+dx)}{d} - \frac{2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d}$$

**Mathematica [A]** time = 0.15, size = 63, normalized size = 0.89

$$\frac{2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2b(c+dx)}{d}\right) - 2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right) + \log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] - 2*S
in[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d
```

**fricas [A]** time = 0.46, size = 85, normalized size = 1.20

$$\frac{\left(\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right) + \operatorname{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right) - 2\,\sin\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right) + \log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] ((cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*cos(-
2*(b*c - a*d)/d) - 2*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d)
+ log(d*x + c))/d
```

**giac [C]** time = 0.32, size = 1118, normalized size = 15.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

```
[Out] (log(abs(d*x + c))*tan(1/2*a)^4*tan(b*c/d)^2 - real_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 - real_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/
d))*tan(1/2*a)^4*tan(b*c/d) - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(1/2*a)^4*tan(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4*tan
(b*c/d) - 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d
)^2 + 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2
 - 8*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d)^2 + log(abs(d*
x + c))*tan(1/2*a)^4 + real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^
```

```
4 + real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 - 8*real_part(co
s_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) - 8*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 2*log(abs(d*x + c))*tan(1
/2*a)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a
)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2
*tan(b*c/d)^2 + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3 - 4
*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3 + 8*sin_integral(2*
(b*d*x + b*c)/d)*tan(1/2*a)^3 - 12*imag_part(cos_integral(2*b*x + 2*b*c/d))
*tan(1/2*a)^2*tan(b*c/d) + 12*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(1/2*a)^2*tan(b*c/d) - 24*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(
b*c/d) + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2
 - 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 + 8*
sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)*tan(b*c/d)^2 + 2*log(abs(d*x + c
))*tan(1/2*a)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2 -
 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2 + 8*real_part(cos
_integral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d) + 8*real_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d) + log(abs(d*x + c))*tan(b*c/d)^2
 - real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 4*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(1/2*a) + 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a
) - 8*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a) + 2*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + log(abs(d*x +
c)) + real_part(cos_integral(2*b*x + 2*b*c/d)) + real_part(cos_integral(-2*
b*x - 2*b*c/d)))/(d*tan(1/2*a)^4*tan(b*c/d)^2 + d*tan(1/2*a)^4 + 2*d*tan(1/
2*a)^2*tan(b*c/d)^2 + 2*d*tan(1/2*a)^2 + d*tan(b*c/d)^2 + d)
```

**maple** [A] time = 0.04, size = 116, normalized size = 1.63

$$-\frac{\ln(dx+c)}{d}+\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}+\frac{2\ln((bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] -ln(d*x+c)/d+2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*
x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d+2*ln((b*x+a)*d-d*a+c*b)/d
```

**maxima** [C] time = 0.41, size = 117, normalized size = 1.65

$$-\frac{\left(E_1\left(\frac{2i\,bdx+2i\,bc}{d}\right)+E_1\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-\left(i\,E_1\left(\frac{2i\,bdx+2i\,bc}{d}\right)-i\,E_1\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(1, -(2*I*b*d
*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (I*exp_integral_e(1, (2*I*b*d*x +
 2*I*b*c)/d) - I*exp_integral_e(1, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c -
 a*d)/d) - log(d*x + c))/d
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,a+3\,b\,x)}{\sin(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] Timed out
```

## 3.373 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=102

$$-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{3\cos^2(a+bx)}{d(c+dx)}$$

```
[Out] -3*cos(b*x+a)^2/d/(d*x+c)-4*b*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2-4*b*Ci
(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^2+sin(b*x+a)^2/d/(d*x+c)
```

**Rubi [A]** time = 0.28, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4431, 3313, 12, 3303, 3299, 3302}

$$-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{3\cos^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (-3*Cos[a + b*x]^2)/(d*(c + d*x)) - (4*b*CosIntegral[(2*b*c)/d + 2*b*x]*Sin
[2*a - (2*b*c)/d])/d^2 + Sin[a + b*x]^2/(d*(c + d*x)) - (4*b*Cos[2*a - (2*b
*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx &= \int \left(\frac{3\cos^2(a+bx)}{(c+dx)^2}-\frac{\sin^2(a+bx)}{(c+dx)^2}\right)dx\\
&= 3\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx-\int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx\\
&= -\frac{3\cos^2(a+bx)}{d(c+dx)}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{(2b)\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx}{d}+\frac{(6b)\int -\frac{\sin(2a+2bx)}{2(c+dx)}\,dx}{d}\\
&= -\frac{3\cos^2(a+bx)}{d(c+dx)}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{b\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d}-\frac{(3b)\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d}\\
&= -\frac{3\cos^2(a+bx)}{d(c+dx)}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}-\frac{\left(3b\cos\right.}{}\\
&= -\frac{3\cos^2(a+bx)}{d(c+dx)}-\frac{4b\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{4b\cos\left(2\right.}{}
\end{aligned}$$

**Mathematica [A]** time = 0.53, size = 81, normalized size = 0.79

$$-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right)+4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+\frac{d(2\cos(2(a+bx))+1)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] -(((d*(1 + 2*Cos[2*(a + b*x)]))/(c + d*x) + 4*b*CosIntegral[(2*b*(c + d*x))
/d]*Sin[2*a - (2*b*c)/d] + 4*b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d
*x))/d])/d^2)
```

**fricas [A]** time = 0.45, size = 131, normalized size = 1.28

$$-\frac{4\,d\cos\left(bx+a\right)^2+4\left(bdx+bc\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\text{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+2\left(\left(bdx+bc\right)\text{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\left(bdx+bc\right)\text{Ci}\left(-\frac{2\,(b}{}\right.\right.}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(4*d*cos(b*x + a)^2 + 4*(b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*sin_integral(2
*(b*d*x + b*c)/d) + 2*((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d
*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) - d)/(d^3
*x + c*d^2)
```

**giac [C]** time = 10.37, size = 5381, normalized size = 52.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*t
an(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*
tan(1/2*a)^4*tan(b*c/d)^2 + 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x
)^2*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b*d*x*real_part(cos_integral(2*b*x + 2*b*
c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) + 4*b*d*x*real_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) - 8*b*d*x*real_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 - 8*b*d*x
*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/
d)^2 + 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)
^4*tan(b*c/d)^2 - 2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^
2*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x
)^2*tan(1/2*a)^4*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*
c/d))*tan(b*x)^2*tan(1/2*a)^4 + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(b*x)^2*tan(1/2*a)^4 - 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*ta
n(b*x)^2*tan(1/2*a)^4 + 16*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*t
an(b*x)^2*tan(1/2*a)^3*tan(b*c/d) - 16*b*d*x*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) + 32*b*d*x*sin_integral(2*(b
*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) + 4*b*c*real_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) + 4*b*c*real_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) - 12*b
*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b
*c/d)^2 + 12*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(1/2*a)^2*tan(b*c/d)^2 - 24*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^
2*tan(1/2*a)^2*tan(b*c/d)^2 - 8*b*c*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 - 8*b*c*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 2*b*d*x*imag_part(cos_
integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 - 2*b*d*x*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b*d*x*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4*tan(b*c/d)^2 + 8*b*d*x*real_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 + 8*b*d*x*real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 - 2*b*c*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 + 2*b*c*imag_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 - 4*b*c*sin_integral(2*(b*d*
x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^4 - 24*b*d*x*real_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) - 24*b*d*x*real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 16*b*c*imag_
part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) - 16
*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(
b*c/d) + 32*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3*tan
(b*c/d) + 4*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan
(b*c/d) + 4*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*ta
n(b*c/d) + 8*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
1/2*a)*tan(b*c/d)^2 + 8*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 12*b*c*imag_part(cos_integral(2*b*x + 2*b
*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 + 12*b*c*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b*c*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 - 8*b*d*x*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 - 8*b*d*x*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 + 2*b*c*im
ag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 - 2*b*c*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b*c*s
in_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4*tan(b*c/d)^2 + d*tan(b*x)^2*tan
(1/2*a)^4*tan(b*c/d)^2 + 12*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*
tan(b*x)^2*tan(1/2*a)^2 - 12*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(1/2*a)^2 + 24*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*
x)^2*tan(1/2*a)^2 + 8*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)
^2*tan(1/2*a)^3 + 8*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^
2*tan(1/2*a)^3 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a
```

```
)^4 + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 - 4*b*
d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4 - 16*b*d*x*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) + 16*b*d*x*imag_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 32*
b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 24
*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b
*c/d) - 24*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2
*a)^2*tan(b*c/d) + 16*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/
2*a)^3*tan(b*c/d) - 16*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
1/2*a)^3*tan(b*c/d) + 32*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3
*tan(b*c/d) + 4*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*t
an(b*c/d) + 4*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*ta
n(b*c/d) + 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(b*c/d)^2 + 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2
 + 8*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan
(b*c/d)^2 + 8*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
1/2*a)*tan(b*c/d)^2 - 12*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan
(1/2*a)^2*tan(b*c/d)^2 + 12*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1
/2*a)^2*tan(b*c/d)^2 - 8*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1
/2*a)^3*tan(b*c/d)^2 - 8*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
1/2*a)^3*tan(b*c/d)^2 - 8*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(b*x)^2*tan(1/2*a) - 8*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(1/2*a) + 12*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(1/2*a)^2 - 12*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*
x)^2*tan(1/2*a)^2 + 24*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1
/2*a)^2 + 8*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3 + 8
*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3 - 2*b*c*imag_
part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4 + 2*b*c*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 - 4*b*c*sin_integral(2*(b*d*x + b*c)/d
)*tan(1/2*a)^4 + d*tan(b*x)^2*tan(1/2*a)^4 + 4*b*d*x*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 4*b*d*x*real_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 16*b*c*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) + 16*b*c*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 32*b*c*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 24*b*d*x*real_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) - 24*b*d*x*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) + 16*b*c*imag_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) - 16*b*c*imag_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 32*b*c*sin_inte
gral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d) + 2*b*c*imag_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b*c*imag_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 4*b*c*sin_integral(2*(b*d*x
+ b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 + 8*b*d*x*real_part(cos_integral(2*b*x +
2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 + 8*b*d*x*real_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 - 12*b*c*imag_part(cos_integral(2*b*x +
2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 + 12*b*c*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b*c*sin_integral(2*(b*d*x + b*c)
/d)*tan(1/2*a)^2*tan(b*c/d)^2 - 14*d*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 -
 16*d*tan(b*x)*tan(1/2*a)^3*tan(b*c/d)^2 - 3*d*tan(1/2*a)^4*tan(b*c/d)^2 -
2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b*d*x*imag_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 4*b*d*x*sin_integral(2*(b
*d*x + b*c)/d)*tan(b*x)^2 - 8*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*
tan(b*x)^2*tan(1/2*a) - 8*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(1/2*a) + 12*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
1/2*a)^2 - 12*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2
+ 24*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2 + 8*b*c*real_part(c
os_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3 + 8*b*c*real_part(cos_integral(-
```

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2*b*x - 2*b*c/d))*tan(1/2*a)^3 + 4*b*c*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(b*c/d) + 4*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(b*c/d) - 16*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))
*tan(1/2*a)*tan(b*c/d) + 16*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(1/2*a)*tan(b*c/d) - 32*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a
)*tan(b*c/d) - 24*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2
*tan(b*c/d) - 24*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2
*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2
 - 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 + 4*b*d*x
*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 8*b*c*real_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 + 8*b*c*real_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 - 2*b*c*imag_part(cos_integral(2
*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(b*x)^2 - 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 - 8*b*d*x*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a) - 8*b*d*x*real_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(1/2*a) + 12*b*c*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(1/2*a)^2 - 12*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(1/2*a)^2 + 24*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2 - 14
*d*tan(b*x)^2*tan(1/2*a)^2 - 16*d*tan(b*x)*tan(1/2*a)^3 - 3*d*tan(1/2*a)^4
+ 4*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 4*b*d*x*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) - 16*b*c*imag_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d) + 16*b*c*imag_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d) - 32*b*c*sin_integral(2*(b*d*x
 + b*c)/d)*tan(1/2*a)*tan(b*c/d) + 2*b*c*imag_part(cos_integral(2*b*x + 2*b
*c/d))*tan(b*c/d)^2 - 2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b
*c/d)^2 + 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + d*tan(b*x)^2
*tan(b*c/d)^2 + 16*d*tan(b*x)*tan(1/2*a)*tan(b*c/d)^2 + 10*d*tan(1/2*a)^2*t
an(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d)) + 2*b*d*x*im
ag_part(cos_integral(-2*b*x - 2*b*c/d)) - 4*b*d*x*sin_integral(2*(b*d*x + b
*c)/d) - 8*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a) - 8*b*c*
real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a) + 4*b*c*real_part(cos_
integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 4*b*c*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*c/d) - 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d)) +
2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d)) - 4*b*c*sin_integral(2*(b*d
*x + b*c)/d) + d*tan(b*x)^2 + 16*d*tan(b*x)*tan(1/2*a) + 10*d*tan(1/2*a)^2
- 3*d*tan(b*c/d)^2 - 3*d)/(d^3*x*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 + c*d
^2*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 + d^3*x*tan(b*x)^2*tan(1/2*a)^4 + 2
*d^3*x*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 + d^3*x*tan(1/2*a)^4*tan(b*c/d)
^2 + c*d^2*tan(b*x)^2*tan(1/2*a)^4 + 2*c*d^2*tan(b*x)^2*tan(1/2*a)^2*tan(b*
c/d)^2 + c*d^2*tan(1/2*a)^4*tan(b*c/d)^2 + 2*d^3*x*tan(b*x)^2*tan(1/2*a)^2
+ d^3*x*tan(1/2*a)^4 + d^3*x*tan(b*x)^2*tan(b*c/d)^2 + 2*d^3*x*tan(1/2*a)^2
*tan(b*c/d)^2 + 2*c*d^2*tan(b*x)^2*tan(1/2*a)^2 + c*d^2*tan(1/2*a)^4 + c*d^
2*tan(b*x)^2*tan(b*c/d)^2 + 2*c*d^2*tan(1/2*a)^2*tan(b*c/d)^2 + d^3*x*tan(b
*x)^2 + 2*d^3*x*tan(1/2*a)^2 + d^3*x*tan(b*c/d)^2 + c*d^2*tan(b*x)^2 + 2*c*
d^2*tan(1/2*a)^2 + c*d^2*tan(b*c/d)^2 + d^3*x + c*d^2)
```

**maple [A]** time = 0.04, size = 169, normalized size = 1.66

$$\frac{1}{d\,(dx+c)}+\frac{b^{2}\left(-\frac{2\cos(2bx+2a)}{((bx+a)d-da+cb)d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}\right)}{d}\right)-\frac{2b^{2}}{((bx+a)d-da+cb)d}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

```
[Out] 1/d/(d*x+c)+4/b*(1/4*b^2*(-2*cos(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d-2*(2*Si(2
*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/
```

```
d)*sin(2*(-a*d+b*c)/d)/d)/d)-1/2*b^2/((b*x+a)*d-d*a+c*b)/d)
```

**maxima [C]** time = 0.42, size = 118, normalized size = 1.16

$$-\frac{\left(E_2\left(\frac{2i\,bdx+2i\,bc}{d}\right)+E_2\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-\left(i\,E_2\left(\frac{2i\,bdx+2i\,bc}{d}\right)-i\,E_2\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+1}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(2, -(2*I*b*d
*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (I*exp_integral_e(2, (2*I*b*d*x +
 2*I*b*c)/d) - I*exp_integral_e(2, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c -
 a*d)/d) + 1)/(d^2*x + c*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,a+3\,b\,x)}{\sin(a+b\,x)\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

## 3.374 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=136

$$-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}+\frac{\sin^2(a+b}{2d(c+dx)}$$

```
[Out] -4*b^2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^3-3/2*cos(b*x+a)^2/d/(d*x+c)^2+
4*b^2*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3+4*b*cos(b*x+a)*sin(b*x+a)/d^2/
(d*x+c)+1/2*sin(b*x+a)^2/d/(d*x+c)^2
```

**Rubi [A]** time = 0.37, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4431, 3314, 31, 3312, 3303, 3299, 3302}

$$-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}+$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (-3*Cos[a + b*x]^2)/(2*d*(c + d*x)^2) - (4*b^2*Cos[2*a - (2*b*c)/d]*CosInte
gral[(2*b*c)/d + 2*b*x])/d^3 + (4*b*Cos[a + b*x]*Sin[a + b*x])/(d^2*(c + d*
x)) + Sin[a + b*x]^2/(2*d*(c + d*x)^2) + (4*b^2*Sin[2*a - (2*b*c)/d]*SinInt
egral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\cos^2(a+bx)}{(c+dx)^3} - \frac{\sin^2(a+bx)}{(c+dx)^3}\right) dx$$

$$= 3\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx$$

$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\int \frac{1}{c+dx}\,dx}{d^2} +$$

$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{2b^2\log(c+dx)}{d^3} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+b}{2d(c+dx)}$$

$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\int \frac{\cos(2a+2bx}{c+dx}}{d^2}$$

$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{\left(b^2\cos\left(2a-\right.\right.}{}$$

$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} - \frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^3} + \frac{4b\cos(a+bx)\sin(a+}{d^2(c+dx)}$$

**Mathematica [A]** time = 0.99, size = 104, normalized size = 0.76

$$-\frac{8b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right) - 8b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + \frac{d(-4b(c+dx)\sin(2(a+bx))+2d\cos(2(a+bx))+d)}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] -1/2*(8*b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + (d*(d + 2
*d*Cos[2*(a + b*x)] - 4*b*(c + d*x)*Sin[2*(a + b*x)]))/(c + d*x)^2 - 8*b^2*
Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^3
```

**fricas [A]** time = 0.46, size = 225, normalized size = 1.65

$$-\frac{4\,d^2\cos(bx+a)^2 - 8\left(bd^2x+bcd\right)\cos(bx+a)\sin(bx+a) - 8\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{2(bc-ad)}{d}\right)\text{Si}\left(\frac{2(b}{}\right.}{2\left(d^5x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*d^2*cos(b*x + a)^2 - 8*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a)
- 8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-2*(b*c - a*d)/d)*sin_integra
l(2*(b*d*x + b*c)/d) - d^2 + 4*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_i
ntegral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_inte
gral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2
*d^3)
```

**giac [C]** time = 10.03, size = 9416, normalized size = 69.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
1/2*a)^4*tan(b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) + 8*b^2*d^2*x^2*
imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d
) - 16*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^4*
tan(b*c/d) + 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(1/2*a)^3*tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 32*b^2*d^2*x^2*sin_in
tegral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 8*b^2*c*d*
x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/
d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(1/2*a)^4*tan(b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(1/2*a)^4 - 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 + 32*b^2*d^2*x^2*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) + 32*b^2*d^2*x^2*real
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) -
16*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)
^4*tan(b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
b*x)^2*tan(1/2*a)^4*tan(b*c/d) - 32*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/
d)*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) - 24*b^2*d^2*x^2*real_part(cos_integr
al(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b^2*d^2*x^2*
real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d
)^2 + 32*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
1/2*a)^3*tan(b*c/d)^2 - 32*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 64*b^2*c*d*x*sin_integral(2*(b*d
*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 4*b^2*d^2*x^2*real_part
(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b^2*d^2*x^2*r
eal_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b^2*
c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*
c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(1/2*a)^4*tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*
c/d))*tan(b*x)^2*tan(1/2*a)^3 + 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 - 32*b^2*d^2*x^2*sin_integral(2*(b*d*
x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3 - 8*b^2*c*d*x*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 - 8*b^2*c*d*x*real_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 + 48*b^2*d^2*x^2*imag_part(cos
_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) - 48*b^2*d^2
*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(
b*c/d) + 96*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*
a)^2*tan(b*c/d) + 64*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*x)^2*tan(1/2*a)^3*tan(b*c/d) + 64*b^2*c*d*x*real_part(cos_integral(-2*b*
```

x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) - 8*b^2*d^2*x^2*imag_part(
cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d) + 8*b^2*d^2*x^2*imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d) - 16*b^2*d^2*
x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4*tan(b*c/d) - 8*b^2*c^2*ima
g_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) +
8*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4
*tan(b*c/d) - 16*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2
*a)^4*tan(b*c/d) - 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*
tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 + 16*b^2*d^2*x^2*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 32*b^2*d^2*x^2*sin_
integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 48*b^2*c*d
*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c
/d)^2 - 48*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*t
an(1/2*a)^2*tan(b*c/d)^2 + 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 + 32*b^2*d^2*x^2*sin_integral(2
*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d)^2 + 16*b^2*c^2*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 - 16*b^2*c^2*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^
2 + 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3*tan(
b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^
4*tan(b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
1/2*a)^4*tan(b*c/d)^2 + 24*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(1/2*a)^2 + 24*b^2*d^2*x^2*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2 - 32*b^2*c*d*x*imag_part(cos_integral(
2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 + 32*b^2*c*d*x*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 - 64*b^2*c*d*x*sin_integral
(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3 - 4*b^2*d^2*x^2*real_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(1/2*a)^4 - 4*b^2*d^2*x^2*real_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 - 4*b^2*c^2*real_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 - 4*b^2*c^2*real_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 - 32*b^2*d^2*x^2*real_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 32*b^2*d^2*x^2*re
al_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) +
96*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)
^2*tan(b*c/d) - 96*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 192*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)
/d)*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 32*b^2*d^2*x^2*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 32*b^2*d^2*x^2*real_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 32*b^2*c^2*real_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) + 32*b^
2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan
(b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^
4*tan(b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1
/2*a)^4*tan(b*c/d) - 32*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a
)^4*tan(b*c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*x)^2*tan(b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(b*c/d)^2 - 32*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 + 32*b^2*c*d*x*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 64*b^2*c*d*x
*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 24*b^
2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^
2 - 24*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*t
an(b*c/d)^2 - 24*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^
2*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b^2*c^2*real_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 + 32*b^2*c*d*x*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 - 32*b^2*c*d*x*imag_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 + 64*b^2*c*d*x*
sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d)^2 + 16*b*d^2*x*tan(

b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 8*b*d^2*x*tan(b*x)*tan(1/2*a)^4*tan(b*c/d)^2 + 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a) - 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a) + 32*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a) + 48*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2 + 48*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3 + 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3 - 32*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3 - 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 + 16*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 - 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3 - 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4 - 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 - 8*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 16*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) - 64*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 64*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) + 48*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) - 48*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) + 96*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(b*c/d) + 48*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) - 48*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 96*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 64*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 64*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) - 8*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d) + 8*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d) - 16*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4*tan(b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 + 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 - 32*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)*tan(b*c/d)^2 - 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 + 16*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 48*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 - 48*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 + 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 - 16*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 + 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d)^2 + 16*b*c*d*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 8*b*c*d*tan(b*x)*tan(1/2*a)^4*tan(b*c/d)^2 + d^2*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 + 32*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a) - 32*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a) + 64*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a) + 24*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2 + 24*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2 + 24*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2 + 24*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2 - 32*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3 + 32*b^2*c*d*

```
x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3 - 64*b^2*c*d*x*sin
_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3 + 16*b*d^2*x*tan(b*x)^2*tan(1/2*a
)^3 - 4*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4 - 4*b
^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 + 8*b*d^2*x*t
an(b*x)*tan(1/2*a)^4 - 16*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*x)^2*tan(b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(b*x)^2*tan(b*c/d) - 32*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*
tan(b*x)^2*tan(b*c/d) - 32*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(1/2*a)*tan(b*c/d) - 32*b^2*d^2*x^2*real_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(1/2*a)*tan(b*c/d) - 32*b^2*c^2*real_part(cos_integral(2*b*x
+ 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 32*b^2*c^2*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) + 96*b^2*c*d*x*im
ag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) - 96*b^2*c*d
*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) + 192*
b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(b*c/d) + 32*b^2*
c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 32*b
^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) +
4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + 4*b^2
*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 + 4*b^2*c^2
*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 4*b^2*c
^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 32*b
^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 +
 32*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/
d)^2 - 64*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)*tan(b*c/d)^2
 - 16*b*d^2*x*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 24*b^2*c^2*real_part(cos
_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b^2*c^2*real_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 - 48*b*d^2*x*ta
n(b*x)*tan(1/2*a)^2*tan(b*c/d)^2 - 16*b*d^2*x*tan(1/2*a)^3*tan(b*c/d)^2 - 8
*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 - 8*b^2*c*d*
x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 + 16*b^2*d^2*x^2*ima
g_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a) - 16*b^2*d^2*x^2*imag_part
(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a) + 32*b^2*d^2*x^2*sin_integral(2
*(b*d*x + b*c)/d)*tan(1/2*a) + 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(1/2*a) - 16*b^2*c^2*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(b*x)^2*tan(1/2*a) + 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d
)*tan(b*x)^2*tan(1/2*a) + 48*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(1/2*a)^2 + 48*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(1/2*a)^2 - 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*
a)^3 + 16*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3 -
32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3 + 16*b*c*d*tan(b*x)
^2*tan(1/2*a)^3 + 8*b*c*d*tan(b*x)*tan(1/2*a)^4 + d^2*tan(b*x)^2*tan(1/2*a)
^4 - 8*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 8*
b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) - 16*b^2*d
^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) - 8*b^2*c^2*imag_part(cos
_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 8*b^2*c^2*imag_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 16*b^2*c^2*sin_integra
l(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) - 64*b^2*c*d*x*real_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d) - 64*b^2*c*d*x*real_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d) + 48*b^2*c^2*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) - 48*b^2*c^2*imag_part
(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) + 96*b^2*c^2*sin_i
ntegral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(b*c/d) + 8*b^2*c*d*x*real_part(
cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 16*b^2*c^2*imag_part(cos_integral(2
*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 + 16*b^2*c^2*imag_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 - 32*b^2*c^2*sin_integral(2*(b
*d*x + b*c)/d)*tan(1/2*a)*tan(b*c/d)^2 - 16*b*c*d*tan(b*x)^2*tan(1/2*a)*tan
(b*c/d)^2 - 48*b*c*d*tan(b*x)*tan(1/2*a)^2*tan(b*c/d)^2 - 14*d^2*tan(b*x)^2
*tan(1/2*a)^2*tan(b*c/d)^2 - 16*b*c*d*tan(1/2*a)^3*tan(b*c/d)^2 - 16*d^2*ta
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n(b*x)*tan(1/2*a)^3*tan(b*c/d)^2 - 3*d^2*tan(1/2*a)^4*tan(b*c/d)^2 - 4*b^2*
d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d)) - 4*b^2*d^2*x^2*real_part(
cos_integral(-2*b*x - 2*b*c/d)) - 4*b^2*c^2*real_part(cos_integral(2*b*x +
2*b*c/d))*tan(b*x)^2 - 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(b*x)^2 + 32*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*
a) - 32*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a) + 64
*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a) - 16*b*d^2*x*tan(b*x)
^2*tan(1/2*a) + 24*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2
*a)^2 + 24*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2 -
 48*b*d^2*x*tan(b*x)*tan(1/2*a)^2 - 16*b*d^2*x*tan(1/2*a)^3 - 16*b^2*c*d*x*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 16*b^2*c*d*x*imag_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) - 32*b^2*c*d*x*sin_integral(2*
(b*d*x + b*c)/d)*tan(b*c/d) - 32*b^2*c^2*real_part(cos_integral(2*b*x + 2*b
*c/d))*tan(1/2*a)*tan(b*c/d) - 32*b^2*c^2*real_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(1/2*a)*tan(b*c/d) + 4*b^2*c^2*real_part(cos_integral(2*b*x + 2
*b*c/d))*tan(b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(b*c/d)^2 + 8*b*d^2*x*tan(b*x)*tan(b*c/d)^2 + 16*b*d^2*x*tan(1/2*a)*tan
(b*c/d)^2 - 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d)) - 8*b^2*c*
d*x*real_part(cos_integral(-2*b*x - 2*b*c/d)) + 16*b^2*c^2*imag_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(1/2*a) - 16*b^2*c^2*imag_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(1/2*a) + 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan
(1/2*a) - 16*b*c*d*tan(b*x)^2*tan(1/2*a) - 48*b*c*d*tan(b*x)*tan(1/2*a)^2 -
 14*d^2*tan(b*x)^2*tan(1/2*a)^2 - 16*b*c*d*tan(1/2*a)^3 - 16*d^2*tan(b*x)*t
an(1/2*a)^3 - 3*d^2*tan(1/2*a)^4 - 8*b^2*c^2*imag_part(cos_integral(2*b*x +
 2*b*c/d))*tan(b*c/d) + 8*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(b*c/d) - 16*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 8*b*c
*d*tan(b*x)*tan(b*c/d)^2 + d^2*tan(b*x)^2*tan(b*c/d)^2 + 16*b*c*d*tan(1/2*a
)*tan(b*c/d)^2 + 16*d^2*tan(b*x)*tan(1/2*a)*tan(b*c/d)^2 + 10*d^2*tan(1/2*a
)^2*tan(b*c/d)^2 - 4*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d)) - 4*b
^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d)) + 8*b*d^2*x*tan(b*x) + 16*
b*d^2*x*tan(1/2*a) + 8*b*c*d*tan(b*x) + d^2*tan(b*x)^2 + 16*b*c*d*tan(1/2*a
) + 16*d^2*tan(b*x)*tan(1/2*a) + 10*d^2*tan(1/2*a)^2 - 3*d^2*tan(b*c/d)^2 -
 3*d^2)/(d^5*x^2*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^
2*tan(1/2*a)^4*tan(b*c/d)^2 + d^5*x^2*tan(b*x)^2*tan(1/2*a)^4 + 2*d^5*x^2*t
an(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 + d^5*x^2*tan(1/2*a)^4*tan(b*c/d)^2 + c
^2*d^3*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2*tan(1/2*
a)^4 + 4*c*d^4*x*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(1/2*a
)^4*tan(b*c/d)^2 + 2*d^5*x^2*tan(b*x)^2*tan(1/2*a)^2 + d^5*x^2*tan(1/2*a)^4
 + c^2*d^3*tan(b*x)^2*tan(1/2*a)^4 + d^5*x^2*tan(b*x)^2*tan(b*c/d)^2 + 2*d^
5*x^2*tan(1/2*a)^2*tan(b*c/d)^2 + 2*c^2*d^3*tan(b*x)^2*tan(1/2*a)^2*tan(b*c
/d)^2 + c^2*d^3*tan(1/2*a)^4*tan(b*c/d)^2 + 4*c*d^4*x*tan(b*x)^2*tan(1/2*a)
^2 + 2*c*d^4*x*tan(1/2*a)^4 + 2*c*d^4*x*tan(b*x)^2*tan(b*c/d)^2 + 4*c*d^4*x
*tan(1/2*a)^2*tan(b*c/d)^2 + d^5*x^2*tan(b*x)^2 + 2*d^5*x^2*tan(1/2*a)^2 +
2*c^2*d^3*tan(b*x)^2*tan(1/2*a)^2 + c^2*d^3*tan(1/2*a)^4 + d^5*x^2*tan(b*c/
d)^2 + c^2*d^3*tan(b*x)^2*tan(b*c/d)^2 + 2*c^2*d^3*tan(1/2*a)^2*tan(b*c/d)^
2 + 2*c*d^4*x*tan(b*x)^2 + 4*c*d^4*x*tan(1/2*a)^2 + 2*c*d^4*x*tan(b*c/d)^2
+ d^5*x^2 + c^2*d^3*tan(b*x)^2 + 2*c^2*d^3*tan(1/2*a)^2 + c^2*d^3*tan(b*c/d
)^2 + 2*c*d^4*x + c^2*d^3)
```

**maple [A]** time = 0.04, size = 207, normalized size = 1.52

$$\frac{1}{2d\left(dx+c\right)^{2}}+\frac{b^{3}\left(-\frac{\cos(2bx+2a)}{((bx+a)d-da+cb)^{2}d}-\frac{-\frac{2\sin(2bx+2a)}{((bx+a)d-da+cb)d}+\frac{\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}}{d}}{d}\right)}{b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

```
[Out] 1/2/d/(d*x+c)^2+4/b*(1/4*b^3*(-cos(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d-(-2*s
in(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2
*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d
)-1/4*b^3/((b*x+a)*d-d*a+c*b)^2/d)
```

**maxima [C]** time = 0.43, size = 130, normalized size = 0.96

$$-\frac{2\left(E_3\left(\frac{2i\,bdx+2i\,bc}{d}\right)+E_3\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-\left(2i\,E_3\left(\frac{2i\,bdx+2i\,bc}{d}\right)-2i\,E_3\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(exp_integral_e(3, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(3, -(2
*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (2*I*exp_integral_e(3, (2*I
*b*d*x + 2*I*b*c)/d) - 2*I*exp_integral_e(3, -(2*I*b*d*x + 2*I*b*c)/d))*sin
(-2*(b*c - a*d)/d) + 1)/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^3),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^3), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

# 3.375 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx$

**Optimal**. Leaf size=205

$$\frac{8b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{8b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)}+\frac{2b^2\cos^2(a+bx)}{d^3(c+dx)}+\frac{4b\,\text{s}}{}$$

```
[Out] -2/3*b^2/d^3/(d*x+c)-cos(b*x+a)^2/d/(d*x+c)^3+2*b^2*cos(b*x+a)^2/d^3/(d*x+c
)+8/3*b^3*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^4+8/3*b^3*Ci(2*b*c/d+2*b*x)*
sin(2*a-2*b*c/d)/d^4+4/3*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^2+1/3*sin(b*x+
a)^2/d/(d*x+c)^3-2/3*b^2*sin(b*x+a)^2/d^3/(d*x+c)
```

**Rubi** [A] time = 0.38, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4431, 3314, 32, 3313, 12, 3303, 3299, 3302}

$$\frac{8b^3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{8b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)}+\frac{2b^2\cos^2(a+}{d^3(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^4,x]
```

```
[Out] (-2*b^2)/(3*d^3*(c + d*x)) - Cos[a + b*x]^2/(d*(c + d*x)^3) + (2*b^2*Cos[a
+ b*x]^2)/(d^3*(c + d*x)) + (8*b^3*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a -
 (2*b*c)/d])/(3*d^4) + (4*b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*(c + d*x)^2)
+ Sin[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Sin[a + b*x]^2)/(3*d^3*(c + d*x
)) + (8*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx = \int \left(\frac{3\cos^2(a+bx)}{(c+dx)^4} - \frac{\sin^2(a+bx)}{(c+dx)^4}\right)dx$$
$$= 3\int \frac{\cos^2(a+bx)}{(c+dx)^4}\,dx - \int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx$$
$$= -\frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} + \frac{\sin^2(a+bx)}{3d(c+dx)^3} - \frac{b^2\int \frac{1}{(c+dx)^2}\,dx}{3d^2} +$$
$$= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2}$$
$$= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2}$$
$$= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2}$$
$$= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{8b^3\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a\right.}{3d^4}$$

**Mathematica [A]** time = 1.14, size = 125, normalized size = 0.61

$$\frac{8b^3\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2b(c+dx)}{d}\right)+8b^3\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)+\frac{d\left(\cos(2(a+bx))\left(4b^2(c+dx)^2-2d^2\right)+d(2b(c+dx)\sin(2(a+bx))-d\right.}{(c+dx)^3}}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^4,x]
```

```
[Out] (8*b^3*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*((-2*d^2 +
4*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + d*(-d + 2*b*(c + d*x)*Sin[2*(a + b*x)
])))/(c + d*x)^3 + 8*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d
])/(3*d^4)
```

**fricas [A]** time = 0.49, size = 343, normalized size = 1.67

$$-\frac{4\,b^2d^3x^2+8\,b^2cd^2x+4\,b^2c^2d-d^3-4\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3\right)\cos\left(bx+a\right)^2-4\left(bd^3x+bcd^2\right)c\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(4*b^2*d^3*x^2 + 8*b^2*c*d^2*x + 4*b^2*c^2*d - d^3 - 4*(2*b^2*d^3*x^2
+ 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 - 4*(b*d^3*x + b*c*d^2)
*cos(b*x + a)*sin(b*x + a) - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d
*x + b^3*c^3)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 4*((b
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(2*(b*d
*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*co
s_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x
^2 + 3*c^2*d^5*x + c^3*d^4)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.04, size = 243, normalized size = 1.19

$$\frac{1}{3d\left(dx+c\right)^3}+\frac{b^4\left(-\frac{2\cos(2bx+2a)}{3((bx+a)d-da+cb)^3d}-\frac{2\left(-\frac{\sin(2bx+2a)}{((bx+a)d-da+cb)^2d}+\frac{-\frac{2\cos(2bx+2a)}{((bx+a)d-da+cb)d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2d\cdots}{}\right)\cdots}{}\right)}{d}}{d}\right)}{3d}\cdots\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x)
```

```
[Out] 1/3/d/(d*x+c)^3+4/b*(1/4*b^4*(-2/3*cos(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^3/d-2
/3*(-sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a)*d-d
*a+c*b)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*
x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)/d)-1/6*b^4/((b*x+a)*d-d*
a+c*b)^3/d)
```

**maxima [C]** time = 0.45, size = 141, normalized size = 0.69

$$-\frac{3\left(E_4\left(\frac{2i\,bdx+2i\,bc}{d}\right)+E_4\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-\left(3i\,E_4\left(\frac{2i\,bdx+2i\,bc}{d}\right)-3i\,E_4\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\cdots\right)\cdots}{3\left(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(exp_integral_e(4, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(4, -(2
*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (3*I*exp_integral_e(4, (2*I
*b*d*x + 2*I*b*c)/d) - 3*I*exp_integral_e(4, -(2*I*b*d*x + 2*I*b*c)/d))*sin
(-2*(b*c - a*d)/d) + 1)/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^4),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^4), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**4,x)
```

```
[Out] Timed out
```

# 3.376 $\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx$

**Optimal**. Leaf size=255

$$-\frac{18id^3\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{18id^3\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}+\frac{24d^3\sin(a+bx)}{b^4}-\frac{18d^2(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{18d^2(c+dx)\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}$$

```
[Out] -6*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b-24*d^2*(d*x+c)*cos(b*x+a)/b^3+4*(d*x
+c)^3*cos(b*x+a)/b+9*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-9*I*d*(d*
x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2-18*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a
)))/b^3+18*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-18*I*d^3*polylog(4,-ex
p(I*(b*x+a)))/b^4+18*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+24*d^3*sin(b*x+a)/
b^4-12*d*(d*x+c)^2*sin(b*x+a)/b^2
```

**Rubi** [A] time = 0.35, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4431, 4408, 3296, 2637, 4183, 2531, 6609, 2282, 6589}

$$-\frac{18d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{18d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{9id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (-6*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (24*d^2*(c + d*x)*Cos[a + b*x
])/b^3 + (4*(c + d*x)^3*Cos[a + b*x])/b + ((9*I)*d*(c + d*x)^2*PolyLog[2, -
E^(I*(a + b*x))])/b^2 - ((9*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b
^2 - (18*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (18*d^2*(c + d*x
)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((18*I)*d^3*PolyLog[4, -E^(I*(a + b*x)
)])/b^4 + ((18*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^3*Sin[a + b*
x])/b^4 - (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^3\cos(a+bx)\cot(a+bx)-(c+dx)^3\sin(a+bx)\right)dx$$

$$= 3\int (c+dx)^3\cos(a+bx)\cot(a+bx)\,dx - \int (c+dx)^3\sin(a+bx)\,dx$$

$$= \frac{(c+dx)^3\cos(a+bx)}{b} + 3\int (c+dx)^3\csc(a+bx)\,dx - 3\int (c+dx)^3$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)^3\cos(a+bx)}{b} - \frac{3d(c+dx)^2}{b}$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(c+dx)^3}{}$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(c+dx)}{}$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(c+dx)}{}$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(c+dx)}{}$$

**Mathematica [A]** time = 1.54, size = 459, normalized size = 1.80

$$\frac{4\cos(bx)\left(b^3c^3\cos(a)+3b^3c^2dx\cos(a)+3b^3cd^2x^2\cos(a)+b^3d^3x^3\cos(a)-3b^2c^2d\sin(a)-6b^2cd^2x\sin(a)-\right.}{b^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (3*(-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] + (3*I)*d*(b^
2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] + (2*I)*b*d*(c + d
*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*PolyLog[4, -Cos[a +
b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x]
+ I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a + b*x] + I*Sin[a +
 b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]])))/b^4 + (4*Cos[b*
x]*(b^3*c^3*Cos[a] - 6*b*c*d^2*Cos[a] + 3*b^3*c^2*d*x*Cos[a] - 6*b*d^3*x*Co
s[a] + 3*b^3*c*d^2*x^2*Cos[a] + b^3*d^3*x^3*Cos[a] - 3*b^2*c^2*d*Sin[a] + 6
*d^3*Sin[a] - 6*b^2*c*d^2*x*Sin[a] - 3*b^2*d^3*x^2*Sin[a]))/b^4 - (4*(3*b^2
*c^2*d*Cos[a] - 6*d^3*Cos[a] + 6*b^2*c*d^2*x*Cos[a] + 3*b^2*d^3*x^2*Cos[a]
+ b^3*c^3*Sin[a] - 6*b*c*d^2*Sin[a] + 3*b^3*c^2*d*x*Sin[a] - 6*b*d^3*x*Sin[
a] + 3*b^3*c*d^2*x^2*Sin[a] + b^3*d^3*x^3*Sin[a])*Sin[b*x])/b^4
```

**fricas [C]** time = 0.56, size = 925, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(18*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 18*I*d^3*polylog(
4, cos(b*x + a) - I*sin(b*x + a)) + 18*I*d^3*polylog(4, -cos(b*x + a) + I*s
in(b*x + a)) - 18*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 8*(b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*
x)*cos(b*x + a) + (-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d)*dil
og(cos(b*x + a) + I*sin(b*x + a)) + (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9
*I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-9*I*b^2*d^3*x^2 - 18
*I*b^2*c*d^2*x - 9*I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (9*
I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*s
in(b*x + a)) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*
log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 3*(b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I
*sin(b*x + a) + 1/2) + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*(b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(
-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) -
I*sin(b*x + a) + 1) + 18*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*si
n(b*x + a)) + 18*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x +
a)) - 18*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 1
8*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 24*(b^2*
d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a))/b^4
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \csc(bx+a)^2 \sin(3\,bx+3\,a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

**maple [B]** time = 0.12, size = 849, normalized size = 3.33

$$-\frac{18icd^2\,\mathrm{polylog}\left(2,\mathrm{e}^{i(bx+a)}\right)x}{b^2}+\frac{18icd^2\,\mathrm{polylog}\left(2,-\mathrm{e}^{i(bx+a)}\right)x}{b^2}+\frac{3d^3\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x^3}{b}+\frac{3d^3\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a^3}{b^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] -18*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4+18*I/b^2*c*d^2*polylog(2,-exp(I*(b
*x+a)))*x-18*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+18*I*d^3*polylog(4,exp
(I*(b*x+a)))/b^4-18/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+18/b^3*c*d^2*polyl
og(3,exp(I*(b*x+a)))+18/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-18/b^3*d^3*poly
log(3,-exp(I*(b*x+a)))*x+2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c
^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^
3)/b^4*exp(I*(b*x+a))+2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-
3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/
b^4*exp(-I*(b*x+a))-6/b*c^3*arctanh(exp(I*(b*x+a)))+6/b^4*d^3*a^3*arctanh(e
xp(I*(b*x+a)))-9/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a+9/b^3*c*d^2*a^2*ln(exp(I*
(b*x+a))+1)-3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^3-18/b^3*c*d^2*a^2*arctanh(exp
(I*(b*x+a)))+18/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))-9/b*c^2*d*ln(exp(I*(b*x
+a))+1)*x+9/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+9/b^2*c^2*d*ln(1-exp(I*(b*x+a)))
*a-9/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+9/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-
9/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+3/b^4*d
^3*ln(1-exp(I*(b*x+a)))*a^3-3/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-9*I/b^2*d^3*po
lylog(2,exp(I*(b*x+a)))*x^2+9*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-9*I/
b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+9*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a))
)
```

**maxima [B]** time = 0.54, size = 602, normalized size = 2.36

$$\frac{c^3\left(8\,\cos\left(bx+a\right)-3\,\log\left(\cos\left(bx\right)^2+2\,\cos\left(bx\right)\cos(a)+\cos(a)^2+\sin\left(bx\right)^2-2\,\sin\left(bx\right)\sin(a)+\sin(a)^2\right)+3\,\mathrm{l}\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 +
 sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)
*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b - 1/2*(3
6*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 36*I*d^3*polylog(4, e^(I*b*x + I*a))
 + (6*I*b^3*d^3*x^3 + 18*I*b^3*c*d^2*x^2 + 18*I*b^3*c^2*d*x)*arctan2(sin(b*
x + a), cos(b*x + a) + 1) + (6*I*b^3*d^3*x^3 + 18*I*b^3*c*d^2*x^2 + 18*I*b^
3*c^2*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 8*(b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a) + (-18*I*
b^2*d^3*x^2 - 36*I*b^2*c*d^2*x - 18*I*b^2*c^2*d)*dilog(-e^(I*b*x + I*a)) +
(18*I*b^2*d^3*x^2 + 36*I*b^2*c*d^2*x + 18*I*b^2*c^2*d)*dilog(e^(I*b*x + I*a
)) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x)*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*b^3*c^2*d*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) +
36*(b*d^3*x + b*c*d^2)*polylog(3, -e^(I*b*x + I*a)) - 36*(b*d^3*x + b*c*d^2
)*polylog(3, e^(I*b*x + I*a)) + 24*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d
 - 2*d^3)*sin(b*x + a))/b^4
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^3)/sin(a + b*x)^2,x)
```

```
[Out] \text{Hanged}
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

# 3.377 $\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx$

**Optimal**. Leaf size=172

$$-\frac{6d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{8d^2\cos(a+bx)}{b^3}+\frac{6id(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{6id(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{8d(c}{}$$

```
[Out] -6*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-8*d^2*cos(b*x+a)/b^3+4*(d*x+c)^2*cos
(b*x+a)/b+6*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-6*I*d*(d*x+c)*polylo
g(2,exp(I*(b*x+a)))/b^2-6*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+6*d^2*polylog(
3,exp(I*(b*x+a)))/b^3-8*d*(d*x+c)*sin(b*x+a)/b^2
```

**Rubi [A]** time = 0.23, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4431, 4408, 3296, 2638, 4183, 2531, 2282, 6589}

$$\frac{6id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{6id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{6d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2\text{PolyLog}\left(3,\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (-6*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (8*d^2*Cos[a + b*x])/b^3 + (4
*(c + d*x)^2*Cos[a + b*x])/b + ((6*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x
))])/b^2 - ((6*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*d^2*Pol
yLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 -
(8*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\csc^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^2\cos(a+bx)\cot(a+bx)-(c+dx)^2\sin(a+bx)\right)dx$$

$$= 3\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx-\int (c+dx)^2\sin(a+bx)$$

$$= \frac{(c+dx)^2\cos(a+bx)}{b}+3\int (c+dx)^2\csc(a+bx)\,dx-3\int (c+dx$$

$$= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{4(c+dx)^2\cos(a+bx)}{b}-\frac{2d(c+d}{}$$

$$= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{2d^2\cos(a+bx)}{b^3}+\frac{4(c+dx)^2\cos(}{b}$$

$$= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{8d^2\cos(a+bx)}{b^3}+\frac{4(c+dx)^2\cos(}{b}$$

$$= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{8d^2\cos(a+bx)}{b^3}+\frac{4(c+dx)^2\cos(}{b}$$

**Mathematica [A]** time = 1.10, size = 223, normalized size = 1.30

$$\frac{4\cos(bx)\left(\cos(a)\left(b^2(c+dx)^2-2d^2\right)-2bd\sin(a)(c+dx)\right)-4\sin(bx)\left(\sin(a)\left(b^2(c+dx)^2-2d^2\right)+2bd\cos(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (3*b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))] - 3*b^2*(c + d*x)^2*Log[1 + E^(
I*(a + b*x))] + (6*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (6*I)*b*
d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] - 6*d^2*PolyLog[3, -E^(I*(a + b*x))
] + 6*d^2*PolyLog[3, E^(I*(a + b*x))] + 4*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)
```

```
^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - 4*(2*b*d*(c + d*x)*Cos[a] + (-2*d^2
+ b^2*(c + d*x)^2)*Sin[a])*Sin[b*x])/b^3
```

**fricas [C]** time = 0.54, size = 562, normalized size = 3.27

$$\frac{6\,d^2\text{polylog}\left(3,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+6\,d^2\text{polylog}\left(3,\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)-6\,d^2\text{polylog}\left(3,-c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 6*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a)) - 6*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 8*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) + (-6*I*b*d^2*x - 6*I*b*c*d)*dil
og(cos(b*x + a) + I*sin(b*x + a)) + (6*I*b*d^2*x + 6*I*b*c*d)*dilog(cos(b*x
 + a) - I*sin(b*x + a)) + (-6*I*b*d^2*x - 6*I*b*c*d)*dilog(-cos(b*x + a) +
I*sin(b*x + a)) + (6*I*b*d^2*x + 6*I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x
 + a)) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b
*x + a) + 1) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I
*sin(b*x + a) + 1) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*
cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2
*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(b^2*d^2*x^
2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) +
 1) - 16*(b*d^2*x + b*c*d)*sin(b*x + a))/b^3
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc\left(bx+a\right)^2 \sin\left(3\,bx+3\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

**maple [B]** time = 0.20, size = 481, normalized size = 2.80

$$\frac{2\left(d^2x^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)\mathrm{e}^{i(bx+a)}}{b^3}+\frac{2\left(d^2x^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2\right)\mathrm{e}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] 2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I*(
b*x+a))+2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/b^3
*exp(-I*(b*x+a))+12/b^2*c*d*a*arctanh(exp(I*(b*x+a)))-6*I/b^2*c*d*polylog(2
,exp(I*(b*x+a)))-6*I/b^2*polylog(2,exp(I*(b*x+a)))*d^2*x-6/b*c^2*arctanh(ex
p(I*(b*x+a)))-6/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))-3/b*d^2*ln(exp(I*(b*x+a
))+1)*x^2+3/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2+6*I/b^2*c*d*polylog(2,-exp(I*(
b*x+a)))+3/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^
2+6*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+6/b*c*d*ln(1-exp(I*(b*x+a)))*x+6
/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-6/b*c*d*ln(exp(I*(b*x+a))+1)*x-6/b^2*c*d*ln
(exp(I*(b*x+a))+1)*a-6*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+6*d^2*polylog(3,e
xp(I*(b*x+a)))/b^3
```

**maxima [B]** time = 0.49, size = 409, normalized size = 2.38

$$\frac{c^2\left(8\,\cos\left(bx+a\right)-3\,\log\left(\cos\left(bx\right)^2+2\,\cos\left(bx\right)\cos(a)+\cos(a)^2+\sin\left(bx\right)^2-2\,\sin\left(bx\right)\sin(a)+\sin(a)^2\right)+3\,\mathrm{l}\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 +
 sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)
*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b - 1/2*(1
2*d^2*polylog(3, -e^(I*b*x + I*a)) - 12*d^2*polylog(3, e^(I*b*x + I*a)) + (
6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) +
 (6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1
) - 8*(b^2*d^2*x^2 + 2*b^2*c*d*x - 2*d^2)*cos(b*x + a) + (-12*I*b*d^2*x - 1
2*I*b*c*d)*dilog(-e^(I*b*x + I*a)) + (12*I*b*d^2*x + 12*I*b*c*d)*dilog(e^(I
*b*x + I*a)) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x)*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 + 2*cos(b*x + a) + 1) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x)*log(cos(b*x + a
)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 16*(b*d^2*x + b*c*d)*sin(b*x +
 a))/b^3
```

**mupad** **[F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^2)/sin(a + b*x)^2,x)
```

```
[Out] \text{Hanged}
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

# 3.378 $\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx$

**Optimal**. Leaf size=95

$$\frac{3id\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{3id\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{4d\sin(a+bx)}{b^2}+\frac{4(c+dx)\cos(a+bx)}{b}-\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

```
[Out] -6*(d*x+c)*arctanh(exp(I*(b*x+a)))/b+4*(d*x+c)*cos(b*x+a)/b+3*I*d*polylog(2
,-exp(I*(b*x+a)))/b^2-3*I*d*polylog(2,exp(I*(b*x+a)))/b^2-4*d*sin(b*x+a)/b^
2
```

**Rubi [A]** time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4431, 4408, 3296, 2637, 4183, 2279, 2391}

$$\frac{3id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{4d\sin(a+bx)}{b^2}+\frac{4(c+dx)\cos(a+bx)}{b}-\frac{6(c+dx)\tanh^{-1}\left(e^{i(\cdot}\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (-6*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (4*(c + d*x)*Cos[a + b*x])/b +
((3*I)*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d*PolyLog[2, E^(I*(a +
b*x))])/b^2 - (4*d*Sin[a + b*x])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
```

```
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx = \int (3(c+dx)\cos(a+bx)\cot(a+bx)-(c+dx)\sin(a+bx))\,dx$$
$$= 3\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx - \int (c+dx)\sin(a+bx)\,dx$$
$$= \frac{(c+dx)\cos(a+bx)}{b} + 3\int (c+dx)\csc(a+bx)\,dx - 3\int (c+dx)\,\text{si}$$
$$= -\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)\cos(a+bx)}{b} - \frac{d\sin(a+bx}{b^2}$$
$$= -\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)\cos(a+bx)}{b} - \frac{4d\sin(a+b}{b^2}$$
$$= -\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)\cos(a+bx)}{b} + \frac{3id\text{Li}_2\left(-e^{i(}\right.}{b^2}$$

**Mathematica [A]** time = 0.30, size = 171, normalized size = 1.80

$$\frac{3d\left(i\left(\text{Li}_2\left(-e^{i(a+bx)}\right)-\text{Li}_2\left(e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)\right)}{b^2}-\frac{4d\sin(a+bx)}{b^2}-\frac{3ad\,\text{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (4*c*Cos[a + b*x])/b + (4*d*x*Cos[a + b*x])/b - (3*c*Log[Cos[(a + b*x)/2]])
/b + (3*c*Log[Sin[(a + b*x)/2]])/b - (3*a*d*Log[Tan[(a + b*x)/2]])/b^2 + (3
*d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(Po
lyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])))/b^2 - (4*d*Sin[
a + b*x])/b^2
```

**fricas [B]** time = 0.48, size = 281, normalized size = 2.96

$$\frac{8(bdx+bc)\cos(bx+a) - 3id\text{Li}_2(\cos(bx+a)+i\sin(bx+a)) + 3id\text{Li}_2(\cos(bx+a)-i\sin(bx+a)) - 3id}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(8*(b*d*x + b*c)*cos(b*x + a) - 3*I*d*dilog(cos(b*x + a) + I*sin(b*x +
a)) + 3*I*d*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*I*d*dilog(-cos(b*x + a
) + I*sin(b*x + a)) + 3*I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*d*
x + b*c)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*d*x + b*c)*log(cos(b
*x + a) - I*sin(b*x + a) + 1) + 3*(b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I
```

```
*sin(b*x + a) + 1/2) + 3*(b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
+ a) + 1/2) + 3*(b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(
b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 8*d*sin(b*x + a))/b^
2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c) \csc (bx + a)^2 \sin (3\,bx + 3\,a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

**maple [B]** time = 0.07, size = 204, normalized size = 2.15

$$\frac{3c\ln(\csc(bx+a)-\cot(bx+a))}{b}+\frac{3d\ln\left(1-e^{i(bx+a)}\right)x}{b}-\frac{3d\ln\left(e^{i(bx+a)}+1\right)x}{b}-\frac{3da\ln(\csc(bx+a)-\cot(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] 3/b*c*ln(csc(b*x+a)-cot(b*x+a))+3/b*d*ln(1-exp(I*(b*x+a)))*x-3/b*d*ln(exp(I
*(b*x+a))+1)*x-3/b^2*d*a*ln(csc(b*x+a)-cot(b*x+a))+3*I*d/b^2*dilog(exp(I*(b
*x+a))+1)-3*I*d/b^2*dilog(1-exp(I*(b*x+a)))+3/b^2*d*ln(1-exp(I*(b*x+a)))*a-
3/b^2*d*ln(exp(I*(b*x+a))+1)*a+4/b*c*cos(b*x+a)+4/b*d*cos(b*x+a)*x-4*d*sin(
b*x+a)/b^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(8\,\cos(bx+a)-3\,\log\left(\cos(bx)^2+2\,\cos(bx)\cos(a)+\cos(a)^2+\sin(bx)^2-2\,\sin(bx)\sin(a)+\sin(a)^2\right)+3\,\mathrm{lo}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] 1/2*c*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + s
in(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)*c
os(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b + (4*b*x*c
os(b*x + a) + 3*b^2*integrate(x*sin(b*x + a)/(cos(b*x + a)^2 + sin(b*x + a)
^2 + 2*cos(b*x + a) + 1), x) + 3*b^2*integrate(x*sin(b*x + a)/(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1), x) - 4*sin(b*x + a))*d/b^2
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x))/sin(a + b*x)^2,x)
```

```
[Out] \text{Hanged}
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

## 3.379 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

**Optimal**. Leaf size=72

$$3\text{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right)-\frac{4\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}-\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] -4*cos(a-b*c/d)*Si(b*c/d+b*x)/d-4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d+3*Unintegrab
le(csc(b*x+a)/(d*x+c),x)
```

**Rubi** [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (-4*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d - (4*Cos[a - (b*c)/d]*Si
nIntegral[(b*c)/d + b*x])/d + 3*Defer[Int][Csc[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx &= \int \left(\frac{3\cos(a+bx)\cot(a+bx)}{c+dx}-\frac{\sin(a+bx)}{c+dx}\right)dx \\
&= 3\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx-\int \frac{\sin(a+bx)}{c+dx}\,dx \\
&= 3\int \frac{\csc(a+bx)}{c+dx}\,dx-3\int \frac{\sin(a+bx)}{c+dx}\,dx-\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,d \\
&= -\frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+3\int \frac{\csc(a+bx)}{c+dx}\,d \\
&= -\frac{4\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+3\int \frac{\csc(a+b}{c+dx}
\end{aligned}$$

**Mathematica** [A] time = 6.23, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]
```

**fricas** [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^2 \sin(3\,bx+3\,a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

**maple [A]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2(bx+a)\right)\sin(3bx+3a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2i\,E_1\left(\frac{i\,bdx+i\,bc}{d}\right)-2i\,E_1\left(-\frac{i\,bdx+i\,bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+3\,d\int\frac{\sin(bx+a)}{(dx+c)\left(\cos(bx+a)^2+\sin(bx+a)^2+2\,\cos(bx+a)+1\right)}\,dx+3\,d\int\frac{}{(dx+c)(c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
[Out] ((2*I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - 2*I*exp_integral_e(1, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 3*d*integrate(sin(b*x + a)/((d*x + c
)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x + a
) + c), x) + 3*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x +
c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + 2*(exp_integr
al_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin
(-(b*c - a*d)/d))/d
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,a+3\,b\,x)}{\sin(a+b\,x)^2\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)),x)
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c),x)
[Out] Timed out
```

# 3.380 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=92

$$3\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)-\frac{4b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4\sin(a+bx)}{d(c+dx)}$$

```
[Out] -4*b*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^2+4*b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2+4*si
n(b*x+a)/d/(d*x+c)+3*Unintegrable(csc(b*x+a)/(d*x+c)^2,x)
```

**Rubi [A]** time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (-4*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 + (4*Sin[a + b*x])/(
d*(c + d*x)) + (4*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + 3*De
fer[Int][Csc[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx &= \int \left(\frac{3\cos(a+bx)\cot(a+bx)}{(c+dx)^2}-\frac{\sin(a+bx)}{(c+dx)^2}\right)dx\\
&= 3\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx-\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}+3\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx-3\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx-\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}\\
&= \frac{4\sin(a+bx)}{d(c+dx)}+3\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx-\frac{(3b)\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}-\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right.}{}\\
&= -\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4\sin(a+bx)}{d(c+dx)}+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}\\
&= -\frac{4b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4\sin(a+bx)}{d(c+dx)}+\frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 6.71, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

**fricas** [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{d^{2}x^{2}+2\,cdx+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

**maple** [A] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{2}\left(bx+a\right)\right)\sin\left(3bx+3a\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2i\,E_{2}\left(\frac{i\,bdx+i\,bc}{d}\right)-2i\,E_{2}\left(-\frac{i\,bdx+i\,bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+3\left(d^{2}x+cd\right)\int\frac{\sin(bx+a)}{(dx+c)^{2}\left(\cos(bx+a)^{2}+\sin(bx+a)^{2}+2\,\cos(bx+a)+1\right)}\,dx+3}{d^{2}x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] ((2*I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - 2*I*exp_integral_e(2, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 3*(d^2*x + c*d)*integrate(sin(b*x +
a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2
 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*
x + a)), x) + 3*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2 + 2*c*d*x + (
d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(b*x
 + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) + 2*(exp_inte
gral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*s
in(-(b*c - a*d)/d))/(d^2*x + c*d)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{\sin\left(a+b\,x\right)^{2}\,\left(c+d\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^2),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

## 3.381 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=115

$$3\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^3},x\right)+\frac{2b^2\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^3}+\frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3}+\frac{2b\cos(a+bx)}{d^2(c+dx)}+\frac{2\sin(a+bx)}{d(c+dx)^2}$$

```
[Out] 2*b*cos(b*x+a)/d^2/(d*x+c)+2*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3+2*b^2*Ci(b*
c/d+b*x)*sin(a-b*c/d)/d^3+2*sin(b*x+a)/d/(d*x+c)^2+3*Unintegrable(csc(b*x+a
)/(d*x+c)^3,x)
```

**Rubi** [A] time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (2*b*Cos[a + b*x])/(d^2*(c + d*x)) + (2*b^2*CosIntegral[(b*c)/d + b*x]*Sin[
a - (b*c)/d])/d^3 + (2*Sin[a + b*x])/(d*(c + d*x)^2) + (2*b^2*Cos[a - (b*c)
/d]*SinIntegral[(b*c)/d + b*x])/d^3 + 3*Defer[Int][Csc[a + b*x]/(c + d*x)^3
, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx &= \int \left(\frac{3\cos(a+bx)\cot(a+bx)}{(c+dx)^3}-\frac{\sin(a+bx)}{(c+dx)^3}\right)dx\\
&= 3\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^3}\,dx-\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx\\
&= \frac{\sin(a+bx)}{2d(c+dx)^2}+3\int \frac{\csc(a+bx)}{(c+dx)^3}\,dx-3\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx-\frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{2d}\\
&= \frac{b\cos(a+bx)}{2d^2(c+dx)}+\frac{2\sin(a+bx)}{d(c+dx)^2}+3\int \frac{\csc(a+bx)}{(c+dx)^3}\,dx+\frac{b^2\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2}-\frac{(3b
\end{aligned}$$

$$\begin{aligned}
&= \frac{2b\cos(a+bx)}{d^2(c+dx)}+\frac{2\sin(a+bx)}{d(c+dx)^2}+3\int \frac{\csc(a+bx)}{(c+dx)^3}\,dx+\frac{\left(3b^2\right)\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2}+\\
&= \frac{2b\cos(a+bx)}{d^2(c+dx)}+\frac{b^2\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{2d^3}+\frac{2\sin(a+bx)}{d(c+dx)^2}+\frac{b^2\cos\left(a-\frac{bc}{d}\right.}{2
\\
&= \frac{2b\cos(a+bx)}{d^2(c+dx)}+\frac{2b^2\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3}+\frac{2\sin(a+bx)}{d(c+dx)^2}+\frac{2b^2\cos\left(a-\right.}{}
\end{aligned}$$

**Mathematica** [A] time = 7.14, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

**fricas** [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{d^{3}x^{3}+3\,cd^{2}x^{2}+3\,c^{2}dx+c^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

**maple** [A] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{2}\left(bx+a\right)\right)\sin\left(3bx+3a\right)}{\left(dx+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

```
[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{{\sin\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^3),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^3), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

# 3.382 $\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx$

**Optimal**. Leaf size=299

$$-\frac{3d^4\text{Li}_5\left(-e^{2i(a+bx)}\right)}{2b^5}+\frac{3d^4\sin^2(a+bx)}{b^5}+\frac{3id^3(c+dx)\text{Li}_4\left(-e^{2i(a+bx)}\right)}{b^4}-\frac{6d^3(c+dx)\sin(a+bx)\cos(a+bx)}{b^4}+\frac{3d^2(c\cdot}{}$$

```
[Out] 6*c*d^3*x/b^3+3*d^4*x^2/b^3-(d*x+c)^4/b-1/5*I*(d*x+c)^5/d+(d*x+c)^4*ln(1+ex
p(2*I*(b*x+a)))/b-2*I*d*(d*x+c)^3*polylog(2,-exp(2*I*(b*x+a)))/b^2+3*d^2*(d
*x+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+3*I*d^3*(d*x+c)*polylog(4,-exp(2*I
*(b*x+a)))/b^4-3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-6*d^3*(d*x+c)*cos(b
*x+a)*sin(b*x+a)/b^4+4*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+3*d^4*sin(b*x+
a)^2/b^5-6*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3+2*(d*x+c)^4*sin(b*x+a)^2/b
```

**Rubi [A]** time = 0.50, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {4431, 4404, 3311, 32, 3310, 4407, 3719, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-3$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (6*c*d^3*x)/b^3 + (3*d^4*x^2)/b^3 - (c + d*x)^4/b - ((I/5)*(c + d*x)^5)/d +
 ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c + d*x)^3*PolyLo
g[2, -E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(
a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^
4 - (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (6*d^3*(c + d*x)*Cos
[a + b*x]*Sin[a + b*x])/b^4 + (4*d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/b
^2 + (3*d^4*Sin[a + b*x]^2)/b^5 - (6*d^2*(c + d*x)^2*Sin[a + b*x]^2)/b^3 +
(2*(c + d*x)^4*Sin[a + b*x]^2)/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^4\cos(a+bx)\sin(a+bx) - (c+dx)^4\sin^2(a+bx)\tan(a+\right.$$

$$= 3\int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)^4\sin^2(a+bx)\tan$$

$$= \frac{3(c+dx)^4\sin^2(a+bx)}{2b} - \frac{(6d)\int (c+dx)^3\sin^2(a+bx)\,dx}{b} + \int (c+dx$$

$$= -\frac{i(c+dx)^5}{5d} + \frac{3d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2} - \frac{9d^2(c+dx)^2\sin}{2b^3}$$

$$= -\frac{3(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{9d^3(c+dx)}{}$$

$$= \frac{9cd^3x}{2b^3} + \frac{9d^4x^2}{4b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}$$

**Mathematica [B]** time = 6.75, size = 2482, normalized size = 8.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] ((I/2)*c^2*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I
)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
- (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E
^(I*a)) + (I/2)*c*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*
a))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*
PolyLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*
x))] - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] - (d^4
*((-4*I)*x^5 - (10*(1 + E^((2*I)*a))*x^4*Log[1 + E^((-2*I)*(a + b*x))])/b +
 (5*(1 + E^((2*I)*a))*((-4*I)*b^3*x^3*PolyLog[2, -E^((-2*I)*(a + b*x))] - 6
*b^2*x^2*PolyLog[3, -E^((-2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[4, -E^((-2*I
)*(a + b*x))] + 3*PolyLog[5, -E^((-2*I)*(a + b*x))]))/b^5)*Sec[a])/(20*E^(I
*a)) + (c^4*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin
[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (2*c^3*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Co
t[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x
)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + P
i*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyL
og[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*
Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(40*b^5) -
 ((I/40)*Sin[2*a + 2*b*x])/b^5)*(-20*b^4*c^4*Cos[a] + (40*I)*b^3*c^3*d*Cos[
```

```
a] + 60*b^2*c^2*d^2*Cos[a] - (60*I)*b*c*d^3*Cos[a] - 30*d^4*Cos[a] - 80*b^4
*c^3*d*x*Cos[a] + (120*I)*b^3*c^2*d^2*x*Cos[a] + 120*b^2*c*d^3*x*Cos[a] - (
60*I)*b*d^4*x*Cos[a] - 120*b^4*c^2*d^2*x^2*Cos[a] + (120*I)*b^3*c*d^3*x^2*C
os[a] + 60*b^2*d^4*x^2*Cos[a] - 80*b^4*c*d^3*x^3*Cos[a] + (40*I)*b^3*d^4*x^
3*Cos[a] - 20*b^4*d^4*x^4*Cos[a] - (20*I)*b^5*c^4*x*Cos[a + 2*b*x] - (40*I)
*b^5*c^3*d*x^2*Cos[a + 2*b*x] - (40*I)*b^5*c^2*d^2*x^3*Cos[a + 2*b*x] - (20
*I)*b^5*c*d^3*x^4*Cos[a + 2*b*x] - (4*I)*b^5*d^4*x^5*Cos[a + 2*b*x] + (20*I
)*b^5*c^4*x*Cos[3*a + 2*b*x] + (40*I)*b^5*c^3*d*x^2*Cos[3*a + 2*b*x] + (40*
I)*b^5*c^2*d^2*x^3*Cos[3*a + 2*b*x] + (20*I)*b^5*c*d^3*x^4*Cos[3*a + 2*b*x]
 + (4*I)*b^5*d^4*x^5*Cos[3*a + 2*b*x] - 10*b^4*c^4*Cos[3*a + 4*b*x] - (20*I
)*b^3*c^3*d*Cos[3*a + 4*b*x] + 30*b^2*c^2*d^2*Cos[3*a + 4*b*x] + (30*I)*b*c
*d^3*Cos[3*a + 4*b*x] - 15*d^4*Cos[3*a + 4*b*x] - 40*b^4*c^3*d*x*Cos[3*a +
4*b*x] - (60*I)*b^3*c^2*d^2*x*Cos[3*a + 4*b*x] + 60*b^2*c*d^3*x*Cos[3*a + 4
*b*x] + (30*I)*b*d^4*x*Cos[3*a + 4*b*x] - 60*b^4*c^2*d^2*x^2*Cos[3*a + 4*b*
x] - (60*I)*b^3*c*d^3*x^2*Cos[3*a + 4*b*x] + 30*b^2*d^4*x^2*Cos[3*a + 4*b*x
] - 40*b^4*c*d^3*x^3*Cos[3*a + 4*b*x] - (20*I)*b^3*d^4*x^3*Cos[3*a + 4*b*x]
 - 10*b^4*d^4*x^4*Cos[3*a + 4*b*x] - 10*b^4*c^4*Cos[5*a + 4*b*x] - (20*I)*b
^3*c^3*d*Cos[5*a + 4*b*x] + 30*b^2*c^2*d^2*Cos[5*a + 4*b*x] + (30*I)*b*c*d^
3*Cos[5*a + 4*b*x] - 15*d^4*Cos[5*a + 4*b*x] - 40*b^4*c^3*d*x*Cos[5*a + 4*b
*x] - (60*I)*b^3*c^2*d^2*x*Cos[5*a + 4*b*x] + 60*b^2*c*d^3*x*Cos[5*a + 4*b*
x] + (30*I)*b*d^4*x*Cos[5*a + 4*b*x] - 60*b^4*c^2*d^2*x^2*Cos[5*a + 4*b*x]
- (60*I)*b^3*c*d^3*x^2*Cos[5*a + 4*b*x] + 30*b^2*d^4*x^2*Cos[5*a + 4*b*x] -
 40*b^4*c*d^3*x^3*Cos[5*a + 4*b*x] - (20*I)*b^3*d^4*x^3*Cos[5*a + 4*b*x] -
10*b^4*d^4*x^4*Cos[5*a + 4*b*x] + 20*b^5*c^4*x*Sin[a + 2*b*x] + 40*b^5*c^3*
d*x^2*Sin[a + 2*b*x] + 40*b^5*c^2*d^2*x^3*Sin[a + 2*b*x] + 20*b^5*c*d^3*x^4
*Sin[a + 2*b*x] + 4*b^5*d^4*x^5*Sin[a + 2*b*x] - 20*b^5*c^4*x*Sin[3*a + 2*b
*x] - 40*b^5*c^3*d*x^2*Sin[3*a + 2*b*x] - 40*b^5*c^2*d^2*x^3*Sin[3*a + 2*b*
x] - 20*b^5*c*d^3*x^4*Sin[3*a + 2*b*x] - 4*b^5*d^4*x^5*Sin[3*a + 2*b*x] - (
10*I)*b^4*c^4*Sin[3*a + 4*b*x] + 20*b^3*c^3*d*Sin[3*a + 4*b*x] + (30*I)*b^2
*c^2*d^2*Sin[3*a + 4*b*x] - 30*b*c*d^3*Sin[3*a + 4*b*x] - (15*I)*d^4*Sin[3*
a + 4*b*x] - (40*I)*b^4*c^3*d*x*Sin[3*a + 4*b*x] + 60*b^3*c^2*d^2*x*Sin[3*a
 + 4*b*x] + (60*I)*b^2*c*d^3*x*Sin[3*a + 4*b*x] - 30*b*d^4*x*Sin[3*a + 4*b*
x] - (60*I)*b^4*c^2*d^2*x^2*Sin[3*a + 4*b*x] + 60*b^3*c*d^3*x^2*Sin[3*a + 4
*b*x] + (30*I)*b^2*d^4*x^2*Sin[3*a + 4*b*x] - (40*I)*b^4*c*d^3*x^3*Sin[3*a
+ 4*b*x] + 20*b^3*d^4*x^3*Sin[3*a + 4*b*x] - (10*I)*b^4*d^4*x^4*Sin[3*a + 4
*b*x] - (10*I)*b^4*c^4*Sin[5*a + 4*b*x] + 20*b^3*c^3*d*Sin[5*a + 4*b*x] + (
30*I)*b^2*c^2*d^2*Sin[5*a + 4*b*x] - 30*b*c*d^3*Sin[5*a + 4*b*x] - (15*I)*d
^4*Sin[5*a + 4*b*x] - (40*I)*b^4*c^3*d*x*Sin[5*a + 4*b*x] + 60*b^3*c^2*d^2*
x*Sin[5*a + 4*b*x] + (60*I)*b^2*c*d^3*x*Sin[5*a + 4*b*x] - 30*b*d^4*x*Sin[5
*a + 4*b*x] - (60*I)*b^4*c^2*d^2*x^2*Sin[5*a + 4*b*x] + 60*b^3*c*d^3*x^2*Si
n[5*a + 4*b*x] + (30*I)*b^2*d^4*x^2*Sin[5*a + 4*b*x] - (40*I)*b^4*c*d^3*x^3
*Sin[5*a + 4*b*x] + 20*b^3*d^4*x^3*Sin[5*a + 4*b*x] - (10*I)*b^4*d^4*x^4*Si
n[5*a + 4*b*x])
```

**fricas [C]** time = 0.64, size = 1644, normalized size = 5.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 - 24*d^4*polylog(5, I*cos(b*x + a) + s
in(b*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) - sin(b*x + a)) - 24*d^4*po
lylog(5, -I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x + a
) - sin(b*x + a)) + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - 2*(2*b^4*d^4*x^4 + 8*
b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*
d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(2*b^3*d^4*x
^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*
x)*cos(b*x + a)*sin(b*x + a) + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x + (4*I*b^3*d
^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(I*c
```

```
os(b*x + a) + sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I
*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-4*
I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*di
log(-I*cos(b*x + a) + sin(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2
 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)
) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*
b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x + a) +
I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4
*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^
2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4
*d^4)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x
^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2
+ 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^4*d
^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*
d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) - sin(
b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) - I*s
in(b*x + a) + I) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a)
 + sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a)
- sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a)
+ sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a)
 - sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3
, I*cos(b*x + a) + sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^
2*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*
c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) + 12*(b^2
*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*
x + a)))/b^5
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \sec(bx+a)\sin(3bx+3a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

**maple [B]** time = 0.13, size = 956, normalized size = 3.20

$$\frac{3c^2d^2\operatorname{polylog}\left(3,-e^{2i(bx+a)}\right)}{b^3}+\frac{3d^4\operatorname{polylog}\left(3,-e^{2i(bx+a)}\right)x^2}{b^3}+\frac{d^4\ln\left(1+e^{2i(bx+a)}\right)x^4}{b}+\frac{2id^4a^4x}{b^4}-\frac{4ic^3d\,a^2}{b^2}+\frac{8ic^2d^2a}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))+
8/5*I/b^5*d^4*a^5-I*c*d^3*x^4-2*I*c^2*d^2*x^3-2*I*c^3*d*x^2+3/b^3*c^2*d^2*p
olylog(3,-exp(2*I*(b*x+a)))+3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2+4/b*
c*d^3*ln(1+exp(2*I*(b*x+a)))*x^3+I*c^4*x+1/b*c^4*ln(1+exp(2*I*(b*x+a)))-1/4
*(2*d^4*x^4*b^4+4*I*b^3*d^4*x^3+8*b^4*c*d^3*x^3+12*I*b^3*c*d^3*x^2+12*b^4*c
^2*d^2*x^2+12*I*b^3*c^2*d^2*x+8*b^4*c^3*d*x+4*I*b^3*c^3*d+2*b^4*c^4-6*b^2*d
^4*x^2-6*I*b*d^4*x-12*b^2*c*d^3*x-6*I*b*c*d^3-6*c^2*d^2*b^2+3*d^4)/b^5*exp(
2*I*(b*x+a))-1/4*(2*d^4*x^4*b^4-4*I*b^3*d^4*x^3+8*b^4*c*d^3*x^3-12*I*b^3*c*
d^3*x^2+12*b^4*c^2*d^2*x^2-12*I*b^3*c^2*d^2*x+8*b^4*c^3*d*x-4*I*b^3*c^3*d+2
*b^4*c^4-6*b^2*d^4*x^2+6*I*b*d^4*x-12*b^2*c*d^3*x+6*I*b*c*d^3-6*c^2*d^2*b^2
```

```
+3*d^4)/b^5*exp(-2*I*(b*x+a))-2/b*c^4*ln(exp(I*(b*x+a)))-1/5*I*d^4*x^5+1/b*
d^4*ln(1+exp(2*I*(b*x+a)))*x^4+2*I/b^4*d^4*a^4*x-4*I/b^2*c^3*d*a^2+8*I/b^3*
c^2*d^2*a^3-6*I/b^4*c*d^3*a^4+8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))+8/b^4*c*d^3*
a^3*ln(exp(I*(b*x+a)))-12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))-8*I/b^3*c*d^3*
a^3*x+12*I/b^2*c^2*d^2*a^2*x-8*I/b*c^3*d*a*x+3*I/b^4*c*d^3*polylog(4,-exp(2
*I*(b*x+a)))+3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x-2*I/b^2*d^4*polylog
(2,-exp(2*I*(b*x+a)))*x^3-2*I/b^2*c^3*d*polylog(2,-exp(2*I*(b*x+a)))+4/b*c^
3*d*ln(1+exp(2*I*(b*x+a)))*x+6/b^3*c*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+6/b
*c^2*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-6*I/b^2*c^2*d^2*polylog(2,-exp(2*I*(b*x
+a)))*x-6*I/b^2*c*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2
```

**maxima [B]** time = 0.52, size = 607, normalized size = 2.03

$$-\frac{c^4\left(2\cos(2bx+2a)-\log\left(\cos(2bx)^2+2\cos(2bx)\cos(2a)+\cos(2a)^2+\sin(2bx)^2-2\sin(2bx)\sin(2\right.\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c^4*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + c
os(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/30*(-
6*I*b^5*d^4*x^5 - 30*I*b^5*c*d^3*x^4 - 60*I*b^5*c^2*d^2*x^3 - 60*I*b^5*c^3*
d*x^2 - 90*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) + (60*I*b^4*d^4*x^4 + 160*I
*b^4*c*d^3*x^3 + 180*I*b^4*c^2*d^2*x^2 + 120*I*b^4*c^3*d*x)*arctan2(sin(2*b
*x + 2*a), cos(2*b*x + 2*a) + 1) - 15*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 - 6*
b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*
b^2*c*d^3)*x)*cos(2*b*x + 2*a) + (-120*I*b^3*d^4*x^3 - 240*I*b^3*c*d^3*x^2
- 180*I*b^3*c^2*d^2*x - 60*I*b^3*c^3*d)*dilog(-e^(2*I*b*x + 2*I*a)) + 10*(3
*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 9*b^4*c^2*d^2*x^2 + 6*b^4*c^3*d*x)*log(cos
(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (180*I*b*d
^4*x + 120*I*b*c*d^3)*polylog(4, -e^(2*I*b*x + 2*I*a)) + 30*(6*b^2*d^4*x^2
+ 8*b^2*c*d^3*x + 3*b^2*c^2*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 30*(2*b
^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 -
 b*d^4)*x)*sin(2*b*x + 2*a))/b^5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(3a+3bx)\,(c+dx)^4}{\cos(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^4)/cos(a + b*x),x)
```

```
[Out] int((sin(3*a + 3*b*x)*(c + d*x)^4)/cos(a + b*x), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

## 3.383 $\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx$

**Optimal**. Leaf size=242

$$\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^3\sin(a+bx)\cos(a+bx)}{2b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{3d^2(c+dx)\sin^2(a+bx)}{b^3}-\frac{3id(c+\dots}{\dots}$$

```
[Out] 3/2*d^3*x/b^3-(d*x+c)^3/b-1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1+exp(2*I*(b*x+a))
)/b-3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c)*poly
log(3,-exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4-3/2
*d^3*cos(b*x+a)*sin(b*x+a)/b^4+3*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-3*d^
2*(d*x+c)*sin(b*x+a)^2/b^3+2*(d*x+c)^3*sin(b*x+a)^2/b
```

**Rubi [A]** time = 0.45, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {4431, 4404, 3311, 32, 2635, 8, 4407, 3719, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+d\dots}{\dots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (3*d^3*x)/(2*b^3) - (c + d*x)^3/b - ((I/4)*(c + d*x)^4)/d + ((c + d*x)^3*Lo
g[1 + E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*
I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2
*b^3) + (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d^3*Cos[a
 + b*x]*Sin[a + b*x])/(2*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])
/b^2 - (3*d^2*(c + d*x)*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^3*Sin[a + b*x]^2
)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^3\cos(a+bx)\sin(a+bx)-(c+dx)^3\sin^2(a+bx)\tan(a+\right.$$

$$= 3\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)^3\sin^2(a+bx)\tan$$

$$= \frac{3(c+dx)^3\sin^2(a+bx)}{2b} - \frac{(9d)\int (c+dx)^2\sin^2(a+bx)\,dx}{2b} + \int (c+dx$$

$$= -\frac{i(c+dx)^4}{4d} + \frac{9d(c+dx)^2\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{9d^2(c+dx)\sin^2(}{4b^3}$$

$$= -\frac{3(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{9d^3\cos(a+}{}$$

$$= \frac{9d^3x}{8b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3id(c+}{}$$

$$= \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3id(c+}{}$$

$$= \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3id(c+}{}$$

$$= \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3id(c+}{}$$

**Mathematica [B]** time = 6.64, size = 1719, normalized size = 7.10

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] ((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*
(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] -
(3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(
I*a)) + (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))*
x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*Poly
Log[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))]
 - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] + (c^3*Sec
[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a
]^2 + Sin[a]^2)) + (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a
]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - A
rcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]]
 - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*
(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2
*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(16*b^4) - ((I/16)*Sin[
2*a + 2*b*x])/b^4)*(-8*b^3*c^3*Cos[a] + (12*I)*b^2*c^2*d*Cos[a] + 12*b*c*d^
2*Cos[a] - (6*I)*d^3*Cos[a] - 24*b^3*c^2*d*x*Cos[a] + (24*I)*b^2*c*d^2*x*Co
s[a] + 12*b*d^3*x*Cos[a] - 24*b^3*c*d^2*x^2*Cos[a] + (12*I)*b^2*d^3*x^2*Cos
[a] - 8*b^3*d^3*x^3*Cos[a] - (8*I)*b^4*c^3*x*Cos[a + 2*b*x] - (12*I)*b^4*c^
2*d*x^2*Cos[a + 2*b*x] - (8*I)*b^4*c*d^2*x^3*Cos[a + 2*b*x] - (2*I)*b^4*d^3
```

```
*x^4*Cos[a + 2*b*x] + (8*I)*b^4*c^3*x*Cos[3*a + 2*b*x] + (12*I)*b^4*c^2*d*x
^2*Cos[3*a + 2*b*x] + (8*I)*b^4*c*d^2*x^3*Cos[3*a + 2*b*x] + (2*I)*b^4*d^3*
x^4*Cos[3*a + 2*b*x] - 4*b^3*c^3*Cos[3*a + 4*b*x] - (6*I)*b^2*c^2*d*Cos[3*a
 + 4*b*x] + 6*b*c*d^2*Cos[3*a + 4*b*x] + (3*I)*d^3*Cos[3*a + 4*b*x] - 12*b^
3*c^2*d*x*Cos[3*a + 4*b*x] - (12*I)*b^2*c*d^2*x*Cos[3*a + 4*b*x] + 6*b*d^3*
x*Cos[3*a + 4*b*x] - 12*b^3*c*d^2*x^2*Cos[3*a + 4*b*x] - (6*I)*b^2*d^3*x^2*
Cos[3*a + 4*b*x] - 4*b^3*d^3*x^3*Cos[3*a + 4*b*x] - 4*b^3*c^3*Cos[5*a + 4*b
*x] - (6*I)*b^2*c^2*d*Cos[5*a + 4*b*x] + 6*b*c*d^2*Cos[5*a + 4*b*x] + (3*I)
*d^3*Cos[5*a + 4*b*x] - 12*b^3*c^2*d*x*Cos[5*a + 4*b*x] - (12*I)*b^2*c*d^2*
x*Cos[5*a + 4*b*x] + 6*b*d^3*x*Cos[5*a + 4*b*x] - 12*b^3*c*d^2*x^2*Cos[5*a
+ 4*b*x] - (6*I)*b^2*d^3*x^2*Cos[5*a + 4*b*x] - 4*b^3*d^3*x^3*Cos[5*a + 4*b
*x] + 8*b^4*c^3*x*Sin[a + 2*b*x] + 12*b^4*c^2*d*x^2*Sin[a + 2*b*x] + 8*b^4*
c*d^2*x^3*Sin[a + 2*b*x] + 2*b^4*d^3*x^4*Sin[a + 2*b*x] - 8*b^4*c^3*x*Sin[3
*a + 2*b*x] - 12*b^4*c^2*d*x^2*Sin[3*a + 2*b*x] - 8*b^4*c*d^2*x^3*Sin[3*a +
 2*b*x] - 2*b^4*d^3*x^4*Sin[3*a + 2*b*x] - (4*I)*b^3*c^3*Sin[3*a + 4*b*x] +
 6*b^2*c^2*d*Sin[3*a + 4*b*x] + (6*I)*b*c*d^2*Sin[3*a + 4*b*x] - 3*d^3*Sin[
3*a + 4*b*x] - (12*I)*b^3*c^2*d*x*Sin[3*a + 4*b*x] + 12*b^2*c*d^2*x*Sin[3*a
 + 4*b*x] + (6*I)*b*d^3*x*Sin[3*a + 4*b*x] - (12*I)*b^3*c*d^2*x^2*Sin[3*a +
 4*b*x] + 6*b^2*d^3*x^2*Sin[3*a + 4*b*x] - (4*I)*b^3*d^3*x^3*Sin[3*a + 4*b*
x] - (4*I)*b^3*c^3*Sin[5*a + 4*b*x] + 6*b^2*c^2*d*Sin[5*a + 4*b*x] + (6*I)*
b*c*d^2*Sin[5*a + 4*b*x] - 3*d^3*Sin[5*a + 4*b*x] - (12*I)*b^3*c^2*d*x*Sin[
5*a + 4*b*x] + 12*b^2*c*d^2*x*Sin[5*a + 4*b*x] + (6*I)*b*d^3*x*Sin[5*a + 4*
b*x] - (12*I)*b^3*c*d^2*x^2*Sin[5*a + 4*b*x] + 6*b^2*d^3*x^2*Sin[5*a + 4*b*
x] - (4*I)*b^3*d^3*x^3*Sin[5*a + 4*b*x])
```

**fricas** [C] time = 0.60, size = 1122, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 6*I*d^3*polylog(4, I*cos(b*x + a) +
sin(b*x + a)) + 6*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 6*I*d^3
*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x
 + a) - sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*
b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2 + 4*
b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3*c^2*d
 - b*d^3)*x + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(I*c
os(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2
*c^2*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*
c*d^2*x - 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (3*I*b^2*d
^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x +
 a)) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a)
 + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a
) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) +
1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3
)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a)
+ I) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*
(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*(b*d^3*x
+ b*c*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^
2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)))/b^4
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sec(bx+a) \sin(3bx+3a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

**maple [B]** time = 0.12, size = 639, normalized size = 2.64

$$\frac{3d^3 \operatorname{polylog}\left(3, -e^{2i(bx+a)}\right) x}{2b^3} + \frac{d^3 \ln\left(1+e^{2i(bx+a)}\right) x^3}{b} + \frac{6ic\, d^2 a^2 x}{b^2} - \frac{6ic^2 dax}{b} + \frac{2d^3 a^3 \ln\left(e^{i(bx+a)}\right)}{b^4} - \frac{3ia^4 d^3}{2b^4} - ic\, d^2 x^3 - \frac{3i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] 6*I/b^2*c*d^2*a^2*x-6*I/b*c^2*d*a*x+3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/
b^4+I*c^3*x+1/b*c^3*ln(1+exp(2*I*(b*x+a)))+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))
-3/2*I/b^4*a^4*d^3-I*c*d^2*x^3-3/2*I*c^2*d*x^2-1/8*(4*d^3*x^3*b^3+6*I*b^2*d
^3*x^2+12*b^3*c*d^2*x^2+12*I*b^2*c*d^2*x+12*b^3*c^2*d*x+6*I*b^2*c^2*d+4*b^3
*c^3-6*b*d^3*x-3*I*d^3-6*c*d^2*b)/b^4*exp(2*I*(b*x+a))-1/8*(4*d^3*x^3*b^3-6
*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2-12*I*b^2*c*d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2
*d+4*b^3*c^3-6*b*d^3*x+3*I*d^3-6*c*d^2*b)/b^4*exp(-2*I*(b*x+a))+3/2/b^3*c*d
^2*polylog(3,-exp(2*I*(b*x+a)))+3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-
1/4*I*d^3*x^4-2/b*c^3*ln(exp(I*(b*x+a)))-3/2*I/b^2*d^3*polylog(2,-exp(2*I*(
b*x+a)))*x^2-3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-6/b^3*c*d^2*a^2*l
n(exp(I*(b*x+a)))-3*I/b^2*c^2*d*a^2-2*I/b^3*a^3*d^3*x+4*I/b^3*c*d^2*a^3+6/b
^2*c^2*d*a*ln(exp(I*(b*x+a)))+1/b*d^3*ln(1+exp(2*I*(b*x+a)))*x^3-3*I/b^2*po
lylog(2,-exp(2*I*(b*x+a)))*c*d^2*x+3/b*c^2*d*ln(1+exp(2*I*(b*x+a)))*x+3/b*c
*d^2*ln(1+exp(2*I*(b*x+a)))*x^2
```

**maxima [B]** time = 0.48, size = 442, normalized size = 1.83

$$-\frac{c^3\left(2\cos(2bx+2a) - \log\left(\cos(2bx)^2 + 2\cos(2bx)\cos(2a) + \cos(2a)^2 + \sin(2bx)^2 - 2\sin(2bx)\sin(2a)\right.\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c^3*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + c
os(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/12*(-
3*I*b^4*d^3*x^4 - 12*I*b^4*c*d^2*x^3 - 18*I*b^4*c^2*d*x^2 + 12*I*d^3*polylo
g(4, -e^(2*I*b*x + 2*I*a)) + (16*I*b^3*d^3*x^3 + 36*I*b^3*c*d^2*x^2 + 36*I*
b^3*c^2*d*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(2*b^3*d^3
*x^3 + 6*b^3*c*d^2*x^2 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(2*b*x +
 2*a) + (-24*I*b^2*d^3*x^2 - 36*I*b^2*c*d^2*x - 18*I*b^2*c^2*d)*dilog(-e^(2
*I*b*x + 2*I*a)) + 2*(4*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x)*log(
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 6*(4*b*
d^3*x + 3*b*c*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 9*(2*b^2*d^3*x^2 + 4*
b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a))/b^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(3a+3bx)\,(c+dx)^3}{\cos(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x),x)
```

```
[Out] int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

# 3.384 $\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx$

**Optimal**. Leaf size=173

$$\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{d^2\sin^2(a+bx)}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}+\frac{2d(c+dx)\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)^2\log(}{b}$$

```
[Out] -2*c*d*x/b-d^2*x^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b-I
*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,-exp(2*I*(b*x
+a)))/b^3+2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-d^2*sin(b*x+a)^2/b^3+2*(d*x
+c)^2*sin(b*x+a)^2/b
```

**Rubi [A]** time = 0.33, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4431, 4404, 3310, 4407, 3719, 2190, 2531, 2282, 6589}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{2d(c+dx)\sin(a+bx)\cos(a+bx)}{b^2}-\frac{d^2\sin^2(a+b}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (-2*c*d*x)/b - (d^2*x^2)/b - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 + E
^((2*I)*(a + b*x))])/b - (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b
^2 + (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (2*d*(c + d*x)*Cos[a
+ b*x]*Sin[a + b*x])/b^2 - (d^2*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^2*Sin[a
+ b*x]^2)/b
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^2\cos(a+bx)\sin(a+bx)-(c+dx)^2\sin^2(a+bx)\tan(\right.$$

$$= 3\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)^2\sin^2(a+bx)$$

$$= \frac{3(c+dx)^2\sin^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)\sin^2(a+bx)\,dx}{b} + \int (c+$$

$$= -\frac{i(c+dx)^3}{3d} + \frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2} - \frac{3d^2\sin^2(a+b}{4b^3}$$

$$= -\frac{3cdx}{2b} - \frac{3d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2d(c}{}$$

$$= -\frac{2cdx}{b} - \frac{d^2x^2}{b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id(c+}{}$$

$$= -\frac{2cdx}{b} - \frac{d^2x^2}{b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id(c+}{}$$

$$= -\frac{2cdx}{b} - \frac{d^2x^2}{b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id(c+}{}$$

**Mathematica [B]** time = 6.56, size = 516, normalized size = 2.98

$$\frac{cd\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))}-\frac{\cot(a)\left(i\text{Li}_2\left(e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan^{-1}(\cot(a))\right.}\right.\right.}{\sqrt{\cot^2(a)+1}}\right.}{b^2\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(
a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (
3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I
*a)) + (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin
[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]
]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] -
 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Lo
g[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2
, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt
[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (Cos[2*b*x]*(2*b^2*c^2*Cos[2*a] - d^2*C
os[2*a] + 4*b^2*c*d*x*Cos[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b*c*d*Sin[2*a]
- 2*b*d^2*x*Sin[2*a]))/(2*b^3) + ((2*b*c*d*Cos[2*a] + 2*b*d^2*x*Cos[2*a] +
2*b^2*c^2*Sin[2*a] - d^2*Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] + 2*b^2*d^2*x^2*Si
n[2*a])*Sin[2*b*x])/(2*b^3) - (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3
```

**fricas [C]** time = 0.55, size = 677, normalized size = 3.91

$$\frac{2\,b^2d^2x^2+4\,b^2cdx-2\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2\right)\cos\left(bx+a\right)^2+2\,d^2\text{polylog}\left(3,i\,\cos\left(bx+a\right)+\sin\left(bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x - 2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c
^2 - d^2)*cos(b*x + a)^2 + 2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a))
+ 2*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos
(b*x + a) + sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)
) + 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) + (2*I*b*d^2*x + 2*I*b*c*
d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(
I*cos(b*x + a) - sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-I*cos(b*
x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) -
sin(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*
x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x
+ a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x
 + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^
2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*
a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 +
 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)
 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I)
+ (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/
b^3
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\sec\left(bx+a\right)\sin\left(3\,bx+3\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

**maple** [B] time = 0.13, size = 377, normalized size = 2.18

$$-\frac{id^2x^3}{3}+\frac{2id^2a^2x}{b^2}+ic^2x-\frac{\left(2d^2x^2b^2+4b^2cdx+2ib\,d^2x+2b^2c^2+2ibcd-d^2\right)\mathrm{e}^{2i(bx+a)}}{4b^3}-\frac{\left(2d^2x^2b^2+4b^2cdx-2ib\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -1/3*I*d^2*x^3-I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+2*I/b^2*d^2*a^2*x-1
/4*(2*d^2*x^2*b^2+2*I*b*d^2*x+4*b^2*c*d*x+2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(
2*I*(b*x+a))-1/4*(2*d^2*x^2*b^2-2*I*b*d^2*x+4*b^2*c*d*x-2*I*b*c*d+2*b^2*c^2
-d^2)/b^3*exp(-2*I*(b*x+a))+1/b*c^2*ln(1+exp(2*I*(b*x+a)))-2/b*c^2*ln(exp(I
*(b*x+a)))-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))-I/b^2*c*d*polylog(2,-exp(2*I*(b
*x+a)))+I*c^2*x+4/3*I/b^3*d^2*a^3+1/b*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-I*c*d*
x^2+1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+4/b^2*c*d*a*ln(exp(I*(b*x+a)))
+2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x-2*I/b^2*c*d*a^2-4*I/b*c*d*a*x
```

**maxima** [A] time = 0.45, size = 301, normalized size = 1.74

$$-\frac{c^2\left(2\,\cos\left(2\,bx+2\,a\right)-\log\left(\cos\left(2\,bx\right)^2+2\,\cos\left(2\,bx\right)\cos\left(2\,a\right)+\cos\left(2\,a\right)^2+\sin\left(2\,bx\right)^2-2\,\sin\left(2\,bx\right)\sin\left(2\right.\right.\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c^2*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + c
os(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/6*(-2
*I*b^3*d^2*x^3 - 6*I*b^3*c*d*x^2 + 3*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) +
 (6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2
*a) + 1) - 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x - d^2)*cos(2*b*x + 2*a) + (-6*I*b
*d^2*x - 6*I*b*c*d)*dilog(-e^(2*I*b*x + 2*I*a)) + 3*(b^2*d^2*x^2 + 2*b^2*c*
d*x)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)
+ 6*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))/b^3
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)\,{\left(c+d\,x\right)}^2}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x),x)
```

```
[Out] int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

# 3.385 $\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx$

**Optimal**. Leaf size=107

$$-\frac{id\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}+\frac{d\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)\sin^2(a+bx)}{b}-\frac{dx}{b}-\frac{i(c+dx)^2}{2d}$$

```
[Out] -d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b-1/2*I*d*polylog(2
,-exp(2*I*(b*x+a)))/b^2+d*cos(b*x+a)*sin(b*x+a)/b^2+2*(d*x+c)*sin(b*x+a)^2/
b
```

**Rubi [A]** time = 0.18, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4431, 4404, 2635, 8, 4407, 3719, 2190, 2279, 2391}

$$-\frac{id\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{d\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)\sin^2(a+bx)}{b}-\frac{dx}{b}-\frac{i}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] -((d*x)/b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 + E^((2*I)*(a + b*x))
])/b - ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (d*Cos[a + b*x]*Sin
[a + b*x])/b^2 + (2*(c + d*x)*Sin[a + b*x]^2)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
```

```
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)\cos(a+bx)\sin(a+bx)-(c+dx)\sin^2(a+bx)\tan(a+\right.$$

$$= 3\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)\sin^2(a+bx)\tan$$

$$= \frac{3(c+dx)\sin^2(a+bx)}{2b} - \frac{(3d)\int \sin^2(a+bx)\,dx}{2b} + \int (c+dx)\cos(a$$

$$= -\frac{i(c+dx)^2}{2d} + \frac{3d\cos(a+bx)\sin(a+bx)}{4b^2} + \frac{2(c+dx)\sin^2(a+bx)}{b}$$

$$= -\frac{3dx}{4b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{d\cos(a+bx)\sin}{b^2}$$

$$= -\frac{dx}{b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{d\cos(a+bx)\sin(}{b^2}$$

$$= -\frac{dx}{b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} +$$

**Mathematica [B]** time = 5.57, size = 254, normalized size = 2.37

$$\frac{d\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))} - \frac{\cot(a)\left(i\text{Li}_2\left(e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan^{-1}\right.}\right.\right.}{\sqrt{\cot^2(a)+1}}\right.}{2b^2\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[
Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^
```

```
((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[S
in[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))])
)/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)])
- (d*Cos[2*b*x]*(2*b*x*Cos[2*a] - Sin[2*a]))/(2*b^2) + (d*(Cos[2*a] + 2*b*x
*Sin[2*a])*Sin[2*b*x])/(2*b^2) + (c*(Log[Cos[a + b*x]] + 2*Sin[a + b*x]^2))
/b - (d*x^2*Tan[a])/2
```

**fricas [B]** time = 0.53, size = 340, normalized size = 3.18

$$\frac{2\,bdx - 4\,(bdx + bc)\cos\left(bx + a\right)^2 + 2\,d\cos\left(bx + a\right)\sin\left(bx + a\right) + i\,d\mathrm{Li}_2\left(i\,\cos\left(bx + a\right) + \sin\left(bx + a\right)\right) - i\,d\mathrm{Li}_2\left(i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x - 4*(b*d*x + b*c)*cos(b*x + a)^2 + 2*d*cos(b*x + a)*sin(b*x +
a) + I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(I*cos(b*x + a) -
sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(-I*co
s(b*x + a) - sin(b*x + a)) + (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a)
+ I) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b*d*x + a*d)*l
og(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*log(I*cos(b*x + a) -
sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) +
 (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*log(-c
os(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(-cos(b*x + a) - I*sin(b
*x + a) + I))/b^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\sec\left(bx + a\right)\sin\left(3\,bx + 3\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

**maple [A]** time = 0.31, size = 177, normalized size = 1.65

$$-\frac{id\,x^2}{2}+icx-\frac{(2bdx+2cb+id)\,\mathrm{e}^{2i(bx+a)}}{4b^2}-\frac{(2bdx+2cb-id)\,\mathrm{e}^{-2i(bx+a)}}{4b^2}+\frac{c\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)}{b}-\frac{2c\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b}-\frac{2idax}{b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -1/2*I*d*x^2+I*c*x-1/4*(2*b*d*x+I*d+2*c*b)/b^2*exp(2*I*(b*x+a))-1/4*(2*b*d*
x-I*d+2*c*b)/b^2*exp(-2*I*(b*x+a))+1/b*c*ln(1+exp(2*I*(b*x+a)))-2/b*c*ln(ex
p(I*(b*x+a)))-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(1+exp(2*I*(b*x+a)))*x-1/2*I*
d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b^2*d*a*ln(exp(I*(b*x+a)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(2\,\cos\left(2\,bx + 2\,a\right) - \log\left(\cos\left(2\,bx\right)^2 + 2\,\cos\left(2\,bx\right)\cos\left(2\,a\right) + \cos\left(2\,a\right)^2 + \sin\left(2\,bx\right)^2 - 2\,\sin\left(2\,bx\right)\sin\left(2\,a\right) + \right.\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos
(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b - 1/2*(2*b*
```

```
x*cos(2*b*x + 2*a) + 4*b^2*integrate(x*sin(2*b*x + 2*a)/(cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) - sin(2*b*x + 2*a))*d/b
^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (3\,a+3\,b\,x)\,\left (c+d\,x\right )}{\cos \left (a+b\,x\right )} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x),x)
[Out] int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x)
[Out] Timed out
```

## 3.386 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

**Optimal**. Leaf size=80

$$-\text{Int}\left(\frac{\tan(a+bx)}{c+dx},x\right)+\frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}$$

```
[Out] 2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d+2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)
/d-Unintegrable(tan(b*x+a)/(d*x+c),x)
```

**Rubi** [A] time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d + (2*Cos[2*a - (2
*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d - Defer[Int][Tan[a + b*x]/(c + d
*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx &= \int \left(\frac{3\cos(a+bx)\sin(a+bx)}{c+dx}-\frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\right)dx \\
&= 3\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx-\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx \\
&= 3\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx+\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx-\int \frac{\tan(a+bx)}{c+dx}\,dx \\
&= \frac{3}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx+\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx-\int \frac{\tan(a+bx)}{c+dx}\,dx \\
&= \frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx+\frac{1}{2}\left(3\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx+\frac{1}{2}\left(3\,\text{si}\right. \\
&= \frac{3\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d}+\frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{1}{2}\cos\left(2a-\right. \\
&= \frac{2\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}-\int \frac{\tan(a+}{c+d}
\end{aligned}$$

**Mathematica** [A] time = 3.23, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x), x]
```

**fricas** [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\sin(3\,bx+3\,a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c), x)
```

**maple [A]** time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(i\,E_1\left(\frac{2i\,bdx+2i\,bc}{d}\right)-i\,E_1\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+2\,d\int\frac{\sin(2\,bx+2\,a)}{(dx+c)\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((I*exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(1, -(2*I
*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*d*integrate(sin(2*b*x + 2*a
)/((d*x + c)*cos(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*(d
*x + c)*cos(2*b*x + 2*a) + c), x) + (exp_integral_e(1, (2*I*b*d*x + 2*I*b*c
)/d) + exp_integral_e(1, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d))/
d
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,a+3\,b\,x)}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)), x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

## 3.387 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=103

$$-\text{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2},x\right)+\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{2\sin(2a+2bx)}{d(c+dx)}$$

```
[Out] 4*b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2-4*b*Si(2*b*c/d+2*b*x)*sin(2*a-2*
b*c/d)/d^2-2*sin(2*b*x+2*a)/d/(d*x+c)-Unintegrable(tan(b*x+a)/(d*x+c)^2,x)
```

**Rubi** [A] time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2 - (2*Sin[2*a
+ 2*b*x])/(d*(c + d*x)) - (4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d +
 2*b*x])/d^2 - Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx &= \int \left(\frac{3\cos(a+bx)\sin(a+bx)}{(c+dx)^2}-\frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\right)dx\\
&= 3\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx-\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx\\
&= 3\int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx+\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx-\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{3}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx+\int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx-\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx\\
&= -\frac{3\sin(2a+2bx)}{2d(c+dx)}+\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx+\frac{(3b)\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d}-\int \frac{\tan(a+
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2\sin(2a+2bx)}{d(c+dx)}+\frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d}+\frac{\left(3b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}-\\
&= \frac{3b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{2\sin(2a+2bx)}{d(c+dx)}-\frac{3b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{d^2}\\
&= \frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{2\sin(2a+2bx)}{d(c+dx)}-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{d^2}
\end{aligned}$$

**Mathematica** [A] time = 4.21, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

**fricas [A]** time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\sin(3\,bx+3\,a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

**maple [A]** time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(i\,E_2\left(\frac{2i\,bdx+2i\,bc}{d}\right)-i\,E_2\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+2\left(d^2x+cd\right)\int\frac{\sin(2\,bx+2\,a)}{(dx+c)^2\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,b}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((I*exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(2, -(2*I
*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(d^2*x + c*d)*integrate(sin
(2*b*x + 2*a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x + 2*
a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 + 2*
c*d*x + c^2)*cos(2*b*x + 2*a)), x) + (exp_integral_e(2, (2*I*b*d*x + 2*I*b*
c)/d) + exp_integral_e(2, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d))
/(d^2*x + c*d)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,a+3\,b\,x)}{\cos(a+b\,x)\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^2), x)
```

**sympy** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.388 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=129

$$-\text{Int}\left(\frac{\tan(a+bx)}{(c+dx)^3},x\right)-\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{2b\cos(2a+2bx)}{d^2(c+dx)}$$

```
[Out] -2*b*cos(2*b*x+2*a)/d^2/(d*x+c)-4*b^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^
3-4*b^2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3-sin(2*b*x+2*a)/d/(d*x+c)^2-U
nintegrable(tan(b*x+a)/(d*x+c)^3,x)
```

**Rubi [A]** time = 0.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (-2*b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)) - (4*b^2*CosIntegral[(2*b*c)/d + 2*
b*x]*Sin[2*a - (2*b*c)/d])/d^3 - Sin[2*a + 2*b*x]/(d*(c + d*x)^2) - (4*b^2*
Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3 - Defer[Int][Tan[a
 + b*x]/(c + d*x)^3, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx &= \int \left(\frac{3\cos(a+bx)\sin(a+bx)}{(c+dx)^3}-\frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^3}\right)dx\\
&= 3\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx-\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^3}\,dx\\
&= 3\int \frac{\sin(2a+2bx)}{2(c+dx)^3}\,dx+\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx-\int \frac{\tan(a+bx)}{(c+dx)^3}\,dx\\
&= \frac{3}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx+\int \frac{\sin(2a+2bx)}{2(c+dx)^3}\,dx-\int \frac{\tan(a+bx)}{(c+dx)^3}\,dx\\
&= -\frac{3\sin(2a+2bx)}{4d(c+dx)^2}+\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx+\frac{(3b)\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{2d}-\int \frac{\tan(}{(c}\\
&= -\frac{3b\cos(2a+2bx)}{2d^2(c+dx)}-\frac{\sin(2a+2bx)}{d(c+dx)^2}-\frac{\left(3b^2\right)\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d^2}+\frac{b\int \frac{\cos(2a+2b}{(c+dx)^2}}{2d}\\
&= -\frac{2b\cos(2a+2bx)}{d^2(c+dx)}-\frac{\sin(2a+2bx)}{d(c+dx)^2}-\frac{b^2\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d^2}-\frac{\left(3b^2\cos\left(2a-\right.\right.}{}\\
&= -\frac{2b\cos(2a+2bx)}{d^2(c+dx)}-\frac{3b^2\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3}-\frac{\sin(2a+2bx)}{d(c+dx)^2}-\frac{3}{}\\
&= -\frac{2b\cos(2a+2bx)}{d^2(c+dx)}-\frac{4b^2\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3}-\frac{\sin(2a+2bx)}{d(c+dx)^2}-\frac{4}{}
\end{aligned}$$

**Mathematica [A]** time = 6.02, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

**fricas [A]** time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\sin(3bx+3a)}{d^3x^3+3cd^2x^2+3c^2dx+c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
 c^3), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

**maple [A]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

```
[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(iE_3\left(\frac{2i\,bdx+2i\,bc}{d}\right)-iE_3\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+2\left(d^3x^2+2cd^2x+c^2d\right)\int\frac{\sin(2bx+2a)}{(dx+c)^3\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2\right.}}{d^3x^2+2cd^2x+c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -((I*exp_integral_e(3, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(3, -(2*I
*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(d^3*x^2 + 2*c*d^2*x + c^2*
d)*integrate(sin(2*b*x + 2*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3 + (d
^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(2*b*x + 2*a)^2 + (d^3*x^3 + 3*c
*d^2*x^2 + 3*c^2*d*x + c^3)*sin(2*b*x + 2*a)^2 + 2*(d^3*x^3 + 3*c*d^2*x^2 +
 3*c^2*d*x + c^3)*cos(2*b*x + 2*a)), x) + (exp_integral_e(3, (2*I*b*d*x + 2
*I*b*c)/d) + exp_integral_e(3, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d
)/d))/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3a+3bx)}{\cos(a+bx)\,(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^3),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^3), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

# 3.389 $\int (c+dx)^3 \sec^2(a+bx) \sin(3a+3bx)\,dx$

**Optimal**. Leaf size=230

$$-\frac{6d^3\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3\sin(a+bx)}{b^4}+\frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}$$

```
[Out] -6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2+24*d^2*(d*x+c)*cos(b*x+a)/b^3-4
*(d*x+c)^3*cos(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^3-6*
I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3-6*d^3*polylog(3,-I*exp(I*(b*x
+a)))/b^4+6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4-(d*x+c)^3*sec(b*x+a)/b-24*d
^3*sin(b*x+a)/b^4+12*d*(d*x+c)^2*sin(b*x+a)/b^2
```

**Rubi [A]** time = 0.33, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4431, 3296, 2637, 4407, 4409, 4181, 2531, 2282, 6589}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{6d^3\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{PolyLog}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] ((-6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 + (24*d^2*(c + d*x)*Cos[
a + b*x])/b^3 - (4*(c + d*x)^3*Cos[a + b*x])/b + ((6*I)*d^2*(c + d*x)*PolyL
og[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(
a + b*x))])/b^3 - (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 + (6*d^3*Pol
yLog[3, I*E^(I*(a + b*x))])/b^4 - ((c + d*x)^3*Sec[a + b*x])/b - (24*d^3*Si
n[a + b*x])/b^4 + (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\sec^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^3\sin(a+bx)-(c+dx)^3\sin(a+bx)\tan^2(a+bx)\right) d$$

$$= 3\int (c+dx)^3\sin(a+bx)\,dx - \int (c+dx)^3\sin(a+bx)\tan^2(a+bx$$

$$= -\frac{3(c+dx)^3\cos(a+bx)}{b} + \frac{(9d)\int (c+dx)^2\cos(a+bx)\,dx}{b} + \int (c$$

$$= -\frac{4(c+dx)^3\cos(a+bx)}{b} - \frac{(c+dx)^3\sec(a+bx)}{b} + \frac{9d(c+dx)^2\sin}{b^2}$$

$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{18d^2(c+dx)\cos(a+bx)}{b^3} - \frac{4(c+}{}$$

$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} - \frac{4(c+}{}$$

$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} - \frac{4(c+}{}$$

$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} - \frac{4(c+}{}$$

**Mathematica [B]** time = 2.60, size = 607, normalized size = 2.64

$$-\frac{\sec(a+bx)\left(2b^3c^3\cos(2(a+bx))+6b^3c^2dx\cos(2(a+bx))+6b^3cd^2x^2\cos(2(a+bx))+ib^3d^3x^3\cos(a+bx)+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] -((Sec[a + b*x]*(3*b^3*c^3 - 12*b*c*d^2 + 9*b^3*c^2*d*x - 12*b*d^3*x + 9*b^
3*c*d^2*x^2 + 3*b^3*d^3*x^3 + I*b^3*d^3*x^3*Cos[a + b*x] + (6*I)*b^2*c^2*d*
ArcTan[Cos[a + b*x] + I*Sin[a + b*x]]*Cos[a + b*x] + 2*b^3*c^3*Cos[2*(a + b
*x)] - 12*b*c*d^2*Cos[2*(a + b*x)] + 6*b^3*c^2*d*x*Cos[2*(a + b*x)] - 12*b*
d^3*x*Cos[2*(a + b*x)] + 6*b^3*c*d^2*x^2*Cos[2*(a + b*x)] + 2*b^3*d^3*x^3*C
os[2*(a + b*x)] + 3*b^2*d^3*x^2*Cos[a + b*x]*Log[1 - I*Cos[a + b*x] - Sin[a
 + b*x]] + 6*b^2*c*d^2*x*Cos[a + b*x]*Log[1 + I*Cos[a + b*x] - Sin[a + b*x]
] - 6*b^2*c*d^2*x*Cos[a + b*x]*Log[1 - I*Cos[a + b*x] + Sin[a + b*x]] - 3*b
^2*d^3*x^2*Cos[a + b*x]*Log[1 - I*Cos[a + b*x] + Sin[a + b*x]] + (6*I)*b*d^
2*(c + d*x)*Cos[a + b*x]*PolyLog[2, I*Cos[a + b*x] - Sin[a + b*x]] - (6*I)*
b*c*d^2*Cos[a + b*x]*PolyLog[2, (-I)*Cos[a + b*x] + Sin[a + b*x]] + (6*I)*b
*d^3*x*Cos[a + b*x]*PolyLog[2, I*Cos[a + b*x] + Sin[a + b*x]] - 6*d^3*Cos[a
 + b*x]*PolyLog[3, I*Cos[a + b*x] - Sin[a + b*x]] + 6*d^3*Cos[a + b*x]*Poly
Log[3, I*Cos[a + b*x] + Sin[a + b*x]] - 6*b^2*c^2*d*Sin[2*(a + b*x)] + 12*d
^3*Sin[2*(a + b*x)] - 12*b^2*c*d^2*x*Sin[2*(a + b*x)] - 6*b^2*d^3*x^2*Sin[2
*(a + b*x)]))/b^4)
```

**fricas [C]** time = 0.56, size = 896, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*c
os(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*
polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, -
I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x +
 a) - sin(b*x + a)) + 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^
2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 - (-6*I*b*d^3*x - 6*I*b*c*d^2
)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - (-6*I*b*d^3*x - 6*I*b
*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) - (6*I*b*d^3*x +
6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - (6*I*b*d^
3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(
b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*
x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*
x + a) - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2
 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^
3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x +
 a) - sin(b*x + a) + 1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^
2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^
2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a)
 - sin(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*l
og(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d
^3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 24*(b^2*d^3*x^2
+ 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a))/(b^4*cos(b*
x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sec(bx+a)^2 \sin(3bx+3a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)^2*sin(3*b*x + 3*a), x)
```

**maple [B]** time = 0.18, size = 677, normalized size = 2.94

$$-\frac{2\left(d^3x^3b^3+3b^3c\,d^2x^2+3ib^2d^3x^2+3b^3c^2dx+6ib^2c\,d^2x+b^3c^3+3ib^2c^2d-6b\,d^3x-6c\,d^2b-6id^3\right)\mathrm{e}^{i(bx+a)}}{b^4}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] -2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d
^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))-2*
(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*
x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-2*ex
p(I*(b*x+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(1+exp(2*I*(b*x+a)))-3/b
^4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))-6/b^3*d^2*c*ln(1+I*exp(I*(b*x+a)))*a-6/b^
2*d^2*c*ln(1+I*exp(I*(b*x+a)))*x-6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*I
/b^2*d*c^2*arctan(exp(I*(b*x+a)))+12*I/b^3*d^2*c*a*arctan(exp(I*(b*x+a)))-6
*I/b^3*d^2*c*polylog(2,I*exp(I*(b*x+a)))+6*I/b^3*d^3*polylog(2,-I*exp(I*(b*
x+a)))*x+3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)))+3/b^2*d^3*ln(1-I*exp(I*(b*x+a
)))*x^2-6*I/b^4*d^3*a^2*arctan(exp(I*(b*x+a)))+6*d^3*polylog(3,I*exp(I*(b*x
+a)))/b^4+6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a+6/b^2*d^2*c*ln(1-I*exp(I*(b*
x+a)))*x-3/b^2*d^3*ln(1+I*exp(I*(b*x+a)))*x^2+6*I/b^3*d^2*c*polylog(2,-I*ex
p(I*(b*x+a)))-6*I/b^3*d^3*polylog(2,I*exp(I*(b*x+a)))*x
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*a
) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x + 3*
a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) +
 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c^3/(b*cos(3*b*x + 3*a)^2
+ 2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + 3*a)
^2 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - 3/2*(4*(cos(a)
^2 + sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) +
 b*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(
b*x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))
*cos(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x*cos(b*x
 + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x + 2*a)*s
in(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*sin(2*b*x +
 3*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a)
)*cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 + (b*x*cos(2*b*
x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x + 3*a)^2 + (b*
x*cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)*cos(b*x + a) +
(b*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 3*a))*cos(3*b
*x + 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*
(b*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a))*sin(b*x + a) + s
in(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (cos(b*x + a)^2 + sin(b*x
+ a)^2)*sin(2*b*x + 3*a))*cos(3*b*x + 4*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(b*
x + a)*cos(a) + 6*b*x*cos(b*x + a)*sin(b*x + 2*a)*sin(a) + b*x*cos(2*b*x +
3*a)^2 + b*x*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*b*x + 2*(3*b*x*cos(
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b*x + 2*a)*cos(b*x + a) + b*x*cos(a))*cos(2*b*x + 3*a) + 2*(3*b*x*cos(b*x +
 a)*sin(b*x + 2*a) + b*x*sin(a))*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + 4*(2*
b*x*cos(b*x + a)*cos(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(b
*x + 2*a) - 2*cos(a)*sin(b*x + a))*cos(2*b*x + 3*a) + 12*(b*x*cos(b*x + a)^
2*cos(a) + b*x*cos(a)*sin(b*x + a)^2)*cos(b*x + 2*a) - ((cos(2*b*x + 3*a)^2
 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x
+ 3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x +
a)^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*cos(b*x + a)^2 + (cos(2*b*
x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*
sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 +
sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*sin(b*x + a)^2 +
 2*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a
) + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a
) + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^
2*cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(cos(2*b*x + 3*a)^2*
sin(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + sin(2*b*x + 3*a)^2*
sin(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^
2)*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(b*x + a)^2*sin(a) + sin(b*x + a)
^2*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*
x + a) + 1) + ((cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 +
 sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a
)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + si
n(a)^2)*cos(b*x + a)^2 + (cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) +
cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3
*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos
(a)^2 + sin(a)^2)*sin(b*x + a)^2 + 2*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*c
os(2*b*x + 3*a)*cos(b*x + a)*cos(a) + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*c
os(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*c
os(3*b*x + 3*a) + 2*(cos(b*x + a)^2*cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b
*x + 3*a) + 2*(cos(2*b*x + 3*a)^2*sin(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*
sin(b*x + a) + sin(2*b*x + 3*a)^2*sin(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x
 + a)*sin(a) + (cos(a)^2 + sin(a)^2)*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(co
s(b*x + a)^2*sin(a) + sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 4*((b*x*sin(2*b*x + 3*a) +
b*x*sin(a) - cos(2*b*x + 3*a) - cos(a))*cos(3*b*x + 3*a)^2 + (b*x*sin(a) -
cos(a))*cos(b*x + a)^2 + (b*x*sin(2*b*x + 3*a) + b*x*sin(a) - cos(2*b*x + 3
*a) - cos(a))*sin(3*b*x + 3*a)^2 + (b*x*sin(a) - cos(a))*sin(b*x + a)^2 + 2
*(b*x*cos(b*x + a)*sin(2*b*x + 3*a) + (b*x*sin(a) - cos(a))*cos(b*x + a) -
cos(2*b*x + 3*a)*cos(b*x + a))*cos(3*b*x + 3*a) - (cos(b*x + a)^2 + sin(b*x
 + a)^2)*cos(2*b*x + 3*a) + 2*(b*x*sin(2*b*x + 3*a)*sin(b*x + a) + (b*x*sin
(a) - cos(a))*sin(b*x + a) - cos(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a
) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*sin(2*b*x + 3*a))*sin(3*b*x +
 4*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(a)*sin(b*x + a) + 6*b*x*sin(b*x + 2*a)*
sin(b*x + a)*sin(a) + 2*(3*b*x*cos(b*x + 2*a)*sin(b*x + a) - cos(a))*cos(2*
b*x + 3*a) - cos(2*b*x + 3*a)^2 - cos(a)^2 + 2*(3*b*x*sin(b*x + 2*a)*sin(b*
x + a) - sin(a))*sin(2*b*x + 3*a) - sin(2*b*x + 3*a)^2 - sin(a)^2)*sin(3*b*
x + 3*a) + 4*(2*b*x*cos(b*x + a)*sin(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b
*x + a)^2)*sin(b*x + 2*a) - 2*sin(b*x + a)*sin(a))*sin(2*b*x + 3*a) + 12*(b
*x*cos(b*x + a)^2*sin(a) + b*x*sin(b*x + a)^2*sin(a))*sin(b*x + 2*a) - 4*(c
os(a)^2 + sin(a)^2)*sin(b*x + a))*c^2*d/((cos(a)^2 + sin(a)^2)*b^2*cos(b*x
+ a)^2 + (cos(a)^2 + sin(a)^2)*b^2*sin(b*x + a)^2 + (b^2*cos(2*b*x + 3*a)^2
 + 2*b^2*cos(2*b*x + 3*a)*cos(a) + b^2*sin(2*b*x + 3*a)^2 + 2*b^2*sin(2*b*x
 + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^2)*cos(3*b*x + 3*a)^2 + (b^2*cos(b
*x + a)^2 + b^2*sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (b^2*cos(2*b*x + 3*a)^
2 + 2*b^2*cos(2*b*x + 3*a)*cos(a) + b^2*sin(2*b*x + 3*a)^2 + 2*b^2*sin(2*b*
x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^2)*sin(3*b*x + 3*a)^2 + (b^2*cos(
b*x + a)^2 + b^2*sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + 2*(b^2*cos(2*b*x + 3*
a)^2*cos(b*x + a) + 2*b^2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a) + b^2*cos(b*
x + a)*sin(2*b*x + 3*a)^2 + 2*b^2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (c
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os(a)^2 + sin(a)^2)*b^2*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(b^2*cos(b*x + a
)^2*cos(a) + b^2*cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b^2*cos(2*b*x
 + 3*a)^2*sin(b*x + a) + 2*b^2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + b^2*s
in(2*b*x + 3*a)^2*sin(b*x + a) + 2*b^2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a)
 + (cos(a)^2 + sin(a)^2)*b^2*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(b^2*cos(b*
x + a)^2*sin(a) + b^2*sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a)) - (6*((b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(2*b*x + 3*a)*cos(b*x
+ 2*a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*sin(2*b*x
+ 3*a)*sin(b*x + 2*a) + (b^3*d^3*x^3*cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 4*b*
d^3*x*cos(a) - 4*b*c*d^2*cos(a))*cos(b*x + 2*a) + (b^3*d^3*x^3*sin(a) + 3*b
^3*c*d^2*x^2*sin(a) - 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*sin(b*x + 2*a))*
cos(3*b*x + 3*a)^2 + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*
d^2)*cos(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*sin(b*x + a))*c
os(2*b*x + 3*a)^2 + 6*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d
^2)*cos(2*b*x + 3*a)*cos(b*x + 2*a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*
d^3*x - 4*b*c*d^2)*sin(2*b*x + 3*a)*sin(b*x + 2*a) + (b^3*d^3*x^3*cos(a) +
3*b^3*c*d^2*x^2*cos(a) - 4*b*d^3*x*cos(a) - 4*b*c*d^2*cos(a))*cos(b*x + 2*a
) + (b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^2*sin(a) - 4*b*d^3*x*sin(a) - 4*b*c
*d^2*sin(a))*sin(b*x + 2*a))*sin(3*b*x + 3*a)^2 + 2*((b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d
^2*x - 2*d^3)*sin(b*x + a))*sin(2*b*x + 3*a)^2 + 2*((b^3*d^3*x^3*cos(a) - 6
*b*c*d^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2
+ 6*(b^2*c*d^2*sin(a) - b*d^3*cos(a))*x + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 -
6*b*d^3*x - 6*b*c*d^2)*cos(2*b*x + 3*a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x -
2*d^3)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a)^2 + (b^3*d^3*x^3*cos(a) - 6*b*c*d
^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2 + 6*(b
^2*c*d^2*sin(a) - b*d^3*cos(a))*x)*cos(b*x + a)^2 + (b^3*d^3*x^3*cos(a) - 6
*b*c*d^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2
+ 6*(b^2*c*d^2*sin(a) - b*d^3*cos(a))*x + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 -
6*b*d^3*x - 6*b*c*d^2)*cos(2*b*x + 3*a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x -
2*d^3)*sin(2*b*x + 3*a))*sin(3*b*x + 3*a)^2 + (b^3*d^3*x^3*cos(a) - 6*b*c*d
^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2 + 6*(b
^2*c*d^2*sin(a) - b*d^3*cos(a))*x)*sin(b*x + a)^2 + 2*((b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(2*b*x + 3*a)*cos(b*x + a) + 3*(b^2*
d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(b*x + a)*sin(2*b*x + 3*a) + (b^3*d^3*x
^3*cos(a) - 6*b*c*d^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3
*sin(a))*x^2 + 6*(b^2*c*d^2*sin(a) - b*d^3*cos(a))*x)*cos(b*x + a))*cos(3*b
*x + 3*a) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*
x + a)^2 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x
+ a)^2)*cos(2*b*x + 3*a) + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x -
6*b*c*d^2)*cos(2*b*x + 3*a)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x -
 2*d^3)*sin(2*b*x + 3*a)*sin(b*x + a) + (b^3*d^3*x^3*cos(a) - 6*b*c*d^2*cos
(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2 + 6*(b^2*c*d
^2*sin(a) - b*d^3*cos(a))*x)*sin(b*x + a))*sin(3*b*x + 3*a) + 3*((b^2*d^3*x
^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(b*x + a)^2 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x -
 2*d^3)*sin(b*x + a)^2)*sin(2*b*x + 3*a))*cos(3*b*x + 4*a) + 2*((cos(a)^2 +
 sin(a)^2)*b^3*d^3*x^3 + 3*(cos(a)^2 + sin(a)^2)*b^3*c*d^2*x^2 - 6*(cos(a)^
2 + sin(a)^2)*b*d^3*x - 6*(cos(a)^2 + sin(a)^2)*b*c*d^2 + (b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(2*b*x + 3*a)^2 + 6*(b^3*d^3*x^3*
cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 4*b*d^3*x*cos(a) - 4*b*c*d^2*cos(a))*cos(
b*x + 2*a)*cos(b*x + a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*
c*d^2)*sin(2*b*x + 3*a)^2 + 6*(b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^2*sin(a)
- 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*cos(b*x + a)*sin(b*x + 2*a) + 2*(b^3
*d^3*x^3*cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 6*b*d^3*x*cos(a) - 6*b*c*d^2*cos
(a) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(b*x + 2
*a)*cos(b*x + a))*cos(2*b*x + 3*a) + 2*(b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^
2*sin(a) - 6*b*d^3*x*sin(a) - 6*b*c*d^2*sin(a) + 3*(b^3*d^3*x^3 + 3*b^3*c*d
^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(b*x + a)*sin(b*x + 2*a))*sin(2*b*x + 3*
a))*cos(3*b*x + 3*a) + 2*(3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4
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*b*c*d^2)*cos(b*x + a)^2 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b
*c*d^2)*sin(b*x + a)^2)*cos(b*x + 2*a) + 2*(b^3*d^3*x^3*cos(a) + 3*b^3*c*d^
2*x^2*cos(a) - 6*b*d^3*x*cos(a) - 6*b*c*d^2*cos(a))*cos(b*x + a) - 6*(b^2*d
^3*x^2*cos(a) + 2*b^2*c*d^2*x*cos(a) - 2*d^3*cos(a))*sin(b*x + a))*cos(2*b*
x + 3*a) + 6*((b^3*d^3*x^3*cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 4*b*d^3*x*cos(
a) - 4*b*c*d^2*cos(a))*cos(b*x + a)^2 + (b^3*d^3*x^3*cos(a) + 3*b^3*c*d^2*x
^2*cos(a) - 4*b*d^3*x*cos(a) - 4*b*c*d^2*cos(a))*sin(b*x + a)^2)*cos(b*x +
2*a) + 2*((cos(a)^2 + sin(a)^2)*b^3*d^3*x^3 + 3*(cos(a)^2 + sin(a)^2)*b^3*c
*d^2*x^2 - 6*(cos(a)^2 + sin(a)^2)*b*d^3*x - 6*(cos(a)^2 + sin(a)^2)*b*c*d^
2)*cos(b*x + a) - ((cos(a)^2 + sin(a)^2)*b^4*cos(b*x + a)^2 + (cos(a)^2 + s
in(a)^2)*b^4*sin(b*x + a)^2 + (b^4*cos(2*b*x + 3*a)^2 + 2*b^4*cos(2*b*x + 3
*a)*cos(a) + b^4*sin(2*b*x + 3*a)^2 + 2*b^4*sin(2*b*x + 3*a)*sin(a) + (cos(
a)^2 + sin(a)^2)*b^4)*cos(3*b*x + 3*a)^2 + (b^4*cos(b*x + a)^2 + b^4*sin(b*
x + a)^2)*cos(2*b*x + 3*a)^2 + (b^4*cos(2*b*x + 3*a)^2 + 2*b^4*cos(2*b*x +
3*a)*cos(a) + b^4*sin(2*b*x + 3*a)^2 + 2*b^4*sin(2*b*x + 3*a)*sin(a) + (cos
(a)^2 + sin(a)^2)*b^4)*sin(3*b*x + 3*a)^2 + (b^4*cos(b*x + a)^2 + b^4*sin(b
*x + a)^2)*sin(2*b*x + 3*a)^2 + 2*(b^4*cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*
b^4*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a) + b^4*cos(b*x + a)*sin(2*b*x + 3*a
)^2 + 2*b^4*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^
4*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(b^4*cos(b*x + a)^2*cos(a) + b^4*cos(a
)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b^4*cos(2*b*x + 3*a)^2*sin(b*x + a)
 + 2*b^4*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + b^4*sin(2*b*x + 3*a)^2*sin(
b*x + a) + 2*b^4*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^
2)*b^4*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(b^4*cos(b*x + a)^2*sin(a) + b^4*
sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))*integrate(6*((d^3*x^2 + 2*c*d^2*x)
*cos(2*b*x + 2*a)*cos(b*x + a) + (d^3*x^2 + 2*c*d^2*x)*sin(2*b*x + 2*a)*sin
(b*x + a) + (d^3*x^2 + 2*c*d^2*x)*cos(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*s
in(2*b*x + 2*a)^2 + 2*b*cos(2*b*x + 2*a) + b), x) + 2*((b^3*d^3*x^3*sin(a)
- 6*b*c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*d^3*cos(a))*x
^2 - 6*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
 - 2*d^3)*cos(2*b*x + 3*a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6
*b*c*d^2)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a)^2 + (b^3*d^3*x^3*sin(a) - 6*b*
c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*d^3*cos(a))*x^2 - 6
*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x)*cos(b*x + a)^2 + (b^3*d^3*x^3*sin(a)
- 6*b*c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*d^3*cos(a))*x
^2 - 6*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
 - 2*d^3)*cos(2*b*x + 3*a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6
*b*c*d^2)*sin(2*b*x + 3*a))*sin(3*b*x + 3*a)^2 + (b^3*d^3*x^3*sin(a) - 6*b*
c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*d^3*cos(a))*x^2 - 6
*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x)*sin(b*x + a)^2 - 2*(3*(b^2*d^3*x^2 +
2*b^2*c*d^2*x - 2*d^3)*cos(2*b*x + 3*a)*cos(b*x + a) - (b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)*sin(2*b*x + 3*a) - (b^3*d^
3*x^3*sin(a) - 6*b*c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*
d^3*cos(a))*x^2 - 6*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x)*cos(b*x + a))*cos(
3*b*x + 3*a) - 3*((b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(b*x + a)^2 + (b
^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*sin(b*x + a)^2)*cos(2*b*x + 3*a) - 2*(3
*(b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(2*b*x + 3*a)*sin(b*x + a) - (b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*sin(2*b*x + 3*a)*sin(b*
x + a) - (b^3*d^3*x^3*sin(a) - 6*b*c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d
^2*sin(a) - b^2*d^3*cos(a))*x^2 - 6*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x)*si
n(b*x + a))*sin(3*b*x + 3*a) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x
- 6*b*c*d^2)*cos(b*x + a)^2 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x -
6*b*c*d^2)*sin(b*x + a)^2)*sin(2*b*x + 3*a))*sin(3*b*x + 4*a) - 6*((cos(a)^
2 + sin(a)^2)*b^2*d^3*x^2 + 2*(cos(a)^2 + sin(a)^2)*b^2*c*d^2*x - 2*(cos(a)
^2 + sin(a)^2)*d^3 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(2*b*x + 3*a)
^2 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*sin(2*b*x + 3*a)^2 - 2*(b^3*d^3*
x^3*cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 4*b*d^3*x*cos(a) - 4*b*c*d^2*cos(a))*
cos(b*x + 2*a)*sin(b*x + a) - 2*(b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^2*sin(a
) - 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*sin(b*x + 2*a)*sin(b*x + a) + 2*(b
```

```
^2*d^3*x^2*cos(a) + 2*b^2*c*d^2*x*cos(a) - 2*d^3*cos(a) - (b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(b*x + 2*a)*sin(b*x + a))*cos(2*b
*x + 3*a) + 2*(b^2*d^3*x^2*sin(a) + 2*b^2*c*d^2*x*sin(a) - 2*d^3*sin(a) - (
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*sin(b*x + 2*a)*sin(b
*x + a))*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) + 2*(2*(b^3*d^3*x^3*sin(a) + 3*
b^3*c*d^2*x^2*sin(a) - 6*b*d^3*x*sin(a) - 6*b*c*d^2*sin(a))*cos(b*x + a) +
3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(b*x + a)^2 +
 (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*sin(b*x + a)^2)*si
n(b*x + 2*a) - 6*(b^2*d^3*x^2*sin(a) + 2*b^2*c*d^2*x*sin(a) - 2*d^3*sin(a))
*sin(b*x + a))*sin(2*b*x + 3*a) + 6*((b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^2*
sin(a) - 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*cos(b*x + a)^2 + (b^3*d^3*x^3
*sin(a) + 3*b^3*c*d^2*x^2*sin(a) - 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*sin
(b*x + a)^2)*sin(b*x + 2*a) - 6*((cos(a)^2 + sin(a)^2)*b^2*d^3*x^2 + 2*(cos
(a)^2 + sin(a)^2)*b^2*c*d^2*x - 2*(cos(a)^2 + sin(a)^2)*d^3)*sin(b*x + a))/
((cos(a)^2 + sin(a)^2)*b^4*cos(b*x + a)^2 + (cos(a)^2 + sin(a)^2)*b^4*sin(b
*x + a)^2 + (b^4*cos(2*b*x + 3*a)^2 + 2*b^4*cos(2*b*x + 3*a)*cos(a) + b^4*s
in(2*b*x + 3*a)^2 + 2*b^4*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b
^4)*cos(3*b*x + 3*a)^2 + (b^4*cos(b*x + a)^2 + b^4*sin(b*x + a)^2)*cos(2*b*
x + 3*a)^2 + (b^4*cos(2*b*x + 3*a)^2 + 2*b^4*cos(2*b*x + 3*a)*cos(a) + b^4*
sin(2*b*x + 3*a)^2 + 2*b^4*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*
b^4)*sin(3*b*x + 3*a)^2 + (b^4*cos(b*x + a)^2 + b^4*sin(b*x + a)^2)*sin(2*b
*x + 3*a)^2 + 2*(b^4*cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*b^4*cos(2*b*x + 3*
a)*cos(b*x + a)*cos(a) + b^4*cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*b^4*cos(b*
x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^4*cos(b*x + a))*co
s(3*b*x + 3*a) + 2*(b^4*cos(b*x + a)^2*cos(a) + b^4*cos(a)*sin(b*x + a)^2)*
cos(2*b*x + 3*a) + 2*(b^4*cos(2*b*x + 3*a)^2*sin(b*x + a) + 2*b^4*cos(2*b*x
 + 3*a)*cos(a)*sin(b*x + a) + b^4*sin(2*b*x + 3*a)^2*sin(b*x + a) + 2*b^4*s
in(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^4*sin(b*x + a
))*sin(3*b*x + 3*a) + 2*(b^4*cos(b*x + a)^2*sin(a) + b^4*sin(b*x + a)^2*sin
(a))*sin(2*b*x + 3*a))
```

**mupad** [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x)^2,x)
```

```
[Out] \text{Hanged}
```

**sympy** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.390 $\int (c+dx)^2 \sec^2(a+bx)\sin(3a+3bx)\,dx$

**Optimal**. Leaf size=147

$$\frac{2id^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}-\frac{2id^2\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}+\frac{8d^2\cos(a+bx)}{b^3}+\frac{8d(c+dx)\sin(a+bx)}{b^2}-\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{4(c}{}$$

```
[Out] -4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2+8*d^2*cos(b*x+a)/b^3-4*(d*x+c)^2*
cos(b*x+a)/b+2*I*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,I*e
xp(I*(b*x+a)))/b^3-(d*x+c)^2*sec(b*x+a)/b+8*d*(d*x+c)*sin(b*x+a)/b^2
```

**Rubi [A]** time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4431, 3296, 2638, 4407, 4409, 4181, 2279, 2391}

$$\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{8d(c+dx)\sin(a+bx)}{b^2}-\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{8d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] ((-4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 + (8*d^2*Cos[a + b*x])/b^3
 - (4*(c + d*x)^2*Cos[a + b*x])/b + ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*
x))])/b^3 - ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((c + d*x)^2*Se
c[a + b*x])/b + (8*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
```

```
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)^2 \sec^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^2\sin(a+bx) - (c+dx)^2\sin(a+bx)\tan^2(a+bx)\right) d$$

$$= 3\int (c+dx)^2\sin(a+bx)\,dx - \int (c+dx)^2\sin(a+bx)\tan^2(a+bx$$

$$= -\frac{3(c+dx)^2\cos(a+bx)}{b} + \frac{(6d)\int (c+dx)\cos(a+bx)\,dx}{b} + \int (c+$$

$$= -\frac{4(c+dx)^2\cos(a+bx)}{b} - \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{6d(c+dx)\sin}{b^2}$$

$$= -\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6d^2\cos(a+bx)}{b^3} - \frac{4(c+dx)^2\cos(}{b}$$

$$= -\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{8d^2\cos(a+bx)}{b^3} - \frac{4(c+dx)^2\cos(}{b}$$

$$= -\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{8d^2\cos(a+bx)}{b^3} - \frac{4(c+dx)^2\cos(}{b}$$

**Mathematica [B]** time = 3.83, size = 364, normalized size = 2.48

$$-4\cos(bx)\left(\cos(a)\left(b^2(c+dx)^2-2d^2\right)-2bd\sin(a)(c+dx)\right)+4\sin(bx)\left(\sin(a)\left(b^2(c+dx)^2-2d^2\right)+2bd\cos($$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + 2*d^2*(2*ArcTan[Cot[a]]*Ar
cTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - (Csc[a]*((b*x - ArcTan[Cot[a]])*(Log[
1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]]))])
+ I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x - Ar
cTan[Cot[a]]))]))/Sqrt[Csc[a]^2]) - b^2*(c + d*x)^2*Sec[a] - 4*Cos[b*x]*((-
2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) + 4*(2*b*d*(c + d
*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] - (b^2*(c + d*x)^2
*Sin[(b*x)/2])/((Cos[a/2] - Sin[a/2])*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])
```

```
) + (b^2*(c + d*x)^2*Sin[(b*x)/2])/((Cos[a/2] + Sin[a/2])*(Cos[(a + b*x)/2]
 + Sin[(a + b*x)/2])))/b^3
```

**fricas [B]** time = 0.54, size = 513, normalized size = 3.49

$$-\frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + i\,d^2\cos\left(bx+a\right)\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right) + i\,d^2\cos\left(bx+a\right)\mathrm{Li}_2\left(i\,\cos\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] -(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x
+ a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x +
a)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(
b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 4*(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - (b*c*d - a*d^2)*cos(b*x + a)*log(cos
(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x
+ a) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a)
 - sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) +
 sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - s
in(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b
*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x +
 a) + I) - 8*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)
)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \sec\left(bx+a\right)^2 \sin\left(3\,bx+3\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)^2*sin(3*b*x + 3*a), x)
```

**maple [B]** time = 0.07, size = 328, normalized size = 2.23

$$-\frac{4c^2\cos\left(bx+a\right)}{b}-\frac{c^2}{b\cos\left(bx+a\right)}-\frac{4d^2\cos\left(bx+a\right)x^2}{b}+\frac{8d^2\sin\left(bx+a\right)x}{b^2}+\frac{8d^2\cos\left(bx+a\right)}{b^3}-\frac{d^2x^2}{b\cos\left(bx+a\right)}-\frac{2d^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] -4*c^2/b*cos(b*x+a)-1/b/cos(b*x+a)*c^2-4*d^2/b*cos(b*x+a)*x^2+8*d^2/b^2*sin
(b*x+a)*x+8*d^2*cos(b*x+a)/b^3-1/b*d^2/cos(b*x+a)*x^2-2/b^2*d^2*ln(1+I*exp(
I*(b*x+a)))*x-2/b^3*d^2*ln(1+I*exp(I*(b*x+a)))*a+2/b^2*d^2*ln(1-I*exp(I*(b*
x+a)))*x+2/b^3*d^2*ln(1-I*exp(I*(b*x+a)))*a-2*I*d^2/b^3*dilog(1-I*exp(I*(b*
x+a)))+2*I*d^2/b^3*dilog(1+I*exp(I*(b*x+a)))-2/b^3*a*d^2*ln(sec(b*x+a)+tan(
b*x+a))-8*c*d/b*cos(b*x+a)*x+8*c*d/b^2*sin(b*x+a)-2/b*c*d/cos(b*x+a)*x+2/b^
2*c*d*ln(sec(b*x+a)+tan(b*x+a))
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x)^2,x)
```

```
[Out] \text{Hanged}
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.391 $\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx$

**Optimal**. Leaf size=57

$$\frac{4d\sin(a+bx)}{b^2}+\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{4(c+dx)\cos(a+bx)}{b}-\frac{(c+dx)\sec(a+bx)}{b}$$

```
[Out] d*arctanh(sin(b*x+a))/b^2-4*(d*x+c)*cos(b*x+a)/b-(d*x+c)*sec(b*x+a)/b+4*d*s
in(b*x+a)/b^2
```

**Rubi [A]** time = 0.09, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4431, 3296, 2637, 4407, 4409, 3770}

$$\frac{4d\sin(a+bx)}{b^2}+\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{4(c+dx)\cos(a+bx)}{b}-\frac{(c+dx)\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (d*ArcTanh[Sin[a + b*x]])/b^2 - (4*(c + d*x)*Cos[a + b*x])/b - ((c + d*x)*S
ec[a + b*x])/b + (4*d*Sin[a + b*x])/b^2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
```

```
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)\sin(a+bx)-(c+dx)\sin(a+bx)\tan^2(a+bx)\right)dx$$
$$= 3\int (c+dx)\sin(a+bx)\,dx - \int (c+dx)\sin(a+bx)\tan^2(a+bx)\,d$$
$$= -\frac{3(c+dx)\cos(a+bx)}{b} + \frac{(3d)\int \cos(a+bx)\,dx}{b} + \int (c+dx)\sin(a$$
$$= -\frac{4(c+dx)\cos(a+bx)}{b} - \frac{(c+dx)\sec(a+bx)}{b} + \frac{3d\sin(a+bx)}{b^2} +$$
$$= \frac{d\tanh^{-1}(\sin(a+bx))}{b^2} - \frac{4(c+dx)\cos(a+bx)}{b} - \frac{(c+dx)\sec(a+b}{b}$$

**Mathematica [A]** time = 0.48, size = 105, normalized size = 1.84

$$-\frac{\sec(a+bx)\left(2b(c+dx)\cos(2(a+bx))-2d\sin(2(a+bx))+d\cos(a+bx)\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+\right.\right.\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] -((Sec[a + b*x]*(3*b*c + 3*b*d*x + 2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Cos[a
 + b*x]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2] +
Sin[(a + b*x)/2]]) - 2*d*Sin[2*(a + b*x)]))/b^2)
```

**fricas [A]** time = 0.44, size = 93, normalized size = 1.63

$$-\frac{2\,bdx+8\,(bdx+bc)\cos(bx+a)^2-d\cos(bx+a)\log(\sin(bx+a)+1)+d\cos(bx+a)\log(-\sin(bx+a)+}{2\,b^2\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + 8*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*log(sin(b*x
 + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) - 8*d*cos(b*x + a)*sin(b
*x + a) + 2*b*c)/(b^2*cos(b*x + a))
```

**giac [B]** time = 1.79, size = 2876, normalized size = 50.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] -1/2*(10*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 10*b*c*tan(1/2*b*x)^4*tan(1/2*
a)^4 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) +
 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*
a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 +
tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1
/2*b*x)^4*tan(1/2*a)^4 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b
*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1
/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2
*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/
```

```
2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 - 12*b*d*x*tan(1/2*b*x)^4*tan(1/2*
a)^2 - 64*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^3 - 12*b*d*x*tan(1/2*b*x)^2*tan(1
/2*a)^4 - 12*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 - 64*b*c*tan(1/2*b*x)^3*tan(1/
2*a)^3 - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*
a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(
1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^
2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*t
an(1/2*b*x)^3*tan(1/2*a)^3 + 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan
(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2
*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)
^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(
tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^3 + 16*d*tan(1/2*b*x)^4*tan(1/
2*a)^3 - 12*b*c*tan(1/2*b*x)^2*tan(1/2*a)^4 + 16*d*tan(1/2*b*x)^3*tan(1/2*a
)^4 + 10*b*d*x*tan(1/2*b*x)^4 + 64*b*d*x*tan(1/2*b*x)^3*tan(1/2*a) + 168*b*
d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 64*b*d*x*tan(1/2*b*x)*tan(1/2*a)^3 + 10*b
*d*x*tan(1/2*a)^4 + 10*b*c*tan(1/2*b*x)^4 - d*log(2*(tan(1/2*b*x)^4*tan(1/2
*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1
/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*
x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(
1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4 + d*log(2*(tan(1/2*b*x)^4*ta
n(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 +
tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1
/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2
*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4 + 64*b*c*tan(1/2*b*x)^3
*tan(1/2*a) - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan
(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2
*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*
b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 +
1))*tan(1/2*b*x)^3*tan(1/2*a) + 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*
tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4
+ 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2
*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1
)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a) - 16*d*tan(1/2*b*x)^4*tan(1
/2*a) + 168*b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 - 96*d*tan(1/2*b*x)^3*tan(1/2*a
)^2 + 64*b*c*tan(1/2*b*x)*tan(1/2*a)^3 - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*
a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/
2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x
)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1
/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^3 + 4*d*log(2*(tan(1
/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan
(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)
^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(
1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^3
- 96*d*tan(1/2*b*x)^2*tan(1/2*a)^3 + 10*b*c*tan(1/2*a)^4 - d*log(2*(tan(1/2
*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1
/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3
 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/
2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a)^4 + d*log(2*(tan(
1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*ta
n(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x
)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan
(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a)^4 - 16*d*tan(1
/2*b*x)*tan(1/2*a)^4 - 12*b*d*x*tan(1/2*b*x)^2 - 64*b*d*x*tan(1/2*b*x)*tan(
1/2*a) - 12*b*d*x*tan(1/2*a)^2 - 12*b*c*tan(1/2*b*x)^2 + 16*d*tan(1/2*b*x)^
3 - 64*b*c*tan(1/2*b*x)*tan(1/2*a) - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2
 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*
x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*ta
n(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a
) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + 4*d*log(2*(tan(1/2*b*x
```

```
)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a
)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2
*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*
x) + 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + 96*d*t
an(1/2*b*x)^2*tan(1/2*a) - 12*b*c*tan(1/2*a)^2 + 96*d*tan(1/2*b*x)*tan(1/2*
a)^2 + 16*d*tan(1/2*a)^3 + 10*b*d*x + 10*b*c + d*log(2*(tan(1/2*b*x)^4*tan(
1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + ta
n(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2
*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*t
an(1/2*a) + 1)/(tan(1/2*a)^2 + 1)) - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 -
 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)
^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(
1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a)
+ 1)/(tan(1/2*a)^2 + 1)) - 16*d*tan(1/2*b*x) - 16*d*tan(1/2*a))/(b^2*tan(1/
2*b*x)^4*tan(1/2*a)^4 - 4*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3 - b^2*tan(1/2*b*x
)^4 - 4*b^2*tan(1/2*b*x)^3*tan(1/2*a) - 4*b^2*tan(1/2*b*x)*tan(1/2*a)^3 - b
^2*tan(1/2*a)^4 - 4*b^2*tan(1/2*b*x)*tan(1/2*a) + b^2)
```

**maple [A]** time = 0.04, size = 87, normalized size = 1.53

$$-\frac{4c\cos(bx+a)}{b}-\frac{c}{b\cos(bx+a)}-\frac{4d\cos(bx+a)x}{b}+\frac{4d\sin(bx+a)}{b^2}-\frac{dx}{b\cos(bx+a)}+\frac{d\ln(\sec(bx+a)+\tan(}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] -4/b*c*cos(b*x+a)-1/b*c/cos(b*x+a)-4/b*d*cos(b*x+a)*x+4*d*sin(b*x+a)/b^2-1/
b*d/cos(b*x+a)*x+1/b^2*d*ln(sec(b*x+a)+tan(b*x+a))
```

**maxima [B]** time = 1.12, size = 3330, normalized size = 58.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*a
) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x + 3*
a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) +
 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c/(b*cos(3*b*x + 3*a)^2 +
2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + 3*a)^2
 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - 1/2*(4*(cos(a)^2
 + sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b
*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*
x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*c
os(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x*cos(b*x +
 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x + 2*a)*sin
(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 3
*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*
cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 + (b*x*cos(2*b*x
+ 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x + 3*a)^2 + (b*x*
cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)*cos(b*x + a) + (b
*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 3*a))*cos(3*b*x
 + 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b
*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a))*sin(b*x + a) + sin
(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (cos(b*x + a)^2 + sin(b*x +
a)^2)*sin(2*b*x + 3*a))*cos(3*b*x + 4*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(b*x
+ a)*cos(a) + 6*b*x*cos(b*x + a)*sin(b*x + 2*a)*sin(a) + b*x*cos(2*b*x + 3*
a)^2 + b*x*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*b*x + 2*(3*b*x*cos(b*
x + 2*a)*cos(b*x + a) + b*x*cos(a))*cos(2*b*x + 3*a) + 2*(3*b*x*cos(b*x + a
```

```
)*sin(b*x + 2*a) + b*x*sin(a))*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + 4*(2*b*
x*cos(b*x + a)*cos(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(b*x
 + 2*a) - 2*cos(a)*sin(b*x + a))*cos(2*b*x + 3*a) + 12*(b*x*cos(b*x + a)^2*
cos(a) + b*x*cos(a)*sin(b*x + a)^2)*cos(b*x + 2*a) - ((cos(2*b*x + 3*a)^2 +
 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x +
3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)
^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*cos(b*x + a)^2 + (cos(2*b*x
+ 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*si
n(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + si
n(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*sin(b*x + a)^2 + 2
*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a)
+ cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a)
+ (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2*
cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(cos(2*b*x + 3*a)^2*si
n(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + sin(2*b*x + 3*a)^2*si
n(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^2)
*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(b*x + a)^2*sin(a) + sin(b*x + a)^2
*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x
+ a) + 1) + ((cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + s
in(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^
2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(
a)^2)*cos(b*x + a)^2 + (cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + co
s(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b
*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a
)^2 + sin(a)^2)*sin(b*x + a)^2 + 2*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos
(2*b*x + 3*a)*cos(b*x + a)*cos(a) + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos
(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos
(3*b*x + 3*a) + 2*(cos(b*x + a)^2*cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x
 + 3*a) + 2*(cos(2*b*x + 3*a)^2*sin(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*si
n(b*x + a) + sin(2*b*x + 3*a)^2*sin(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x +
 a)*sin(a) + (cos(a)^2 + sin(a)^2)*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(
b*x + a)^2*sin(a) + sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x +
a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 4*((b*x*sin(2*b*x + 3*a) + b*
x*sin(a) - cos(2*b*x + 3*a) - cos(a))*cos(3*b*x + 3*a)^2 + (b*x*sin(a) - co
s(a))*cos(b*x + a)^2 + (b*x*sin(2*b*x + 3*a) + b*x*sin(a) - cos(2*b*x + 3*a
) - cos(a))*sin(3*b*x + 3*a)^2 + (b*x*sin(a) - cos(a))*sin(b*x + a)^2 + 2*(
b*x*cos(b*x + a)*sin(2*b*x + 3*a) + (b*x*sin(a) - cos(a))*cos(b*x + a) - co
s(2*b*x + 3*a)*cos(b*x + a))*cos(3*b*x + 3*a) - (cos(b*x + a)^2 + sin(b*x +
 a)^2)*cos(2*b*x + 3*a) + 2*(b*x*sin(2*b*x + 3*a)*sin(b*x + a) + (b*x*sin(a
) - cos(a))*sin(b*x + a) - cos(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a)
+ (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*sin(2*b*x + 3*a))*sin(3*b*x + 4
*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(a)*sin(b*x + a) + 6*b*x*sin(b*x + 2*a)*si
n(b*x + a)*sin(a) + 2*(3*b*x*cos(b*x + 2*a)*sin(b*x + a) - cos(a))*cos(2*b*
x + 3*a) - cos(2*b*x + 3*a)^2 - cos(a)^2 + 2*(3*b*x*sin(b*x + 2*a)*sin(b*x
+ a) - sin(a))*sin(2*b*x + 3*a) - sin(2*b*x + 3*a)^2 - sin(a)^2)*sin(3*b*x
+ 3*a) + 4*(2*b*x*cos(b*x + a)*sin(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b*x
 + a)^2)*sin(b*x + 2*a) - 2*sin(b*x + a)*sin(a))*sin(2*b*x + 3*a) + 12*(b*x
*cos(b*x + a)^2*sin(a) + b*x*sin(b*x + a)^2*sin(a))*sin(b*x + 2*a) - 4*(cos
(a)^2 + sin(a)^2)*sin(b*x + a))*d/((cos(a)^2 + sin(a)^2)*b^2*cos(b*x + a)^2
 + (cos(a)^2 + sin(a)^2)*b^2*sin(b*x + a)^2 + (b^2*cos(2*b*x + 3*a)^2 + 2*b
^2*cos(2*b*x + 3*a)*cos(a) + b^2*sin(2*b*x + 3*a)^2 + 2*b^2*sin(2*b*x + 3*a
)*sin(a) + (cos(a)^2 + sin(a)^2)*b^2)*cos(3*b*x + 3*a)^2 + (b^2*cos(b*x + a
)^2 + b^2*sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (b^2*cos(2*b*x + 3*a)^2 + 2*
b^2*cos(2*b*x + 3*a)*cos(a) + b^2*sin(2*b*x + 3*a)^2 + 2*b^2*sin(2*b*x + 3*
a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^2)*sin(3*b*x + 3*a)^2 + (b^2*cos(b*x +
a)^2 + b^2*sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + 2*(b^2*cos(2*b*x + 3*a)^2*c
os(b*x + a) + 2*b^2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a) + b^2*cos(b*x + a)
*sin(2*b*x + 3*a)^2 + 2*b^2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^
2 + sin(a)^2)*b^2*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(b^2*cos(b*x + a)^2*co
```

```
s(a) + b^2*cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b^2*cos(2*b*x + 3*a
)^2*sin(b*x + a) + 2*b^2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + b^2*sin(2*b
*x + 3*a)^2*sin(b*x + a) + 2*b^2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (co
s(a)^2 + sin(a)^2)*b^2*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(b^2*cos(b*x + a)
^2*sin(a) + b^2*sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))
```

**mupad [B]** time = 1.31, size = 150, normalized size = 2.63

$$e^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}\left(\frac{-2\,b\,c+d\,2\mathrm{i}}{b^2}-\frac{2\,d\,x}{b}\right)-e^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\left(\frac{2\,b\,c+d\,2\mathrm{i}}{b^2}+\frac{2\,d\,x}{b}\right)-\frac{d\,\ln\left(e^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}-\mathrm{i}\right)}{b^2}+\frac{d\,\ln\left(e^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{b^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x)^2,x)
```

```
[Out] exp(- a*1i - b*x*1i)*((d*2i - 2*b*c)/b^2 - (2*d*x)/b) - exp(a*1i + b*x*1i)*
((d*2i + 2*b*c)/b^2 + (2*d*x)/b) - (d*log(exp(a*1i + b*x*1i) - 1i))/b^2 + (
d*log(exp(a*1i + b*x*1i) + 1i))/b^2 - (exp(a*1i + b*x*1i)*(c + d*x)*2i)/(b*
(exp(a*2i + b*x*2i)*1i + 1i))
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.392 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

**Optimal**. Leaf size=78

$$-\text{Int}\left(\frac{\tan(a+bx)\sec(a+bx)}{c+dx},x\right)+\frac{4\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}+\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] -CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)+4*cos(a-b*c/d)*Si(b*c/d+b
*x)/d+4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d
```

**Rubi** [A] time = 0.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (4*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d + (4*Cos[a - (b*c)/d]*Sin
Integral[(b*c)/d + b*x])/d - Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*
x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx &= \int \left(\frac{3\sin(a+bx)}{c+dx}-\frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\right)dx\\
&= 3\int \frac{\sin(a+bx)}{c+dx}\,dx-\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx\\
&= \left(3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\left(3\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\\
&= \frac{3\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}+\frac{3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin}{}\\
&= \frac{4\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}+\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}-\int \frac{\sec(a+bx)\tan(}{c+dx}
\end{aligned}$$

**Mathematica** [A] time = 13.73, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]
```

**fricas** [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)^2\sin(3\,bx+3\,a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)^2 \sin(3\,bx+3\,a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

**maple [A]** time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^2(bx+a)\right) \sin(3bx+3a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,a+3\,b\,x)}{\cos(a+b\,x)^2\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] Timed out
```

## 3.393 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=98

$$-\mathrm{Int}\left(\frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)+\frac{4b\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{4b\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{4\sin(a+bx)}{d(c+dx)}$$

```
[Out] -CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)+4*b*Ci(b*c/d+b*x)*cos(a
-b*c/d)/d^2-4*b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2-4*sin(b*x+a)/d/(d*x+c)
```

**Rubi** [A] time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (4*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 - (4*Sin[a + b*x])/(d
*(c + d*x)) - (4*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 - Defer
[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx=\int\left(\frac{3\sin(a+bx)}{(c+dx)^2}-\frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\right)dx$$

$$=3\int\frac{\sin(a+bx)}{(c+dx)^2}\,dx-\int\frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

$$=-\frac{3\sin(a+bx)}{d(c+dx)}+\frac{(3b)\int\frac{\cos(a+bx)}{c+dx}\,dx}{d}+\int\frac{\sin(a+bx)}{(c+dx)^2}\,dx-\int\frac{\sec(a+bx)\tan}{(c+dx)}$$

$$=-\frac{4\sin(a+bx)}{d(c+dx)}+\frac{b\int\frac{\cos(a+bx)}{c+dx}\,dx}{d}+\frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}-\frac{(3b\sin}{}$$

$$=\frac{3b\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{4\sin(a+bx)}{d(c+dx)}-\frac{3b\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+$$

$$=\frac{4b\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{4\sin(a+bx)}{d(c+dx)}-\frac{4b\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d^2}-$$

**Mathematica** [A] time = 16.58, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

**fricas** [**A**] time = 1.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{d^{2}x^{2}+2\,cdx+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**giac** [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

**maple** [**A**] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^{2}\left(bx+a\right)\right)\sin\left(3bx+3a\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

**maxima** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

**mupad** [**A**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{\cos\left(a+b\,x\right)^{2}\,\left(c+d\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^2),x)
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^2), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**2,x)
[Out] Timed out
```

# 3.394 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=121

$$-\text{Int}\left(\frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^3},x\right)-\frac{2b^2\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^3}-\frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3}-\frac{2b\cos(a+bx)}{d^2(c+dx)}-$$

```
[Out] -CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^3,x)-2*b*cos(b*x+a)/d^2/(d*x
+c)-2*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3-2*b^2*Ci(b*c/d+b*x)*sin(a-b*c/d)/d
^3-2*sin(b*x+a)/d/(d*x+c)^2
```

**Rubi [A]** time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (-2*b*Cos[a + b*x])/(d^2*(c + d*x)) - (2*b^2*CosIntegral[(b*c)/d + b*x]*Sin
[a - (b*c)/d])/d^3 - (2*Sin[a + b*x])/(d*(c + d*x)^2) - (2*b^2*Cos[a - (b*c
)/d]*SinIntegral[(b*c)/d + b*x])/d^3 - Defer[Int][(Sec[a + b*x]*Tan[a + b*x
])/(c + d*x)^3, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx &= \int \left(\frac{3\sin(a+bx)}{(c+dx)^3}-\frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^3}\right)dx\\
&= 3\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx-\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^3}\,dx\\
&= -\frac{3\sin(a+bx)}{2d(c+dx)^2}+\frac{(3b)\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{2d}+\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx-\int \frac{\sec(a+bx)\tan}{(c+dx)}\\
&= -\frac{3b\cos(a+bx)}{2d^2(c+dx)}-\frac{2\sin(a+bx)}{d(c+dx)^2}-\frac{\left(3b^2\right)\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2}+\frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{2d}-\int\\
&= -\frac{2b\cos(a+bx)}{d^2(c+dx)}-\frac{2\sin(a+bx)}{d(c+dx)^2}-\frac{b^2\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2}-\frac{\left(3b^2\cos\left(a-\frac{bc}{d}\right)\right)\int}{2d^2}\\
&= -\frac{2b\cos(a+bx)}{d^2(c+dx)}-\frac{3b^2\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{2d^3}-\frac{2\sin(a+bx)}{d(c+dx)^2}-\frac{3b^2\cos\left(a\right.}{}\\
&= -\frac{2b\cos(a+bx)}{d^2(c+dx)}-\frac{2b^2\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3}-\frac{2\sin(a+bx)}{d(c+dx)^2}-\frac{2b^2\cos\left(a\right.}{}
\end{aligned}$$

**Mathematica [A]** time = 19.13, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

**fricas** [**A**] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

**giac** [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

**maple** [**A**] time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^2\left(bx+a\right)\right)\sin\left(3bx+3a\right)}{\left(dx+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

```
[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

**maxima** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad** [**A**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{{\cos\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^3),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^3), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

# 3.395 $\int x\cos(2x)\sec(x)\,dx$

**Optimal**. Leaf size=57

$$-i\text{Li}_2\left(-ie^{ix}\right)+i\text{Li}_2\left(ie^{ix}\right)+2x\sin(x)+2\cos(x)+2ix\tan^{-1}\left(e^{ix}\right)$$

```
[Out] 2*I*x*arctan(exp(I*x))+2*cos(x)-I*polylog(2,-I*exp(I*x))+I*polylog(2,I*exp(
I*x))+2*x*sin(x)
```

**Rubi [A]** time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {4431, 3296, 2638, 4407, 4181, 2279, 2391}

$$-i\text{PolyLog}\left(2,-ie^{ix}\right)+i\text{PolyLog}\left(2,ie^{ix}\right)+2x\sin(x)+2\cos(x)+2ix\tan^{-1}\left(e^{ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x]*Sec[x],x]
```

```
[Out] (2*I)*x*ArcTan[E^(I*x)] + 2*Cos[x] - I*PolyLog[2, (-I)*E^(I*x)] + I*PolyLog
[2, I*E^(I*x)] + 2*x*Sin[x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\int x\cos(2x)\sec(x)\,dx &= \int (x\cos(x)-x\sin(x)\tan(x))\,dx\\ &= \int x\cos(x)\,dx-\int x\sin(x)\tan(x)\,dx\\ &= x\sin(x)+\int x\cos(x)\,dx-\int x\sec(x)\,dx-\int \sin(x)\,dx\\ &= 2ix\tan^{-1}\left(e^{ix}\right)+\cos(x)+2x\sin(x)+\int \log\left(1-ie^{ix}\right)dx-\int \log\left(1+ie^{ix}\right)dx-\int \text{s}\\ &= 2ix\tan^{-1}\left(e^{ix}\right)+2\cos(x)+2x\sin(x)-i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{ix}\right)+i\,\text{Subst}\Big(\int\\ &= 2ix\tan^{-1}\left(e^{ix}\right)+2\cos(x)-i\text{Li}_2\left(-ie^{ix}\right)+i\text{Li}_2\left(ie^{ix}\right)+2x\sin(x)\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 77, normalized size = 1.35

$$-i\left(\text{Li}_2\left(-ie^{ix}\right)-\text{Li}_2\left(ie^{ix}\right)\right)-x\left(\log\left(1-ie^{ix}\right)-\log\left(1+ie^{ix}\right)\right)+2x\sin(x)+2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x]*Sec[x],x]
```

```
[Out] 2*Cos[x] - x*(Log[1 - I*E^(I*x)] - Log[1 + I*E^(I*x)]) - I*(PolyLog[2, (-I)
*E^(I*x)] - PolyLog[2, I*E^(I*x)]) + 2*x*Sin[x]
```

**fricas [B]** time = 1.23, size = 106, normalized size = 1.86

$$-\frac{1}{2}x\log(i\cos(x)+\sin(x)+1)+\frac{1}{2}x\log(i\cos(x)-\sin(x)+1)-\frac{1}{2}x\log(-i\cos(x)+\sin(x)+1)+\frac{1}{2}x\log(-i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x),x, algorithm="fricas")
```

```
[Out] -1/2*x*log(I*cos(x) + sin(x) + 1) + 1/2*x*log(I*cos(x) - sin(x) + 1) - 1/2*
x*log(-I*cos(x) + sin(x) + 1) + 1/2*x*log(-I*cos(x) - sin(x) + 1) + 2*x*sin
(x) + 2*cos(x) + 1/2*I*dilog(I*cos(x) + sin(x)) + 1/2*I*dilog(I*cos(x) - si
n(x)) - 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dilog(-I*cos(x) - sin(x))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(2x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x),x, algorithm="giac")
```

```
[Out] integrate(x*cos(2*x)*sec(x), x)
```

**maple [A]** time = 0.06, size = 66, normalized size = 1.16

$$x\ln\left(1+ie^{ix}\right)-x\ln\left(1-ie^{ix}\right)-i\,\text{dilog}\left(1+ie^{ix}\right)+i\,\text{dilog}\left(1-ie^{ix}\right)+2\cos(x)+2x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x)*sec(x),x)
```

```
[Out] x*ln(1+I*exp(I*x))-x*ln(1-I*exp(I*x))-I*dilog(1+I*exp(I*x))+I*dilog(1-I*exp
(I*x))+2*cos(x)+2*x*sin(x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$2\,x\sin(x) + 2\,\cos(x) - 2\int \frac{x\cos\left(2\,x\right)\cos(x) + x\sin\left(2\,x\right)\sin(x) + x\cos(x)}{\cos\left(2\,x\right)^{2} + \sin\left(2\,x\right)^{2} + 2\,\cos\left(2\,x\right) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x),x, algorithm="maxima")
```

```
[Out] 2*x*sin(x) + 2*cos(x) - 2*integrate((x*cos(2*x)*cos(x) + x*sin(2*x)*sin(x)
+ x*cos(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x)
```

**mupad [B]** time = 2.21, size = 46, normalized size = 0.81

$$2\,\cos(x) + 2\,x\,\sin(x) - \mathrm{polylog}\left(2, -\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i}\right)\,1\mathrm{i} + \mathrm{polylog}\left(2, \mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i}\right)\,1\mathrm{i} + x\,\mathrm{atan}\left(\mathrm{e}^{x\,1\mathrm{i}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(2*x))/cos(x),x)
```

```
[Out] 2*cos(x) - polylog(2, -exp(x*1i)*1i)*1i + polylog(2, exp(x*1i)*1i)*1i + x*a
tan(exp(x*1i))*2i + 2*x*sin(x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(2x\right)\sec\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x),x)
```

```
[Out] Integral(x*cos(2*x)*sec(x), x)
```

# 3.396 $\int x\cos(2x)\sec^2(x)\,dx$

**Optimal**. Leaf size=14

$$x^2 - x\tan(x) - \log(\cos(x))$$

[Out] x^2-ln(cos(x))-x*tan(x)

**Rubi [A]** time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4431, 3720, 3475, 30}

$$x^2 - x\tan(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[x*Cos[2*x]*Sec[x]^2,x]

[Out] x^2 - Log[Cos[x]] - x*Tan[x]

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\int x\cos(2x)\sec^2(x)\,dx &= \int \left(x - x\tan^2(x)\right)\,dx\\ &= \frac{x^2}{2} - \int x\tan^2(x)\,dx\\ &= \frac{x^2}{2} - x\tan(x) + \int x\,dx + \int \tan(x)\,dx\\ &= x^2 - \log(\cos(x)) - x\tan(x)\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 14, normalized size = 1.00

$$x^2 - x\tan(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x]*Sec[x]^2,x]
```

```
[Out] x^2 - Log[Cos[x]] - x*Tan[x]
```

**fricas [A]** time = 0.56, size = 26, normalized size = 1.86

$$\frac{x^2\cos(x) - \cos(x)\log(-\cos(x)) - x\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="fricas")
```

```
[Out] (x^2*cos(x) - cos(x)*log(-cos(x)) - x*sin(x))/cos(x)
```

**giac [B]** time = 1.55, size = 118, normalized size = 8.43

$$\frac{2x^2\tan\left(\frac{1}{2}x\right)^2 - \log\left(\frac{4\left(\tan\left(\frac{1}{2}x\right)^4 - 2\tan\left(\frac{1}{2}x\right)^2 + 1\right)}{\tan\left(\frac{1}{2}x\right)^4 + 2\tan\left(\frac{1}{2}x\right)^2 + 1}\right)\tan\left(\frac{1}{2}x\right)^2 - 2x^2 + 4x\tan\left(\frac{1}{2}x\right) + \log\left(\frac{4\left(\tan\left(\frac{1}{2}x\right)^4 - 2\tan\left(\frac{1}{2}x\right)^2 + 1\right)}{\tan\left(\frac{1}{2}x\right)^4 + 2\tan\left(\frac{1}{2}x\right)^2 + 1}\right)}{2\left(\tan\left(\frac{1}{2}x\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*x^2*tan(1/2*x)^2 - log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/
2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - 2*x^2 + 4*x*tan(1/2*x) + log(4
*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/
(tan(1/2*x)^2 - 1)
```

**maple [A]** time = 0.06, size = 15, normalized size = 1.07

$$x^2 - \ln(\cos(x)) - x\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x)*sec(x)^2,x)
```

```
[Out] x^2-ln(cos(x))-x*tan(x)
```

**maxima [B]** time = 0.42, size = 111, normalized size = 7.93

$$\frac{2x^2\cos(2x)^2 + 2x^2\sin(2x)^2 + 4x^2\cos(2x) + 2x^2 - \left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)\log\left(\cos(2x)^2 +\right.}{2\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^2*cos(2*x)^2 + 2*x^2*sin(2*x)^2 + 4*x^2*cos(2*x) + 2*x^2 - (cos(2*
x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x
) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

**mupad [B]** time = 2.35, size = 31, normalized size = 2.21

$$x^2 - \ln\left(e^{x\,2i} + 1\right) + x\,2i - \frac{x\,2i}{e^{x\,2i} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(2*x))/cos(x)^2,x)
```

```
[Out] x*2i - log(exp(x*2i) + 1) - (x*2i)/(exp(x*2i) + 1) + x^2
```

**sympy [B]** time = 5.74, size = 144, normalized size = 10.29

$$x^2+\frac{2x\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}-\frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}+\frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{\tan^2\left(\frac{x}{2}\right)-1}-\frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}+\frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)**2,x)
```

```
[Out] x**2 + 2*x*tan(x/2)/(tan(x/2)**2 - 1) - log(tan(x/2) - 1)*tan(x/2)**2/(tan(
x/2)**2 - 1) + log(tan(x/2) - 1)/(tan(x/2)**2 - 1) - log(tan(x/2) + 1)*tan(
x/2)**2/(tan(x/2)**2 - 1) + log(tan(x/2) + 1)/(tan(x/2)**2 - 1) + log(tan(x
/2)**2 + 1)*tan(x/2)**2/(tan(x/2)**2 - 1) - log(tan(x/2)**2 + 1)/(tan(x/2)*
*2 - 1)
```

# 3.397 $\int x \cos(2x) \sec^3(x)\, dx$

**Optimal**. Leaf size=67

$$\frac{3}{2} i\text{Li}_2\left(-ie^{ix}\right) - \frac{3}{2} i\text{Li}_2\left(ie^{ix}\right) - 3ix\tan^{-1}\left(e^{ix}\right) + \frac{\sec(x)}{2} - \frac{1}{2}x\tan(x)\sec(x)$$

```
[Out] -3*I*x*arctan(exp(I*x))+3/2*I*polylog(2,-I*exp(I*x))-3/2*I*polylog(2,I*exp(
I*x))+1/2*sec(x)-1/2*x*sec(x)*tan(x)
```

**Rubi [A]** time = 0.14, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4431, 4181, 2279, 2391, 4413, 4185}

$$\frac{3}{2} i\text{PolyLog}\left(2,-ie^{ix}\right) - \frac{3}{2} i\text{PolyLog}\left(2,ie^{ix}\right) - 3ix\tan^{-1}\left(e^{ix}\right) + \frac{\sec(x)}{2} - \frac{1}{2}x\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x]*Sec[x]^3,x]
```

```
[Out] (-3*I)*x*ArcTan[E^(I*x)] + ((3*I)/2)*PolyLog[2, (-I)*E^(I*x)] - ((3*I)/2)*P
olyLog[2, I*E^(I*x)] + Sec[x]/2 - (x*Sec[x]*Tan[x])/2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
```

```
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\int x\cos(2x)\sec^3(x)\,dx &= \int \left(x\sec(x) - x\sec(x)\tan^2(x)\right)\,dx\\ &= \int x\sec(x)\,dx - \int x\sec(x)\tan^2(x)\,dx\\ &= -2ix\tan^{-1}\left(e^{ix}\right) - \int \log\left(1-ie^{ix}\right)\,dx + \int \log\left(1+ie^{ix}\right)\,dx + \int x\sec(x)\,dx - \int x\,\text{s}\\ &= -4ix\tan^{-1}\left(e^{ix}\right) + \frac{\sec(x)}{2} - \frac{1}{2}x\sec(x)\tan(x) + i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{ix}\right) - i\,\text{S}\\ &= -3ix\tan^{-1}\left(e^{ix}\right) + i\text{Li}_2\left(-ie^{ix}\right) - i\text{Li}_2\left(ie^{ix}\right) + \frac{\sec(x)}{2} - \frac{1}{2}x\sec(x)\tan(x) + i\,\text{Subst}\left(\int\right.\\ &= -3ix\tan^{-1}\left(e^{ix}\right) + 2i\text{Li}_2\left(-ie^{ix}\right) - 2i\text{Li}_2\left(ie^{ix}\right) + \frac{\sec(x)}{2} - \frac{1}{2}x\sec(x)\tan(x) - \frac{1}{2}i\,\text{Subst}\\ &= -3ix\tan^{-1}\left(e^{ix}\right) + \frac{3}{2}i\text{Li}_2\left(-ie^{ix}\right) - \frac{3}{2}i\text{Li}_2\left(ie^{ix}\right) + \frac{\sec(x)}{2} - \frac{1}{2}x\sec(x)\tan(x)\end{aligned}$$

**Mathematica [B]** time = 0.28, size = 146, normalized size = 2.18

$$\frac{1}{4}\left(6i\text{Li}_2\left(-ie^{ix}\right) - 6i\text{Li}_2\left(ie^{ix}\right) + 6x\log\left(1-ie^{ix}\right) - 6x\log\left(1+ie^{ix}\right) + \frac{x}{\sin(x)-1} + \frac{x}{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2} + \frac{}{\cos}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x]*Sec[x]^3,x]
```

```
[Out] (6*x*Log[1 - I*E^(I*x)] - 6*x*Log[1 + I*E^(I*x)] + (6*I)*PolyLog[2, (-I)*E^
(I*x)] - (6*I)*PolyLog[2, I*E^(I*x)] + (2*Sin[x/2])/(Cos[x/2] - Sin[x/2]) +
 x/(Cos[x/2] + Sin[x/2])^2 - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2]) + x/(-1 + S
in[x]))/4
```

**fricas [B]** time = 2.07, size = 144, normalized size = 2.15

$$\frac{3\,x\cos(x)^2\log\left(i\,\cos(x)+\sin(x)+1\right) - 3\,x\cos(x)^2\log\left(i\,\cos(x)-\sin(x)+1\right) + 3\,x\cos(x)^2\log\left(-i\,\cos(x)+\text{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*x*cos(x)^2*log(I*cos(x) + sin(x) + 1) - 3*x*cos(x)^2*log(I*cos(x) -
sin(x) + 1) + 3*x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - 3*x*cos(x)^2*log(-
I*cos(x) - sin(x) + 1) - 3*I*cos(x)^2*dilog(I*cos(x) + sin(x)) - 3*I*cos(x)
^2*dilog(I*cos(x) - sin(x)) + 3*I*cos(x)^2*dilog(-I*cos(x) + sin(x)) + 3*I*
cos(x)^2*dilog(-I*cos(x) - sin(x)) - 2*x*sin(x) + 2*cos(x))/cos(x)^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(2\,x)\sec(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cos(2*x)*sec(x)^3, x)
```

**maple [B]** time = 0.21, size = 102, normalized size = 1.52

$$\frac{i\left(x\,\mathrm{e}^{3ix}-x\,\mathrm{e}^{ix}-i\mathrm{e}^{3ix}-i\mathrm{e}^{ix}\right)}{\left(\mathrm{e}^{2ix}+1\right)^{2}}-\frac{3x\ln\left(1+i\mathrm{e}^{ix}\right)}{2}+\frac{3x\ln\left(1-i\mathrm{e}^{ix}\right)}{2}+\frac{3i\,\mathrm{dilog}\left(1+i\mathrm{e}^{ix}\right)}{2}-\frac{3i\,\mathrm{dilog}\left(1-i\mathrm{e}^{ix}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x)*sec(x)^3,x)
```

```
[Out] I/(exp(2*I*x)+1)^2*(x*exp(3*I*x)-x*exp(I*x)-I*exp(3*I*x)-I*exp(I*x))-3/2*x*
ln(1+I*exp(I*x))+3/2*x*ln(1-I*exp(I*x))+3/2*I*dilog(1+I*exp(I*x))-3/2*I*dil
og(1-I*exp(I*x))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(x\sin(3x)-x\sin(x)-\cos(3x)-\cos(x))\cos(4x)-(2x\sin(2x)+2\cos(2x)+1)\cos(3x)-2(x\sin(x)+\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="maxima")
```

```
[Out] -((x*sin(3*x) - x*sin(x) - cos(3*x) - cos(x))*cos(4*x) - (2*x*sin(2*x) + 2*
cos(2*x) + 1)*cos(3*x) - 2*(x*sin(x) + cos(x))*cos(2*x) - 3*(2*(2*cos(2*x)
+ 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x
) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)*integrate((x*cos(2*x)*cos(x) + x*sin(2*x
)*sin(x) + x*cos(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x) - (x*co
s(3*x) - x*cos(x) + sin(3*x) + sin(x))*sin(4*x) + (2*x*cos(2*x) + x - 2*sin
(2*x))*sin(3*x) + 2*(x*cos(x) - sin(x))*sin(2*x) - x*sin(x) - cos(x))/(2*(2
*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*
x)*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

**mupad [B]** time = 2.32, size = 63, normalized size = 0.94

$$\frac{1}{2\cos(x)}+x\,\mathrm{atanh}\left(\mathrm{e}^{x\,1i}\,1i\right)-\frac{x\sin(x)}{2\cos(x)^{2}}+\frac{\mathrm{polylog}\left(2,-\mathrm{e}^{x\,1i}\,1i\right)\,3i}{2}-\frac{\mathrm{polylog}\left(2,\mathrm{e}^{x\,1i}\,1i\right)\,3i}{2}-x\,\mathrm{atan}\left(\mathrm{e}^{x\,1i}\right)\,4i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(2*x))/cos(x)^3,x)
```

```
[Out] (polylog(2, -exp(x*1i)*1i)*3i)/2 - (polylog(2, exp(x*1i)*1i)*3i)/2 + 1/(2*c
os(x)) - x*atan(exp(x*1i))*4i + x*atanh(exp(x*1i)*1i) - (x*sin(x))/(2*cos(x
)^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(2x)\sec^{3}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)**3,x)
```

```
[Out] Integral(x*cos(2*x)*sec(x)**3, x)
```

# Chapter 4

# Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

### 4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```